optical constructions 2

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optical constructions 1

laws of geometrical optics
single interface
planparallel plate
prism and wedge

(thin) lens:
  magnifying glass
  microscope
  telescope
  lensmeter

matrix calculus

optical constructions 2

thick lens

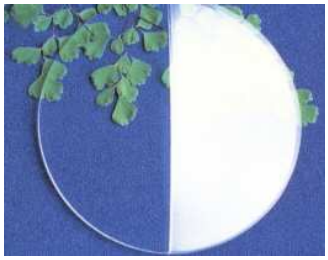
antireflex coating
camera obscura
conversion lenses
extension tubes

achromatised doublet

aberrations

web: **http://www.physics.muni.cz/~hemzal/vyuka/optometry**

anti-reflection coating

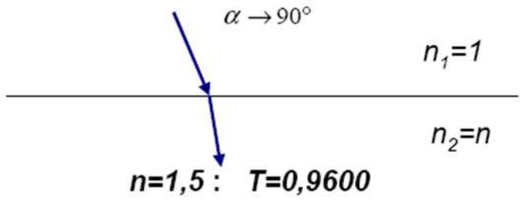


$$n_2 = \sqrt{n_1}$$

transmission over glass-air interface

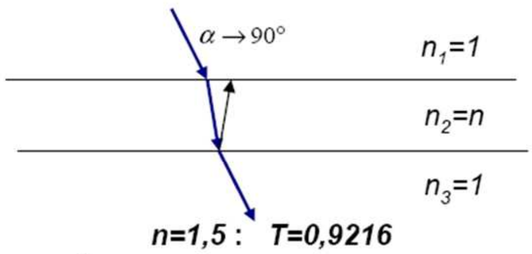
single interface

$$T = \frac{4n}{(1+n)^2}$$



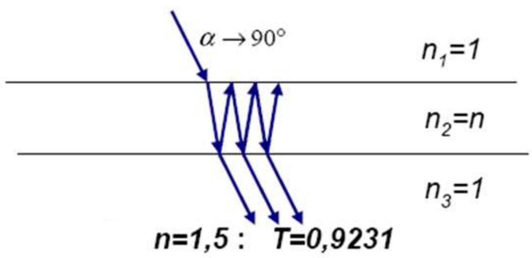
plan-parallel plate:
(without multiple reflections)

$$T = \left[\frac{4n}{(1+n)^2}\right]^2$$

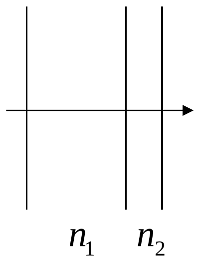


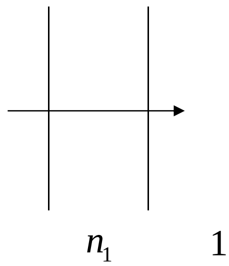
plan-parallel plate:
(with multiple reflections)

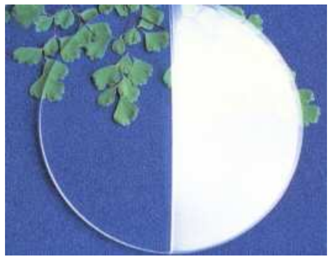
$$T = \frac{2n}{n^2+1}$$

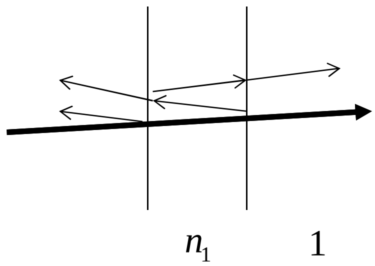


anti-reflex coating







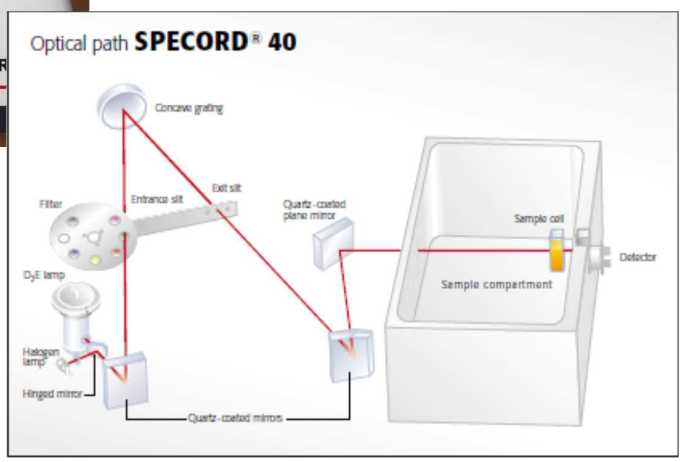


the more light should pass through the system,
the less can get reflected

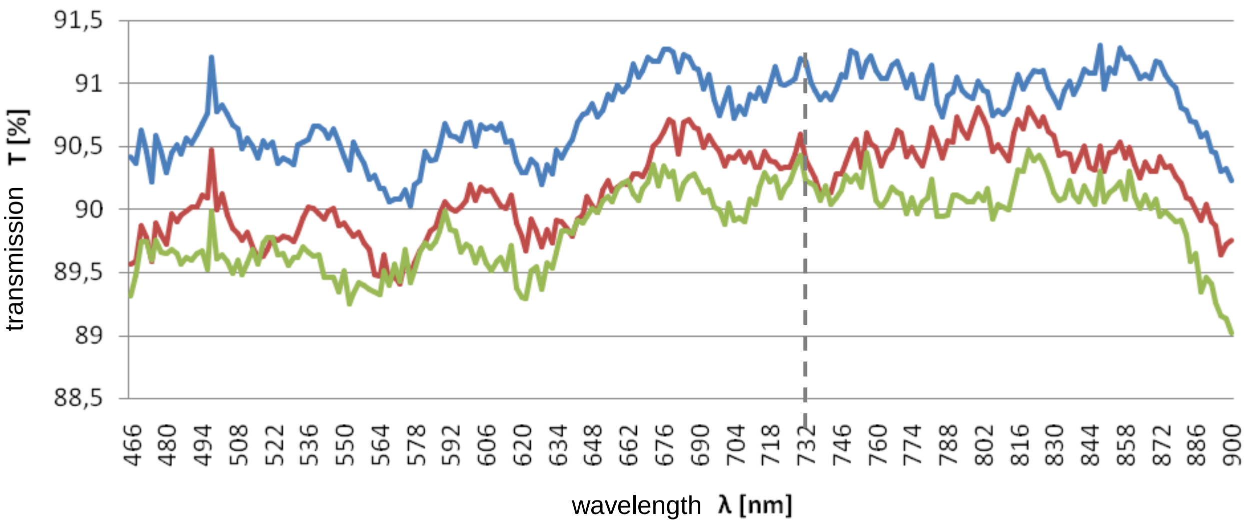
for a single layer AR coating, the best working layer is

$$n_2 = \sqrt{n_1}$$

spectrofotometric measurements

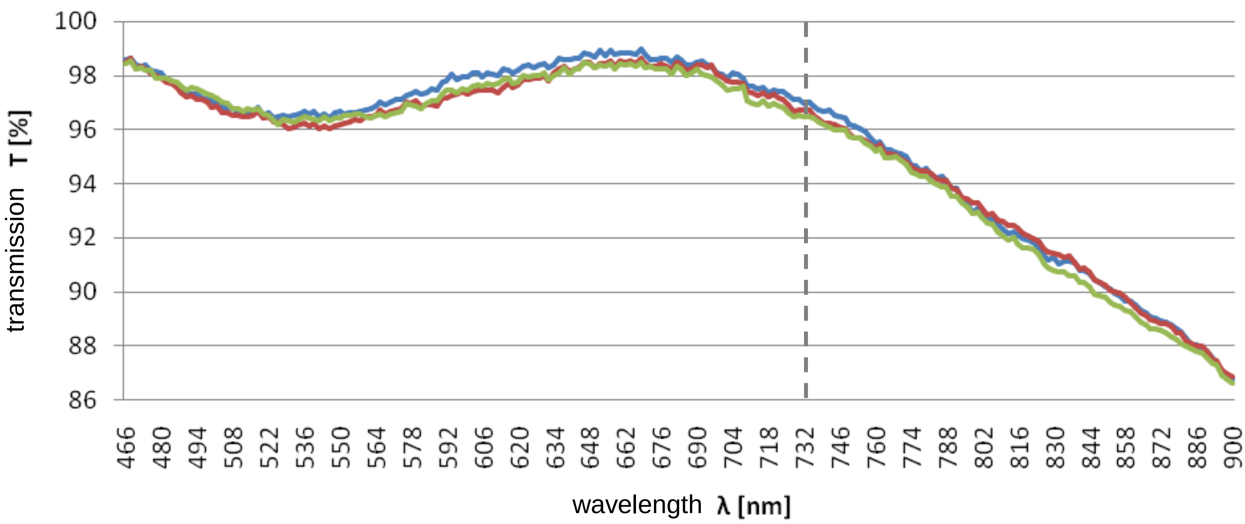


Eco Hard 1,5 no AR coating



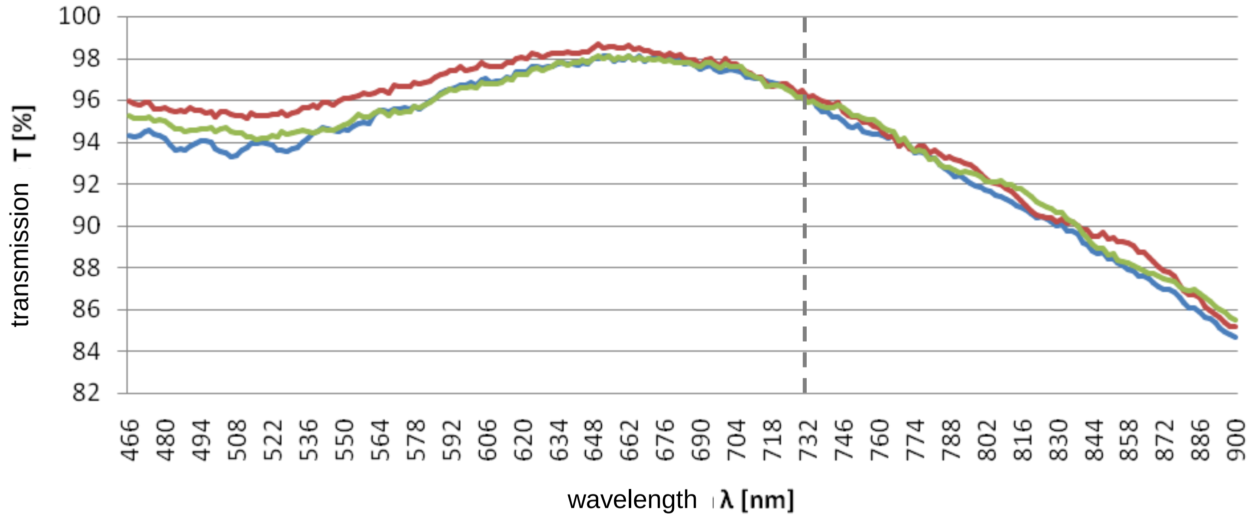
	plan	-0,5 D	-1,0 D
T_{max} [%]	**91,4**	90,8	90,5
T_{pvg} [%]	**90,7**	90,3	89,8

Eco organic 1,5 HMC. 3 layer AR coating



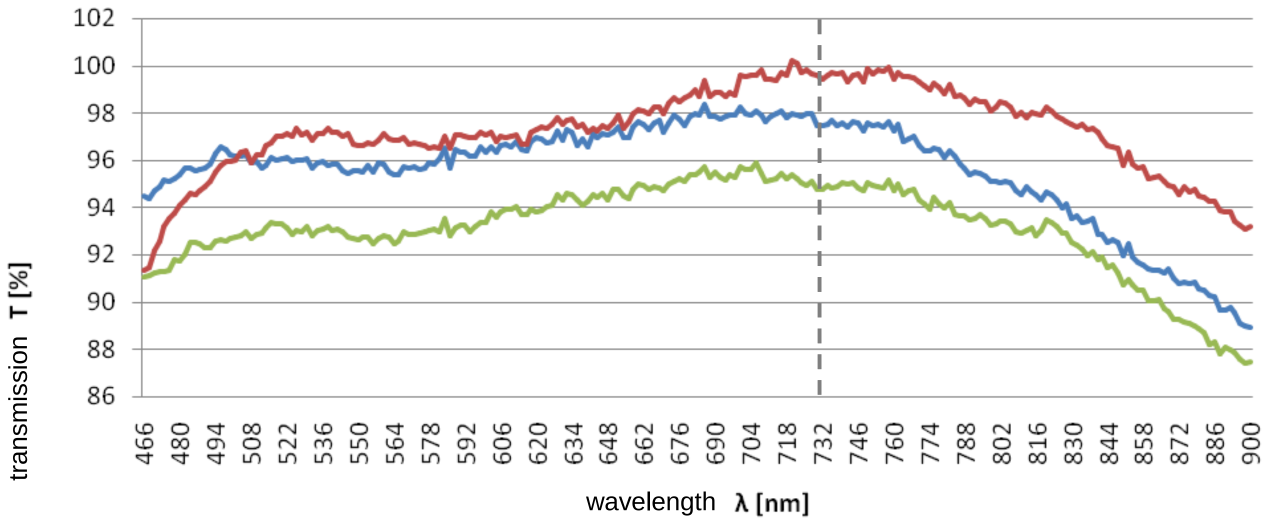
	plan	-0,5 D	-1,0 D
T_{max} [%]	**99,0**	98,8	98,8
T_{avg} [%]	**97,8**	97,5	87,5

Orma 1,5 trio   5 layer AR coating



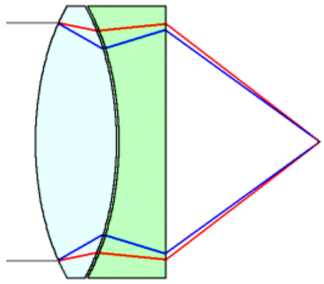
	plan	-0,5 D	-1,0 D
T_{max} [%]	98,0	**98,5**	98
T_{avg} [%]	96,0	**97,0**	96,5

Orma 1,5 Crizal Forte.   7 layer AR coating



	plan	-0,5 D	-1,0 D
T_{max} [%]	98,2	**99,9**	96,0
T_{avg} [%]	96,3	**96,5**	93,0

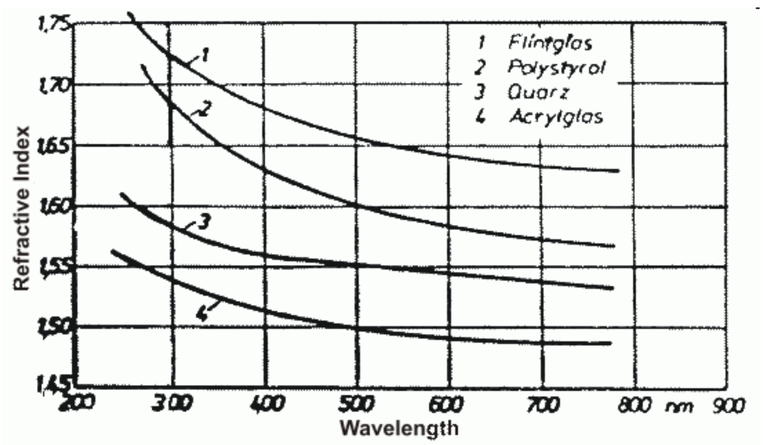
achromatised doublet



chromatic aberration of a lens

dispersion of refractive index:

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$ (Cauchy's formula)

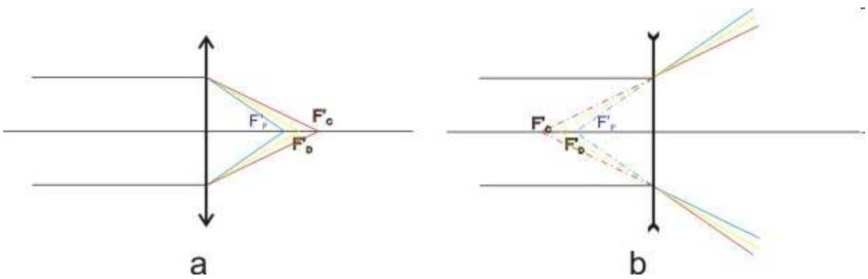


in result: the **parameters of a lens**

$$n'\varphi' = \frac{n_L - n}{r_1} + \frac{n' - n_L}{r_2}$$

(here focus of a thin lens) **depend on the wavelength**

the light with different wavelength is focused into different points and with different magnification. The image gets blurred the image gets blurred with characteristic colouring of its contour:



this **chromatic aberration** is independent of refractive aberrations
(it is present even for aspheric surfaces)

chromatic aberrations of a lens

Introduce the **Abbe's number**

$$\nu = \frac{n_d - 1}{n_f - n_c}$$

refractive indices:

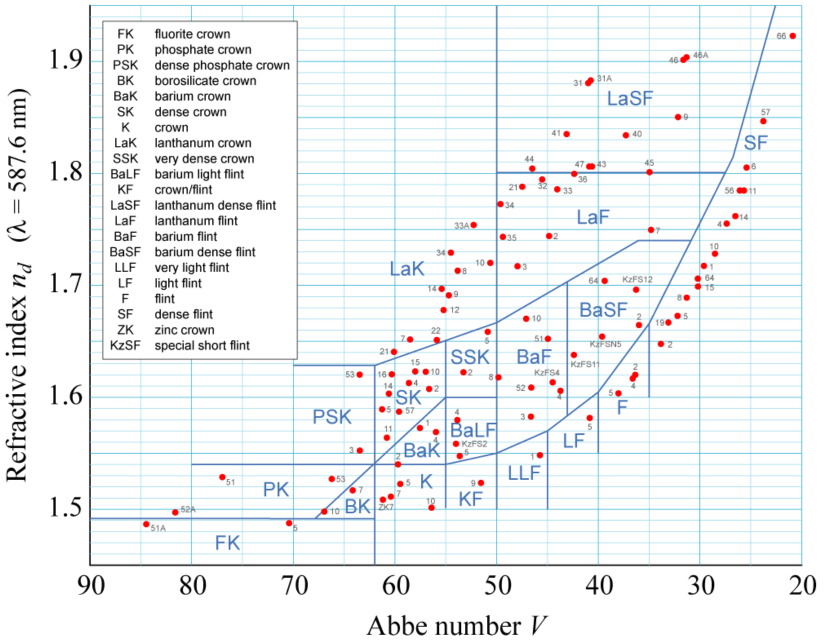
n_F *for* blue line of hydrogen, *λ =486,1 nm*
n_D *for yellow line of natrium, λ =589,3 nm*
n_C *for red line of hydrogen, λ=656,3 nm*

flint glasses
contain potassium-lead admixture

higher refractive index (*n=1,7*)
lower Abbe's number (v=35)

crown glasses
potassium-calcium admixture

lower index of refraction (*n=1,5*)
higher Abbe's number (v=65)



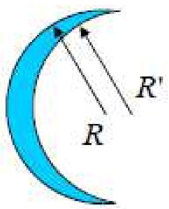
an optimal material:
high refractive index (thin lenses)and high Abbe's number (small chromatic aberration)

lens bulgyness

for thin submersed lens $$n\varphi' = \frac{n_L - n}{R} + \frac{n - n_L}{R'}$$ .. **lens maker equation**

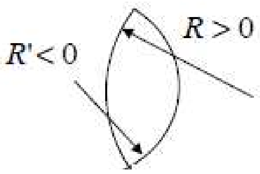
we introduce the **bulgyness** ρ $$n\varphi' = (n_L - n)\left(\frac{1}{R} - \frac{1}{R'}\right) := (n_L - n)\rho$$

(so that, in this case, the geometrical and material parameters get separated)

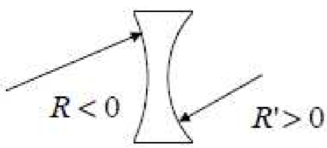


menisces $R = R' \longrightarrow \rho = 0,\ f \to \infty$

for *r1 ≠ r2* power of meniscus depends on its orientation



positive lenses $$\rho = \frac{1}{|R|} + \frac{1}{|R'|} > 0$$



negative lenses $$\rho = -\left(\frac{1}{|R|} + \frac{1}{|R'|}\right) < 0$$

whether lens is positive or negative depends also on surrounding media

chromatic aberrations of a lens

consider **two adjacent lenses**: $\phi' = \phi_1' + \phi_2'$

using the bulginess, $\phi' = (n_1 - 1)\rho_1 + (n_2 - 1)\rho_2$ for some λ

$\tilde{\phi}' = (\tilde{n}_1 - 1)\rho_1 + (\tilde{n}_2 - 1)\rho_2$ for some $\tilde{\lambda}$

if we want to suppress the chromatic aberration for these two wavelengths, it must hold $\phi' = \tilde{\phi}'$, ie.

$$\frac{\rho_1}{\rho_2} = -\frac{n_2' - n_2}{n_1' - n_1} = U$$

from Cauchy's formula this fraction of refractive indices is positive, so that we need a **combination of a positive and a negative lens**

adjacent lenses share the inner radius of curvature, r_{z1}, the outer ones are free (r_{z1}, r_{z2})

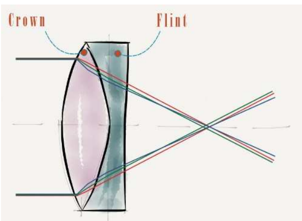
the condition for the surfaces gets
$$\frac{1}{r_z} = \frac{\frac{1}{r_{z1}} + U\frac{1}{r_{z2}}}{1+U}$$

by fulfilling it, we correct the **chromatic aberration of position**

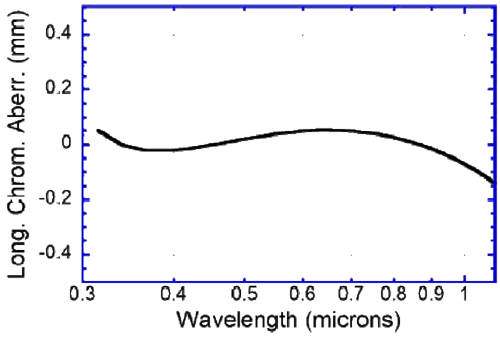
usually, we choose $\lambda = \lambda_C$, $\lambda = \lambda_F$

we still have one degree of freedom, we use it to correct the **chromatic aberration of magnification**

achromatic doublet



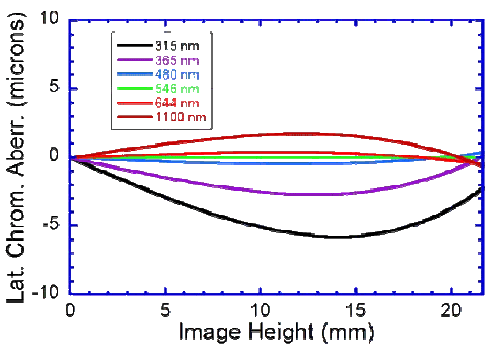
a doublet gets achromatized when both chromatic error of position and magnigication get corrected



chromatic aberration of magnification is corrected when

$$\frac{f_{D1}}{f_{D2}} = -\frac{\nu_2}{\nu_1}$$

this brings a need to use one flint and one crown lens

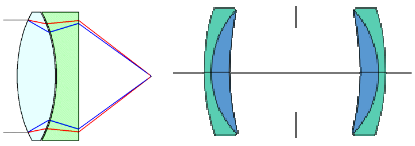


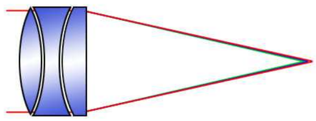
alltogether, for prescribed focal power and chosen glass we have a system of three equations for three unknown radii, which gets solved uniquely:

$$\frac{1}{f_D} = (n_{D1} - 1)\rho_1 + (n_{D2} - 1)\rho_2 \qquad \frac{(n_{D2} - 1)\rho_2}{(n_{D1} - 1)\rho_1} = -\frac{\nu_2}{\nu_1} \qquad \frac{\rho_1}{\rho_2} = U$$

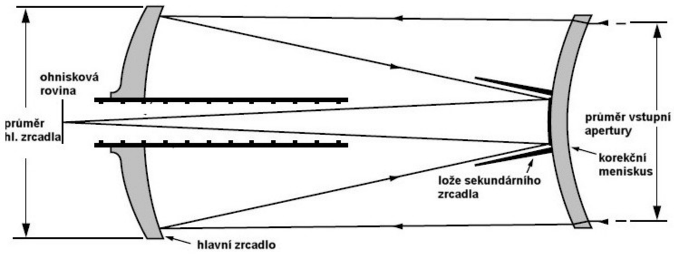
construction of optical systems

usually, the system are composed from groups of achromatized doublets:

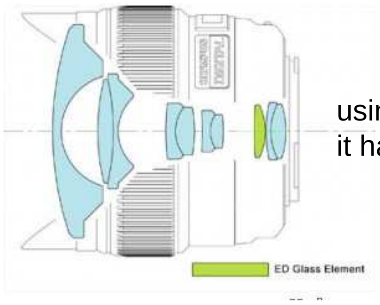




apochromat triplet

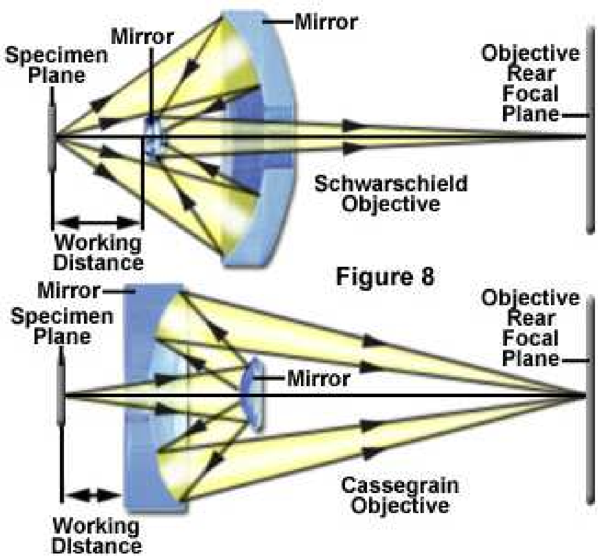


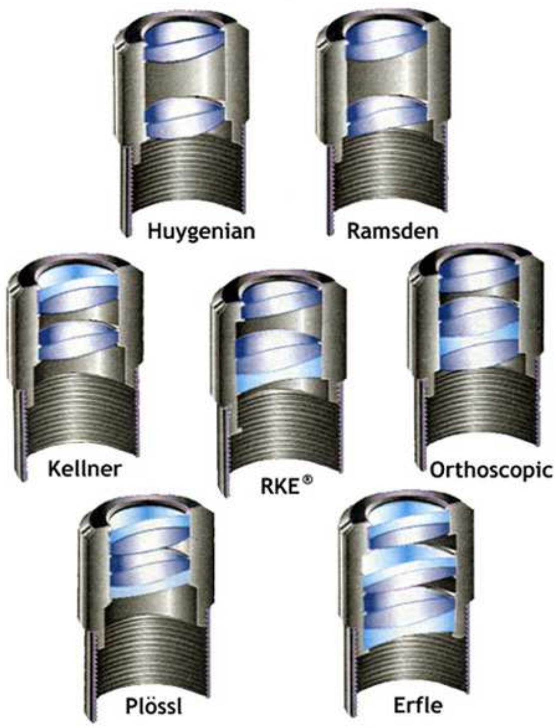
reflective systems show no chromatic aberration at all



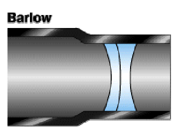
using fluorite (CaF_2) instead of glass is popular: it has very low dispersion

eyepieces

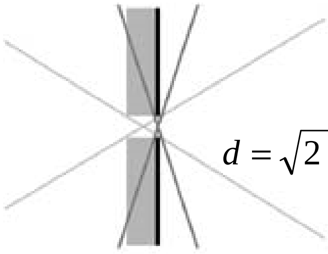




Barlowov lens

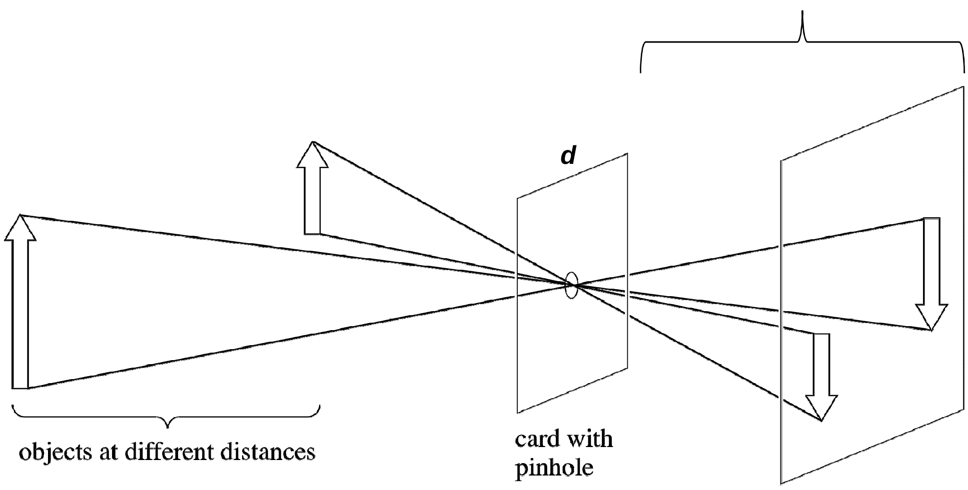


pinhole camera



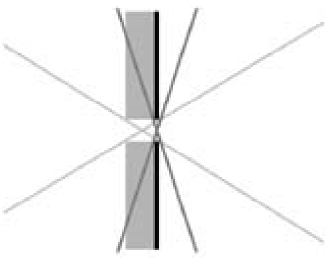
$$d = \sqrt{2f\lambda}$$

pinhole camera

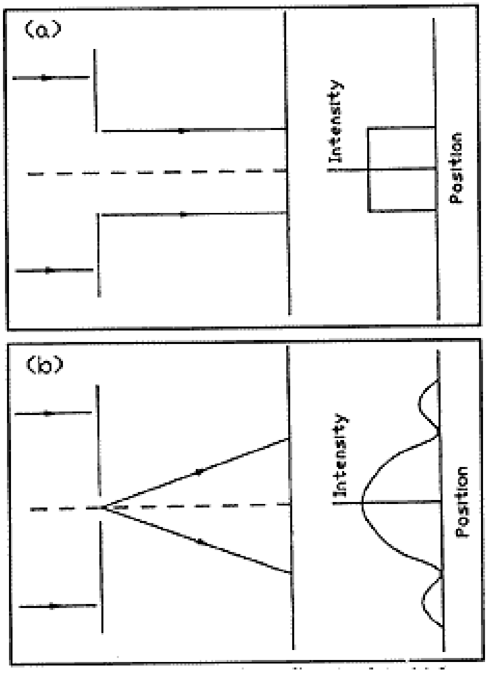


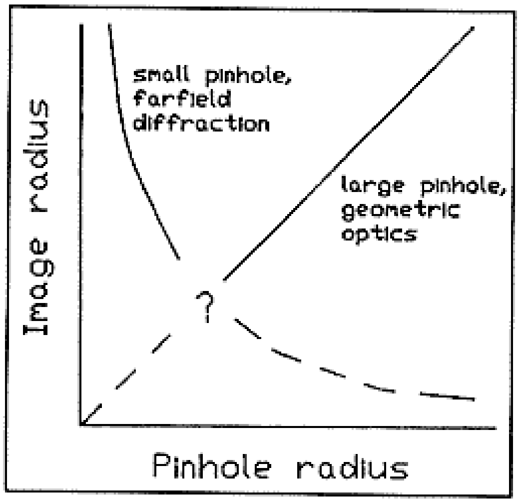
simplest imaging instrument

later: fill the hole with lens (camera lens)



pinhole camera





for optimal performance

$$d = \sqrt{2 f\lambda}$$

pinhole camera

high aperture values:

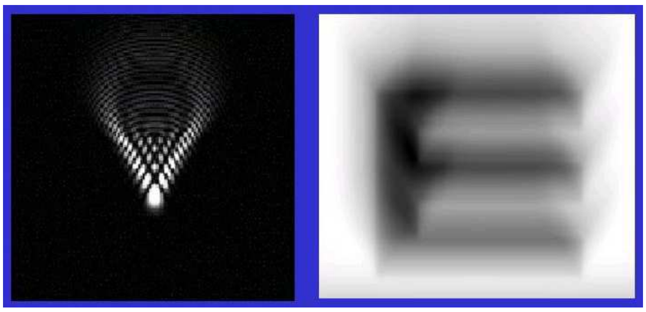
- long expositions
- vignetting
+ high field depth

www.paladix.cz

Pinhole camera picture: Telc main square, negative 6x9,
pinhole 0.25mm, focal distance 20mm … **c=80**

pinhole camera

Seidel aberrations

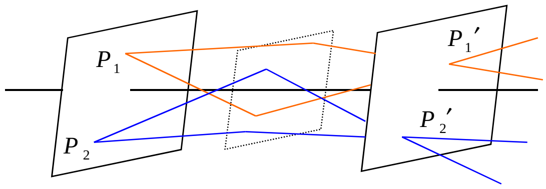


paraxial (Gauss) optics

within paraxial space, the points are imaged into (perfect) points

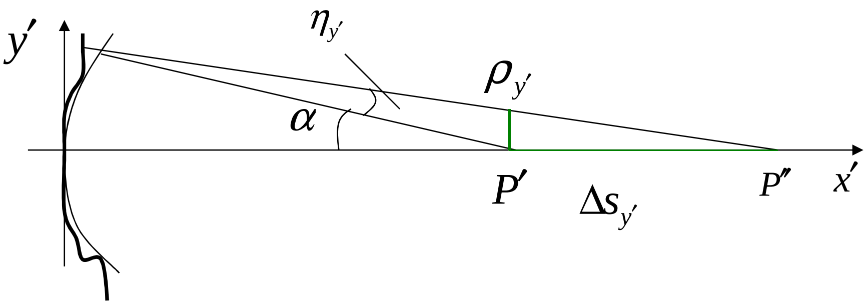
in addition, points from a same plane image into same planes again:

we talk about **focal planes**.



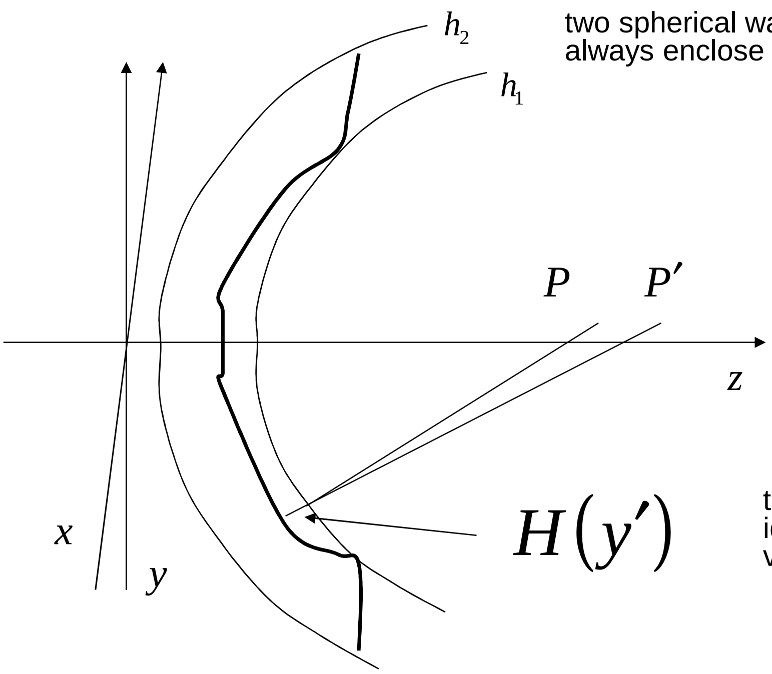
in real systems, the perfect imaging is distorted by **aberrations** (longitudinal, transversal, etc.)

the images get **blur**red.



(we will not consider the chromatic aberrations here)

wave aberration



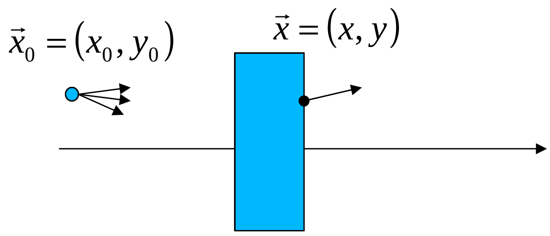
two spherical wavefronts from *P* always enclose the real wavefront

the difference between real and ideal wavefront can be given via an expansion series

the **aberration coefficients** (of the expansion) can be minimised by the optical design

we talk about producing the **circle of least confusion**

wave aberrations of axially symmetric systems



for description of optical systems we choose
position of (the point) **source** and
the coordinates of selected **ray at the exit pupil**

we can always put $y_0 = 0$

thanks to axial symmetry, rotation must not matter,

the only important terms are the following: $\vec{x}_0 \cdot \vec{x}_0, \vec{x} \cdot \vec{x}_0, \vec{x} \cdot \vec{x}$

the wavefront difference gets $H(x_0, y_0, x, y) \to H(x_0^2, x_0 x, x^2 + y^2)$ ($y_0 = 0$)

we use polar coordinates $x = \rho \cos\vartheta \quad y = \rho \sin\vartheta$ (ρ is the aperture)

$$H(x_0^2, x_0\rho\cos\vartheta, \rho^2) = \sum_{k,l,m} W_{klm} x_0^k \rho^l \cos^m \vartheta =$$

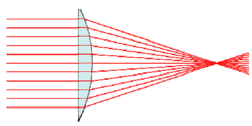
$$= W_{000} + W_{200}x_0^2 + W_{111}x_0\rho\cos\theta + W_{002}\rho^2 +$$

$$+ W_{400}x_0^4 + W_{040}\rho^4 + W_{131}x_0\rho^3\cos\theta + W_{222}x_0^2\rho^2\cos^2\theta +$$

$$+ W_{220}x_0^2\rho^2 + W_{311}x_0^3\rho\cos\theta + \ldots$$

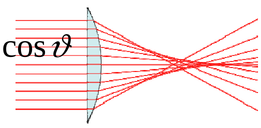
note the special case of axial source, $x_0 = 0$: only W_{040} remains from green lines

Seidel aberrations 1856 (**axially symmetric systems**)

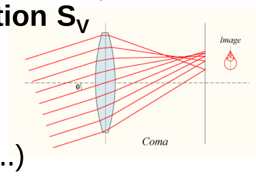


lowestorder terms can be organized into five (Seidel) aberrations:

$$H = \frac{1}{8}S_I\rho^4 + \frac{1}{2}S_{II}x_0\rho^3\cos\vartheta + \frac{1}{2}S_{III}x_0^2\rho^2\cos^2\vartheta + \frac{1}{4}(S_{III} + S_{IV})x_0^2\rho^2 + \frac{1}{2}S_V x_0^3\rho\cos\vartheta$$



spherical aberration S_I, coma S_{II}, astigmatism S_{III}, curvature S_{IV}, distortion S_V



the piston (W_{200}), tilt (W_{111}) and defocus (W_{002}) do not belong here
(they can be removed by moving the system)

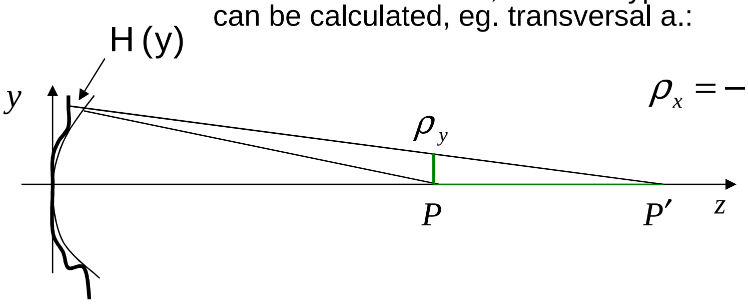
higher order aberrations bring more subtle behavior (secondary coma etc..)

good thing about **Seidel coefficients**:

the **overall aberration** can be found as a **sum of aberrations from individual surfaces**

$$S_I = S_I^1 + S_I^2 + S_I^3 + S_I^4 + :::$$

from wave aberration, all other types of aberrations
can be calculated, eg. transversal a.:



$$\rho_x = -f\frac{dH(y)}{dx} \quad \rho_y = -f\frac{dH(y)}{dy}$$

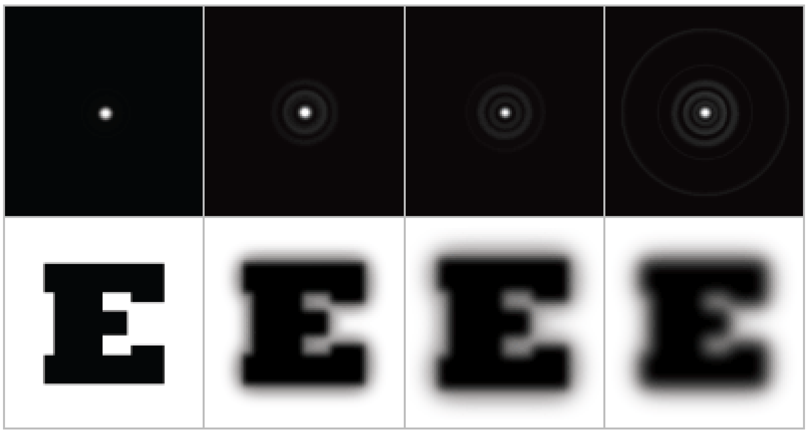
spherical aberration

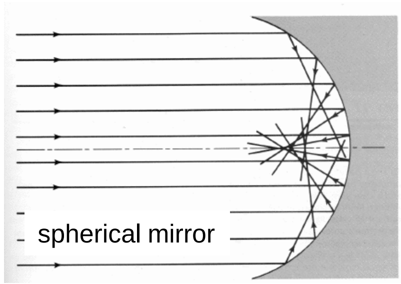
$$H(y) = \frac{1}{8} S_I \rho^4$$

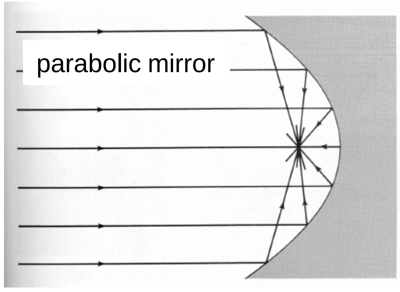
the blur is symmetrically shaped (circles)

for aperture ρ, the blur extents to $\frac{1}{2} f S_I \rho^3$ (*f* is the focal distance)

- it is very effective to fight spherical aberration using small apertures $\rho \to 0$
- spherical aberration is present even for axial sources
- systems with spherical aberration repaired are called *stigmatic at the axis*







coma

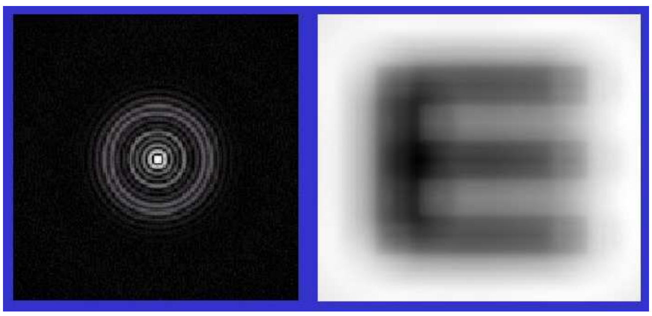
$$H(y) = \frac{1}{2} S_{II} x_0 \rho^3 \cos \vartheta$$

coma consists from circles

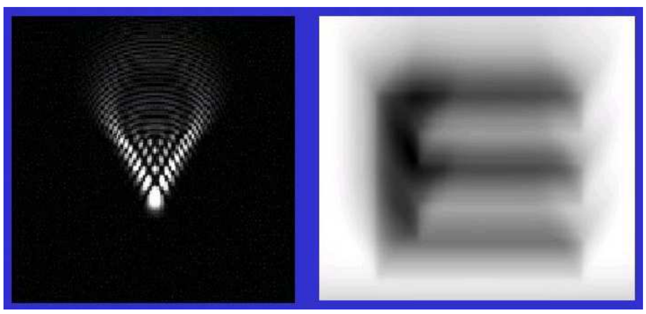
of radii $\frac{1}{2} f S_{II} x_0 \rho^2$

and offset $f S_{II} x_0 \rho^2$

- coma is observed only for nonaxial sources, small apertures help
- the circles fill up an angle of 60°, length : width of the whole blurr is 3:2
- systems without both spherical aberration and coma are called *aplanatic*



spherical aberration blur



coma blur

astigmatism

$$H(y) = \frac{1}{4} S_{III} x_0^2 \left(3x^2 + y^2\right)$$

the aberration has a form of ellipses with axes ratio 1:3

- the sagital a meridional rays are each focused perfectly, but in different planes

- big aperture is not that harmful, but astigmatism rises steeply with nonaxial rays (ie. with field of view)