Wind-driven Ekman flow

What is the response of ocean to the wind forcing?

Equation of wind-driven motion

x:

$$Acceleration\,(\frac{du}{dt}) = Coriolis\ \ force\,(fv)$$

$$-\,pressure\ gradient\,(\frac{1}{\rho}\frac{dp}{dx}) + Friction\,(A_z\,\frac{d^2u}{dz^2})$$

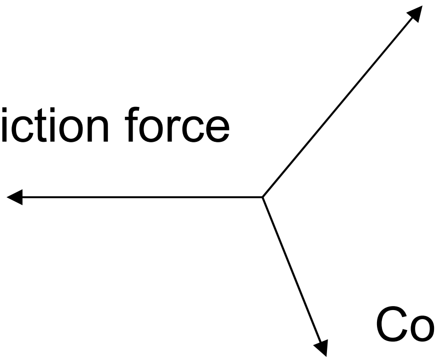
y:

$$\frac{dv}{dt} = -fu - \frac{1}{\rho}\frac{dp}{dy} + A_z\,\frac{d^2v}{dz^2}$$

For steady state:

$$fv - \frac{1}{\rho}\frac{dp}{dx} + A_z \frac{d^2u}{dz^2} = 0,$$

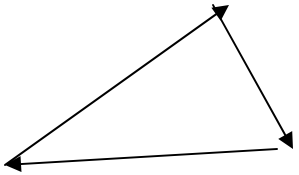
$$-fu - \frac{1}{\rho}\frac{dp}{dy} + A_z \frac{d^2v}{dz^2} = 0.$$



Pressure force

Net friction force

Coriolis force



Vector diagram

Ekman solution

- Consider velocity as two parts, one associated with the horizontal pressure gradient (**Vg)** and one with vertical friction (**Ve)**

$$fv = f(v_g + v_e) = \frac{1}{\rho}\frac{\partial p}{\partial x} - A_z \frac{d^2}{dz^2}(u_g + u_e)$$

The Ekman equations

$$fv_e + A_z \frac{d^2}{dz^2} u_e = 0,$$

$$-fu_e + A_z \frac{d^2}{dz^2} v_e = 0,$$

Coriolis +Friction=0.

Solution of Ekman equations

$$v_e = \pm V_0 \sin(\frac{\pi}{4} + \frac{\pi}{D_e})\exp(\frac{\pi}{D_e} z)$$ ('+' for NH)

$$u_e = \pm V_o \cos(\frac{\pi}{4} + \frac{\pi}{D_e})\exp(\frac{\pi}{D_e} z)$$

where $$V_o = (\frac{2^{0.5}\tau_y}{D_e \rho \mid f \mid})$$

Solution of Ekman equations (Cont.)

- ***Vo*** is the total Ekman surface current,
- Tauy=magnitude of the wind stress on the sea surface (fr. ?) Tau=Cd$\rho_a W^2$
- |*f*|=the magnitude of Coriolis parameter
- *De* the Ekman depth or depth of frictional influence

$$De = \pi \left(\frac{2\ Az}{|f|} \right)^{\frac{1}{2}}$$

Solution interpretation:

- At *z*=0, the solution become

$$u = \pm V_0 \cos 45^o$$

$$v = \pm V_0 \sin 45^o$$

Surface current flows at 45 degree to the right (left) of the wind direction in the NH (SH).

Solution interpretation (Cont.)

- Below surface,

$$\overrightarrow{V} = (u^{2} + v^{2})^{\frac{1}{2}} = Vo\exp(\frac{\pi z}{De})$$

V ↓ as *z* decrease (i.e. water depth increase, note: *z*<0)

From **ue** and **ve** equations, angle of **v**

$$angle = \pm\arctan(\frac{\pi}{4} + \frac{\pi}{De}z)$$

Current direction changes clockwise (anticlockwise) as water depth increase in the NH (SH).

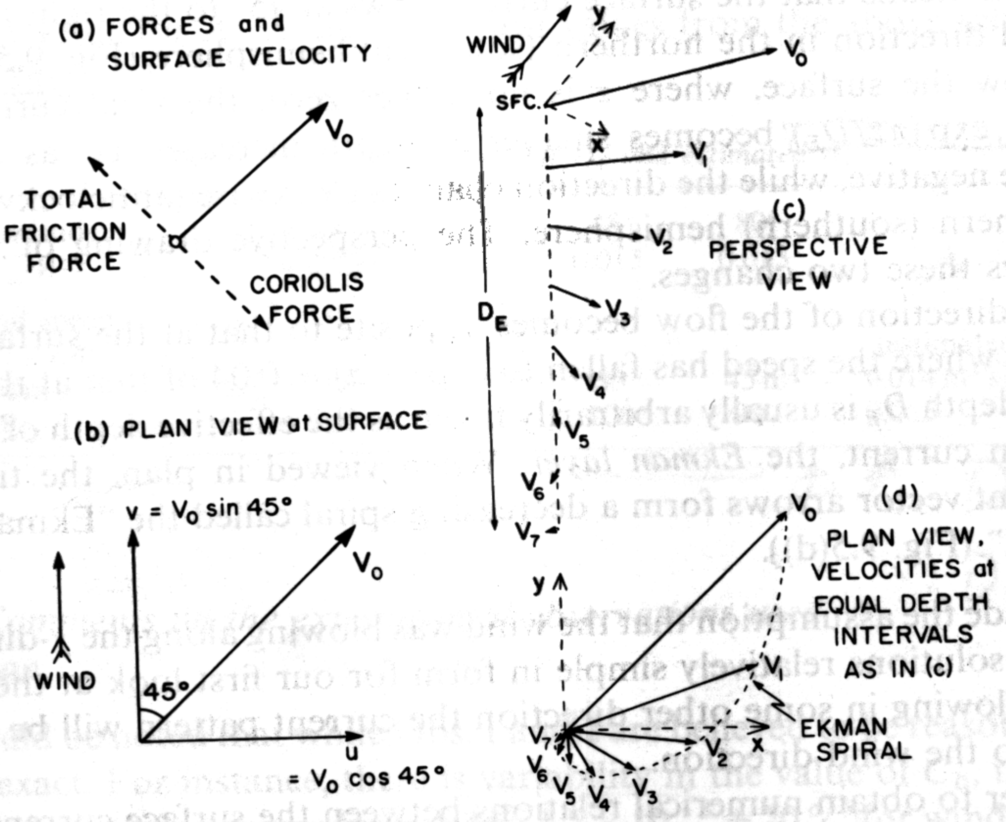


FIG. 9.5 *Wind-driven currents from Ekman analysis: (a) net frictional stress balances Coriolis force with surface current* $\mathbf{V}_0$ *perpendicular to both; (b) wind in y-direction, surface velocity*

Typical values for De and Az

- Latitude 10:

De=100m for 10m/s wind speed

De=200m for 20m/s wind speed

- Latitude 45:

De=50m for 10m/s wind speed

De=100m for 20m/s wind speed

Az:0.012 m^2/s, 0.055 m^2/s

Similarly, ***the water beneath the surface***, although not dragged directly by the wind, is dragged by the surface water (45^o to the right) and causes the flow to the right of surface current, i,.e at an angle > 45^o.

This process continues downward … and forms the **Ekman spiral.**

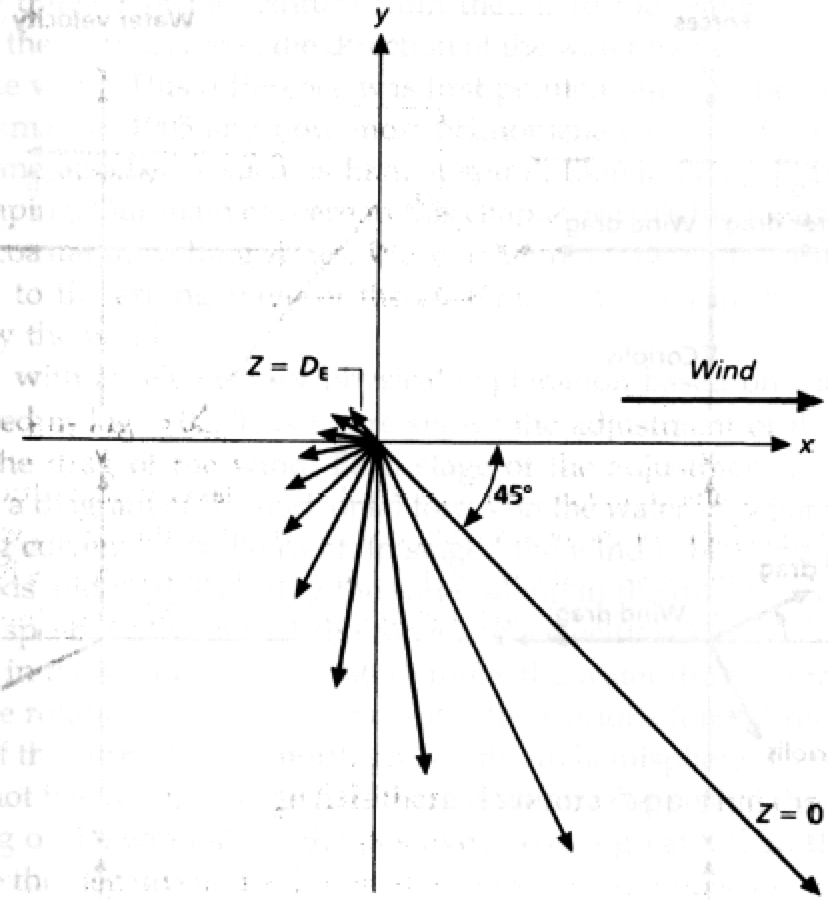


Fig. 5.03 A horizontal projection of the currents at 11 equally spaced levels from the surface to the bottom of the Ekman layer (D_E). The currents are generated by a wind blowing parallel to the positive x axis. For explanation, see text.

At depth De, the direction of velocity is opposite to the surface velocity. De is called Ekman depth

$$D_E \approx 4.4W/(\sin\varphi)^{1/2}$$

W is the wind speed and φ is the latitude.

Ekman drift and coastal upwelling

What is the net transport in the wind-driven Ekman layer and its consequence in the coastal region?

Bottom friction and shallow water effects

- Bottom friction generates an Ekman spiral current pattern.
- Assume $u=u_g$ and $v=0$ away from BL and $u=v=0$ at the bottom, the solution in the NH

$$ue = ug[1 - \exp(-\pi z / De)\cos(\pi z / De)]$$

$$ve = ug\exp(-\pi z / De)\sin(\pi z / De)$$

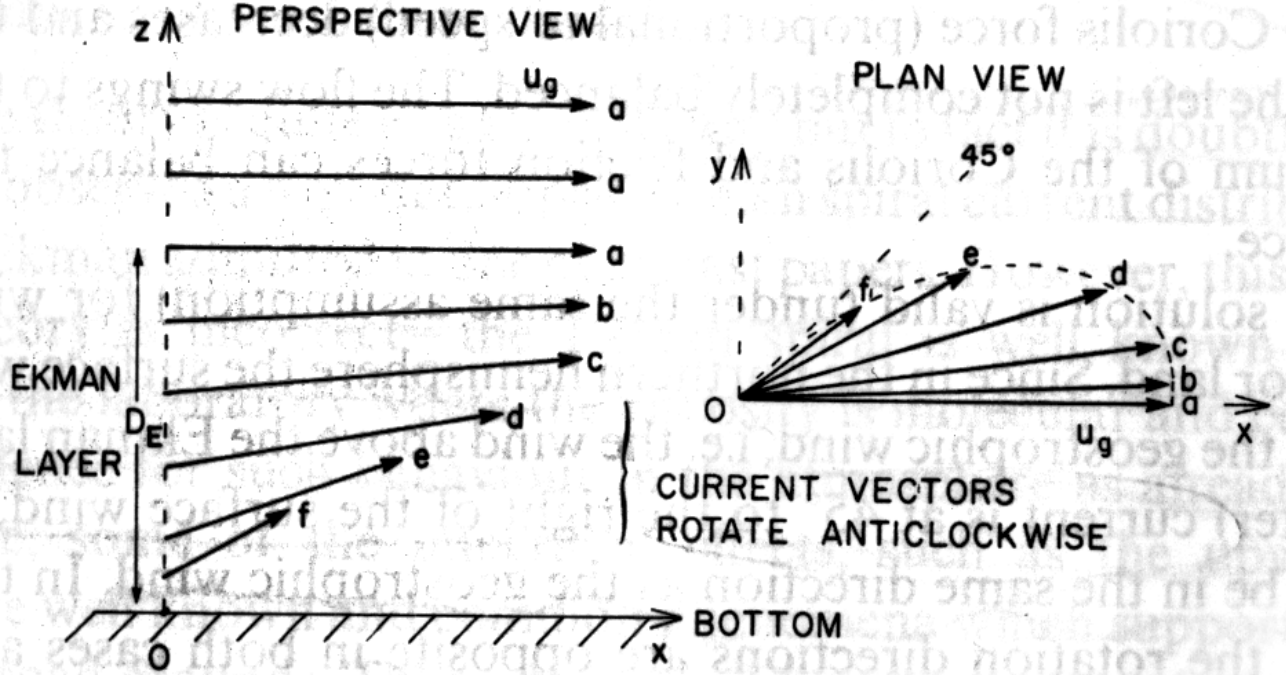


FIG. 9.7 *Frictional effects on a geostrophic current near the bottom of the ocean (northern hemisphere).*

The flow swings to the left as sea floor is approaching due to the unbalance among PGF, Coriolis force and bottom friction, i.e. due to the decrease of Coriolis F by the bottom friction.

Similarly, surface current will be in the same direction as the geostrophic winds.

The De is the atmos. Is 10 times that in the ocean, Az is typically 10 times of Az for ocean.

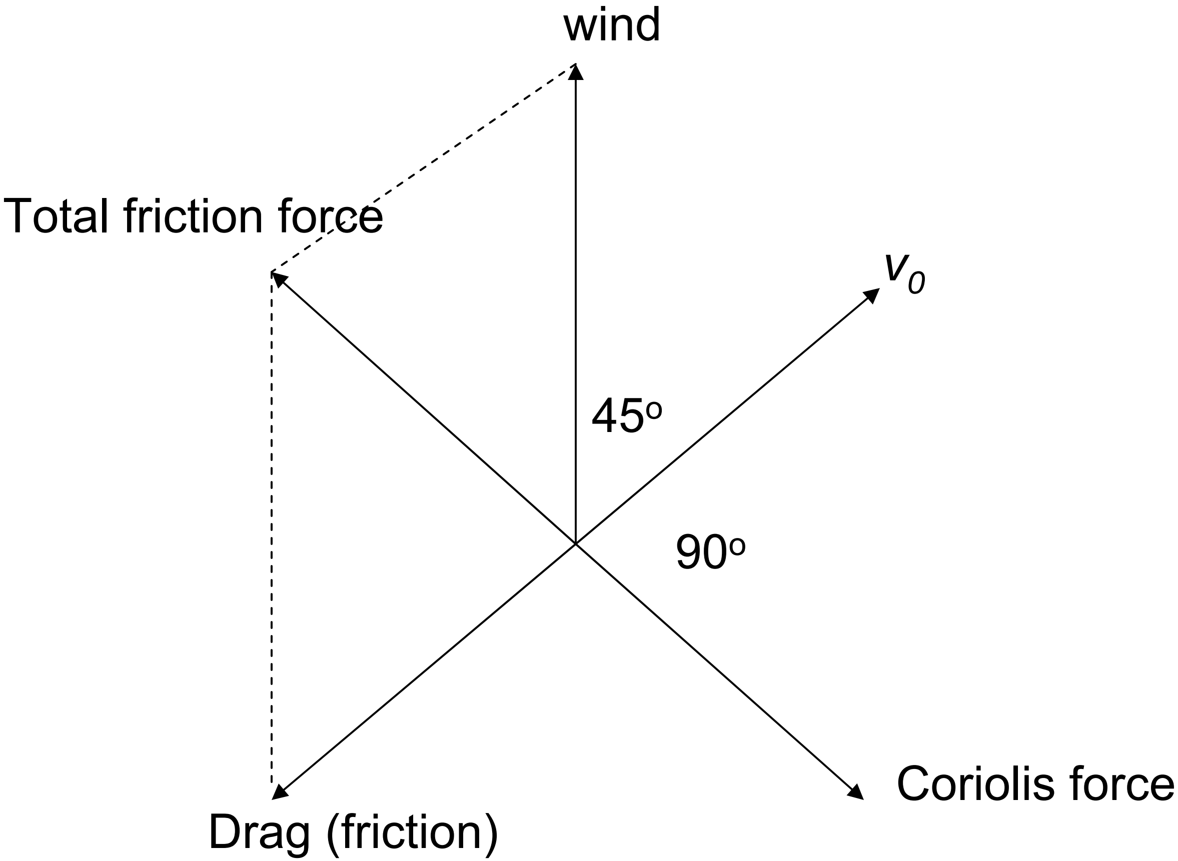
Larger De is atmos. is due to greater wind speed and therefore greater vertical velocity shears.

Bottom friction and shallow water effects

- Surface and bottom Ekman layers overlap
- Two spirals cancel each other (Can this occur in the continental shelf?)
- For water depth deceases to about De/10, the effect of Coriolis force being swamped by the friction.

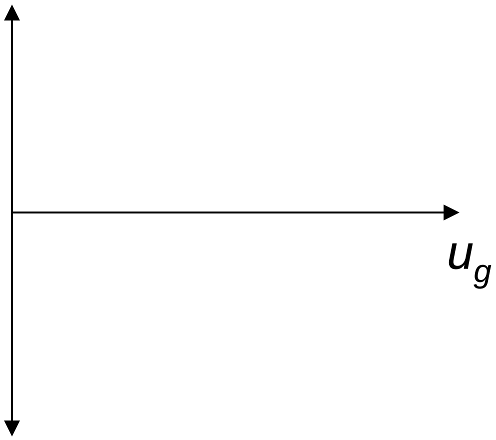
- Balance of Forces in Boundary Layers

Surface flow and balance of forces



Bottom flow and balance of forces

Away from bottom frictional layer

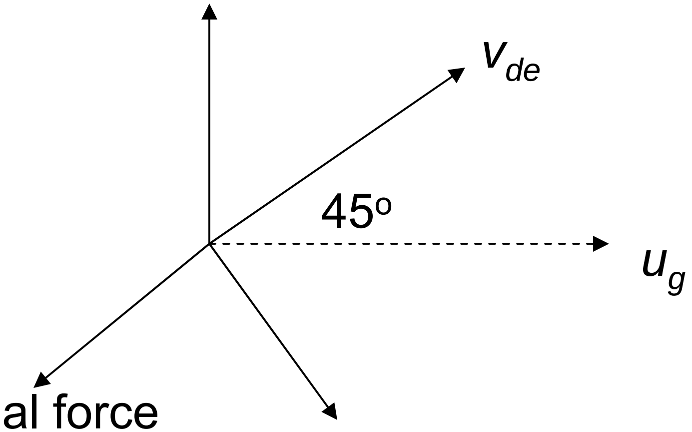


Pressure force

u_g

Coriolis force

In the bottom of bottom Ekman layer



Pressure force

v_{de}

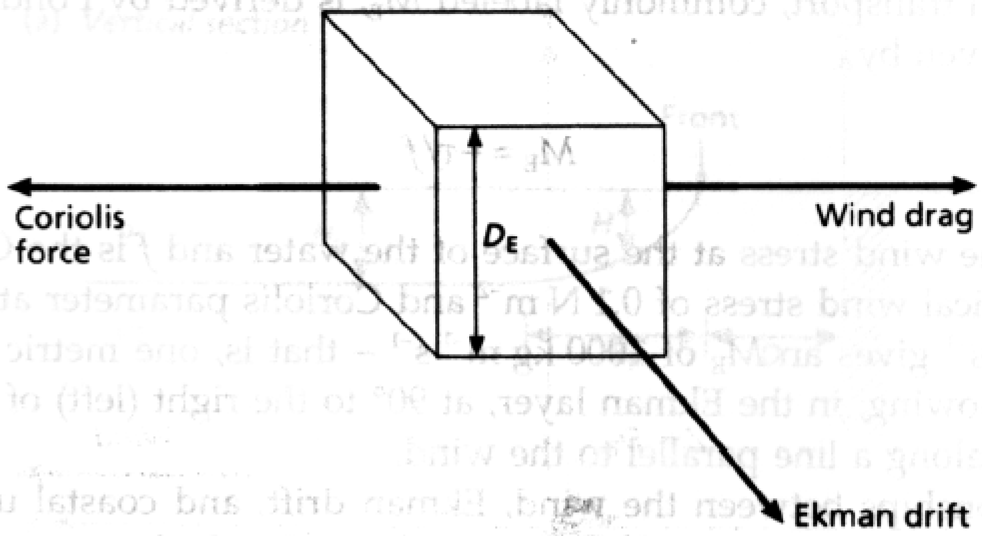
45^o

u_g

Frictional force

Coriolis force

The net movement of the wind-driven flow, after averaging over the Ekman layer, is 90° to the right of the wind.



(averaging effect over De)

Fig. 5.04 A cube of water in the Ekman layer treated as a layer without frictional coupling to the remainder of the ocean lying below. The force due to the wind drag is assumed to act on the whole cube rather than just at the surface and is balanced by the Coriolis force that is generated by the Ekman drift moving perpendicular to both forces.

Transport by Ekman drift, i.e. Ekman transport (Me) is given by

$$M_E = -\frac{\tau}{f}$$

τ is the wind stress at the surface , f is Coriolis parameter.

Relationships between the wind, Ekman drift and coast, as illustrated below, lead to the formation of **Coastal Upwelling**

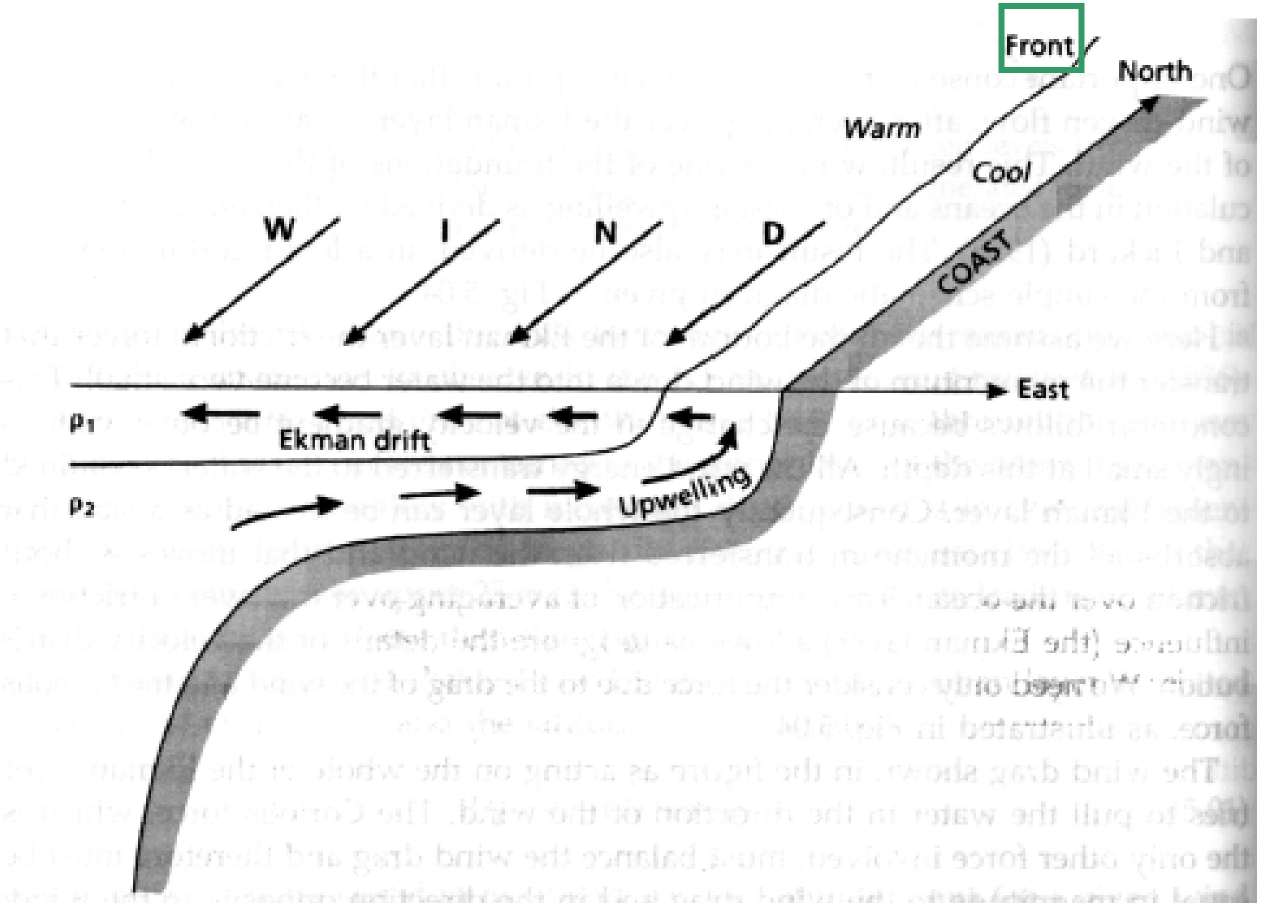


Fig. 5.05 A perspective drawing through an upwelling region illustrating the offshore Ekman drift in the upper layer being replaced near the coast by upward-moving water from the lower layer. The upwelling water, usually cool, is separated from the offshore warm water by a surface front parallel to the coast. The wind blows from north to south.

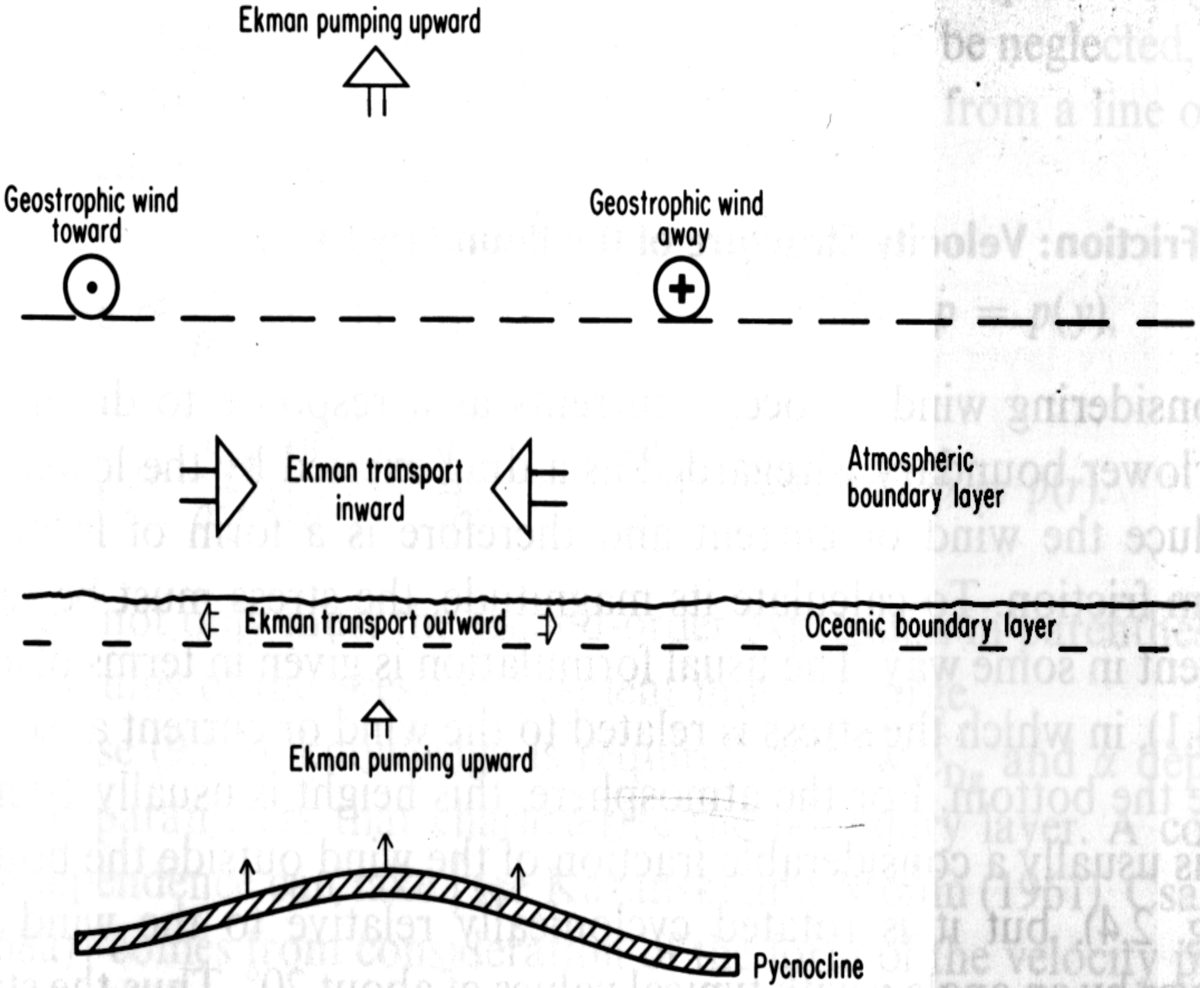


Fig. 9.4. Section through a cyclone over the ocean showing the adjustments due to Ekman transports. The geostrophic wind gives, as shown, a cyclonic rotation around the low-pressure center. Consequently, the Ekman transport in the atmospheric boundary layer is inward, bringing mass in to "fill" the low, and the associated vertical "pumping" velocity is therefore upward. The Ekman mass transport in the oceanic boundary layer is equal and opposite to that in the atmosphere, so there is an outward mass transport and upward pumping velocity in the ocean. This tends to raise the thermocline and create a low-pressure center in the ocean.