Diplomarbeitspräsentationen der Fakultät für Informatik

# Dialogue Games for Fuzzy Logics

Masterstudium
Computational Intelligence

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## Giles's Game

### Overview & Motivation

- dialogue game introduced by Robin Giles in the 1970s
- models reasoning in physical theories
- asserting a proposition means committing oneself to pay a certain amount of money if the associated experiment(s) fail(s)
- seperates evaluation of atomic formulas from decomposing compound formulas

**Betting for Positive Results:**

- each atomic proposition $a$ is associated with a binary (yes/no) experiment $E_a$
- experiments may be probabilistic, i.e. show dispersion
- for each assertion of an atomic proposition an experiment is made
- each player places bets on positive outcomes of experiments corresponding to his claims

**Decomposing Compound Formulas:**

- arguments about complex formulas are systematically reduced to arguments about less complex formulas
- dialogue rules have already been introduced by Lorenzen for Intuitionistic Logic
- these rules characterize the *meaning* of logical connectives, independently of the underlying betting scheme

### Rules

**Atomic Evaluation:** *Let $a$ be an atomic proposition. He who asserts $a$ agrees to pay his opponent €1 if a trial of the experiment associated with $a$ yields the outcome "no".*

**Implication:** *He who asserts $A \to B$ agrees to assert $B$ if his opponent will assert $A$.*

**Negation:** *He who asserts $\neg A$ agrees to assert $\bot$ if his opponent will assert $A$ where $\bot$ is associated with an experiment that always evaluates to "no".*

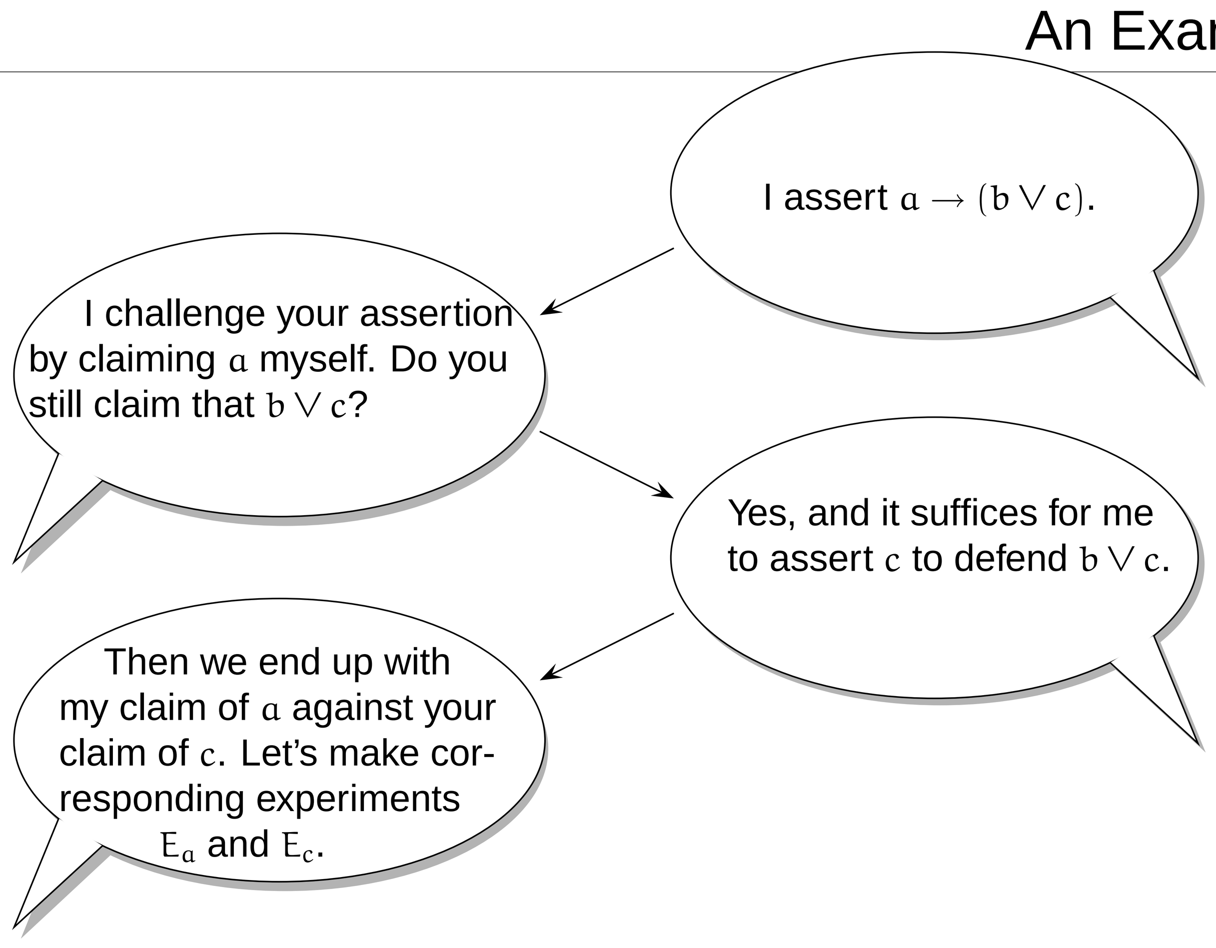
**Disjunction:** *He who asserts $A \vee B$ commits himself to assert either $A$ or $B$ at his own choice.*

**Conjunction:** *He who asserts $A \wedge B$ commits himself to assert either $A$ or $B$ at his opponent's choice.*

**Strong conjunction:** *He who asserts $A \& B$ commits himself either to assert both $A$ and $B$ or to admit falsity by asserting $\bot$.*

After being attacked, a formula is being deleted from the game.

### An Example Dialogue



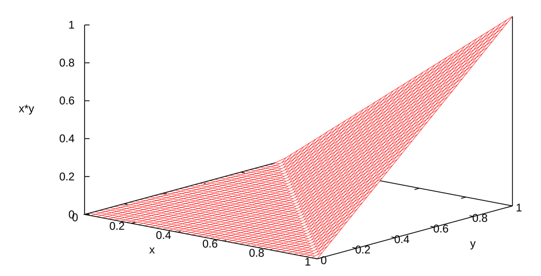

## Łukasiewicz Logic

### T-Norm Based Fuzzy Logics

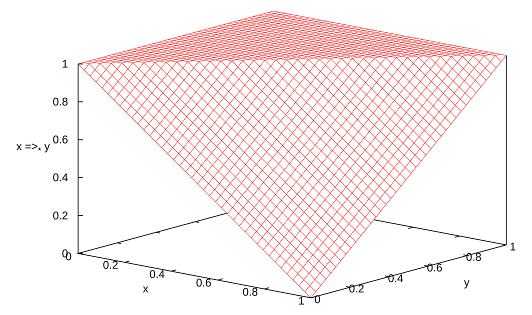
- many valued logics: $0$ stands for absolute falsity, $1$ for truth, but inifinitely many intermediate degrees of truth between $0$ and $1$
- truth function for (strong) conjunction $\&$ is a continuous t-norm
- a t-norm is a commutative, associative function $*: [0,1]^2 \to [0,1]$ with unit $1$ which is order preserving
- truth function for implication $\to$ is the residuum of a t-norm
- the residuum $\Rightarrow_*$ of a t-norm $*$ is determined by $x \Rightarrow_* y := \sup\{z \mid x * z \leqslant y\}$
- other connectives $\wedge$, $\vee$, and $\neg$ are derived from $\&, \to$, and $\bot$

### Łukasiewicz Logic

- one of three *fundamental* t-norm based fuzzy logics
- originally J. Łukasiewicz defined a three-valued logic for modelling future contingents, which has later been extended to infinitely many truth values
- Łukasiewicz t-norm: $x *_L y = \max(0, x + y - 1)$
- associated residuum: $x \Rightarrow_L y = \min(1, 1 - x + y)$
- the *unique* fuzzy logic where all truth functions are continuous
- all connectives can be derived from $\to$ and $\bot$



Łukasiewicz t-Norm $*_L$



Residuum $\Rightarrow_L$

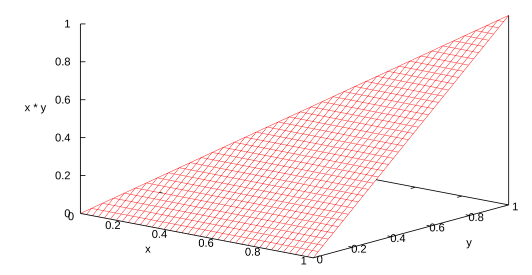
## Other Fuzzy Logics

### Gödel Logic

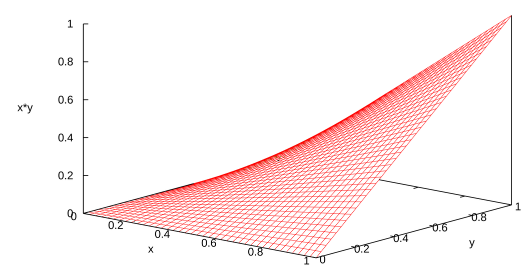
- also known as *Intuitionistic Fuzzy Logic*
- based on the Gödel t-norm $x *_G y = \min(x, y)$
- associated residuum: $x \Rightarrow_G y = y$ if $x > y$, and is $1$ otherwise
- only the order of truth values is relevant for evaluating formulas

### Product Logic

- introduced in 1996 by Hajek, Godo, and Esteva
- based on the Product t-norm $x *_\Pi y = x \cdot y$
- associated residuum: $x \Rightarrow_\Pi y = y/x$ if $x > y$, and is $1$ otherwise



Gödel t-Norm $*_G$



Product t-Norm $*_\Pi$

## Adequateness of Giles's Game

### For Łukasiewicz Logic

- Already proved by Giles in the 1970s:
- A formula $F$ is valid in Łukasiewicz Logic iff I have a strategy to avoid risk (expected loss) in a game starting with me asserting $F$ for any assignment of probability values to experiments.
- Moreover: given a fixed interpretation, my expected loss of money from asserting a formula in the game directly corresponds to a valuation in Łukasiewicz Logic.

### For Gödel & Product Logic

- Variants of Giles's Game presented by Fermüller recently,
- alternative betting schemes: *selecting representatives* (Gödel Logic) and *joint bets* (Product Logic)
- dialogue rule for implication has to be extended as well,
- dialogue rules correspond to the logical rules of an analytic proof system based on relational hypersequents.

### Alternative Dialogue Rules

- Presented in this thesis,
- another way to adapt the dialogue rule for implication for Gödel Logic and Product Logic,
- game gets simpler compared to the other approach,
- connection to the hypersequent calculus is lost.

## Accompanying Implementation

### Webgame

- Web-based application which allows playing Giles's Game interactively,
- simulates evaluation by dispersive experiments.
- see `http://logic.at/people/roschger/thesis/webgame`

### Giles

- Small Haskell-program to display game trees of Giles's game,
- given a formula, computes a game tree of the corresponding game and outputs the tree as a dot-Graph specification.

### Hypseq

- Utility to find derivations of hypersequents in the relational hypersequent calculus **rH**,
- computes all possible derivations and outputs the one with the smallest height.

### TCGame

- Utility to find a winning strategy for the proponent **P** in a Truth Comparison Game,
- for Gödel Logic,
- winning strategy for **P** can be seen as a proof of the starting formula.

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