Atti del Convegno Internazionale di Storia della Logica, *Le teorie delle modalità*; San Gimignano 5-8 dicembre 1987. CLUEB, Bologna (Italy) © 1989.

THE MODAL LOGIC OF JOHN BURIDAN

George E. Hughes

Emeritus Professor, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand

This paper will be almost wholly expository. My aim in it is to give an outline, though I fear a very incomplete one, of the system of modal logic developed by one of the greatest of mediaeval logicians, the 14th. century French philosopher John Buridan. I shall base my account on two of his logical works. One is his *Consequentiae*, a work on inferences in general, about a third of which is devoted to modal logic. There is an excellent modern edition of this work by Hubert Hubien, in the Introduction to which Hubien argues, to my mind convincingly, that it was written about 1335. The other is his lengthy and comprehensive work on logic, the *Summulae de Dialectica*, which contains two substantial sections on modal logic. There are serious problems involved in dating this work, but I shall not try to discuss these here. Most of it, including all the modal material, still exists, unfortunately, only in manuscript form. The modal material is arranged differently in these two works, each is fuller than the other on certain topics, and there are a few discrepancies between them on points of detail; but substantially they present the same ideas, and for the most part I shall not try to distinguish between them here.

Like most modern modal logicians, Buridan builds up his modal logic as an extension of an underlying, simpler, non-modal logic, and to make his modal logic intelligible I shall have to give a sketch of that first. I shall concentrate on those elements in it which are specially relevant to the modal developments, though inevitably this will mean omitting much that is of considerable intrinsic interest.

Buridan's logic is a term-logic, in roughly the Aristotelian tradition. In such a logic the simplest kind of proposition consists of a pair of (categorematic) terms, known as subject and predicate respectively, joined by a copula ('is' or 'is-not'), and perhaps preceded by a sign of quantity such as 'every', 'some' or 'no'. Terms are of two kinds: singular (or discrete) and common. We can construct a proposition out of two singular terms, or one singular and one common term, in either order, or two common terms; and if the subject is a common term it may be prefixed by 'every' or 'some'. (If no sign of quantity is used, the proposition is interpreted as if 'some' had occurred.) 'Every A is-not B' is frequently written as 'No A is B'; but I shall usually use the former form, since it will be more useful when we come to modal logic.

It is important to see what Buridan takes the truth-conditions of propositions of these forms to be. His basic idea is that each term, when it occurs in a proposition, stands for (*supponit pro*) each member of a certain class of things (possibly empty), and the truth-conditions of the proposition are stated in terms of the relations between these classes. Moreover, affirmative propositions are interpreted as having 'existential import', and negative ones are not. Thus, using X and Y as variables for singular terms and A and B as variables for common ones, 'X is Y' is true iff X stands for something and Y stands for that same thing; 'X is-not Y' is true iff either X does not stand for anything at all or it stands for something that Y does not stand for. 'X is (an) A' is true iff X stands for something which is one of the things that Y stands for. 'X is-not (an) A' is true iff either X does not stand for anything or it stands for something which is not among the things that A stands for. 'Every A is (a) B' is true iff A stands for at least one thing (is not an 'empty term') and everything it stands for B also stands for. 'Some A is (a) B' is true iff there is something for which A and B both stand. 'Every A is-not (a) B' is true iff there is nothing for which A and B both stand; clearly this is so when A does not stand for anything, as well as when it stands for something but not for anything that B stands for. 'Some A is-not (a) B' is true when either A stands for nothing or else it stands for something for which B does not stand. These last four forms are of course the A, E, I and O propositions of 'traditional formal logic', and there has been so much discussion about their truth-conditions that it is important to be quite clear what Buridan takes these to be. It may perhaps help if I offer the following analogies from modern predicate calculus:

Every A is B (AaB): $\exists xAx \wedge \sim\exists x(Ax \wedge \sim Bx)$

Every A is-not B (AeB): $\sim\exists x(Ax \wedge Bx)$

Some A is B (AiB): $\exists x(Ax \wedge Bx)$

Some A is-not B (AoB): $\sim\exists xAx \vee \exists x(Ax \wedge \sim Bx)$

These were, I think, the usual interpretations given to such propositions by mediaeval logicians, and are not peculiar to Buridan. They are certainly in accordance with the traditional Square of Opposition. In addition Buridan recognizes two weaker forms of negative propositions, which he calls the 'unaccustomed' forms or modes. His Latin formulations of them are difficult to translate: they are 'Omne A B non est' and 'Aliquod A B non est'. The former means that every A is distinct from (non-identical with) at least one B, and the latter that some A is distinct from at least one B. He uses these forms to show that certain conclusions can be drawn from pairs of syllogistic premisses from which no ordinary A, E, I or O conclusion can be drawn, and also to provide converses for O propositions, which are usually said not to have any valid converses at all. I shall, however, for the most part ignore these unaccustomed forms.

In addition, before we proceed we need the rules of conversion, which can be summarized thus:

AeB iff BeA; AiB iff BiA; and if (not iff) AaB then BiA.

These are certainly valid given the above interpretation.

All this applies to the most straightforward cases. But there are complications, of which I'll mention one in particular because of its importance for modal logic later on.

This arises from the fact that propositions for Buridan are tensed. In the propositional forms we have so far considered, the copulas are genuine present tense verbs meaning 'is now' and 'is not now' respectively. And what 'A' and 'B' stand for in such propositions are those and only those presently existing things to which 'A' and 'B' apply. But we can equally well have past tense and future tense copulas such as 'was' or 'will be' or more determinate ones like 'was in 1978' or 'will be in 2001'; and the presence of such copulas systematically changes what the terms stand for. Briefly, what Buridan holds is that in a past tense proposition 'A was B', the subject term 'A' is ampliated to stand for all present A's together with all past A's; but the predicate term 'B' stands for past B's only; and in a future tense proposition 'A will be B', 'A' stands for all present and future A's, but 'B' stands for future B's only. Thus he would take 'Every human being will die' to mean that every human being that either now exists or will exist in the future will be one of the things that will die in the future. And he would take 'A New Zealand professor visited Kathmandu in 1978' to mean that someone who either now is or was in 1978 a New Zealand professor was one of the people who visited Kathmandu in 1978.

Tense also has repercussions on the rules for conversion. For example, although 'Something white is square' is equivalent to 'Something square is white', 'Something white will be square' is not equivalent to 'Something square will be white', but rather to 'Something which will be square is or will be white'.

Buridan recognizes, however, that we may very well want to have a past or future tense proposition with a non-ampliated subject; e.g. we may want to say of present A's only that they were B, or will be B. His way of achieving this is to insert the phrase *quod est* - 'which is' - before the subject. E.g., in 'Something which is A was B' the insertion cancels the ampliation and makes A stand only for what it would stand for in a present tense proposition. Similarly, we could shift the supposition of A to the past only by saying 'Something which was A was B'. And the other variations are dealt with similarly.

Ampliation of the subject also occurs when, even though the copula is in the present tense, the predicate has a past or future meaning. Buridan's favourite example of such a predicate is 'dead'. 'Some man is dead' does not mean that some presently existing man is (now) dead, but that something which is or was a man is (now) dead. And as far as conversion is concerned, we must beware of inferring from 'Some Prime Ministers are now dead' to 'Some dead person is now a Prime Minister'.

The topic of ampliation will be important in connection with modal logic; but before I come to that I must say something about syllogisms. A syllogism consists of a pair of subject-predicate propositions having a term in common, from which a conclusion is drawn whose terms are the other two terms which occur in the premisses. The classical theory of the syllogism, which has its roots in the *Prior Analytics*, is formulated for cases in which all the terms are common, no ampliation occurs, and in which there are no negative propositions of the unaccustomed form. Buridan has much of interest to say about what difference it makes to the validity of syllogisms when we have singular or ampliated terms, or negative propositions of the unaccustomed kind, and in fact his account of the basic principles underlying syllogistic reasoning starts from propositions with singular terms. But for brevity I must ignore all that here. His setting out of the classical theory goes like this. A pair of syllogistic premisses must have their terms arranged in one or other of four patterns, known as Figures. These are:

(I)	(II)	(III)	(IV)
B–A	A–B	B–A	A–B
C–B	C–B	B–C	B–C

In each case the terms in the conclusion must be A and C, in some order. If the order is C-A, the conclusion is said to be direct; if it is A-C, the conclusion is said to be indirect. There are thus eight possible patterns of terms in premisses and conclusion. However, of these we need consider only four. For if we transpose the premisses in Figure II with an indirect conclusion, we obtain a relettered version of the same Figure with a direct conclusion; and the same applies to Figure III; so we can ignore the indirect conclusions in these cases. By transposing the premisses in Figure IV with an indirect conclusion we obtain (relettered) Figure I with a direct conclusion; and direct Figure IV similarly gives us indirect Figure I; so we need keep only two of these four cases. Buridan chooses to keep both direct and indirect Figure I. Later on (from the 17th. century) it was more usual to keep direct I and direct IV; but nothing of substance turns on this.

If we specify each premiss as an A,E,I or O proposition, we obtain 16 premiss-pairs in each figure - 64 in all. The list of 19 valid moods immortalized in the well-known verses beginning 'Barbara Celarent . . .' records all the pairs of premisses from which conclusions can be validly drawn - subject, that is, to the conditions I have mentioned, that there are no singular or ampliated terms, and that we ignore negative propositions of the 'unaccustomed' kind.

I turn at last to modal logic. Modal propositions, according to Buridan, are of two kinds, composite and divided. A composite modal proposition is one which affirms or denies of a certain proposition that it has a certain modal characteristic, such as possibility, necessity, impossibility or contingency. Such composite propositions, he says, are not in the strict sense modal propositions at all. They are simply 'assertoric' subject-predicate propositions in which one term is a modal expression and the other stands for a proposition. Nevertheless a modal logic must deal with them. Examples of composite modal propositions are 'That there are no snakes in New Zealand is contingent' and 'It is impossible for both candidates to win the election'. According to Buridan, these mean, respectively, 'The proposition "There are no snakes in New Zealand" is a contingent proposition', and 'Some impossible proposition is the proposition "Both candidates win the election"'; and here the 'is' is the ordinary copula. Divided modal propositions, on the other hand, are ones in which the modal expression comes between the subject and the predicate ('divides' them). Examples are: 'No pigs can fly' and 'Some logicians can't speak Italian'. Such propositions, Buridan says, are genuinely modal. The difference between the two kinds can be illustrated by an example frequently used by mediaeval logicians: the divided modal proposition 'Some white thing can be black' is true, but the corresponding composite one, 'It is possible that some white thing is black', is false.

So we have a variety of things to consider: not merely the relations of composite modal propositions among themselves and the relations of divided ones among themselves, but the relations of composite propositions to divided ones containing the same terms, as well as the relations of each kind to non-modal propositions.

Let's begin with divided propositions. The first point is that Buridan insists that in such propositions the modality belongs to, or is part of, the copula. Modal copulas are expressions like *necesse est esse* or *possibile est esse:* literally 'is-necessary-to-be' and 'is-possible-to-be', though 'is necessarily' and 'is possibly', or 'must be' and 'can be', make more idiomatic English. So divided modal propositions will have the structure:

(Sign of quantity)/subject/modal copula/predicate.

The next point concerns ampliation. Buridan says that the presence of a modal copula - any modal copula - in a proposition ampliates the subject to stand for not only the actual things but also the possible things that fall under that term. This is parallel to the way in which a past tense copula ampliates the subject to the past as well as the present. Just as he takes 'Every A was B' to mean that everything that is or was A was B, so he takes 'Every A can be B' to mean that not merely every actual A but every possible A can be B, or more briefly, everything that is or can be A can be B. Since, however, what is actual counts as possible, we can shorten this further to 'Everything that can be A can be B'. Of course, just as in the parallel case of ampliation to the past or the future, we may want to have modal propositions with non-ampliated subjects; we may, e.g., want merely to say that every actual A can be B; and here he uses a similar device: we prefix *quod est* to the subject, saying, e.g., 'Everything which is A can be B'.

A short digression seems in order here. For a long time I was puzzled about what Buridan could mean by talking about possible but non-actual things of a certain kind. Did he mean by a 'possible A', I wondered, an actual object which is not in fact A but might have been, or might become, A? My house, e.g., is in this sense a possible green thing because, although it is not in fact green, it could become green by being painted. But this interpretation won't do; for Buridan wants to talk, e.g., about possible horses; and it seems quite clear that he does not believe that there are, or even could be, things which are not in fact horses but which might become horses. What I want to suggest here, very briefly, is that we might understand what he says in terms of modern 'possible world semantics'. Possible world theorists are quite accustomed to talking about possible worlds in which there are more horses than there are in the actual world. And then, if Buridan assures us that by 'Every horse can sleep' he means 'Everything that is or can be a horse can sleep', we could understand this to mean that for everything that is a horse in any possible world, there is a (perhaps other) possible world in which it is asleep. It seems to me, in fact, that in his modal logic he is implicitly working with a kind of possible worlds semantics throughout.

To get back to my exposition: My next point concerns negation. For the moment let us ignore singular terms, and let us also confine our attention to the modalities *necessary, possible,* and *impossible,* ignoring *contingent,* which raises more complicated issues. Then the general form of a divided modal proposition will be sign of quantity/subject/modal copula/predicate.

Now we have three signs of quantity, 'every', 'some' and 'no', and three modal copulas, 'is necessary to be', 'is possible to be' and 'is impossible to be'. This gives us nine propositional forms for a given subject and predicate. In each there are three places at which we can insert a negation: the very beginning, just before the modal term in the copula, and just before 'to be' at the end of the copula. This gives us 72 formulae in all. But by various rules of equivalence, such as that 'is not necessary to be' is equivalent to 'is possible not to be' or that 'No – is possible to be . . .' is equivalent to 'Every – is necessary not to be . . .', Buridan is able to show that these 72 formulae fall into 8 groups containing 9 equivalent formulae apiece. To make all this more manageable, he introduces a kind of canonical notation, choosing from each group a form which has either 'every' or 'some' as its sign of quantity, 'necessary' or 'possible' as its modality, and either no sign of negation at all or one occurring in the copula immediately after the modal expression (i.e. just before 'to be'). This gives us only 8 formulae to deal with, each of the other 64 being equivalent to one or other of these. They are:

(1) Every A is-necessary-to-be B (2) Every A is-necessary-not-to-be B

(3) Every A is-possible-to-be B (4) Every A is-possible-not-to-be B

(5) Some A is-necessary-to-be B (6) Some A is-necessary-not-to-be B

(7) Some A is-possible-to-be B (8) Some A is-possible-not-to-be B

I shall use an abbreviated notation for these, based on 'AaB' etc, together with 'L' and 'M' as symbols for necessity and possibility. This will enable me to write (1) - (8) as:

(1) $A\overset{L}{a}B$ (2) $A\overset{L}{e}B$

(3) $A\overset{M}{a}B$ (4) $A\overset{M}{e}B$

(5) $A\overset{L}{i}B$ (6) $A\overset{L}{o}B$

(7) $A\overset{M}{i}B$ (8) $A\overset{M}{o}B$

(In following some of Buridan's arguments it is important to keep clearly in mind that (2) – which he calls a universal negative *de necessario* – means 'Every A is necessarily not B' and not 'No A is necessarily B'; and similarly, (4) – a universal negative *de possibili* – means 'Every A is possibly not B', not 'No A is possibly B'.)

With these formulae Buridan constructs an analogue of the square of opposition. But whereas in the square of opposition for non-modal propositions we had only four formulae and thus six pairs to consider, here

we have 8 formulae and hence 28 pairs. In one section of the *Summulae* Buridan examines each of these 28 pairs in detail, and in some of the manuscripts of this work there is a full-page diagram summarizing his results. This contains 8 boxes, in each of which is written one of the forms I have numbered (1) - (8), together with the other 8 equivalent forms, and for each pair of boxes the logical relation between them is spelled out, with a brief explanation, on a line joining them. It turns out that in the case of four pairs, (3)-(4), (3)-(5), (4)-(6) and (5)-(6), the propositions are independent in the sense that neither implies either the other or its negation, but that in the case of each of the other 24 pairs one of the relations on the ordinary square of opposition holds - contradiction, contrariety, subcontrariety or subalternation. We might call this the octagon of opposition. In the Appendix I give a transcription of this diagram from one of the manuscripts, and an abbreviated diagram summarizing its results.

We now want to see what the rules of conversion are for divided modal propositions. Briefly, they are like those for non-modal propositions, but hold only for certain modalities. I-I conversion holds only for possibility, E-E conversion only for necessity, and A-I conversion only when the I proposition has a possibility copula. That is, the rules are:

$A\overset{M}{i}B$ iff $B\overset{M}{i}A$ $A\overset{L}{e}B$ iff $B\overset{L}{e}A$

If $A\overset{L}{a}B$ then $B\overset{M}{i}A$ If $A\overset{M}{a}B$ then $B\overset{M}{i}A$

But we no not have $A\overset{L}{i}B$ iff $B\overset{L}{i}A$, or $A\overset{M}{e}B$ iff $B\overset{M}{e}A$, or if $A\overset{L}{a}B$ then $B\overset{L}{a}A$. As an example of the failure of the first of these we might use a slightly adapted Quinean example: Some number which is the number of the planets is bound to be greater than 7; but there is no number greater than 7 which is bound to be the number of the planets.

I now want to say something about Buridan's treatment of contingency. To be contingent is to be possible but not necessary, or, equivalently, to be both possibly so and possibly not so. We should therefore be able to give an analysis of contingency in terms of possibility and negation. In the case of composite propositions this works out very simply: 'It is contingent that p' is equivalent to 'It is possible that p and it is possible that not-p' - a conjunction of two composite possibility propositions. But with divided propositions matters are a little more complicated. There is first of all a problem of translation, at least as far as English is concerned. When Buridan puts a contingency copula between terms A and B, he usually writes *'A contingit esse B'*, and the translation that one first thinks of is 'A is contingently B'. But in English this strongly suggests that A is in fact B, though only contingently so; and that certainly is not what Buridan means. What he means is that for A it is a contingent matter whether or not it is B. There is, I think, no short, idiomatic and unambiguous way of saying this in English; so I shall use the unidiomatic form 'A is contingent to be B' to mean precisely what Buridan means by '*A contingit esse B*'', without carrying any suggestion that A is in fact B. Buridan's account is that the contingency copula has to be analysed as a conjunctive possibility copula: 'is contingent to be' means 'possibly-is-and-possibly-is-not'. He stresses that not every divided contingency proposition is equivalent to a conjunction of possibility propositions. More precisely, a universal one is but a particular one is not. For although (1) 'Some A is contingent to be B' does indeed imply (2) 'Some A is-possibly B and some A is-possibly-not B', it is not equivalent to it. For suppose that the A's are exhaustively divided into those that are necessarily B (and therefore possibly B) and those that are necessarily-not B (and therefore possibly-not B) (as, e.g., the natural numbers are divided into those that are necessarily even and those that are necessarily-not even). Then (2) will be true but (1) will be false. This shows, according to Buridan, that we can't dispense with conjunctive copulas in favour of conjunctive propositions with simple or non-conjunctive copulas.

The other main point that he makes about divided contingency propositions is that 'is contingent to be' and 'is contingent not to be' are equivalent. In this way 'contingent' differs strikingly from 'necessary' or 'possible'. So if I extend my notation for divided modal propositions by writing '$A\overset{C}{a}B$' for 'A is contingent to be B' etc., we shall find that the two forms $A\overset{C}{a}B$ and $A\overset{C}{i}B$ will be equivalent to the corresponding E and O forms, which will therefore not be needed.

Buridan gives a parallel account of non-contingency propositions. To say that it is non-contingent that p is to say that either it is necessary that p or it is impossible that p. For divided non-contingency propositions - which Buridan often calls contingency propositions *de modo negato* ('with a negated modality') - we have the copula '*non contingit esse*', which I shall again unidiomatically translate as 'is not contingent to be'; and he says this is to be analysed as the disjunctive copula 'is necessarily or is necessarily not'. Moreover, just as some contingency propositions are equivalent to conjunctions of possibility propositions and others are not, so some non-contingency propositions are, he says, equivalent to a disjunction of necessity propositions and others are not. But this time it is particular propositions which are so equivalent and universal ones which are not. 'Every A is either necessarily or necessarily not B' does not entail 'Either every A is necessarily B or every A is necessarily not B', though it is entailed by it.

As with the contingency copula, 'is not contingent to be' is equivalent to 'is not contingent not to be', so again only two new forms are needed.

As far as conversion rules are concerned, Buridan says that no contingency proposition converts to another contingency proposition: briefly, even if an A both may and may not be a B, it might still be the case that any B would be bound to be A, and hence not be contingently A. However, since every contingency proposition entails the corresponding affirmative possibility proposition, and an affirmative possibility

proposition converts to a particular possibility proposition, we do have the result that if $A\overset{C}{a}B$ (or $A\overset{C}{i}B$) then $B\overset{M}{i}A$.

I shall next say something about the logical relations between modal and non-modal propositions with the same terms. These relations, according to Buridan, are governed by the principles that whatever is necessarily so is so, that whatever is so is possibly so, and that consequently whatever is necessarily so is possibly so; principles that nowadays we associate with the system T. We have, however, to be careful about how this works out.

For composite propositions everything is straightforward. 'It is necessary that p' implies p itself, which in turn implies 'It is possible that p'. (At least, Buridan is careful to add, the latter holds on the hypothesis of the existence of p.)

But with divided propositions the ampliation of the subject to the possible brings certain complications. Suppose, e.g., that there are no actual A's but that every possible A is bound to be B. Then 'Every A is B' has an empty subject and is therefore false; but 'Everything that can be A is necessarily B' is true. Thus $A\overset{L}{a}B$ does not entail AaB. So in general a divided necessity proposition does not entail the corresponding non-modal proposition, The exception, he tells us, is that $A\overset{L}{e}B$ ('Every A is-necessarily-not B') does entail AeB ('Every A is-not B'), and therefore of course AoB. The reason is that if every A, actual or possible, is bound not to be B, then either there are no actual A's at all or else there are some but none of them are B; and in either case AeB is true.

The position with possibility propositions and their non-modal counterparts is the mirror-image of this. In general a non-modal proposition does not entail the corresponding divided possibility proposition. 'Every A is B', for example, does not entail 'Every A is-possibly B', because it might be the case that every actual A is B but that some possible A's could not possibly be B. But again there is an exception, parallel to the one we had in the case of necessity: 'Some A is B' does entail 'Some A is-possibly B', and therefore so does 'Every A is B'.

However, when necessity propositions are restricted by the 'which is' insertion before the subject, and thus do not have ampliated subjects, they do entail the corresponding non-modal propositions; and similarly, non-modal propositions entail the corresponding possibility propositions when these have the 'which is' restriction. The reason is that in such cases the subject has the same supposition, i.e. stands for the same things, in the modal and the non-modal proposition. The general principle is that a necessity proposition entails the corresponding non-modal one, and this in turn entails the corresponding possibility proposition, whenever their subjects have the same supposition or range of reference. As we have seen, we can achieve this by removing the ampliation in modal propositions by the 'which is' qualification. Another way of achieving it is to introduce an 'ampliation to the possible' into non-modal propositions. to produce forms such as 'Everything which can be A is B' - which would be entailed by 'Every A is-necessarily B'.

A necessity proposition, however, although it does not in general entail the corresponding non-modal proposition, always does entail the corresponding possibility proposition, since in these the subjects are ampliated in just the same way. (This result was already contained in the 'octagon of opposition'.)

Since propositions with the 'which is' qualification will play an important role later, I shall find it convenient to extend my notation to cover them. To do so I shall use a superscripted 'Q' (the initial letter of the Latin *'quod est'*) before the subject term. Thus '$^{Q}A\overset{L}{a}B$' will mean 'Everything which is A is-necessarily B', and so forth.

We now have all the material necessary to approach the most elaborate part of Buridan's modal logic, his theory of modal syllogisms, i.e. syllogisms in which at least one premiss is a modal proposition. His fullest account is of syllogisms in which all the modal propositions are of the divided kind, so I shall deal mainly with these. I shall assume, as he does in his own exposition, that all terms are common terms and that we are ignoring negative propositions of the unaccustomed kind. In the *Summulae* he also restricts his consideration to cases in which no ampliation occurs except the ampliation to the possible which is brought about by modal copulas. In the *Consequentiae* , however, he explores some of the cases in which the occurrence of other kinds of ampliation would affect validity.

Since a modal proposition, like a non-modal one, has a subject, a copula and a predicate, we have the same four figures as for non-modal syllogisms. Direct Figure I, e.g., will as before be

B–A

C–B

∴ C–A

In the non-modal case this gave us only 16 pairs of premisses to consider for this figure, since each premiss must be either of the A, E, I or O form. But now the first premiss may not only be BaA, BeA, BiA or BoA as before, but may instead have a necessity, possibility, contingency or non-contingency copula in place of the non-modal one, and in each of these cases it may or may not have the 'which is' qualification. This gives us not 4 but 28 possibilities for each premiss, and so 784 premiss-pairs; and if we subtract the 16 purely non-modal pairs, we are left with 768. Since the same holds for each figure, we have 3072 premiss-pairs in all. Of each of these we can ask whether it yields any of the 28 possible conclusions, and if so, which. So if we define a syllogistic mood in what is probably the most usual way, in terms of the forms of premisses and conclusion, we shall have 3072X28 (86016) modal syllogistic moods, including of course all the invalid ones, as compared with the modest number of 256 for non-modal syllogisms. (The arithmetic, I should perhaps say, is mine, not Buridan's own.)

Buridan, understandably, does not attempt the daunting task of examining each of these moods for validity or invalidity. What he does do,

and it is certainly arguable that it is sufficient for his purposes, is to give us a detailed examination of a considerable number of carefully selected groups of them in such a way as to display the principles by which their validity can be assessed, and to leave it to us to deal with the others in the same way. In fact, his closing words in this section of the *Summulae*, after admitting that he has not discussed all possible cases, are: ' . . . and if anyone, in order to make his intellect more subtle, should wish to look into these matters in finer detail, the above results will give him the way to do so' (this sounds much better in his Latin than in my English). As far as I can see, he is correct in his claim that the cases he does not discuss can be dealt with by the methods he states and illustrates.

I shall indicate the range of cases he does examine. First, he deals with only three of the four figures, direct I, II and III, leaving us to deal with indirect I for ourselves. Next, in each of these figures, he confines himself to those pairs of premises where the unmodalized forms would yield an (unmodalized) conclusion. In direct Figure I, e.g., it is uncontroversial that, if we ignore propositions with singular terms and negative propositions of the unaccustomed kind, the only four pairs that yield a conclusion in non-modal syllogistic are

BaA	BeA	BaA	BeA
CaB	CaB	CiB	CiB

and the corresponding results for Figures II and III are equally well-established (four pairs for Figure II and six for Figure III). This gives him 14 distinct (unmodalized) pairs of premisses. For each of these he surveys the pairs we obtain my modalizing the propositions in each of 20 ways which I shall list in a moment, showing in each case which propositions, if any, can be drawn as conclusions. In drawing up the list I use, for brevity, 'M' to indicate a possibility proposition, '^{Q}M' to indicate a possibility proposition with a 'which is' restriction, and analogously with 'L' and 'C', and '–' for an unmodalized proposition. Where the modalizations of the two propositions differ, I give the one for the first premiss first. The list is:

1. M, M	2. ^{Q}M, ^{Q}M	3. L, L	4. ^{Q}L, ^{Q}L
5. C, C	6. ^{Q}C, ^{Q}C	7. M, –	8. –, M
9. L, –	10. –, L	11. C, –	12. –, C
13. M, L	14. L, M	15. M, C	16. C, M
17. L, C	18. C, L	19. ^{Q}M, ^{Q}L	20. ^{Q}L, ^{Q}M

The last two groups are examined in the *Consequentiae* but not in the *Summulae;* the opposite holds for group 6. In addition, in the *Consequentiae,* though again not in the *Summulae,* he gives us some discussion of syllogisms containing non-contingency propositions, though he does not deal with them in great detail.

Perhaps in all this Buridan is making the assumption that a modal syllogism is never valid unless its unmodalized counterpart - the syllogism we get by 'demodalizing' all the propositions in it - is valid. But he doesn't say explicitly that he is, and it may be that he is merely running through what seem to him to be the most likely candidates for validity. If he *is* making such an assumption, I have to admit that is isn't clear to me that it is a correct one, and I'd like to see a proof of it, or at least some argument for it. On the other hand, it is not, I think, devoid of all intuitive plausibility. Perhaps a partial parallel might be found in modern propositional modal logic, where, apart from some systems which it is difficult to interpret in terms of any ordinary ideas of necessity and possibility, a formula is valid only when its non-modal propositional calculus 'skeleton' is also valid.

It would obviously be impossible for me to take you through Buridan's treatment of all the cases I have mentioned. What I shall do is to run through the way he deals with one group of moods, and then list the additional principles he uses for the others. The group I shall choose is 'M,M' - the moods in which each premiss is a possibility proposition. He maintains that in Figures I and III these premisses yield a possibility conclusion of the same form (A,E,I or O) as the corresponding non-modal premisses would yield, but that in Figure II no conclusion follows at all.

The Figure I moods, he says, are valid by *dici de omni* and *dici de nullo*. These were established phrases in mediaeval discussions of first figure syllogisms. Literally they mean, respectively, 'to be said of every' and 'to be said of no'. Buridan first explains them in connection with non-modal syllogisms, in this way. In a universal affirmative proposition we have a 'being said of every' in the sense that what is asserted is that the predicate is truly affirmed of everything of which the subject is truly affirmed. Now take the first figure mood BaA, CaB, ∴ CaA. The first premiss says that the term A is truly affirmed of everything of which B is truly affirmed; the second premiss adds the information that the things of which B is truly affirmed include all those of which C is truly affirmed; so obviously A is truly affirmed of all these, which is what the conclusion states. And no other proof of the validity of this mood is needed. The *dici de nullo* analysis is used analogously when we have a universal negative first premiss. 'Every B is-not A' is taken to mean that A is truly *denied* of everything of which B is truly affirmed, and the argument then proceeds as before.

Buridan claims that the validity of the Figure I M-M moods can be made obvious in the same way. Take, e.g., the mood

$$B\overset{M}{a}A,\ C\overset{M}{a}B\ \therefore\ C\overset{M}{a}A.$$

The first premiss, we may recall, means, in virtue of the ampliation of the subject, that every possible B is possibly A. By the *dici de omni* analysis, this means that of everything of which B *could possibly* be truly affirmed, A *could possibly* also be truly affirmed. The second premiss then adds that among the things of which B could possibly be truly affirmed are all those of which C could possibly be truly affirmed; so obviously A could possibly be truly affirmed of all these - which is what the conclusion states. The other three Figure I moods are dealt with similarly, using *dici de nullo* when the first premiss is negative.

For Figure III he gives two proofs, one applicable to five of the six standard moods, and the other applicable to all six. The first proof assumes the results for Figure I as already established, and uses some of the conversion rules. E.g., take the mood

$$B\overset{M}{a}A,\ B\overset{M}{a}C\ \therefore\ C\overset{M}{i}A$$

Here the second premiss converts to $C\overset{M}{i}B$, and the first premiss together with this gives us the required conclusion by a Figure I syllogism. The one Figure III mood which cannot be proved in this way is the O, A ∴ O pattern (Bocardo). The other method is by what in mediaeval logic was called *exposition*. The idea here is that one way of showing that some C is A is to exhibit some individual object which both is A and is C. E.g., if Socrates is bald and that self-same Socrates is snub-nosed, that shows that some snub-nosed person is bald. This amounts to constructing a third figure syllogism with a singular term as the middle term (a syllogism with a singular middle term was known as an *expository syllogism*). By extension, if the premisses of a syllogism entail that there is at least one object which is both A and C, even though they do not actually identify it, that is equally sufficient to show that some C is A. To show how this works out for the Figure III M-M moods, I shall run through the way Buridan applies it to Bocardo, the case that resisted treatment by the previous type of proof. This is the mood

$$B\overset{M}{o}A,\ B\overset{M}{a}C\ \therefore\ C\overset{M}{o}A$$

Here the second premiss says that every possible B is possibly C, and the first premiss says that at least one of these possible B's is possibly not A. Hence there must be at least one thing (one of the possible B's) which is both possibly C and possibly-not A; and hence some possible C is-possibly-not A, which is what the conclusion states.

For establishing the invalidity of a mood he uses the time-honoured method of producing a counter-example; i.e. he describes an actual, or imaginary but self-consistent, case in which an instance of each of the premisses is true but the corresponding instance of the conclusion is false. Sometimes he is able to use a single case of this kind to reject several moods simultaneously. To show the invalidity of the Figure II M,M moods he produces two counter-instances. One is theological. Let us assume, as Buridan did, and as at least seems consistent, that God is necessarily identical with the First Cause, and that God (or the First Cause) has both the ability to create and the ability to refrain from creating. Then in Figure II the following premisses will be true: 'Every God is-possibly-not creating' and 'Every First Cause is-possibly creating'. But the negative possibility conclusion 'Every First Cause is-possibly-not God' will be false. And this example can be adapted to show that no negative conclusion follows in any of the Figure II M,M moods. Then for good measure he shows that no affirmative conclusion follows either, by another counter-instance: 'Every human being is-possibly-not running' and 'Every horse is-possibly running' are both true, but 'Every (or even some) horse is-possibly a human being' is false.

That, in outline is how Buridan deals with the M,M moods. For the other groups he uses, as the case makes appropriate, the basic arguments by dici de omni et nullo and exposition, and supplements them as required by some of the other principles which I have already mentioned. These consist of the relations on the 'octagon of opposition'; the conversion rules, both for non-modal and for modal propositions; the rules relating contingency and possibility propositions; and those relating modal and non-modal propositions. In addition, of course, he can use the principles of propositional logic to transpose premisses and to transform an inference of the form 'X and Y, ∴ Z' into one of the form 'X and not-Z, ∴ not-Y'. By these methods he is able to determine the validity or invalidity of all the modal syllogistic moods he considers. (His results are summarized in the Appendix.) If I understand him correctly, he claims that all the moods he does not consider could be settled by the same methods. As far as I can see, this claim is correct.

It is worth noting here that in the case of a very few of the moods we find something extremely rare - a genuine contradiction between the *Summulae* and the *Consequentiae*. For example, in the *Summulae* he says that the mood

$$BeA,\ \overset{L}{CaB}\ \therefore\ CeA$$

is invalid, and offers a counter-example. But in the *Consequentiae* he gives an elaborate argument to show that it is valid. (It seems to me that the latter view is correct, and that the counter-example is not a genuine one; but I shall not go into this here.)

That is all I have time to say about syllogisms with divided modal propositions. I now want to say something - much more briefly - about syllogisms containing composite modal propositions. In the *Summulae* Buridan simply says that since so-called composite modal propositions are really ordinary 'assertoric' propositions in which the terms are either modal expressions like 'necessary' and 'possible' or expressions which stand for propositions, the ordinary rules for non-modal syllogisms apply to them; and with a few remarks about how this works out, he leaves the matter there. But in the *Consequentiae* he has something else to say; for there he considers cases in which the (non-modal) propositions embedded in the composite modal propositions would themselves yield a conclusion. In order to state briefly what he has to say here, I shall write 'Lp', 'Mp' and 'Cp' for the composite propositions 'It is necessary that p', 'It is possible that p' and 'It is contingent that p' respectively. Then what he says is that when p and q together yield r by a valid syllogism, Lp and Lq together entail Lr, but Mp and Mq (or Cp and Cq) do not entail even Mr. This is undoubtedly correct, and for the reasons he gives. But I think it is worth pointing out that there is something he seems to have missed, and that is that Lp and Mq do together entail Mr.

In the *Consequentiae*, too, though again not in the *Summulae*, he has some comments about syllogisms in which one premiss is a divided modal proposition and the other is a composite one. He does not develop this theme exhaustively, but he considers the cases where one premiss is a divided necessity proposition and the other a composite necessity proposition. Confining himself, as usual, to the cases where the non-modal forms would yield a conclusion by a valid syllogism, he reaches the result that in Figures I and II we can always draw a divided necessity conclusion, no matter which of the premisses is divided and which is composite, but that in Figure III we can do this only when the first premiss is the divided one and the second the composite one.

This will have to be the end of my survey of Buridan's modal logic. He himself admits that his treatment of the topic is incomplete, and at several places he makes remarks like 'Anyone who wishes may look into this matter in more detail'. So I want to conclude by mentioning some ways in which his discussion might be filled out or developed.

1. There would first of all, of course, be the task of examining the syllogistic forms, both for divided and for composite propositions, which he does not explicitly deal with, trying to determine their validity or invalidity on his own principles. But this, although it might be time-consuming, would probably be merely a matter of detail.

2. In his non-modal logic Buridan has much to say about negative terms ('infinite' terms, as they were called by mediaeval logicians), though I did not mention these in my sketch. Corresponding to each categorematic common term A we can have a negative term non-A, which can be a term in a proposition whenever A can, and which stands precisely for everything that A does not stand for. It is important to distinguish between a proposition with a negative predicate and one with a negative copula. In spite of what later textbooks had to say about 'obversion', 'Some A is-not a B' is not equivalent to 'Some A is a non-B'. At least Buridan says this quite explicitly. The former is a negative proposition, and is therefore true if there are no A's; but the latter is an affirmative proposition with a negative predicate, and is false if there are no A's. Now in his non-modal syllogistic Buridan gives rules for syllogisms in which the middle term appears as a negative term in one premiss and as a non-negative one in the other, and the moods we thus obtain are ones which are not in the traditional list. But when he comes to deal with modal logic he makes no mention of negative terms, and this is a gap one might try to fill in. It would not, I think, be very complicated to do so.

3. In his non-modal logic Buridan has a great deal to say about singular terms. In fact he regards syllogisms with singular middle terms as somehow more basic than those with common terms; and he goes into some detail to show that there are certain moods which are valid when certain of their terms are singular, but which would be invalid if those terms were replaced by common ones. One would expect that something of the same kind would hold in modal logic; but although he sometimes does use modal propositions with singular terms as examples, he does not give us any specific rules for modal syllogisms containing such propositions. This again is a gap which one might try to fill.

4. As I mentioned, Buridan says in the *Summulae* that in dealing with modal syllogisms he is going to ignore any ampliation of terms other than the ampliation to the possible which is brought about by the copula in a divided modal proposition. This implicitly recognises that terms in modal propositions might have other kinds of ampliation instead or as well; and, as I also mentioned, in the *Consequentiae* he does deal with some cases of this type. One would like, however, to have a more systematic theory of them than the *Consequentiae* provides.

5. A more drastic elaboration of his system would be to try to work out, again using his own general principles, the rules for a wider range of inferences than he considers. There are, after all, many other patterns of inference than syllogistic ones, even if one confines oneself to taking subject-predicate propositions as one's units, and it is not obvious that all of these can be reduced to syllogisms or strings of syllogisms. One might, for example, try to graft on to Buridan's system a complete range of truth-functional and modal operators.

6. A much more elaborate project still would be to try to give a Kripke-style possible worlds semantics for Buridan's modal system and then an axiomatic basis for it. I think this could probably be done, and would be worth doing; but it would take us well into the twentieth century.

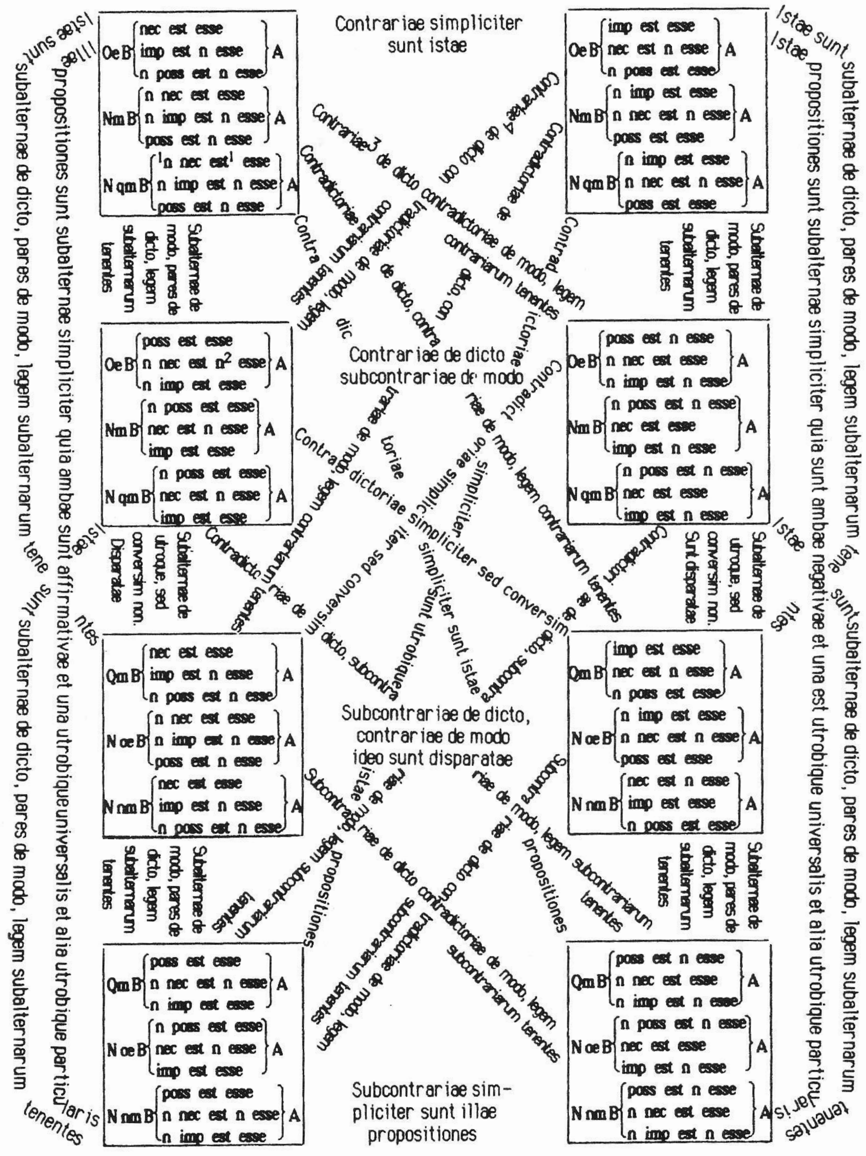
APPENDIX

(1) Transcription of ms. Cracow BJ662, fol.10r.

(2) Summary of relations in Buridan's diagram: the "Octagon of Opposition".

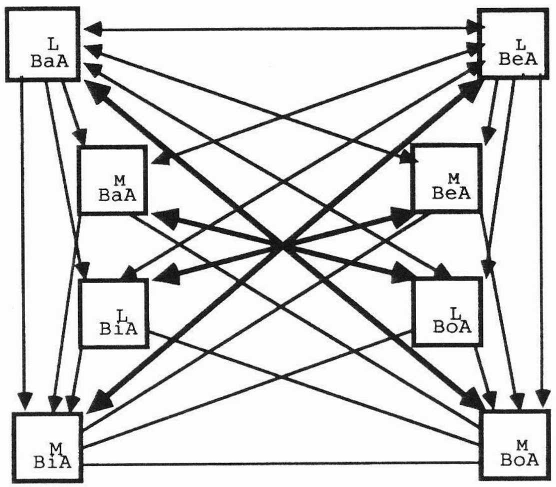
(3) Valid modal syllogistic moods according to the *Summulae* and the *Consequentiae*.

(1) Transcription of ms. Cracow BJ662. fol.10r. [Abbreviations: n = non; oe = omne; nm = nullum; qm = quoddam; nec = necesse; poss = possibile; imp = impossibile.]



1-1 necesse est non *ms.* [2]non *om. ms.* [3,4]contradictoriae *ms.*

(2) Summary of relations in Buridan's diagram: the "Octagon of Opposition"



Key:

- ——▶ represents subalternation
- ◀——▶ represents contrariety
- ◀━━▶ represents contradiction
- ——— represents subcontrariety

(For explanation of notation see text.)

---

(3) Valid modal syllogistic moods according to the *Summulae (S)* and the *Consequentiae (C)*.

('M', L' and 'C' indicate (divided) possibility, necessity and contingency propositions respectively; '—' indicates a non-modal proposition. The order throughout is: first premiss, second premiss, conclusion.)

In *S* the following moods are said to be valid:

Figure I [Pattern of terms: B-A, C-B ∴ C-A. Valid non-modal moods: a,a,a; e,a,e; a,i,i; e,i,o.]

All four moods when modalized in any of the following ways: M,M,M; L,L,L; ^{Q}L,^{Q}L,^{Q}L; C,C,C; —,L,M; C,—,C; M,L,M; L,M,L; M,C,M; C,M,C; L,C,L; C,L,C. In addition the following: For M,—,M and L,—,L: a,a,i; e,a,o; a,i,i; e,i,o. For M,—,^{Q}M and L,—,^{Q}L: a,a,a; e,a,e.

Figure II [Pattern of terms: A-B, C-B ∴ C-A. Valid non-modal moods: e,a,e; a,e,e; e,i,o; a,o,o.]

All four moods when modalized in any of the following ways: L,L,L; —,L,M; M,L,L; L,M,L; L,C,L; C,L,L. In addition the following: For L,—,L: e,a,o; a,e,o; e.i.o. For L,—,—: a,o,o. For L,—,^{Q}L: e,a,e; a,e,e.

Figure III [Pattern of terms: B-A, B-C ∴ C-A. Valid non-modal moods: a,a,i; i,a,i; a,i,i; e,a,o; o,a,o; e,i,o.]

All six moods when modalized in any of the following ways: M,M,M; ^{Q}M,^{Q}M,M; L,L,L; ^{Q}L,^{Q}L,^{Q}L; C,C,C; M,L,M; L,M,L; M,C,M; C,M,C; L,C,L; C,L,C. In addition the following: For M,—,M, L,—,L and C,—,C: a,a,i; a,i,i; e,a,o; e,i,o. For —,M,M, —,L,— and —,C,M: a,a,i; i,a,i.

C adds the following:

Figure I: For all four moods: ^{Q}M,^{Q}L,^{Q}M, —,^{Q}L,— and ^{Q}M,—,M (not considered in *S*).
In addition, for —,L,—: e,a,e (explicitly denied in *S*).

Figure II: For all four moods: ^{Q}L,^{Q}M,^{Q}M, and ^{Q}L,^{Q}L,— (implicitly denied in *S* ?) In addition, for —,L,—: a,e,e; a,o,o.

Figure III: For all six moods: ^{Q}L,^{Q}L,L, ^{Q}L,^{Q}M,L, ^{Q}M,^{Q}L,^{Q}M, ^{Q}M,^{Q}L,M (not considered in *S*)

C omits the following, though does not explicitly deny their validity:

Figures I and II: —,L,M (all 8 moods). L,—,^{Q}L for Figure I a,a,a and e,a,e and for Figure II e,a,e and a,e,e (though it gives the weaker L,—,— for these). Figure III: —,L,— for a,a,i and i,a,i.

C denies the validity of Figure II L,—,L for a,e,o, though this is asserted in *S* . It also denies the validity of Figure I M,—,^{Q}M for a,a,a and e,a,e, but only when ampliative terms occur.