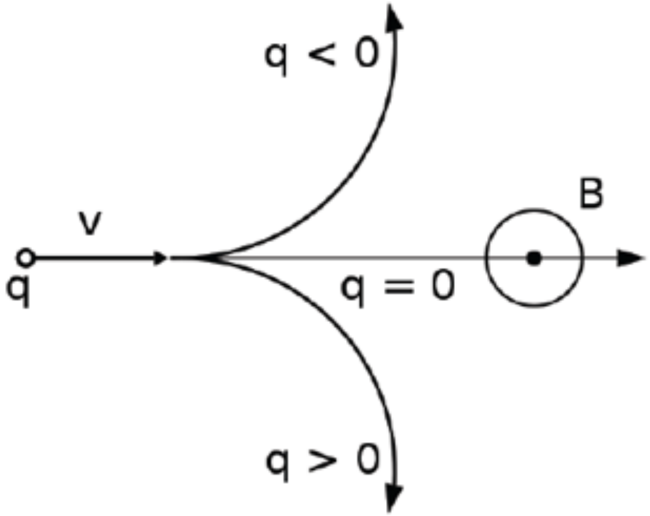
Tracking and Vertex Reconstruction

The determination of the momentum of charged particles can be performed by measuring the bending of a particle trajectory (track) in a magnetic field



$$\vec{F} = q\vec{v} \times \vec{B}$$

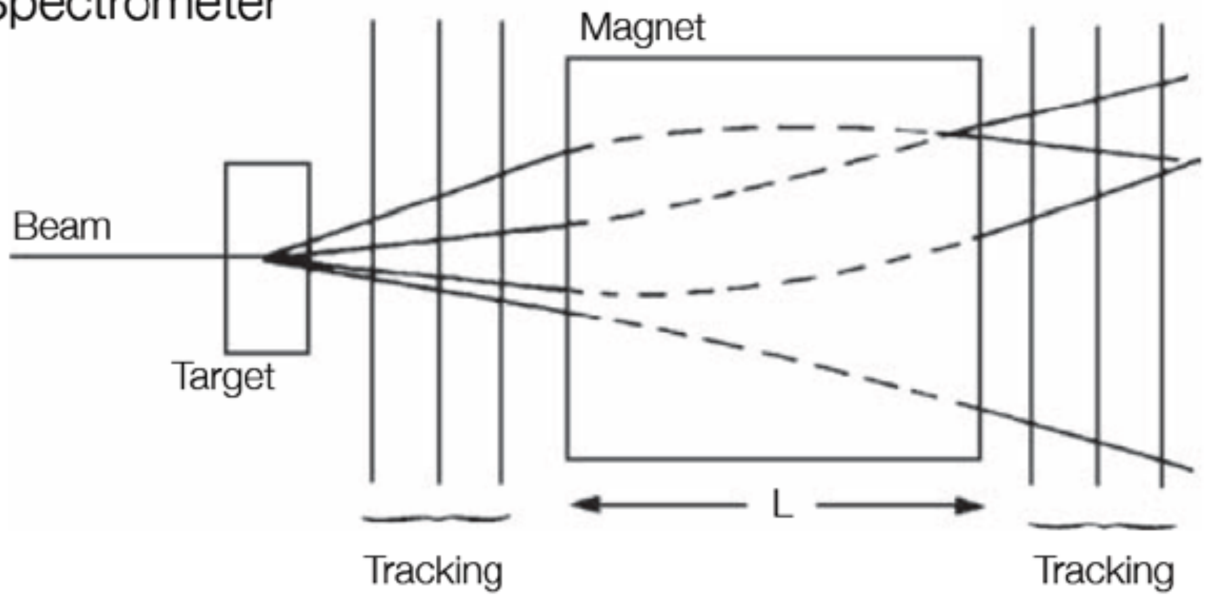
Lorentz force: is the force on a point charge due to electromagnetic fields

$$\frac{mv^2}{r} = qvB$$

… for a particle in motion perpendicular to a constant B field

In practice:
- use layers of position sensitive detectors before and after (or inside) a magnetic field to measure a trajectory
- determine the bending radius

Schematics of a Spectrometer

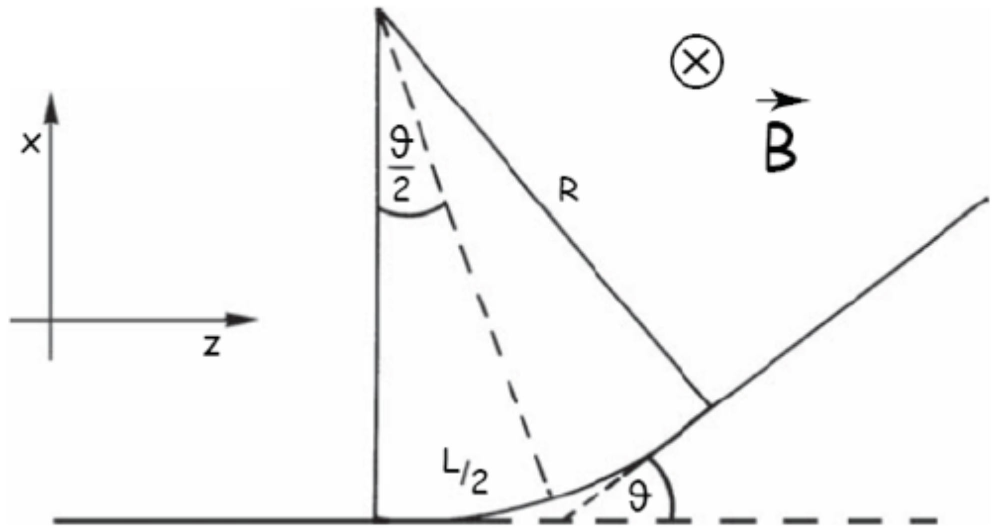


Fixed Target Experiments

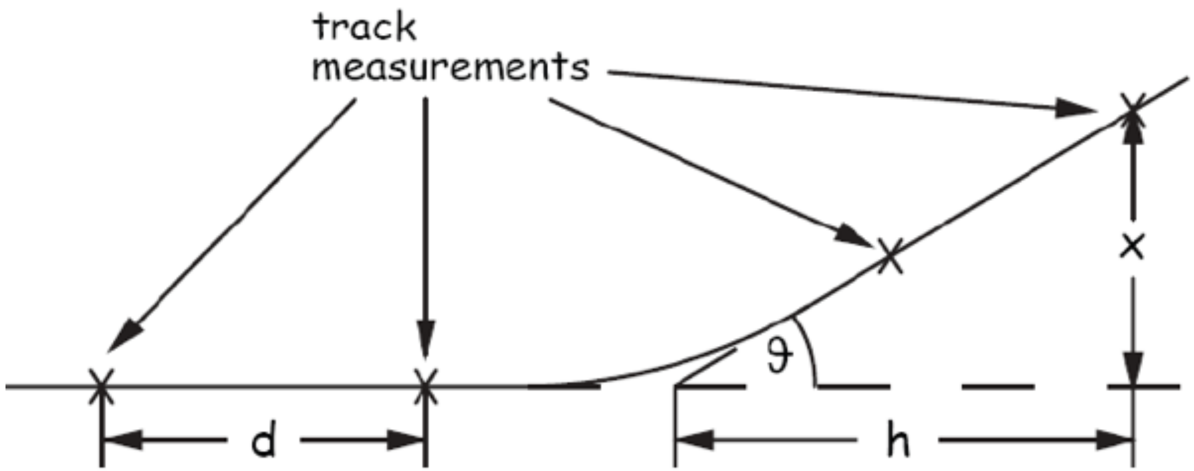
Momentum determination

$$p = eRB \qquad \vartheta = {}^L/_R$$
$$= {}^L/_p \cdot eB$$

$$p = eB \cdot {}^L/_\vartheta$$



Momentum resolution: $\rightarrow \quad \frac{\sigma_p}{p} = \frac{\sigma_\vartheta}{\vartheta}$ with $\sigma_\vartheta \sim \sigma_x$



Determination of σ_p/p:

$$\vartheta = \frac{x}{h} \qquad \sigma_\vartheta = \frac{\sigma_x}{h}$$

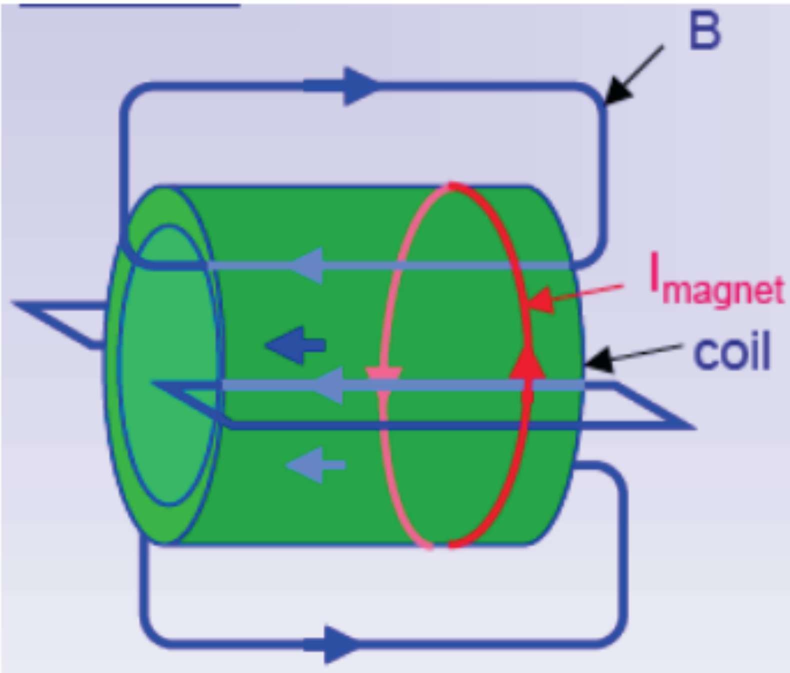
$$\frac{\sigma_p}{p} = \frac{\sigma_\vartheta}{\vartheta} = \frac{\sigma_x}{h} \cdot \frac{p}{eBL}$$

Long lever arm improves momentum resolution ...

Magnets for 4π Detectors

Solenoid

+ Large homogeneous field inside
- Weak opposite field in return yoke
- Size limited by cost
- Relatively large material budget

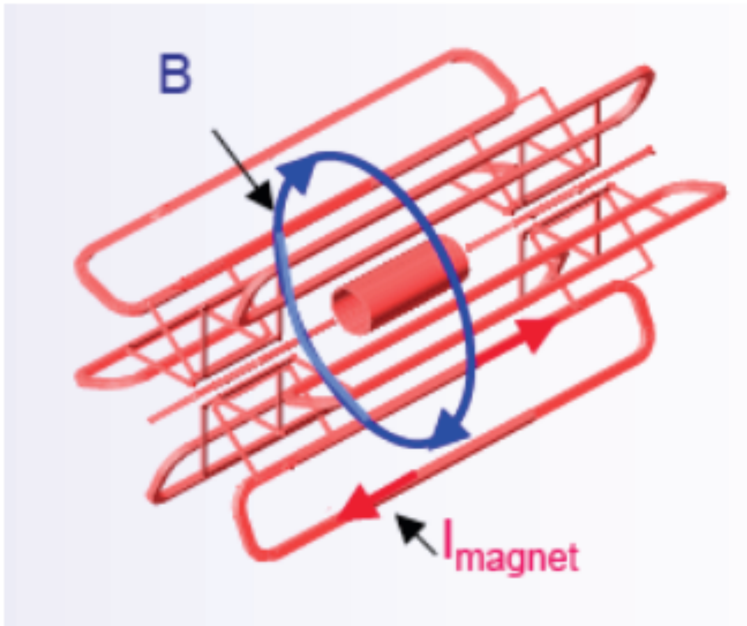


Examples:

•Delphi: SC, 1.2 T, 5.2 m, L 7.4 m
•L3: NC, 0.5 T, 11.9 m, L 11.9 m
•CMS: SC, 4 T, 5.9 m, L 12.5 m

Toroid

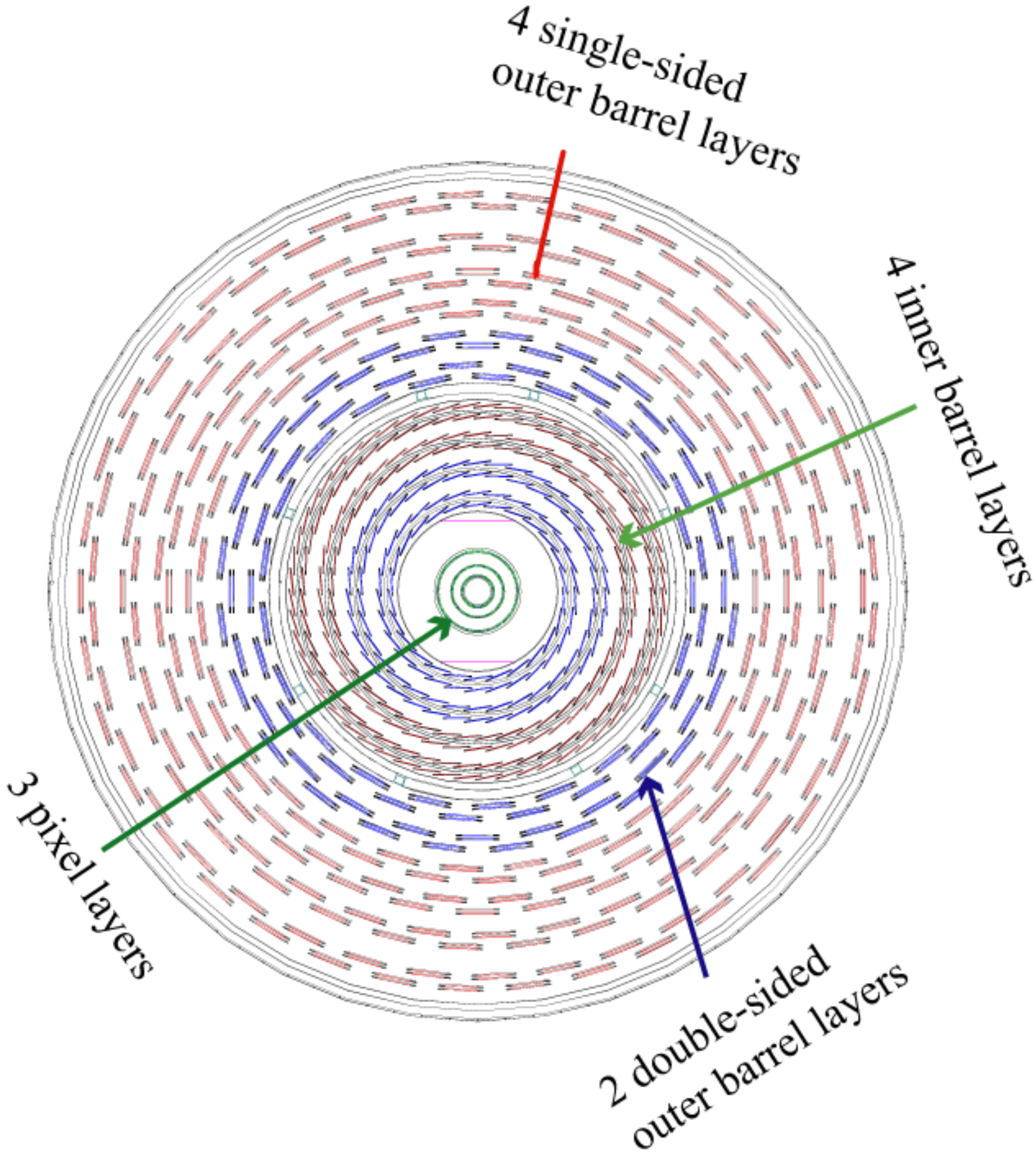
+ Field always perpendicular to p
+ Rel. large fields over large volume
+ Rel. low material budget
- Non-uniform field
- Complex structural design



Example:

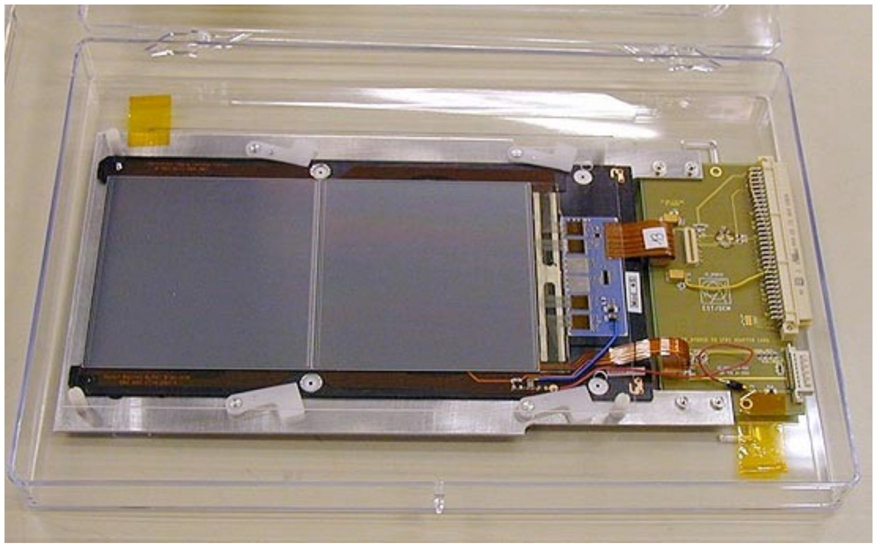
•ATLAS: Barrel air toroid, SC, ~1 T, 9.4 m, L 24.3 m

The CMS Tracker

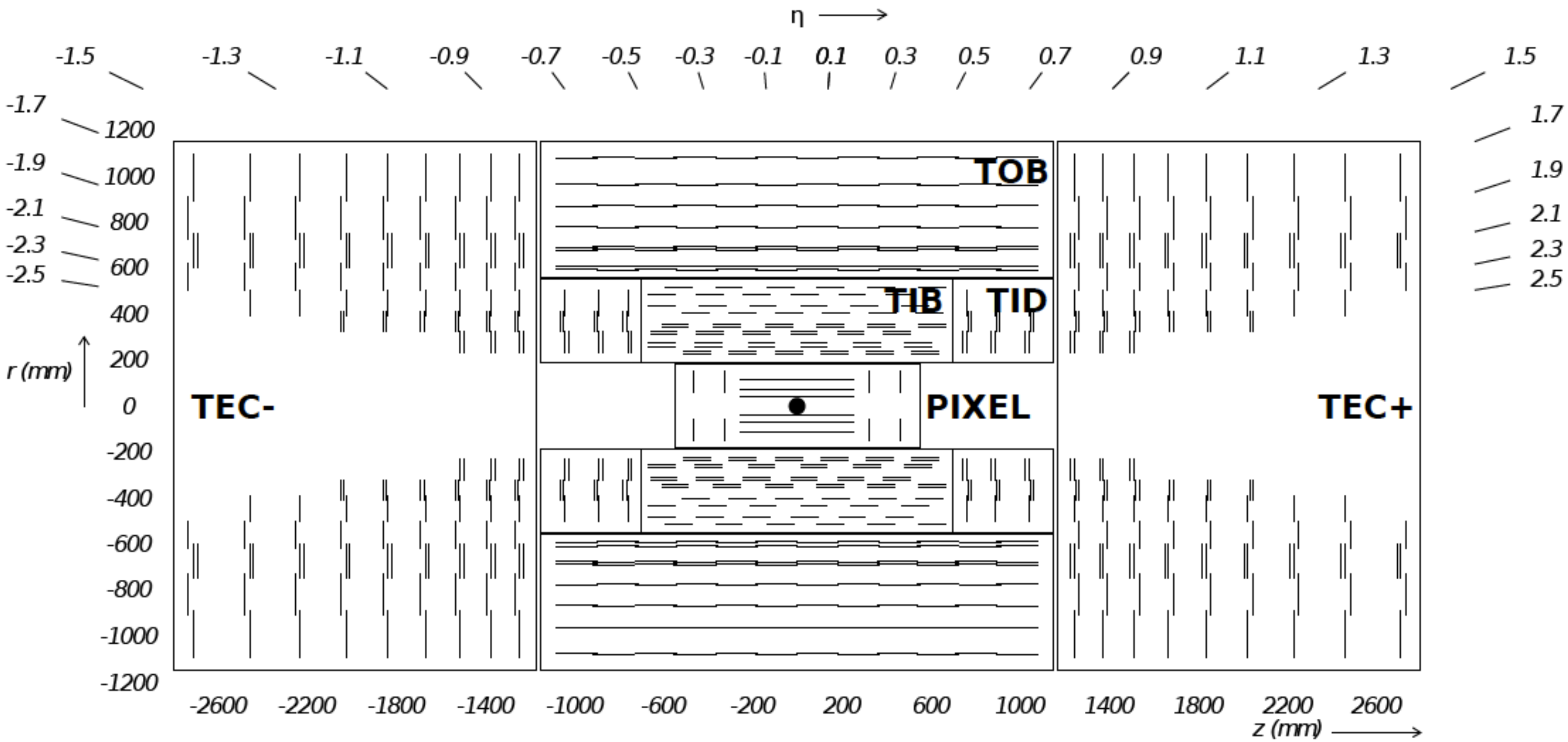


- 10 barrel layers
- 9+3 endcap layers (next slide)
- radius 1.1 m, length 5.8 m
- 200 m² active silicon (largest silicon tracker ever built)
- acceptance up to |η|<2.5
- 500 people, 15 years design development and construction

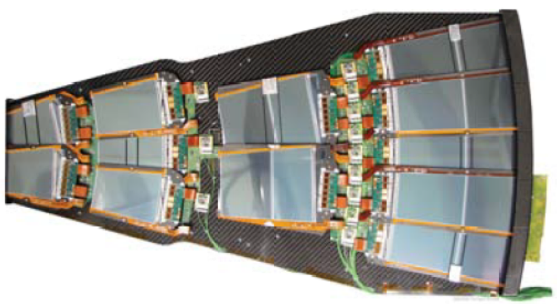
strip module in CMS



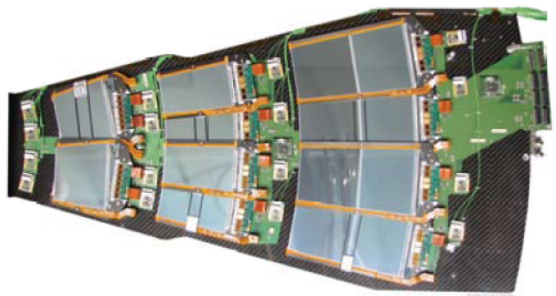
The CMS Tracker



front



back



endcap (TEC)
"petal"

The Helix Equation

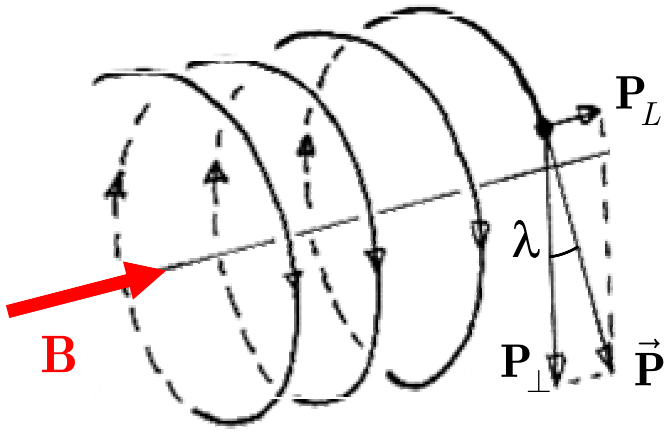
The helix is described in **parametric** form

$$x(s) = x_o + R\left[\cos\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_o\right]$$

$$y(s) = y_o + R\left[\sin\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_o\right]$$

$$z(s) = z_o + s\sin\lambda$$

$$R(m) = \frac{p_\perp(GeV)}{0.3B(T)}$$

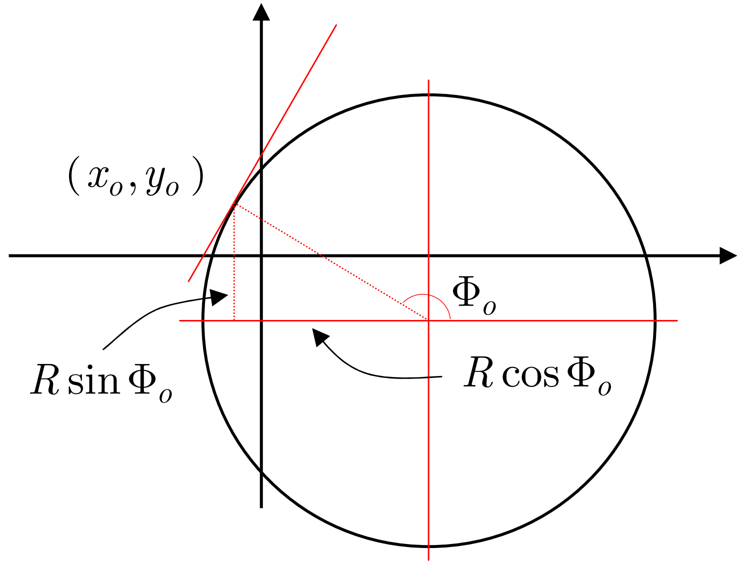


λ is the **dip angle**

$h = \pm 1$ is the **sense of rotation** on the helix

The **projection** on th x-y **plane** is a circle

$$(x - x_o + R\cos\Phi_o)^2 + (y - y_o + R\sin\Phi_o)^2 = R^2$$



x_o and y_o the coordinates at $s = 0$

Φ_o is also related to the **slope** of the tangent to the circle at $s = 0$

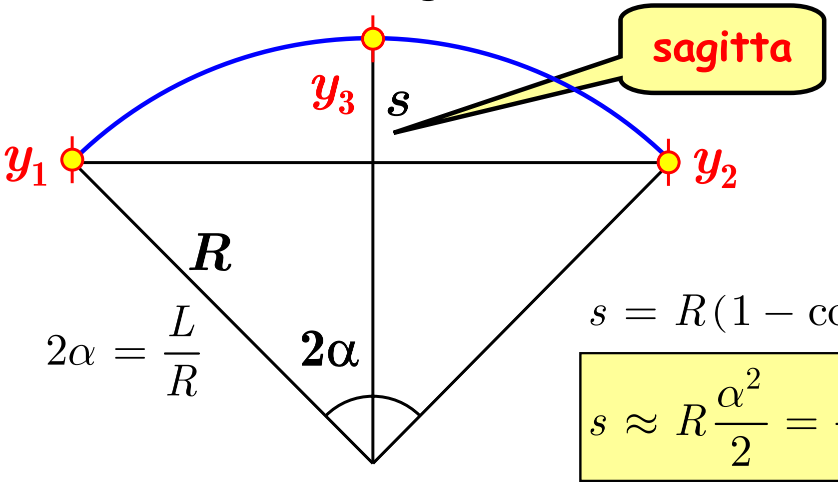
Uncertainty on Momentum Measurement

To introduce the problem of **momentum measurement** let's go back to the **sagitta**

a particle moving in a plane perpendicular to a **uniform magnetic field** B

$$R = \frac{p}{0.3B} \qquad \frac{\delta p}{p} = \frac{\delta R}{R}$$

the trajectory of the particle is an arc of **radius** R of **length** L



$$2\alpha = \frac{L}{R}$$

$$s = R(1 - \cos\alpha)$$

$$s \approx R\frac{\alpha^2}{2} = \frac{L^2}{8R}$$

Momentum Resolution

We stress again that a good momentum resolution call for a long track

$$\frac{\delta p}{p^2} \sim \frac{1}{L^2}$$

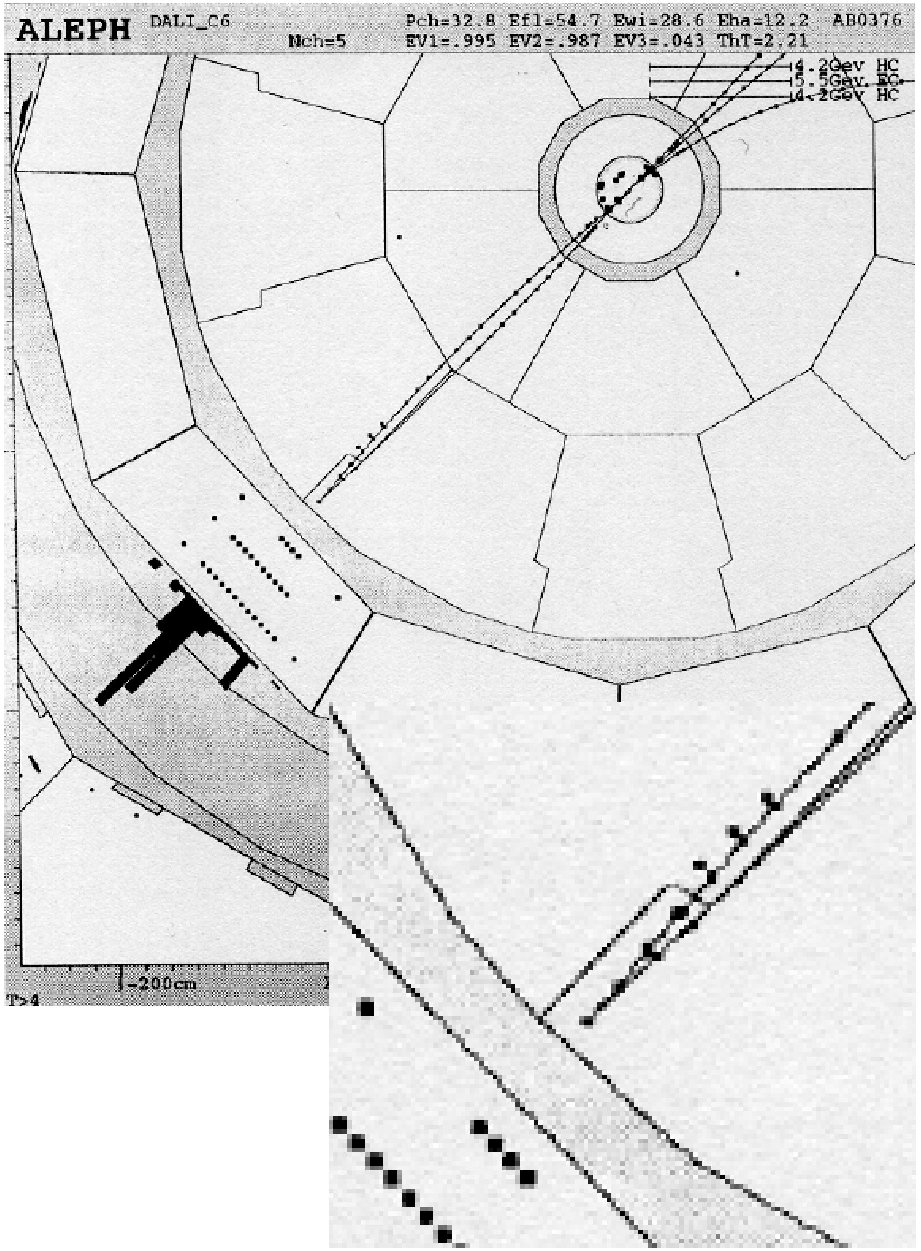
any trick that can extend the track length can produce significant improvements on the momentum resolution

the use of the vertex can also improve momentum resolution:

- the common vertex from which all the tracks originate can be fitted
- the point found can be added to every track to extend the track length at $R_{min} \rightarrow 0$

the position of the beam spot can also be used as constraint

Extending R_{max} can be very expensive



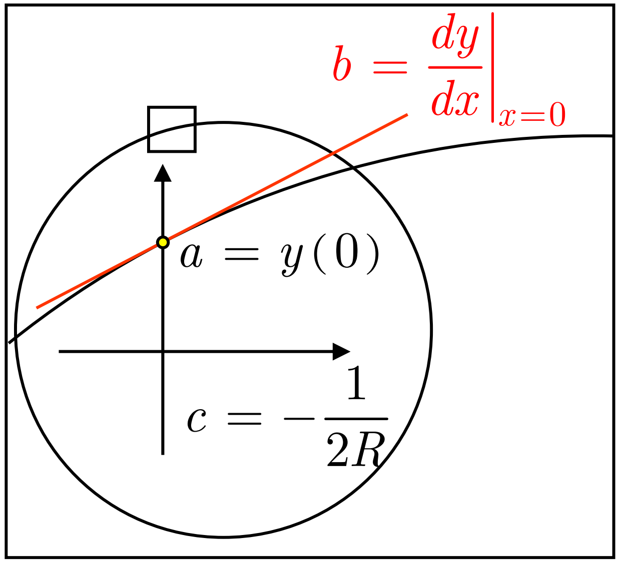
Tracking in a Magnetic Field

The previous example showed the basic principle of a track fit.

Let's now turn to a more complete treatement of the measurement of the charged particle trajectory

We have already seen that for an homogeneus magnetic field the trajectory projected on a plane perpendicular to the magnetic field is a circle

$$(y - y_o)^2 + (x - x_o)^2 = R^2$$



for not too low momenta we can use a linear approximation

$$y = y_o + \sqrt{R^2 - (x - x_o)^2}$$

$$y \approx y_o + R\left(1 - \frac{(x - x_o)^2}{2R^2}\right)$$

$$y = \left(y_o + R - \frac{x_o^2}{2R}\right) + \frac{x_o}{R}x - \frac{1}{2R}x^2$$

$R^2 >> (x - x_o)^2$

we are led to the parabolic approximation of the trajectory

$$y = a + bx + cx^2$$

let's stress that as far as the track parameters is concerned the dependence is linear

The parameters a,b,c are

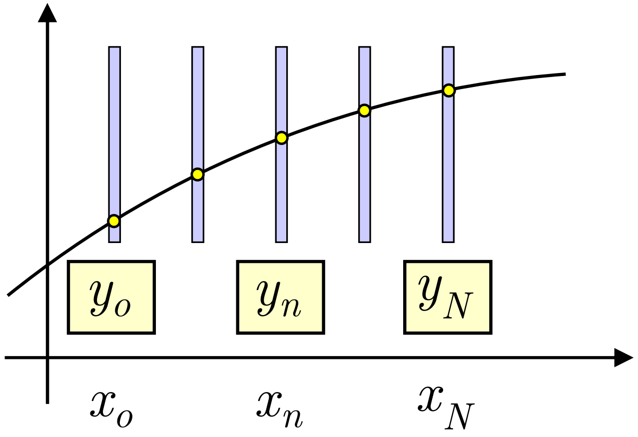
intercept at the origin

slope at the origin

radius of curvature (momentum)

Quadratic Fit

Assume N detectors measuring the y coordinate [Gluckstern 63]



The detectors are placed at positions $x_o, \ldots, x_n, \ldots, x_N$

A track crossing the detectors

gives the measurements $y_0, \ldots, y_n, \ldots, y_N$

Each measurement has an error σ_n

Using the parabola approximation, the track parameters are found by minimizing the χ^2

$$\chi^2 = \sum_{n=0}^{N} \frac{\left(y_n - a - bx_n - cx_n^2\right)^2}{\sigma_n^2}$$

The result is [4: Avery 1991, Blum-Rolandi 1993 p.204, Gluckstern 63]

$$a = \frac{\sum y_n G_n}{\sum G_n} \quad b = \frac{\sum y_n G_n}{\sum x_n G_n} \quad c = \frac{\sum y_n G_n}{\sum x_n^2 G_n}$$

and finally the momentum error

$$\frac{\delta p}{p^2} = \frac{\sigma}{0.3BL^2}\sqrt{4C_N}$$

the formula shows the same basic features we noticed in the sagitta discussion

we have also found the dependence on the number of measurements (weak)

$$C_N = \frac{180N^3}{(N-1)(N+1)(N+2)(N+3)}$$

for N > 10: $C_N \approx \frac{720}{N+4} \rightarrow \frac{\delta p}{p^2} \sim \frac{1}{\sqrt{N}}$

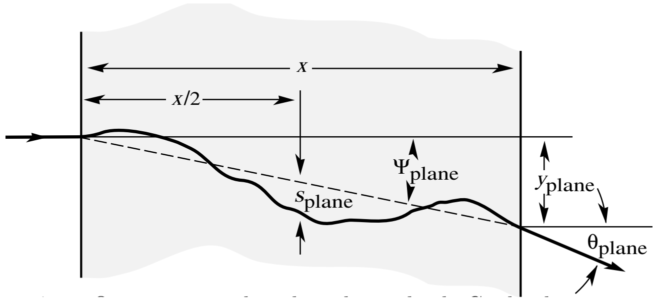
Tracking resolution and multiple scattering

We had the momentum resolution: $\left(\frac{\Delta P}{P}\right)^2 = \left(\frac{\Delta R}{R}\right)^2 + (\tan\lambda\Delta\lambda)^2$

rewrite it using $P_\perp = 0.3BR$ and $\lambda = \pi/2 - \theta$ → $\tan\lambda = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

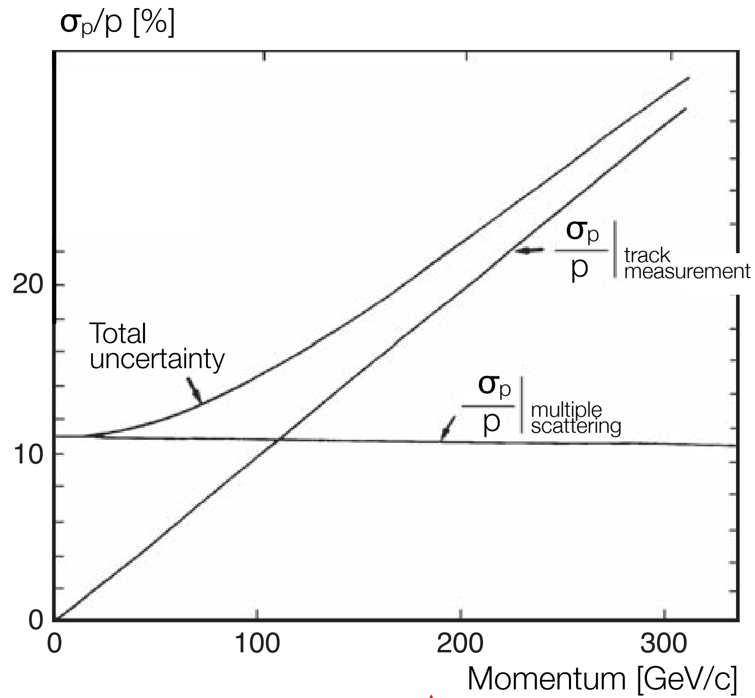
we get $$\left(\frac{\Delta P}{P}\right)^2 = \left(\frac{\Delta P_\perp}{P_\perp}\right)^2 + \left(\frac{\Delta\theta}{\tan\theta}\right)^2$$

Multiple scattering contribution:



$$\Delta\theta = \frac{13.6\text{MeV}}{\beta c p} z\sqrt{x/X_0}\,[1 + 0.038\ln(x/X_0)]$$

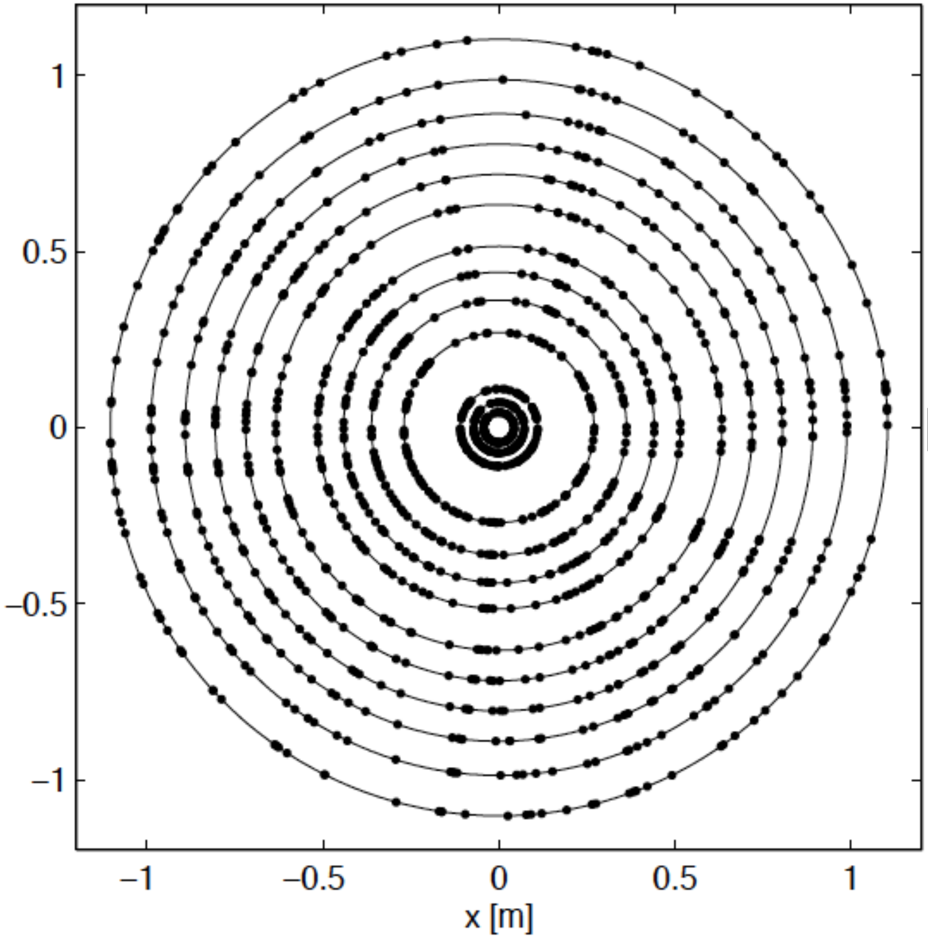
$$\Delta\theta = \sigma_\theta \approx \frac{14\ \text{MeV/c}}{p}\sqrt{\frac{L}{X_0}}$$ with: $L = R\theta$

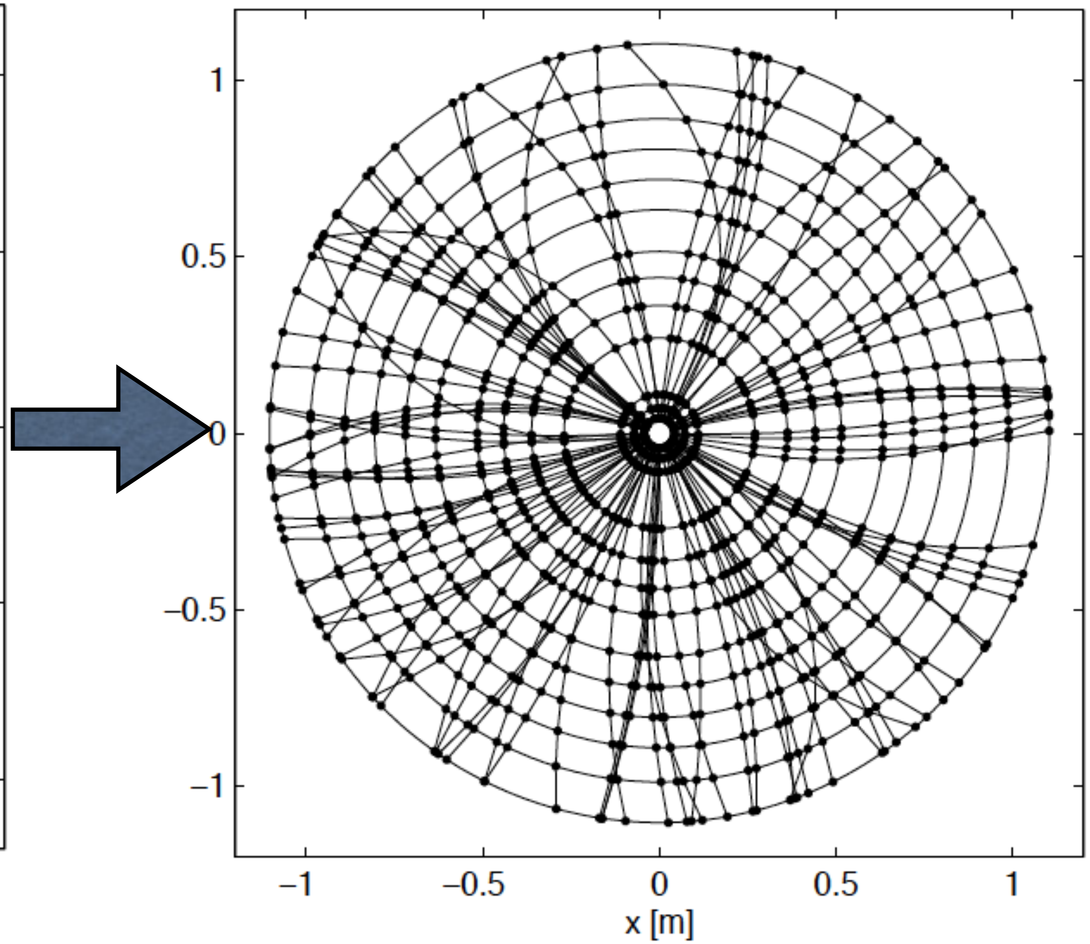


$$\frac{\sigma_\theta}{\theta} \approx \frac{14\ \text{MeV/c}}{p}\sqrt{\frac{L}{X_0}} \cdot \frac{R}{L} \approx \frac{50\ \text{MeV/c}}{p}\sqrt{\frac{1}{LX_0}} \cdot \frac{p}{B} \sim \frac{1}{\sqrt{LX_0}B}$$

and we get: $$\left(\frac{\sigma_{p_t}}{p_t}\right)^2 = \text{const}\cdot\left(\frac{p_t}{BL^2}\right)^2 + \text{const}\cdot\left(\frac{1}{B\sqrt{LX_0}}\right)^2$$

Track Finding





- classification or pattern recognition problem
- multiple ambiguous hypotheses possible
- supposed to be conservative (discarded hypothesis cannot be recovered later)

Track Finding

examples for “global“ track finding approaches

- global track fit
  - ★taking into account all possible combinations of hits
  - ★number of possible combinations from thousands of hits is immense, track candidates need to be validated → computationally too expensive

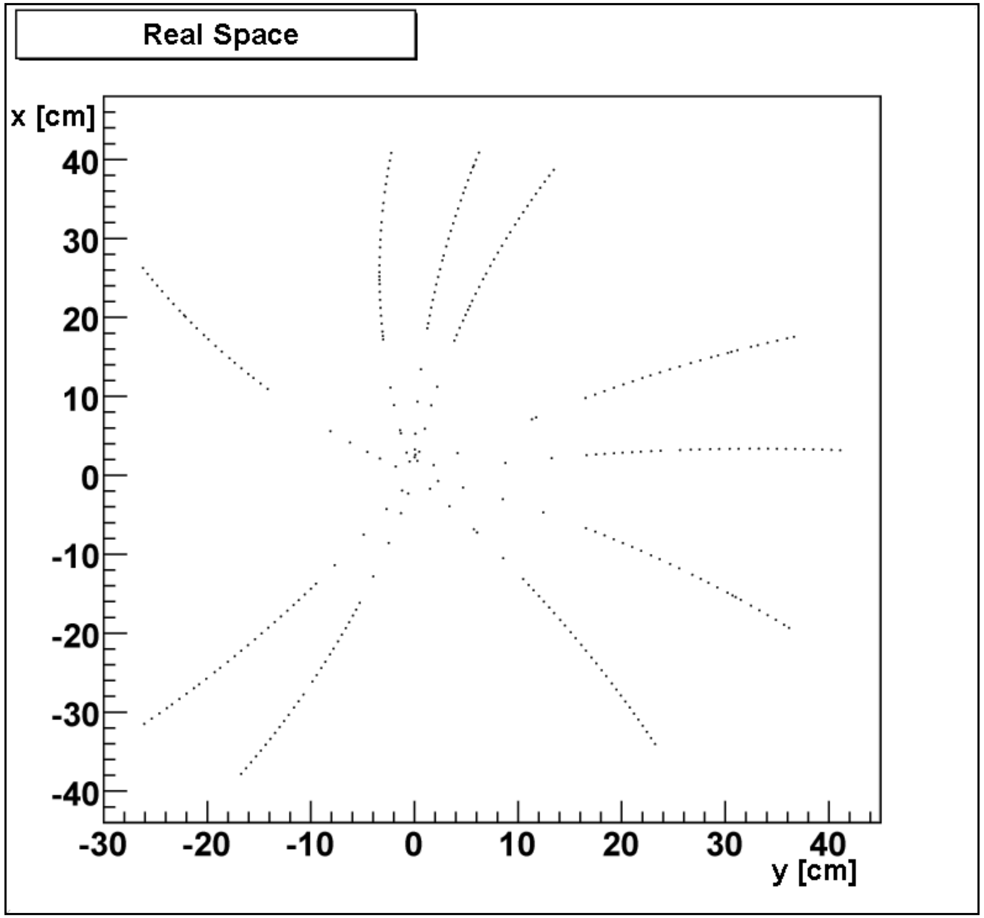
- conformal mapping:
  - ★circles (tracks) through the origin in a 2D x-y-coordinate system map to straight lines in u-v system by the transformation

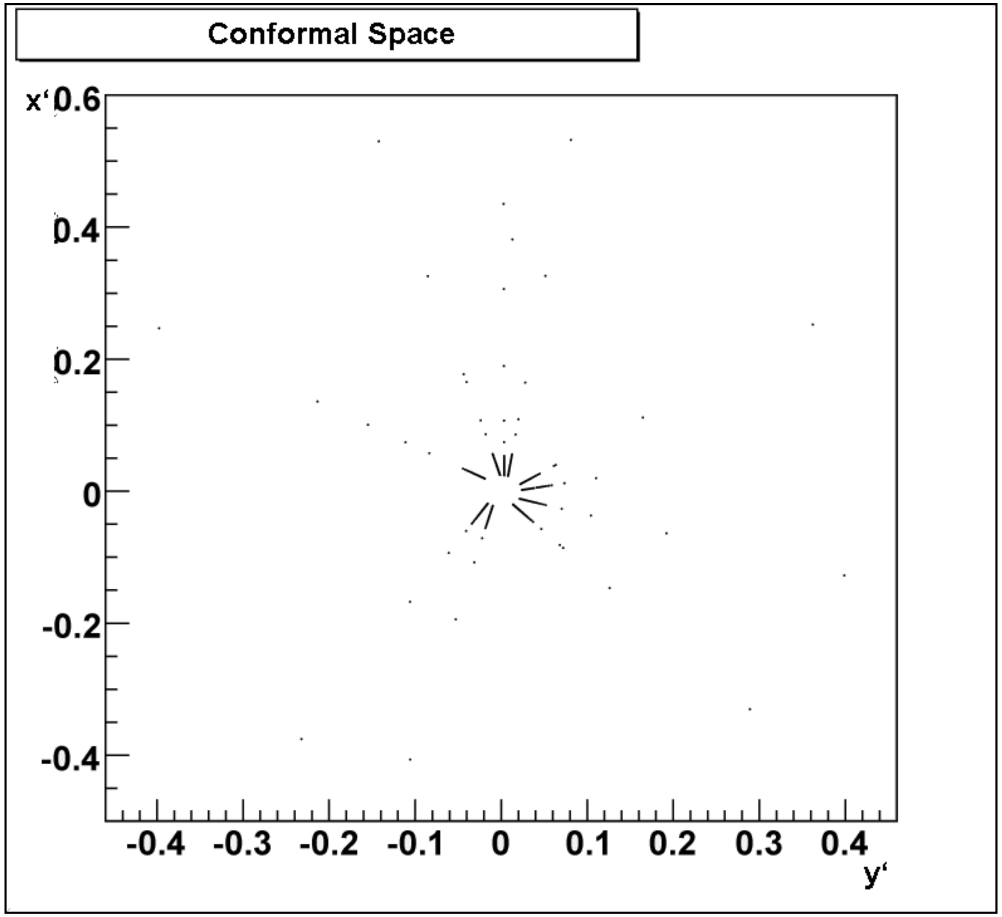
$$u = \frac{x}{x^2 + y^2}, \qquad v = \frac{y}{x^2 + y^2}$$

    where the circle equation is given by $(x-a)^2 + (y-b)^2 = r^2 = a^2 + b^2$
  - ★scan along azimuthal angle to find accumulation of hits along the straight line (peaks in the histogram indicate tracks)
  - ★works for high-pt tracks passing close to the origin

Conformal Mapping Example

- Real space
- Conformal space



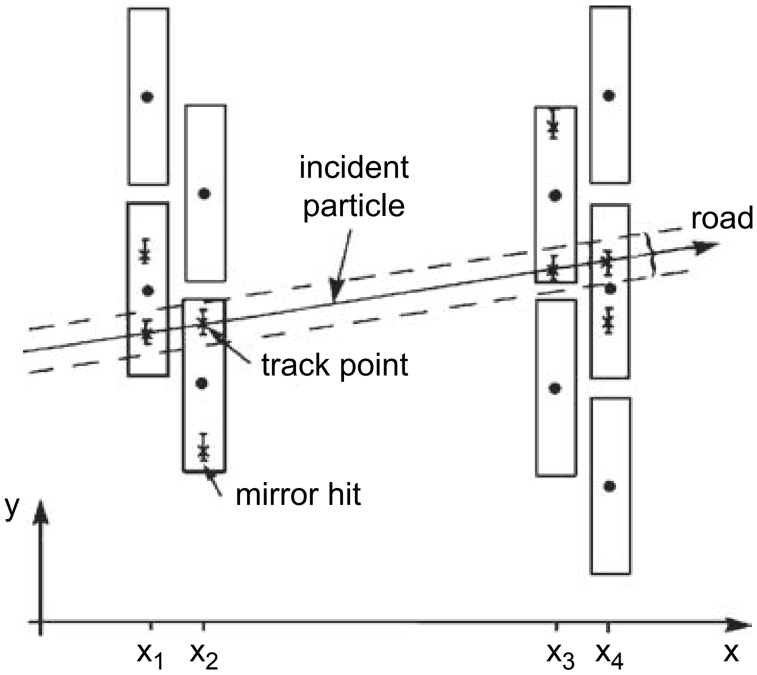


- Angle preserving, not length preserving
- Reference point must be on the circle
- Re-iterate with each hit point as seed

Track Finding

example for "local" track finding approaches

- track road:
  - ★ initiated with a set of measurements that could come from the same particle
  - ★ use a model (shape of the trajectory) to interpolate between the measurements and create a "road" around the trajectory
  - ★ measurements inside the road boundaries constitute the track candidate
  - ★ subsequent track fit can evaluate the correctness



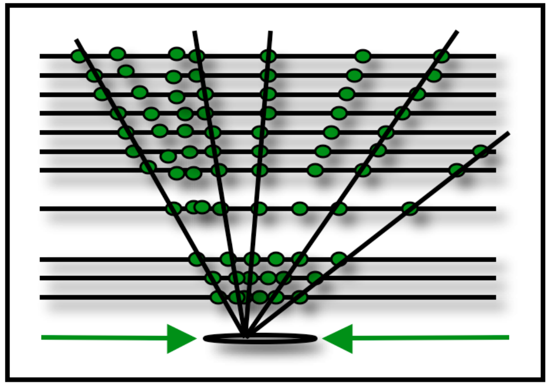
Iterative Tracking in CMS

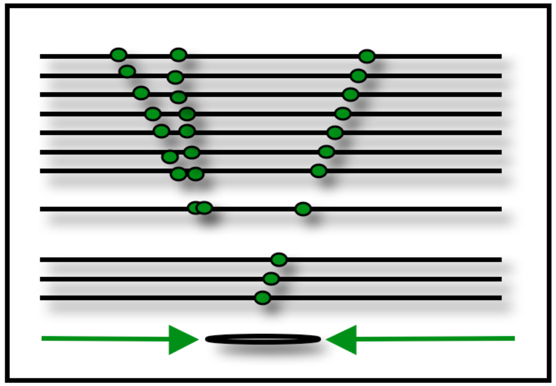
six iterations:

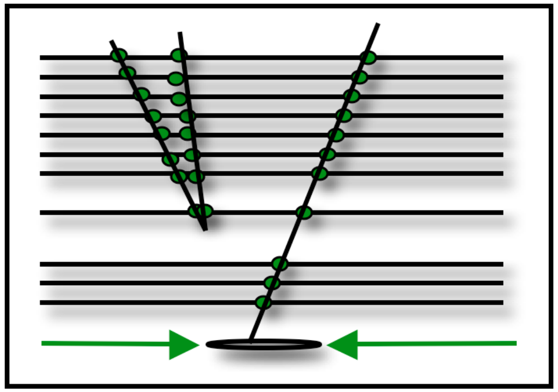
- propagate seed outwards and search for new hits
- unambiguously assigned hits are removed from the list
- filter track collection to remove fakes or bad tracks
- repeat with remaining hits

differences in seeding:

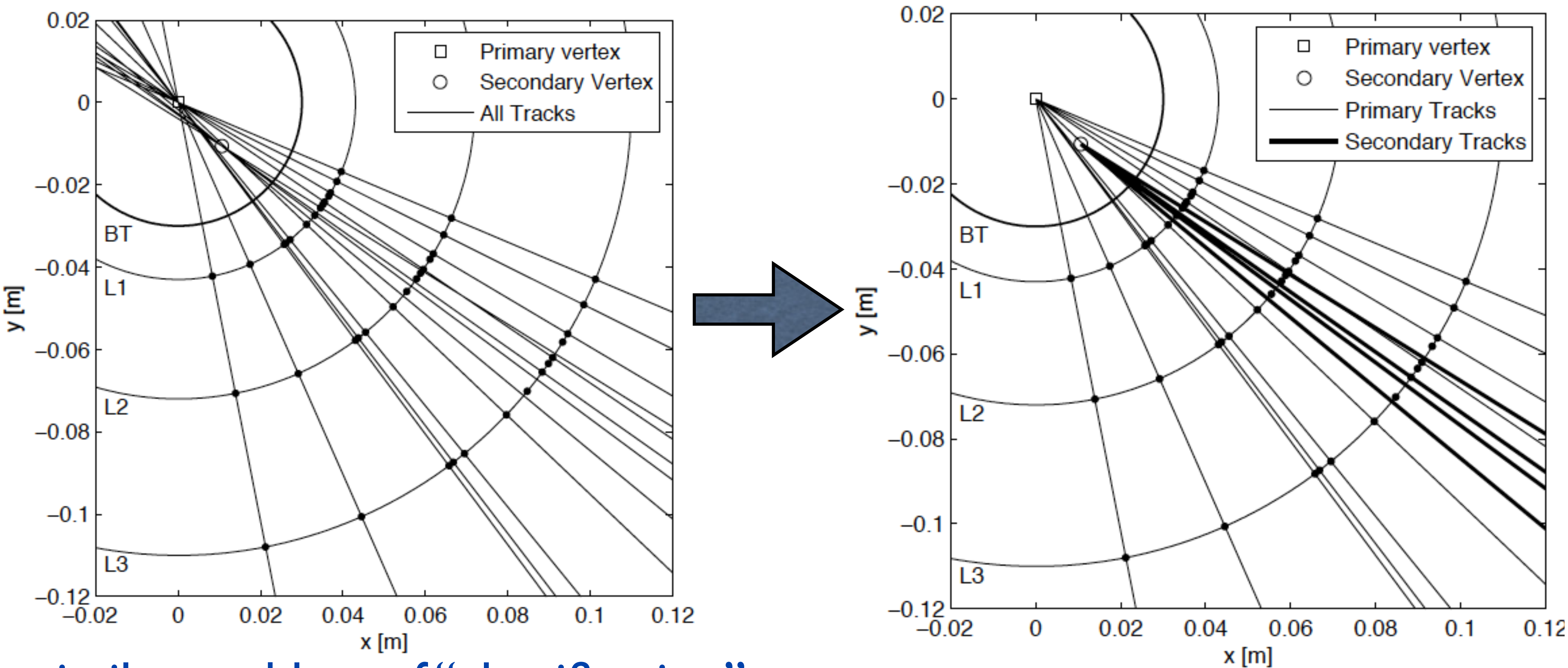
- first two iterations: pixel pairs or pixel triplets, p_t>0.9GeV
- third iteration: pixel triplets, low momentum tracks
- fourth iteration: pixel + strip layers as seeds (find displaced tracks)
- fifth, sixth iterations: strip pairs (for tracks lacking pixel hits)







Vertex Finding



similar problem of "classification" or pattern recognition

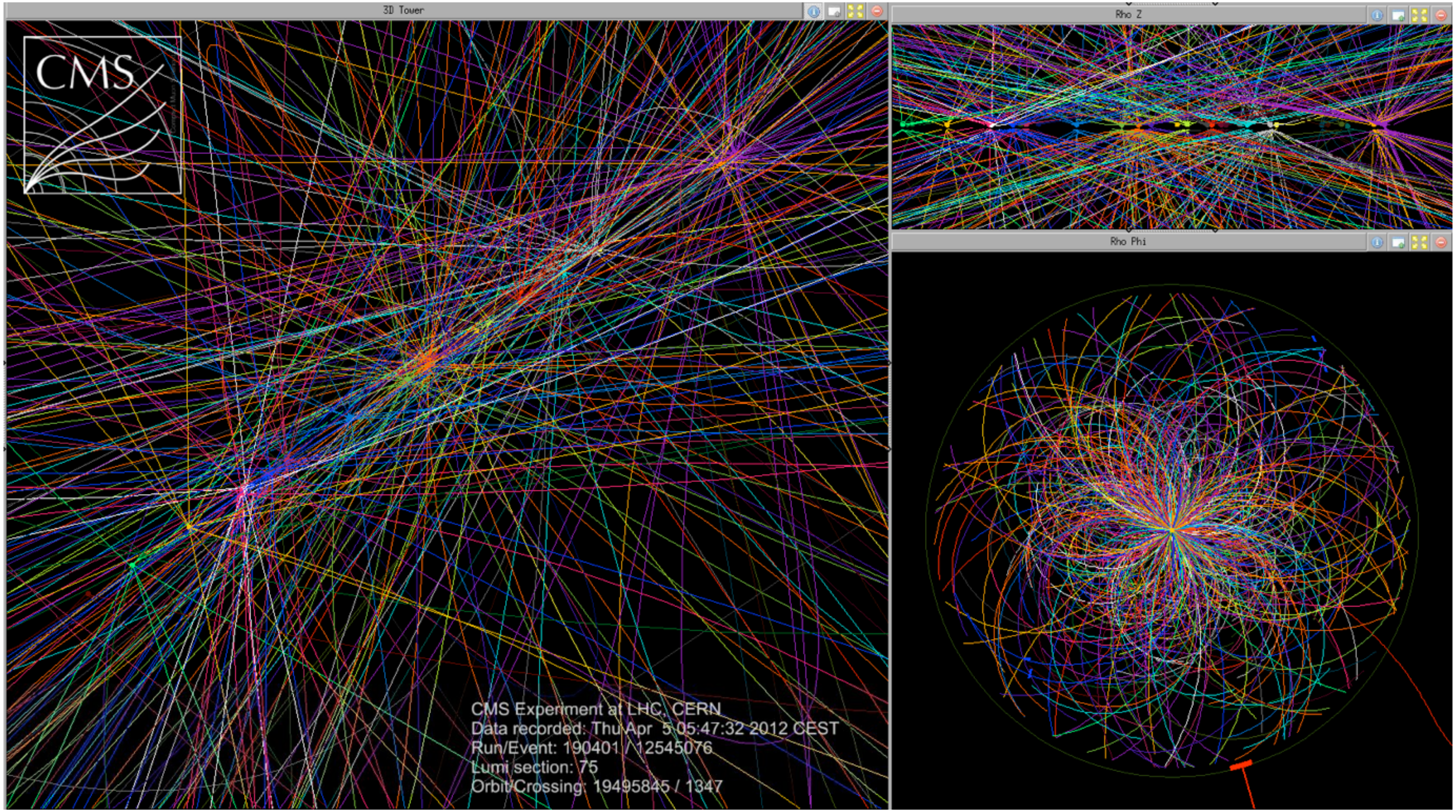
- find points in space where tracks originate (and associated uncertainties)
- example: proton collisions, decays of long-lived particles

two main steps:

- vertex finding
- vertex fitting

Vertex Finding

- need to identify all proton-proton interactions from one bunch crossing
- identify points along the beam line where tracks are **intersecting**
- simplest algorithm: cluster finding



Track Reconstruction Performance

- a helix is fully defined with **5 parameters**. In CMS the parameters are chosen for practical reasons as:
  - transverse momentum: $\mathbf{p_t}$
  - azimuthal angle: $\boldsymbol{\phi}$
  - polar angle: $\cot\theta = \tan\lambda$
  - transverse impact parameter at the point of closest approach to PV: $\mathbf{d_0}$
  - longitudinal impact parameter: $\mathbf{z_0}$