The Logic of Categorematic and Syncategorematic Infinity

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Treatises on syncategorematic terms

- Logical treatises on the so-called syncategorematic terms date from the late 12th/early 13th C.
- Syncategorematic terms: *omnis*, *totum*, *decem*, *infinita*, *qualislibet*, *uterque*, *nullus*, *nihil*, *neutrum*, *praeter*, *solus*, *tantum*, *est*, *necessario*, *contingenter*, *incipit*, *desinit*, *si*, *nisi*, *quin*, *et*, *vel*, *an*, *ne*, *sive* (William of Sherwood).
- Discussions of syncategorematic terms can also be found in *tractati de exponibilia*, *de distinctionibus terminorum*, *de sophismatibus*.

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Infinita sunt finita

The most common *sophisma* (logical puzzle or paradox) involving *infinita* is:

Infinita sunt finita. (1)

This sentence is taken to be ambiguous between

Infinite things are finite. (2)

and

Infinitely many things are finite. (3)

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(2) is false, as it predicates "an opposite of an opposite" (William of Sherwood).

(3) is true, since there are infinitely many finite things, e.g., numbers.

The distinction (1): Syntactic

> *Any word that can be used alone as a subject term or as a predicate term is classifiable as a categorematic word; all other words are classifiable as syncategorematic words, those that can occur in a proposition, whether categorical or hypothetical, only along with at least one properly matched pair of categorematic words [Kretzmann, 1982, p. 211].*

Paul of Venice calls this 'the common definition':

> *A syncategorematic term is that which, taken as significant, cannot be the subject or the predicate, or a part of the distributed subject or predicate, of a categorical proposition [of Venice, 1979, p. 7].*

This approach dates back to Priscian's *Institutiones grammaticae*, which was retained in the grammatical tradition.

Paul's objections to the syntactic definition

1. In "Everything seeing every man is an animal" or "You are not seeing every man" the syncategorematic term 'every' occurs as part of a distributed subject and as part of a distributed predicate, respectively. (The assumption is that the status of 'every' as a syncategorematic term is not in question.)
2. Kretzmann's syntactic characterization "produces mutually exclusive and jointly exhaustive classes"; but Paul points out that "there is a simple term that is neither categorematic nor syncategorematic":
    1. terms such as *nihil* 'nothing',
    2. the copula,
    3. material terms 'A', 'B', 'C', which stand for terms but are not significative in themselves [of Venice, 1979, pp. 6–7].

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The distinction (2): Semantic

Henry of Ghent (c. 1260) who says:

> *They are called syncategorematic as if to say 'consignificant'—i.e., significant together with others, namely, with categoremata—not because they signify nothing on their own, but because they have a signification that is not definite but indefinite, a signification whose definiteness they derive from those [words] that are adjoined to them [Kretzmann, 1982, p. 213].*

Syncategorematic terms do signify, but in an improper and indeterminate way.

The *use* of a term

- Syncategorematic terms vs. categorematic terms.
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The question:

Is infinita in the two readings of infinita sunt finita that same word used in different ways, or different, but homophonic, words?

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Relation to other distinctions

- Actual vs. potential infinity.
- Wide vs. narrow scope.
- Divided vs. composite readings.

Actual and potential infinity (1)

Some have argued that the syncategorematic/categorematic distinction is the same as the Aristotelian potential/actual distinction:

> *[Anneliese Maier] believed that it was merely a matter of terminology that a categorematic infinite corresponded to an actual infinite, whereas a syncategorematic infinite was equivalent to a potential infinity [Murdoch and Thijssen, 2001, p. 129].*

> *One particularly troublesome case here for a modern English speaker is the word 'infinite', which can be taken to refer to an actual infinite (used categorically) or a merely potential infinite (used syncategorematically) [Longeway, 2010, §3.4].*

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Actual and potential infinity (2)

This is a mistake. Geach accurately diagnoses the problem:

> *The distinction between actual and potential infinity is a distinction between two ways in which outside things, res extra, could be said to be infinite. 'Categorematic' and 'syncategorematic' on the other hand are words used to describe (uses of) words in a language; an infinite multitude, say, can no more be syncategorematic than it can be pronominal or adverbial. . . [while] the confusion is explicable. . . this does not make the confusion excusable—especially as there is no such close connexion between the potentially infinite and the syncategorematic use of 'infinite'" [Geach, 1967, p. 41].*

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Additionally, Peter of Spain has explicitly rejected any coincidence between the two distinctions [Moore, 1990, p. 51].

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Wide and narrow scope

Moore offers an alternative explanation of the distinction:

> *Roughly: to use 'infinite' categorematically is to say that there is something which has a property that surpasses any finite measure; to use it syncategorematically is to say that, given any finite measure, there is something which has a property that surpasses it [Moore, 1990, p. 51].*

On this view, the problem is a matter of scope, and one can solve paralogisms involving *infinita* simply by keeping track of the appropriate scoping.

Divided and composite readings (1)

But describing the distinction in terms of 'wide scope' and 'narrow scope' is rather anachronistic. Instead, for sentences with modal operators, medieval logicians distinguished between the divided and composite readings of sentences. This distinction has likewise often been conflated with the syncategorematic/ categorematic one:

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For example, Murdoch [Murdoch, 1982, pp. 567–568]:

infinita	categorematic	syncategorematic
	collective	distributive
	composite	divided
	actual infinity	potential infinity
	follows the subject	precedes the subject

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A better rule is given by Heytesbury:

> *When the word 'infinite' is placed at the beginning of a sentence and belongs to the subject, it has to be interpreted as a syncategorematic term; in any other case, it is usually interpreted as a categorematic term (Sophismata, sophisma xviii, fol.130va).*
>
> *It must be said generally that when that syncategorematic term comes first in any proposition, with nothing determinable with respect to what is preceding it, it is interpreted syncategorematically. But when a term determinable with respect to it precedes it when it occurs in the subject, it is interpreted categorematically, just as when it occurs in the predicate" [Heytesbury, 1988, pp. 421–422].*

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The logic of syn/categorematic infinity

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  - Heytesbury's rule
  - *Infinita* as a distributive sign
  - Suppositional rules for conversion

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Heytesbury's rule

Heytesbury gives nine modes of compounding and dividing. The fourth "is dependent on a term that is sometimes taken categorematically and other times syncategorematically. [Heytesbury, 1988, pp. 416–417]. He elaborates:

> *With respect to terms that are sometimes taken categorematically and other times syncategorematically, however, the consequence is fallacious when one infers the compounded sense from the divided sense. . . Thus in general, when the term 'infinite' or any syncategorematic term of that sort precedes the proposition entirely, so that there is no term ahead of it which is a determination in respect of that term standing syncategorematically, the divided sense occurs and [the proposition] signifies dividedly [Heytesbury, 1988, pp. 421–422].*

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Infinita as a distributive sign

Paul of Venice's definition of syncategorematic terms gives a further avenue for research:

Definition

A syncategorematic term is a sign that carries out a function and in the absence of a new imposition is significant *per se* of nothing other than itself and what is equiform to it. . . by carrying out a function I mean having the force of distributing, confusing, conjoining, disjoining, conditionalizing, and determinating [of Venice, 1979, p. 5].

William of Sherwood: A universal sign is that which "divides the subject with respect to the predicate".

Gregory of Rimini: "For the term 'infinite', taken syncategorematically and with respect to the subject, makes a proposition universal; but taken categorematically it does not" (trans. from [Thakkar, 2004].)

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Standard account of simple conversion (1)

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Definition

Simple conversion is when the predicate of a categorical proposition is made the subject of a new categorical proposition, and the subject of the first the predicate of the second.

Rule

In a proposition which has been simply converted nothing more nor less than the predicate would become the subject.

Standard account of simple conversion (2)

If distributive terms such as ‘every’ can be part of the predicate of a sentence, then

Socrates is every man. (5)

would be converted into

Every man is Socrates. (6)

But this violates the Rule, because in (5), the predicate is ‘every man’, but in (6), the subject is just ‘man’, not ‘every man’, since when ‘every’ precedes a subject term, it functions syncategorematically and is not a part of the subject.

Suppositional account of simple conversion (1)

Paul rejects the Rule, and gives another one:

Rule

What is required in simple conversion is that the terms in the converse and the convertend supposit for precisely the same thing or things, and in the same way [of Venice, 1979, p. 35].

Corollary

Not all universal negative propositions can be simply converted [of Venice, 1979, pp. 34–35].

Suppositional account of simple conversion (1)

Examples [of Venice, 1979, pp. 34–35]:

No man is running. (7)

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- The fascinating *sophismata* of Richard Kilvington, e.g.,

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  - "Socrates is infinitely whiter than Plato begins to be white."
  - "It is infinitely easier to make C be true than to make D be true."
  - "It is infinitely easier for B to make it be the case that the proposition 'Infinitely many parts of A have been traversed' is true than to make it the case that the proposition 'All of A has been traversed' is true."
  - "Infinitely sooner will A be true than B will be true."

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