10 Binary Stars and Stellar Masses

Quite often, two stars may appear to be close together in the sky, although they are really at very different distances. Such chance pairs are called *optical binary stars*. However, many close pairs of stars really are at the same distance and form a physical system in which two stars are orbiting around each other. Less than half of all stars are single stars like the Sun. More than 50 % belong to systems containing two or more members. In general, the multiple systems have a hierarchical structure: a star and a binary orbiting around each other in triple systems, two binaries orbiting around each other in quadruple systems. Thus most multiple systems can be described as binaries with several levels.

Binaries are classified on the basis of the method of their discovery. This classification has nothing to do with the physical properties of the stars. *Visual binaries* can be seen as two separate components, i.e. the separation between the stars is larger than about 0.1 arc seconds. The relative position of the components changes over the years as they move in their orbits (Fig. 10.1). In *astrometric binary stars* only one component is seen, but its variable proper motion shows that a second invisible component must be present. The *spectroscopic binary stars* are discovered on the basis of their spectra. Either two sets of spectral lines are seen or else the Doppler shift of the observed lines varies periodically, indicating an invisible companion. The fourth class of binaries are the *photometric binary stars* or *eclipsing variables*. In these systems the components of the pair regularly pass in front of each other, causing a change in the total apparent magnitude.

Binary stars can also be classified on the basis of their mutual separation. In *distant binaries* the separation between the components is tens or hundreds of astronomical units and their orbital periods are from tens to thousands of years. In *close binaries* the separation is from about one au down to the radius of the stars. The orbital period ranges from a few hours to a few years. The components of *contact binaries* are so close that they are touching each other.

The stars in a binary system move in an elliptical orbit around the centre of mass of the system. In Chap. 6 it was shown that the relative orbit, too, is an ellipse, and thus the observations are often described as if one component remained stationary and the other orbited around it.

10.1 Visual Binaries

We consider a visual binary, assuming initially that the brighter primary component is stationary and the fainter secondary component is orbiting around it. The angular separation of the stars and the angular direction to the secondary can be directly observed. Making use of observations extending over many years or decades, the relative orbit of the secondary can be determined. The first binary orbit to be determined was that of ξ UMa in 1830 (Fig. 10.2).

The observations of visual binaries only give the projection of the relative orbital ellipse on the plane of the sky. The shape and position of the

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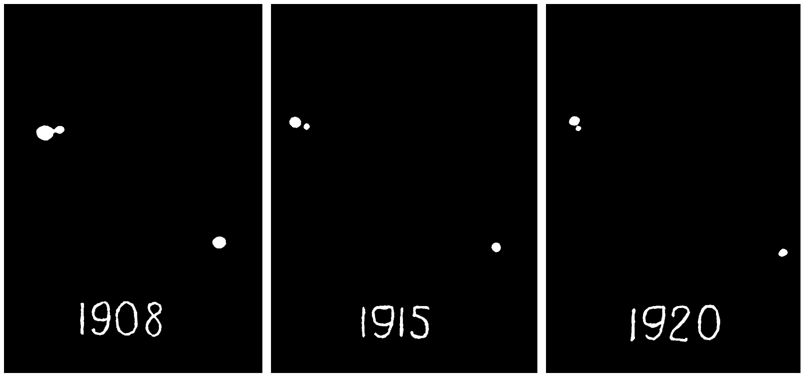


Fig. 10.1 When a visual binary is followed for a long time, the components can be seen to move with respect to each other. Picture of Krüger 60. (Yerkes Observatory)

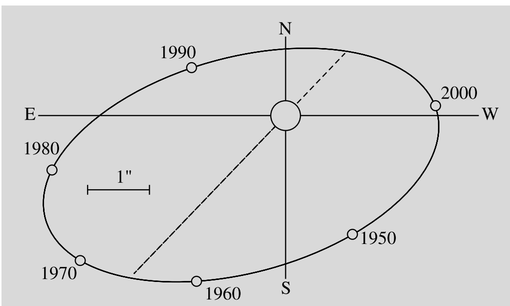


Fig. 10.2 In 1830 the orbit of ξ Ursae Majoris was the first binary orbit determined observationally

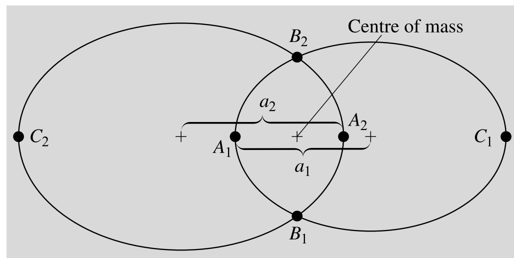


Fig. 10.3 The components of a binary system move around their common centre of mass. A_1, A_2 denote the positions of the stars at a given time A, and similarly for B and C

true orbit are not known. However, they can be calculated if one makes use of the fact that the primary should be located at a focal point of the relative orbit. The deviation of the projected position of the primary from the focus of the projected relative orbit allows one to determine the orientation of the true orbit.

The absolute size of the orbit can only be found if the distance of the binary is known. Knowing this, the total mass of the system can be calculated from Kepler's third law.

The masses of the individual components can be determined by observing the motions of both components relative to the centre of mass (Fig. 10.3). Let the semimajor axes of the orbital ellipses of the primary and the secondary be a_1 and a_2. Then, according to the definition of the centre of mass,

$$\frac{a_1}{a_2} = \frac{m_2}{m_1}, \tag{10.1}$$

where m_1 and m_2 are the component masses. The semimajor axis of the relative orbit is

$$a = a_1 + a_2. \tag{10.2}$$

For example, the masses of the components of ξ UMa have been found to be 1.3 and 1.0 solar masses.

10.2 Astrometric Binary Stars

In astrometric binaries, only the orbit of the brighter component about the centre of mass can be observed. If the mass of the visible component is estimated, e.g. from its luminosity, the mass of the invisible companion can also be estimated.

The first astrometric binary was Sirius, which in the 1830's was observed to have an undulating proper motion. It was concluded that it had a small companion, which was visually discovered a few decades later (Figs. 10.4 and 15.1).

The companion, Sirius B, was a completely new type of object, a white dwarf (Sect. 15.1).

The proper motions of nearby stars have been carefully studied in the search for planetary systems. Although e.g. Barnard's star may have unseen companions, the existence of planetary systems around other stars was not established by proper motion studies but with spectroscopic observations (Sect. 21.8).

10.3 Spectroscopic Binaries

The spectroscopic binaries (Fig. 10.5) appear as single stars in even the most powerful telescopes, but their spectra show a regular variation. The first spectroscopic binary was discovered in the 1880's, when it was found that the spectral lines of ζ UMa or Mizar split into two at regular intervals.

The Doppler shift of a spectral line is directly proportional to the radial velocity. Thus the separation of the spectral lines is largest when one component is directly approaching and the other is receding from the observer. The period of the variation is the orbital period of the stars. Unfortunately, there is no general way of determining the position of the orbit in space. The observed velocity v is related to the true velocity v_0 according to

$$v = v_0 \sin i, \tag{10.3}$$

where the inclination i is the angle between the line of sight and the normal of the orbital plane.

Consider a binary where the components move in circular orbits about the centre of mass. Let the radii of the orbits be a_1 and a_2. From the definition of the centre of mass $m_1 a_1 = m_2 a_2$, and writing $a = a_1 + a_2$, one obtains

$$a_1 = \frac{a m_2}{m_1 + m_2}. \tag{10.4}$$

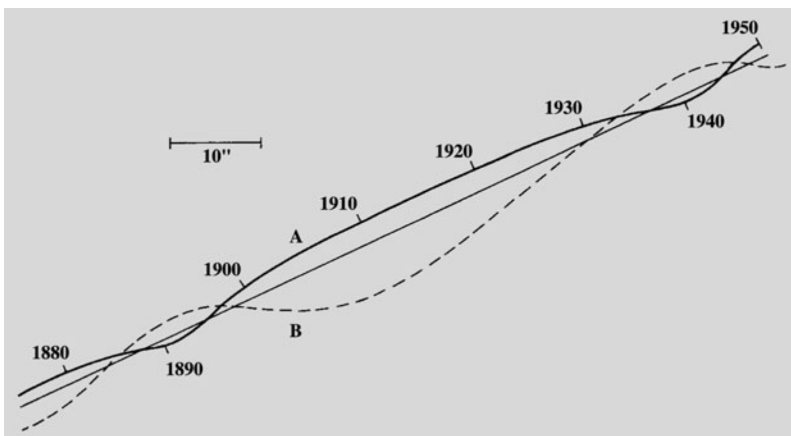


Fig. 10.4 The apparent paths of Sirius and its companion in the sky

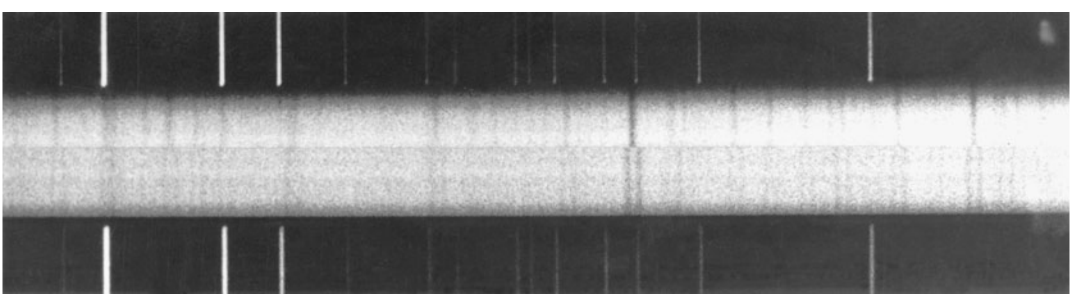


Fig. 10.5 Spectrum of the spectroscopic binary κ Arietis. In the upper spectrum the spectral lines are single, in the lower one doubled. (Lick Observatory)

The true orbital velocity is

$$v_{0,1} = \frac{2\pi a_1}{P},$$

where P is the orbital period. The observed orbital velocity according to (10.3) is thus

$$v_1 = \frac{2\pi a_1 \sin i}{P}. \tag{10.5}$$

Substituting (10.4), one obtains

$$v_1 = \frac{2\pi a}{P} \frac{m_2 \sin i}{m_1 + m_2}.$$

Solving for a and substituting it in Kepler's third law, one obtains the *mass function equation*:

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{v_1^3 P}{2\pi G}. \tag{10.6}$$

If one component in a spectroscopic binary is so faint that its spectral lines cannot be observed, only P and v_1 are observed. Equation (10.6) then gives the value of the mass function, which is the expression on the left-hand side. Neither the masses of the components nor the total mass can be determined. If the spectral lines of both components can be observed, v_2 is also known. Then (10.5) gives

$$\frac{v_1}{v_2} = \frac{a_1}{a_2}$$

and furthermore the definition of the centre of mass gives

$$m_1 = \frac{m_2 v_2}{v_1}.$$

When this is substituted in (10.6), the value of $m_2 \sin^3 i$, and correspondingly, $m_1 \sin^3 i$, can be determined. However, the actual masses cannot be found without knowing the inclination.

The size of the binary orbit (the semimajor axis a) is obtained from (10.5) apart from a factor $\sin i$. In general the orbits of binary stars are not circular and the preceding expressions cannot be applied as they stand. For an eccentric orbit, the shape of the velocity variation departs more and more from a simple sine curve as the eccentricity increases. From the shape of the velocity variation, both the eccentricity and the longitude of the periastron can be determined. Knowing these, the mass function or the individual masses can again be determined to within a factor $\sin^3 i$.

10.4 Photometric Binary Stars

In the photometric binaries, a periodic variation in the total brightness is caused by the motions of the components in a double system. Usually the photometric binaries are *eclipsing variables*, where the brightness variations are due to the components passing in front of each other. A class of photometric binaries where there are no actual eclipses are the *ellipsoidal variables*. In these systems, at least one of the components has been distorted into an ellipsoidal shape by the tidal pull of the other one. At different phases of the orbit, the projected surface area of the distorted component varies. The surface temperature will also be lower at the ends of the tidal bulges. Together these factors cause a small variation in brightness.

The inclination of the orbit of an eclipsing binary must be very close to 90°. These are the only spectroscopic binaries for which the inclination is known and thus the masses can be uniquely determined.

The variation of the magnitude of eclipsing variables as a function of time is called the *lightcurve*. According to the shape of the lightcurve, they are grouped into three main types: *Algol*, *β Lyrae* and *W Ursae Majoris* type (Fig. 10.6).

Algol Stars The Algol-type eclipsing variables have been named after β Persei or Algol. During most of the period, the lightcurve is fairly constant. This corresponds to phases during which the stars are seen separate from each other and the total magnitude remains constant. There are two different minima in the lightcurve, one of which, the primary minimum, is usually much deeper than the other one. This is due to the brightness difference of the stars. When the larger star, which is usually a cool giant, eclipses the smaller and hotter component, there is a deep minimum in the lightcurve. When the small,

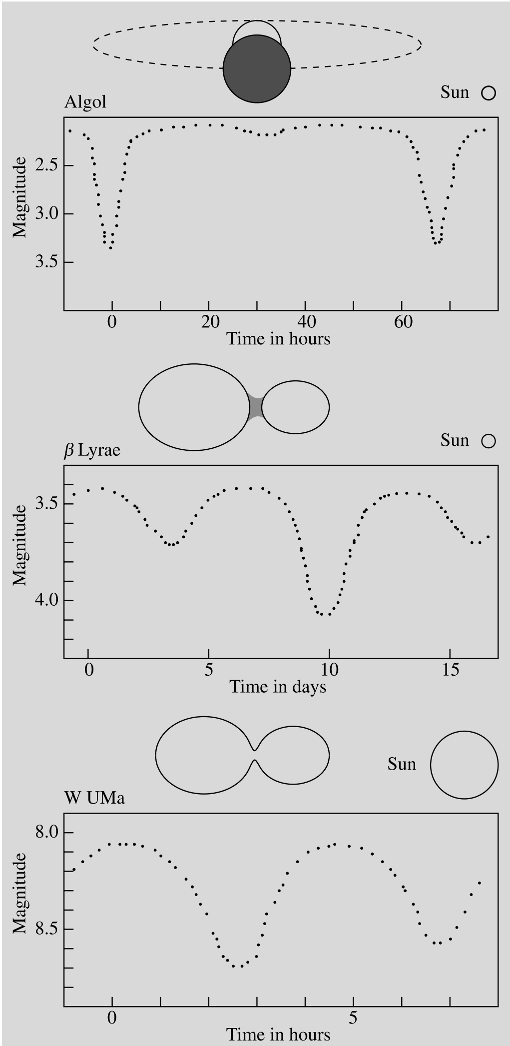


Fig. 10.6 Typical lightcurves and schematic views of Algol, β Lyrae and W Ursae Majoris type binary systems. The size of the Sun is shown for comparison

bright star passes across the disk of the giant, the total magnitude of the system does not change by much.

The shape of the minima depends on whether the eclipses are partial or total. In a partial eclipse the lightcurve is smooth, since the brightness changes smoothly as the depth of the eclipse varies. In a total eclipse there is an interval during which one component is completely invisible. The total brightness is then constant and the lightcurve has a flat bottomed minimum. The shape of the minima in Algol variables thus gives information on the inclination of the orbit.

The duration of the minima depends on the ratio of the stellar radii to the size of the orbit. If the star is also a spectroscopic binary, the true dimensions of the orbit can be obtained. In that case the masses and the size of the orbit, and thus also the radii can be determined without having to know the distance of the system.

β Lyrae Stars In the β Lyrae-type binaries, the total magnitude varies continuously. The stars are so close to each other that one of them has been pulled into ellipsoidal shape. Thus the brightness varies also outside the eclipses. The β Lyrae variables can be described as eclipsing ellipsoidal variables. In the β Lyrae system itself, one star has overfilled its Roche lobe (see Sect. 12.6) and is steadily losing mass to its companion. The mass transfer causes additional features in the lightcurve.

W UMa Stars In W UMa stars, the lightcurve minima are almost identical, very round and broad. These are close binary systems where both components overfill their Roche lobes, forming a contact binary system.

The observed lightcurves of photometric binaries may contain many additional features that confuse the preceding classification.

- The shape of the star may be distorted by the tidal force of the companion. The star may be ellipsoidal or fill its Roche surface, in which case it becomes drop-like in shape.
- The limb darkening (Sects. 9.6 and 13.2) of the star may be considerable. If the radiation from the edges of the stellar disk is fainter than that from the centre, it will tend to round off the lightcurve.
- In elongated stars there is gravity darkening: the parts most distant from the centre are cooler and radiate less energy.
- There are also reflection phenomena in stars. If the stars are close together, they will heat the sides facing each other. The heated part of the surface will then be brighter.
- In systems with mass transfer, the material falling onto one of the components will change the surface temperature.

All these additional effects cause difficulties in interpreting the lightcurve. Usually one computes a theoretical model and the corresponding lightcurve, which is then compared with the observations. The model is varied until a satisfactory fit is obtained.

So far we have been concerned solely with the properties of binary systems in the optical domain. Recently many double systems that radiate strongly at other wavelengths have been discovered. Particularly interesting are the binary pulsars, where the velocity variation can be determined from radio observations. Many different types of binaries have also been discovered at X-ray wavelengths. These systems will be discussed in Chap. 15.

The binary stars are the only stars with accurately known masses. The masses for other stars are estimated from the mass-luminosity relation (Sect. 9.7), but that is valid only for main-sequence stars and has to be calibrated by means of binary observations.

10.5 Examples

Example 10.1 (The Mass of a Binary Star) The distance of a binary star is 10 pc and the largest angular separation of the components is $7''$ and the smallest is $1''$. The orbital period is 100 years. The mass of the binary is to be determined, assuming that the orbital plane is normal to the line of sight.

From the angular separation and the distance, the semimajor axis is

$$a = 4'' \times 10\ \mathrm{pc} = 40\ \mathrm{au}.$$

According to Kepler's third law

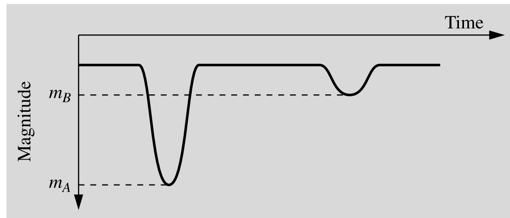
$$m_1 + m_2 = \frac{a^3}{P^2} = \frac{40^3}{100^2} M_\odot = 6.4\, M_\odot.$$

Let the semimajor axis of one component be $a_1 = 3''$ and for the other $a_2 = 1''$. Now the masses of the components can be determined separately:

$$m_1 a_1 = m_2 a_2 \quad \Rightarrow \quad m_1 = \frac{a_2}{a_1} m_2 = \frac{m_2}{3},$$

$$m_1 + m_2 = 6.4 \quad \Rightarrow \quad m_1 = 1.6, \qquad m_2 = 4.8.$$

Example 10.2 (The Lightcurve of a Binary) Let us suppose that the line of sight lies in the orbital plane of an Algol type binary, where both components have the same radius. The lightcurve is essentially as shown in the figure. The primary minimum occurs when the brighter component is eclipsed. The depth of the minima will be calculated.



If the effective temperatures of the stars are T_A and T_B and their radius is R, their luminosities are given by

$$L_A = 4\pi R^2 \sigma T_A^4, \qquad L_B = 4\pi R^2 \sigma T_B^4.$$

The flat part of the curve corresponds to the total luminosity

$$L_{\mathrm{tot}} = L_A + L_B.$$

The luminosities may be expressed as absolute bolometric magnitudes by means of (4.13). Since the distance moduli of the components are the same, the apparent bolometric magnitude at the primary minimum will be

$$\begin{aligned} m_A - m_{\mathrm{tot}} &= M_A - M_{\mathrm{tot}} \\ &= -2.5 \lg \frac{L_A}{L_{\mathrm{tot}}} = +2.5 \lg \frac{L_{\mathrm{tot}}}{L_A} \\ &= 2.5 \lg \frac{4\pi R^2 \sigma T_A^4 + 4\pi R^2 \sigma T_B^4}{4\pi R^2 \sigma T_A^4} \\ &= 2.5 \lg \left(1 + \left(\frac{T_B}{T_A}\right)^4\right). \end{aligned}$$

Similarly the depth of the secondary minimum is

$$m_B - m_{\mathrm{tot}} = 2.5 \lg \left(1 + \left(\frac{T_A}{T_B}\right)^4\right).$$

Let the effective temperatures of the stars be $T_A = 5000$ K and $T_B = 12{,}000$ K. The depth of