Models of Sea Shells

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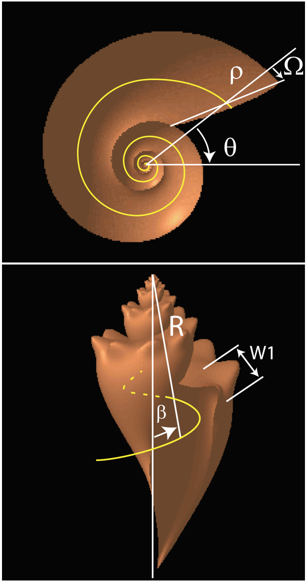
These notes give a brief explanation of the significance of the parameters used in the `Sea_Shells` MATLAB GUI

Sea shells as surfaces

- A sea shell is a surface in 3-space, which can be thought of as resulting from the motion of a *generating curve* along a *structural curve*. The latter describes the global shape of the shell while the former models the shape of the shell aperture.
- These notes describe a 13-parameter model of sea shells, based on [1]. The equations we use are a slight modification of those presented in [1], but the notation is similar.
- The parameters values used for predefined shells in the MATLAB GUI `Sea_Shells` are taken from [1] and [2].

1. M.B. Cortie, *Models for mollusc shell shape*, S. Afr. J. Sci. **85**, 454-460 (1989).
2. M.B. Cortie, *Modelling the surface bumps and spikes of molluscan shells*, in the Proceedings of the First International Conchology Conference, Edited by C.R. Illert, Hadronic Press, Palm Harbor, 1995; pp. 46-65.

Model parameters



- The basic structure of a shell is defined by a curve in 3-space, the structural curve, shown in yellow on the figures.
- Seen from above, this curve looks like a logarithmic spiral of equation

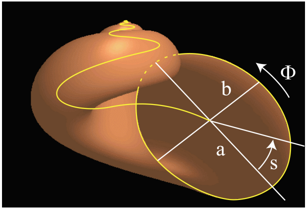
$$\rho = A \sin(\beta) \exp\left(\theta \cot(\alpha)\right),$$

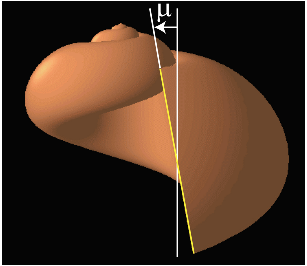
where A, α and β are parameters of the model.

- The distance R illustrated in the lower figure is given by

$$R = A \exp\left(\theta \cot(\alpha)\right).$$

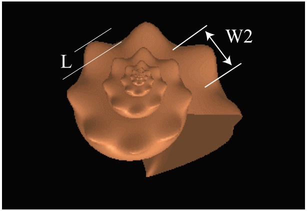
Model parameters (continued)

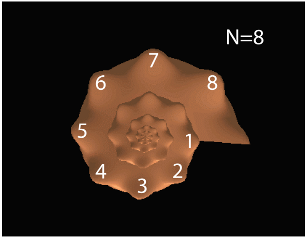




- A second curve, whose basic shape is an ellipse parametrized by s (see figure), is used to generate the outer surface of the shell.
- The parameters a and b are the half lengths of respectively the major and minor axes of the ellipse.
- The ellipse is further rotated by and angle μ about its major axis, by an angle Ω about the vertical axis, and by an angle Φ about a vector normal to its plane.

Model parameters (continued)





For shells with “bumps”, five extra parameters are needed:

1. P is an angle measuring the position of the bump along the ellipse;
2. L measures the height of each bump;
3. $W1$ measures the width of each bump along the ellipse;
4. $W2$ measures the width of each bump along the logarithmic spiral;
5. N is the number of bumps encountered as the angle θ is rotated by 2π.

Model equations

Based on the above description, the parametric equations describing the shell surface are as follows.

$$
\begin{aligned}
x &= \exp\left(\theta \cot(\alpha)\right)\left[A \sin(\beta)\cos(\theta) + h(s,\theta)\right. \\
&\quad \left.\left(\cos(s+\Phi)\,\cos(\Omega+\theta) - \sin(s+\Phi)\,\sin(\mu)\,\sin(\theta+\Omega)\right)\right] \\
y &= \exp\left(\theta \cot(\alpha)\right)\left[-A \sin(\beta)\sin(\theta) - h(s,\theta)\right. \\
&\quad \left.\left(\cos(s+\Phi)\,\sin(\Omega+\theta) + \sin(s+\Phi)\,\sin(\mu)\,\cos(\theta+\Omega)\right)\right] \\
z &= \left[-A\cos(\beta) + h(s,\theta)\sin(s+\Phi)\,\cos(\mu)\right]\exp\left(\theta\cot(\alpha)\right),
\end{aligned}
$$

$$
\begin{aligned}
h(s,\theta) &= \left(\left(\frac{\cos(s)}{a}\right)^2 + \left(\frac{\sin(s)}{b}\right)^2\right)^{-1/2} \\
&\quad + L\,\exp\left(-\left(\frac{2(s-P)}{W1}\right)^2 - \left(\frac{2\,l(\theta)}{W2}\right)^2\right) \\
l(\theta) &= \frac{2\pi}{N}\left(\frac{N\theta}{2\pi} - \mathrm{int}\left(\frac{N\theta}{2\pi}\right)\right).
\end{aligned}
$$