

Estimation of Operational Serviceability of Constructions with Regard to the Faults Arising During Erection

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Abstract: Presented are results of a research into the elaboration of the general method for numeric modelling of the stress-deformation state of reinforced concrete constructions during prolonged operating stresses and exposures. The method is presented in the form of an algorithm on the basis of which computer software has been developed for estimating reinforced concrete constructions operating in a difficult environment. For the numeric solution of the plane problem a method of terminal elements was used; this method employed the displacement pattern. In recording the defining relations taken into account were the main factors of the physical non-linearity of reinforced concrete, i.e. the acquired orthotropy of the concrete at an operating stage prior to formation of cracks in the concrete, crazing and development of plastic deformations in the reinforcement. Considered, by way of example of the numeric realization of the method, is an estimation of pre-stressed reinforced beam experimentally studied by other authors. The choice for the estimation of this construction was determined by the fact that in the course of investigations the functional dependencies of the rheological properties of the concrete were experimentally established in the required amount, including those in the area of non-linear deformations. The results obtained in the estimation tallied well with the theoretical data as well as with those produced using the proposed method and experimental ones.

Key words: Reinforced constructions • Physically non-linear estimation • Flat stressed state • Terminal element method

INTRODUCTION

The faults arising in the structural elements during manufacture or erection may include all the changes in the materials' strength and resilience capabilities that affect the stress-deformed state in the course of operation. As a rule, such changes are inseparable from the history of the stressed state. The dependence of the concrete's strength and deformation properties on the factors of stress-deformed state was considered in the research papers [1-5]. The use of the terminal elements method (TEM) in estimating the reinforced mediums with a discrete arrangement of reinforcement was studied in research papers [6-9]; and with reinforcement spread over the plane – in research paper [10-11]. This research work

deals with the method of numeric modelling of enduring processes in reinforced concrete. The method is based on transformation of the stress time (technique $[\tau]_{\tau}$), which significantly simplifies the solution of the creep problem in difficult load conditions.

MATERIALS AND METHODS

The main investigating method is the numeric modeling of the construction's stress-deformed state based on the terminal elements method (TEM). An algorithmic calculation was performed for the stress-deformed state of the element. Following the changes in the stress-deformed state, three calculation cases were considered. In order to check out the

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proposed method and to study its computer-aided application, a numerical experiment is used normally employed for calculating the sagging curves of the beam being investigated.

The Main Part: The essence of the approach is in that the actual stepwise mode of growing stresses in the concrete $[\sigma]_1, [\sigma]_2, \dots, [\sigma]_i$ is successively replaced by a single-step one in which the duration of the step application $[\sigma]_i$ - transformed the load time $[\tau]_{\tau_i}$ ($[\tau]_{i,1} < [\tau]_{\tau_i} < [\tau]_{i,2}$) - determined based on the equation's solution

$$\sigma_i C(t, \tau_{m,i}) = \sigma_{i-1} C(t, \tau_{m,i-1}) + (\sigma_i - \sigma_{i-1}) C(t, \tau_i) \quad (1)$$

where $C(t, [\tau])$ - is a measure of creep.

For a mature concrete the demand for the minimum discrepancy of the creep curves of the actual and transformed modes is ensured for $t \geq [\tau]_i + \Delta t$, $\Delta t \geq 1$ day (24 hours), in which case the transformation equation at each stress stage is solved only once, at $t = [\tau]_i + \Delta t$, $\Delta t = 1$ day (24 hours).

Many authors' investigations of the young concrete creep made with the use of experimental data showed that the accuracy of solution of the creep problem by the $[\tau]_{\tau}$ method is in this case assured by the functional dependence of the parameter Δt on the concrete's age in the transformation equation (1). The expression for Δt obtained through numerical experiments assumes the following form:

$$\Delta t = \frac{A}{28} \left[e^{-\gamma(A-\tau)} - e^{-\gamma A} \right] \leq 1 \text{ day} \quad (2)$$

where $[\gamma] = 7.5 \times 10^{-2} \text{ day}^{-1}$; $A = 28$ days (of 24 h).

Considering the practical purpose of the task, the method of estimating the residual deformations is formulated as a computing algorithm. To simplify the mathematical expressions, a particular case of a reinforced plate (disc) is considered where the intensity and direction of reinforcement $[\mu]$ coincides with the direction of the coordinate axis x along which normal stresses $[\sigma]$ are applied to the disc faces. In this case, the equation of the element balance in the direction of the x axis assumes the form of:

$$\sigma = E'_b (\varepsilon - \varepsilon_0) + \mu E_s \varepsilon; \quad (3)$$

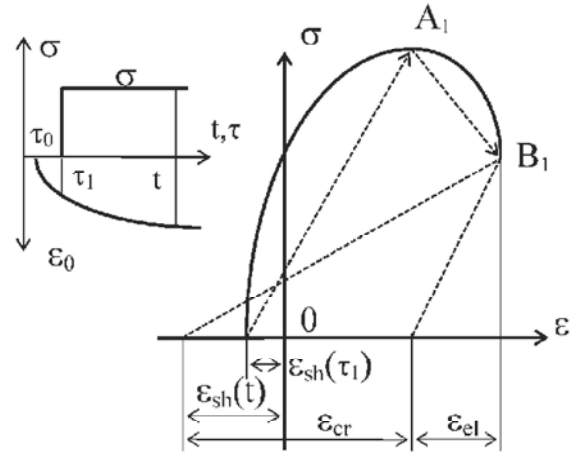


Fig. 1: Conventional chart of concrete deformation

where E'_b is a secant module of concrete deformation; $[\varepsilon]$ - axial element deformations; $[\varepsilon]_0$ - residual deformations in concrete.

In the concrete age $[\tau]_1$ the element is charged with tensions $[\sigma]_1$, which remain constant throughout the time $(t - \tau_1)$ Fig. 1 shows the conventional chart of concrete deformation in coordinates $(\sigma_b - \varepsilon)$ ($[\varepsilon]$ - element deformation) where the point A_1 corresponds to the stress-deformed state at the moment $[\tau]_1$ after application of the external stress, whilst the point B_1 - is presented at the moment t .

Let us formulate the algorithm of calculating the stress-deformed state of the element at the first load stage.

- Let us assume the equation (3) $E_b = E_b(\tau_1)$. The residual deformation in the concrete is equal to that of the free shrinkage at the moment τ_1 : $\varepsilon_0 = \varepsilon_{sh}(\tau_1)$
- From equation (3) we define the element deformation $[\varepsilon]$ at the moment $[\tau]_1$.
- Then we define the in-concrete strain at the moment $[\tau]_1$:

$$\sigma_b = E'_b (\varepsilon - \varepsilon_0) \quad (4)$$

- Let us define the complete deformation of the concrete at the moment t via a measure of creep and the function of non-linear deformations expressed through a level of stresses:

$$\varepsilon_b = \sigma_b / E_b(t) + \sigma_b [C(t, \tau) + C_n(\eta, t, \tau)]; \quad (5)$$

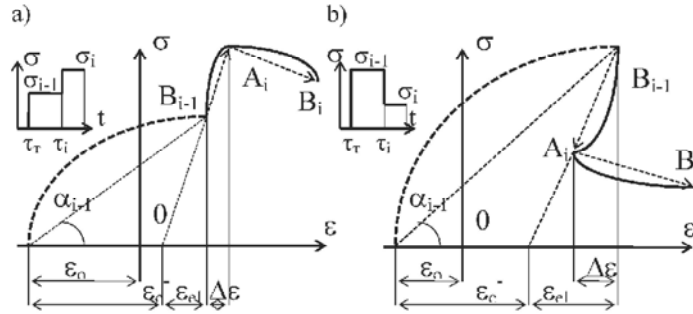


Fig. 2: Conventional charts of concrete deformation in extra stress and unloading

where $\eta = \sigma_i/R_b(t)$.

- Let us define the secant module of the concrete deformation:

$$E'_b = \sigma_b / \epsilon_b \quad (6)$$

- Let us assume $E'_b = E'_b(t)$, $\epsilon_0 = \epsilon_{sh}(t)$. For a convergence criterion we accept the deformation discrepancy $(\epsilon - \epsilon_0) - \epsilon_b$. Let us consider the loading of the element with a subsequent load stage $\Delta\sigma_i$ at the moment $[\tau]_i$. Generally, towards the moment $[\tau]_i$ a certain history of the stressed state takes place which is transformed into a single-step mode with τ_i, σ_{i-1} parameters.

At the moment of the load change, in the conventional charts of concrete deformations, Fig. 2, the strained state is characterized by the point B_{i-1} . It is believed that at the moment of load application the element resiliently transforms to the value:

$$\Delta\epsilon = \frac{\Delta\sigma}{E_b(\tau_i) + \mu_s E_s}$$

Following the change of the strained-deformed state, the following three predicted cases may occur.

Case 1: The sign $\Delta\epsilon$ coincides with the sign $\sigma_b(\tau_i)$ before the load is changed, i.e. extra loading (Fig. 2a). At the moment of the load change the stressed state of the concrete is characterized by the transition from point B_{i-1} to point A_i .

The algorithm of estimating the stress-deformed state of the element will be as follows.

- Let us assume in equation (3)

$$\sigma = \sigma_i; \quad \epsilon_0 = \epsilon_c^+(\tau_i) + \epsilon_{sh}(\tau_i); \quad E'_b = tg\alpha_i;$$

$$\text{where } tg\alpha_i = \frac{\sigma_b(\tau_i) + E_b(\tau_i)\Delta\epsilon}{\sigma_b(\tau_i)/E_b(\tau_i) + \epsilon_c^-(\tau_i) + \Delta\epsilon}$$

- From equation (3) let us define the element deformation [epsilon] at the moment t .
- Based on formula (4) let us determine the in-concrete stress at the moment t .
- Let us determine the complete concrete deformations at the moment t based on the formula below.

$$\epsilon_b = \sigma_b / E_b(t) + \sigma_b^- C(t, \tau_m^-) + \sigma_b^+ C(t, \tau_m^+) + \epsilon_n^-(t),$$

$$\text{where } \epsilon_n^-(t) = \epsilon_n^-(\tau_i) + \sigma_b [C_n(\eta_i, t, \tau_i) - C_n(\eta_{i-1}, t, \tau_i)] \quad (7)$$

- Let us determine the concrete deformation module $E'_b(t)$ by the formula (4).
- Let us assume that $E'_b = E'_b(t)$, $\epsilon_0 = \epsilon_c^+(t) + \epsilon_{sh}(t)$.

And so on.

Case 2: The sign $\Delta\epsilon$ is opposite to the sign $\sigma_b(\tau_i)$ prior to the change of the load. The unloading in this instance will be $|\Delta\epsilon| < |\sigma_b/E_b(\tau_i)|$ (Fig. 2b). The comparison of the charts shown in Fig.2 reveals that from the methodological position the case 2 differs from 1 only slightly. So, the computing algorithm and the formulas it contains will be true, excepting formula (7) which in case of the unloading takes the form of:

$$\epsilon_n^-(t) = \epsilon_n^-(\tau_i) + \sigma_b [C_n(\eta_i, t, \tau_m) - C_n(\eta_i, t, \tau_i)] \quad (8)$$

Case 3: The sign $\Delta\epsilon$ is opposite to the sign $\sigma_b(\tau_i)$ prior to the load change, where $|\Delta\epsilon| < |\sigma_b/E_b(\tau_i)|$ is the load with a variable sign. The change of the stressed state is characterised by a transition of the chart via the axis [epsilon] and by an establishment of a different sign in the

stressed concrete. According to the above stated terms, following the load change we get $\sigma_b(\tau_i) > 0$. In this case, the residual concrete deformations in point A_i include creep deformations on both stress branches, which is why in item 1 of the algorithm the following is accepted as true:

$$\varepsilon_o = \varepsilon_c^+(\tau_i) + \varepsilon_c^-(\tau_i) + \varepsilon_{sh}(\tau_i); \quad E'_b = E_b(\tau_i).$$

In other aspects the calculating procedure does not differ from that in case 1.

In the event of the flat stressed state the equation (3) is recorded with regard to all the components of the stress-deformed state in the form of:

$$\{\sigma\} = [D_{rc}]\{\varepsilon\} - [D_c]\{\varepsilon_o\} \quad (9)$$

The rigidity parameters of the concrete and reinforced concrete $D_{ij,c}$, $D_{ij,cr}$ are determined based on the formulas of research work [12] via the secant modules of the concrete deformations $E'_{b,n}$, $E'_{b,t}$ in direction of the main stress axes n, t. The creep deformations in direction of the main stress axes are calculated according to the dependence:

$$\varepsilon_{c,\alpha} = \sigma_{b,\alpha} [k_\alpha C(t, \tau) + C_n(\eta_\alpha, t, \tau)], \quad \alpha = n, t, \quad (10)$$

where $C(t, \tau)$ is a measure of creep which is recorded with regard to the influence of the element's shape and that of the ambient temperature and humidity. The consideration of the said factors is presented, specifically, in research paper [13]; k_α is an empirical ratio taking into account the influence of the complex stressed state on deformation of the linear creep, which is determined by the formulas:

in the area "compression-compression" ($[\sigma]_{b,n} < 0$, $[\sigma]_{b,t} < 0$):

$$k_n = k_t = 1 - \sqrt{|\sigma_{b,n}| / R_b(\tau)} < 1; \quad (11)$$

in the area "extension-compression" ($[\sigma]_{b,n} > 0$, $[\sigma]_{b,t} < 0$):

$$k_n = [1 + 0,9\sigma_{b,t} / R_b(\tau)]^{-1} > 1; \quad (12)$$

$k_t = 1$;

in the area "extension-extension" ($[\sigma]_{b,n} > 0$, $[\sigma]_{b,t} > 0$) accepted $k_n = k_t = 1$.

The turn of the main stress axes during a complex disproportionate load is taken into consideration based on the approximation method. The calculation registers a certain direction of the main axes $[\alpha]_{con}$, to which the data arrays are "linked" concerning the history of the stressed state. Such arrays that are formed in transformation of the load time. If, during the loading, the main axes deflect from the fixed direction to the angle

$$|[\alpha] - [\alpha]_{con}| < [\pi] / 4,$$

then the turn of the main stress axes is neglected. Otherwise a new fixed direction of the main axes is introduced

$$[\alpha]_{con} = [\alpha]_{con} + [\pi] / 2,$$

and, accordingly, the "link" of the data array concerning the history of the stressed state also changes.

The external load is set as vectors of concentrated forces P and F applied to TE nodes in the axes direction x, y (Fig. 3a). All of the external load's vector components are linked to the concrete age, i.e. vector $\{\tau\} = \{[\tau]_1, [\tau]_2, \dots, [\tau]_n\}$, where $[\tau]_1, [\tau]_2, \dots, [\tau]_n$ - are moments of time when load changes occur and for which a solution of the non-linear task is found (Fig. 3b). Together with the external load, as a function of the concrete age, one also sets free shrinkage curves, prismatic strength and axial extension strength, instantly resilient deformation module and specific deformations of linear and non-linear concrete creep. The possibility of considering the influence of the reinforcement corrosion is dealt with in research works [14-16] as part of the general calculation method.

The Numeric Experiment: To check out the proposed method and to study the feasibility of its computer-aided implementation, a pre-stressed beam was estimated and considered. The initial data, based on which the calculations were made, are accepted as proposed by research work [17-18].

The load scheme and the construction reinforcement are shown in Fig. 4. The reinforcement extension method is mechanical, with the use of stopping rests. The reinforcement pre-stress is applied with regard to the losses that became evident before the concrete reduction. The registered pressure was 596 MPa. The reinforcement tempering was done at the concrete age of 20 days and prismatic strength of $R_b = 38.5$ MPa. The pre-stress was transferred in four stages.

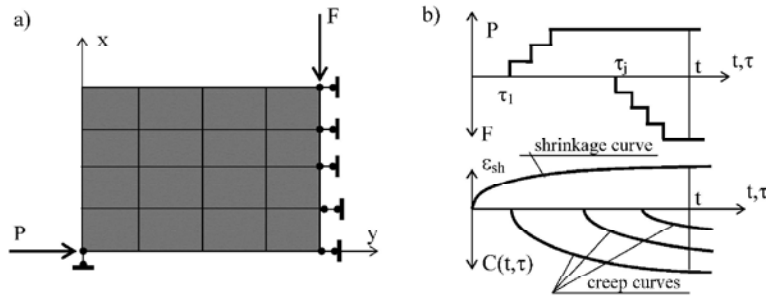


Fig. 3: Chart of complex loading

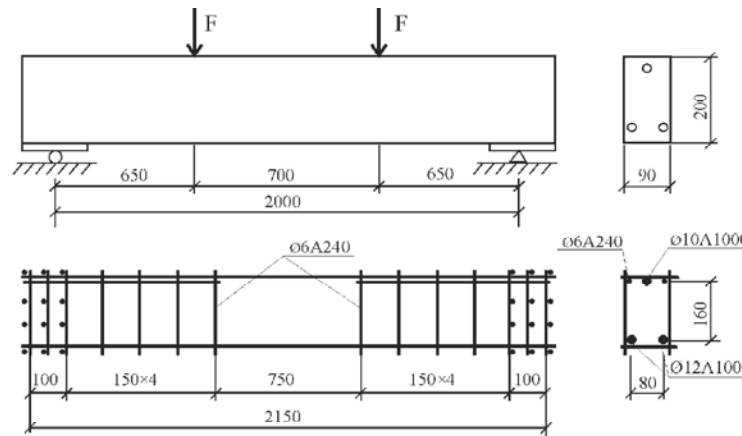


Fig. 4: Beam loading and reinforcing chart

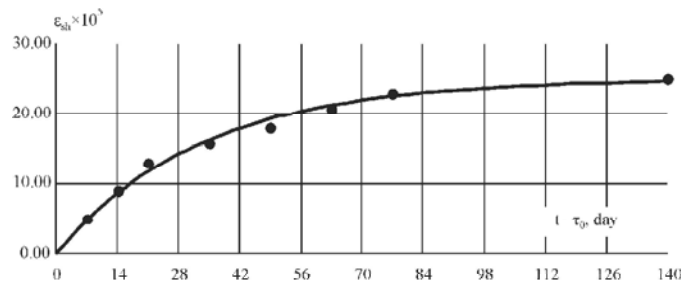


Fig. 5: Curve of relative deformations of free concrete shrinkage

The creep deformation functions were accepted based on research work [18] for non-linear deformations

$$C_n(\eta, t, \tau) = \eta(\tau) A [1 - e^{-\alpha_1 \sqrt{t-\tau}}] \quad (13)$$

where $A = 3.8 \times 10^{-5}$ 1/MPa;
 $[\alpha]_1 = 0,25$;
 for linear deformations

$$C(t, \tau) = B [C + e^{-\alpha_2 \sqrt{t}}] [1 - e^{-\alpha_2 \sqrt{t-\tau}}] \quad (14)$$

where $B = 8.4 \times 10^{-5}$;
 $C = 0.1$;
 $[\alpha]_2 = 0.25$.

The relative deformation curve of the free shrinkage at the level of the non-reinforced beam's gravity centre is shown in Fig. 5. For calculation purposes the experimental data are approximated by the dependence (solid line in Fig. 5).

$$\varepsilon_{sh} = D [1 - e^{-\alpha_3 (\tau - \tau_0)}] \quad (15)$$

where $D = 25 \times 10^{-5}$;
 $[\alpha]_3 = 0.03$ 1/day (24 h);
 $[\tau]_0 = 20$ days (24 h).

The data on the change in the concrete prismatic strength and instantly resilient deformations are shown in Tables 1 and 2. For calculation, the experimental data are approximated by dependences:

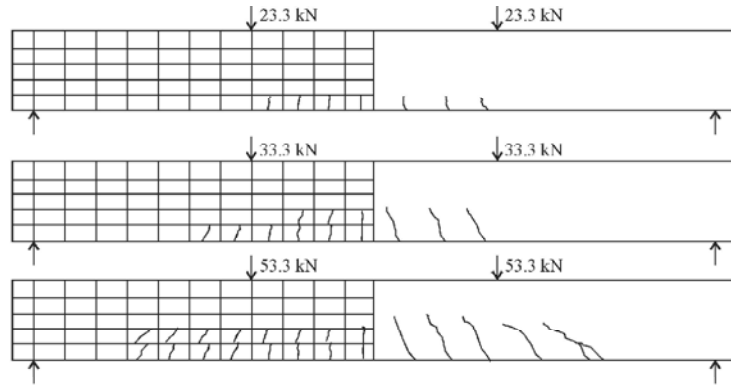


Fig. 6: Crack formation chart

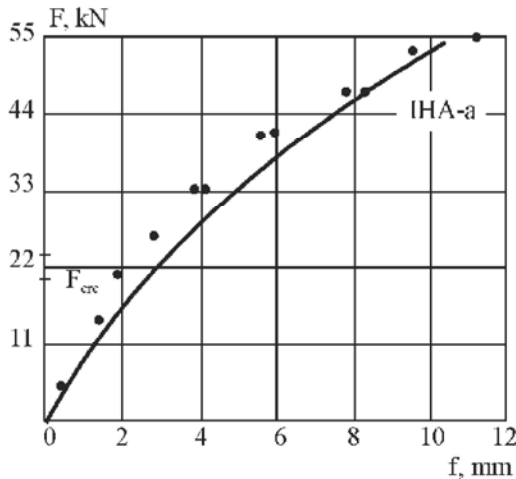


Fig. 7: Beam sagging chart

Table 1: Changes in concrete prismatic strength

Concrete age, days (of 24 h)	Concrete prismatic strength, MPa	
	Experimental data [10]	By formula (16)
21	39.8	39.6
28	42.0	42.2
260	44.7	44.7

Table 2: Changes in instantly resilient concrete deformation module

Concrete age, days (of 24 h)	Instantly resilient concrete deformation module, MPa	
	Experimental data [10]	By formula (17)
21	35.7×10^3	35.7×10^3
28	36.0×10^3	36.0×10^3
260	36.1×10^3	36.1×10^3

$$R_b(\tau) = R_{b,\infty}(1 - e^{-\alpha_4 \tau}) \quad (16)$$

where $R_{b,\infty} = 44,7$ MPa;
 $[\alpha_4]_4 = -0,103$ 1/day (24 h);

$$E_b(\tau) = E_{b,\infty}(1 - e^{-\alpha_5 \tau}), \quad (17)$$

where $E_{b,\infty} = 36.1 \times 10^3$ MPa;
 $[\alpha_5]_4 = -0.215$ 1/day (24 h).

Concrete axial extension strength was determined by the empiric formula

$$R_{bt}(\tau) = 0.05 \left[\frac{40}{3} R_b(\tau) \right]^{2/3}. \quad (18)$$

In the tests, the twin beams were subjected to a short-time stress to destruction. The stress was applied as the concrete was 160 days old and proceeded in stages of 3.3 kN each with a 15 minute cure time at each stage. The destructive stress for the beam IHA-A was 53.3 kN and for IHA-b 57.8 kN. The crack forming stress was, respectively, 19 and 18.8 kN. The designed crack forming stress was 23.3 kN. Both in the experiment and calculation the formation of the first cracks did not result in an abrupt sagging. Fig.6 shows the experimental and designed cracking patterns. Fig.7 presents the experimental and designed beam sagging curves.

CONCLUSIONS

The correlation of experimental and theoretical data shows that the calculation by the proposed method adequately reflects the changes in the construction's stress-deformed state in the event of a complex disproportionate stress by a strain of a previous reduction and with external load.

The proposed method makes it possible to investigate in detail the stress-deformed state of all structural elements and to quantitatively estimate the influence of technical faults, that occurred in erection, on the construction's operational capabilities.

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