CHARTS , PROJECTION

The original problem of map making is still with us even in the 21st century, how can you represent the curved surface of the earth on a flat piece of paper without distortion?

The answer is IT CANNOT BE DONE!! It's the same as trying to flatten out a Orange peel, it too cannot be done.

Charts which are produced by conic projections are used widely in aviation – mainly because conic projections.

1. preserve true shapes
2. preserve angular relationships (called conformal or orthomorphic)
3. have a reasonably constant scale over the whole chart
4. show great circle as straight lines.

Lets now look at the chart projections and properties that we as pilots are interested in:

ORTHOMORPHISM

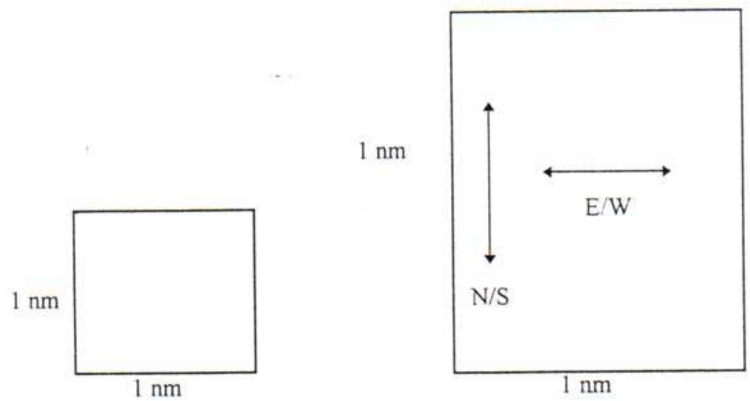
Orthomorphism means true shape. In theory a cartographer starts with a 'reduced earth' which is the earth reduced by the required scale. The 'reduced earth' is a true undistorted representation of the earth. Details, such as Parallels of Latitude, Meridians and topographical features are 'projected' from the reduced earth onto a cylinder (Mercator's Projection), a cone (Lambert's Projection) or a flat sheet of paper (Polar Stereographic Projection). The ideal chart would possess the following features.

Scale, both correct and constant
Bearings correct
Shapes correctly shown
Areas correctly shown
Parallels of Latitude and Meridians will intersect at 90°

Unfortunately to reproduce a spherical surface on a flat sheet of paper is impossible. Distortions will occur. Only one of the above features can be shown correctly.

If shapes and areas are approximately correct to enable map reading, then slight distortions can be tolerated.

Bearings and scale must be correct, but we cannot have both.



1 nm square on reduced earth   1 nm square projected onto a cylinder

The 1 nm square on the reduced earth is correct, the diagonal of a square is 45° and bearings are correct.

The 1 nm square of the reduced earth projected onto a cylinder becomes a rectangle. Bearings are no longer correct. The scale has been expanded in the North/South direction to a greater degree than the East/West case. To overcome this problem the scale expansion North/South is reduced mathematically to equal the scale expansion East/West. The rectangle becomes a square and the diagonal is 45° Bearings are now correct. Meridians and Parallels of Latitude intersect at 90° Scale is expanded, but by the same amount in all directions over short distances. Shapes and areas are approximately correct and the chart is orthomorphic. On the Mercator, Lambert and Polar Stereographic charts the Parallels of Latitude are adjusted in the above manner. Bearings are correct but the scale is variable.

SCALE

Scale is the ratio of a line drawn on a chart to the corresponding distance on the surface of the earth.

STATEMENT IN WORDS 1 inch equals 40 nm

Usually found on radio facility charts. 1 inch on the chart equals 40 nm.

GRADUATED SCALE LINE

0 10 20 30 40 50 60 70 80 90 100
1_____1_____1_____1_____!_____1_____1_____1_____i___________1

REPRESENTATIVE FRACTION

$$\frac{1}{1000\ 000} \quad \text{or } 1/1000000 \quad \text{or} \quad 1:1000000$$

1 Unit on the chart equals 1 000 000 units on the earth
1 Centimetre on the chart equals 1 000 000 centimetres on the earth .
1 Inch on the chart equals 1 000 000 inches on the earth

SCALE FACTOR

Due to the inherent difficulty of presenting a spherical object (the earth) on a flat sheet of paper. there is no such thing as a constant scale chart. Scale expansion or contraction will occur. Usually scale will be correct at a certain Latitude but expands elsewhere. For example :-

Mercator Chart Scale 1:1 000 000 at the Equator Scale factor 1.3054 at 40°N

$$\frac{1}{1\ 000\ 000} \times \text{Scale factor } 1.3054 = \text{Scale at } 40°\text{'N} \quad \frac{1}{766\ 049}$$

Q1 A chart has a scale of 1:2 500 000. How many nautical miles are represented by 4 cm on the chart?

$$\text{Scale} = \frac{\text{CL Chart Length}}{\text{ED Earth Distance}} \qquad \frac{1}{2\ 500\ 000} \qquad \frac{4\text{ cm}}{\text{ED}}$$

ED = 2500000 x 4 cms

$$\frac{2500000 \times 4\text{ cms}}{2.54 \times 12 \times 6080} = 53.96\text{nm}$$

Divide by 2.54 = Inches
Divide by 12 = Feet
Divide bv 6080 = Nautical Mile;

Q2 32 centimetres on a chart represents 468 nm. The scale of the chart is :

$$\text{Scale} = \frac{\text{CL} \quad 32\text{ cms}}{\text{ED } 468\text{ nm} \times 6080 \times 12 \times 2.54} = \frac{1}{2710282}$$

Q3 The scale of a chart is 1: 3 500 000. The length of a line that represents 105 nm is :-

$$\text{Scale} \quad \frac{CL}{ED} \qquad \frac{1}{3500000} \qquad \frac{CL}{105\text{ nm} \times 6080 \times 12 \times 2.54}$$

$$3500000 \times CL = 105\text{ nm} \times 6080 \times 12 \times 2.54$$

$$CL = \frac{105\text{ nm} \times 6080 \times 12 \times 2.54}{3\,500\,000} \qquad = 5.56\text{ cms}$$

Q4 Chart A has a scale of 1:2 500 000
Chart B has a scale of 1:1750 000

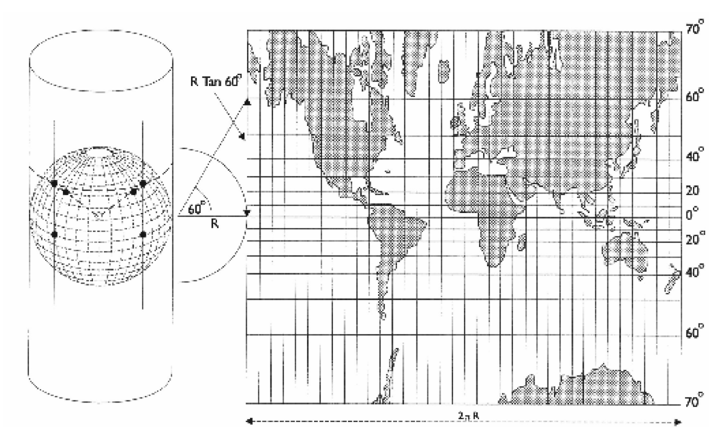
Which chart has the larger scale?

Chart B has the larger scale $\qquad \frac{1}{2} > \frac{1}{4}$

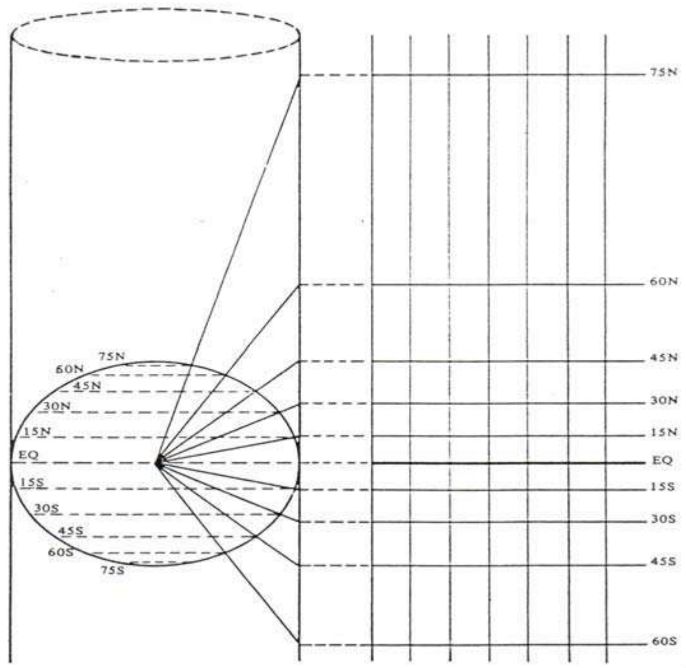
The smaller denominator is the larger scale (half a cake is larger than quarter of a cake)

MERCATOR CHART

Before the advent of Inertial Navigation, and GPS computers aircraft flew constant headings. They flew Rhumb Lines. The Mercator chart was constructed so that Rhumb Lines are straight lines and the headings flown were easily plotted.

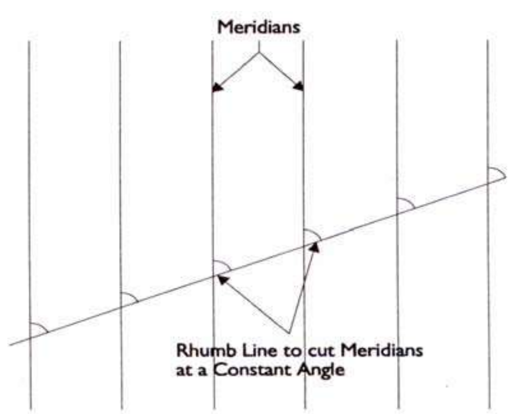


A cylinder is positioned over the reduced earth tangential to the Equator. A light source at the centre of the reduced earth projects details of the reduced earth onto the cylinder and we have a Geometric Cylindrical Projection. After adjusting the Parallels of Latitude so that the scale expansion North/South equals the scale expansion East/West it becomes a Mercator chart.

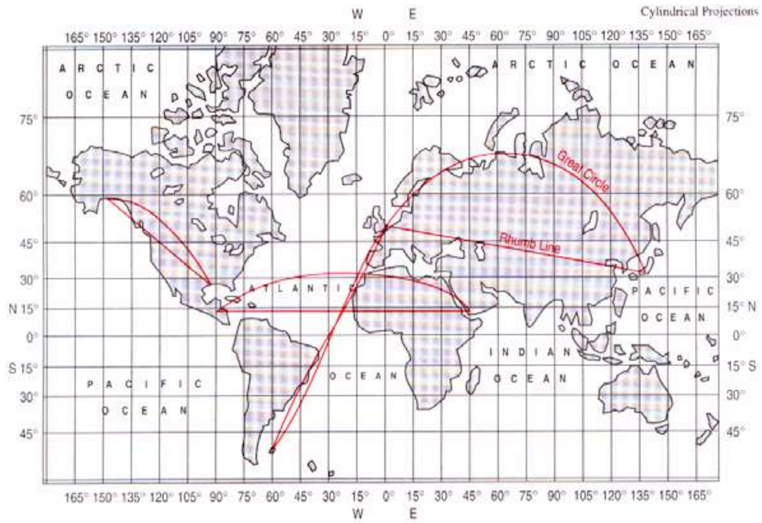


MERCATOR CHART PROPERTIES

POINT OF PROJECTION	Centre of the reduced earth
POINT OF TANGENCY	Equator
PARALLELS OF LATITUDE	Parallel straight lines, unequally spaced
MERIDIANS	Parallel straight lines, equally spaced
CONVERGENCY	Constant Value Zero Correct at the Equator
SCALE	Correct at the Equator Expands as the secant of the Latitude
RHUMB LINES	Straight Lines



GREAT CIRCLES — Complex curves towards the nearer Pole Convex to the Pole, Concave to the Equator



SHAPES & AREAS — Approximately correct, excellent between 12°N and 12°S becoming distorted with increasing Latitude. The chart has a limit of 70°N and 70°S.

CHART FIT — Charts of the same equatorial scale will fit N/S. E/W and diagonally.

USES — Plotting and Met charts topographical maps between 12°N and 12°S

ADVANTAGES — Rhumb Lines are straight lines - plotting easy

DISADVANTAGES — Great Circles (radio bearings) are complex curves great care must be taken measuring distances due to rapidly changing scale.

SCALE

Scale is correct at the Equator and expands North and South as the secant of the Latitude. Every Parallel of Latitude has its own scale.

Equator	1:2 000 000
5°S	1:1 992 389
10°S	1:1 969 615
30°S	1:1 732 051
60°S	1:1 000 000

Great care must be taken when measuring distances on a Mercator chart due to the variable scale. Use the Latitude scale at the mid point between the two positions.

SCALE PROBLEMS

Scale problems are easily solved by use of **ABBA**

SCALE DENOMINATOR A x COS B = SCALE DENOMINATOR B x COS A

Q1 The scale of a Mercator chart is l:2500000 at 15°S. What is the scale at 45°N?

15°S = A
45°N = B

SCALE DENOMINATOR A x COS B = SCALE DENOMINATOR B x COS A

$$2\ 500\ 000 \times \cos 45 = \text{Scale B} \times \cos 15$$

$$\frac{2\ 500\ 000 \times \cos 45}{\cos 15} = 1\ 830\ 127$$

Scale at 45°N 1:1 830 127

Q2 The scale of a Mercator chart is 1:3 500 000 at 10°N. At what Latitude is the scale 1:2 500 000?

10°N = A
Lat X = B

SCALE DENOMINATOR A x COS B **=** SCALE DENOMINATOR B x COS A

$$3\ 500\ 000 \times \cos X = 2\ 500\ 000 \times \cos 10$$

$$\cos X = \frac{2\ 500\ 000 \times \cos 10}{3\ 500\ 000} = 0{:}7034$$

$(0{,}7034)^{\cos^{-1}} = 45°\ 17'49''$ N/S

Q3 The Meridian spacing on a Mercator chart is 2.7 cms. The scale at 30°S is **:-**
If ABBA cannot solve the problem, then revert to:-

$$\text{Scale} = \frac{CL}{ED} = \frac{2.7 \text{ cms}}{1 \text{ Long} \times 60 \cos 30 \times 6080 \times 12 \times 2.54} = 1{:}3566454$$

(Departure)

Q4 The scale at 200 N is 1: 250000 What is the scale at 50 N?

Always work latitude - equator - latitude.

SCALE AT O° - $\frac{1}{250000} \times \frac{\text{COS } 200}{1}$

$$\frac{\text{COS } 20°}{250000}$$

$$= \frac{1}{266044}$$

SCALE AT 5O°N = $\frac{1}{266044} \times \frac{SEC\ 50^{\circ}}{1}$

$$= \frac{1}{266044} \times \frac{1}{COS\ 500}$$

$$= \frac{1}{171010}$$

This particular problem can also be solved in one step:

$$\text{SCALE AT } 50^{\circ}\text{ N} \quad - \quad \frac{\text{SCALE AT } 20^{\circ}\text{ N}}{COS\ 2^{\circ}} \times \frac{COS\ 50^{\circ}}{1}$$

$$\frac{250000}{COS\ 200} \times \frac{COS\ 50^{\circ}}{1}$$

$$= \frac{1}{171010}$$

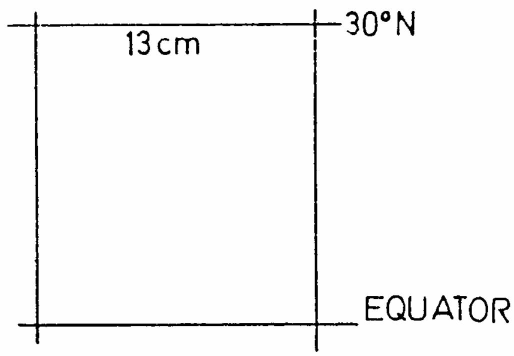
CALCULATING DISTANCE AND dLONG ON A MERCATOR CHART

When calculating distance and dLONG on a Mercator chart, remember that between any two given meridians:

- the chart length remains the same regardless of latitude change.
- the dLONG remains the same regardless of latitude change.
- the scale varies with latitude (use the Mercator scale formula).
- the earth distance varies with latitude (use the departure formula).

EXAMPLE:

Two meridians at latitude 30° N measure 13 cm apart on a Mercator chart. What is the dLONG between these two meridians if the scale is $\frac{1}{250000}$ at 30° N?



At 30° N :

$$SC = \frac{CL}{ED}$$

$$\frac{1}{250000} = \frac{13\ CM}{ED}$$

ED = 13 CM x 250000

ED = 3250000 CM

ED = 17.539 nm

DEP (nm's) = dLONG' x COS LAT

17.539 nm's = dLONG' x COS 30°

$$dLONG' = \frac{17.539\ nm}{COS\ 30°}$$

dLONG' = 20.252'

Because chart length is constant regardless of latitude and dLONG is constant regardless of latitude, this question could also have been calculated at the equator, or any other latitude, provided that the scale is calculated at that latitude.

At the equator:

$$SCALE\ AT\ 0° = \frac{1}{250000} \times \frac{COS\ 30°}{1}$$

$$= \frac{1}{288675}$$

At 0° N:

$$SC = \frac{CL}{ED}$$

$$\frac{1}{288675} = \frac{13\ CM}{ED}$$

ED = 13 CM x 288675

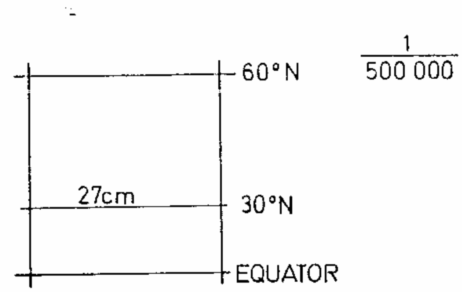
ED = 3752777 CM

ED = 20.252 nm

There is no departure at the equator, **therefore 20.252 nm = 20.252' dLONG**.

EXAMPLE:

Two meridians at 30° N are 27 cm apart. What is the earth distance between these two meridians if the scale at 60° N is $\frac{1}{500000}$?



Again, apply the scale at the latitude where the work is being done:

$$\text{SCALE AT } 30° \text{ N} = \frac{\text{SCALE AT } 60° \text{ N}}{\text{COS } 60°} \times \frac{\text{COS } 30°}{1}$$

$$= \frac{1}{866025}$$

$$\text{SC} = \frac{\text{CL}}{\text{ED}}$$

$$\frac{1}{866025} = \frac{27 \text{ CM}}{\text{ED}}$$

ED = 27 CM x 866025

ED = 23382686 CM

ED = 126 nm

Note that this question must be solved at 30° N. The earth distance at 30° N is required and, unlike dLONG, earth distance is *not* a constant regardless of latitude.

PLOTTING RADIO BEARINGS ON A MERCATOR

When plotting radio bearings, the final goal is always to plot a QTE, and on a Mercator chart, specifically a rhumb line QTE, because this is a straight line.

STEPS TO PLOTTING ON A MERCATOR CHART

a) Always draw a sketch.

b) Orientate the hemisphere (to determine which way the great circle will curve).

c) Take the given information and make it true.

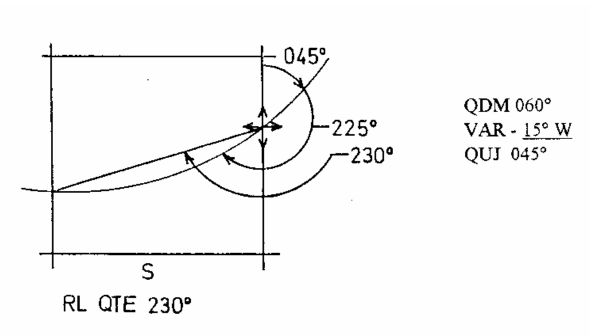
d) Plot this great circle.

e) Measure the bearing where the work was done.

f) Apply the conversion angle to convert the GC to a RL.

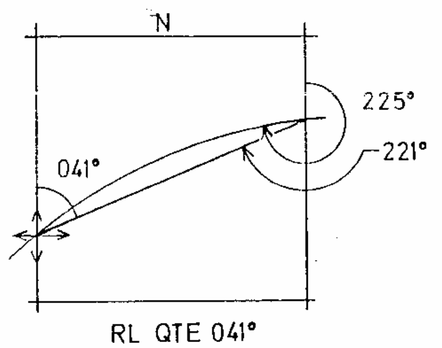
VDF BEARING EXAMPLE

ATC passes an aircraft a QDM of 060°. The variation at the station is 15° W. The variation at the aircraft is 20° W. The deviation is 5° E. The convergency between the aircraft and the station is 10°. Southern hemisphere.



RBI EXAMPLE

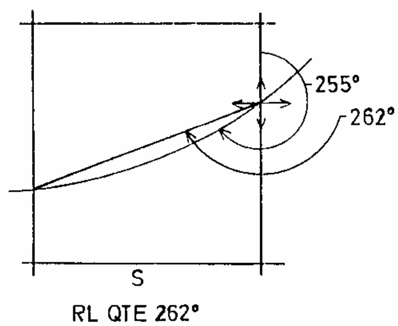
Aircraft compass heading 200°. Relative bearing to an NDB station 40°. Aircraft variation 20°W. Station variation 15° W. Deviation 5° E. Convergency between the aircraft and the station is 8°. Northern hemisphere.



HDG	200°
RB	+ 40°
QDM	240°
DEV	+ 5°E
	245°
VAR	- 20°W
QUJ	225°

VOR NEEDLE ON THE RMI EXAMPLE

The VOR needle at the RMI indicates a radial of 270° (tail of the needle). The variation at the aircraft is 20° W. The variation at the station is 15° W. The deviation is 5° E. The convergency between the aircraft and the station is 14°. Southern hemisphere.



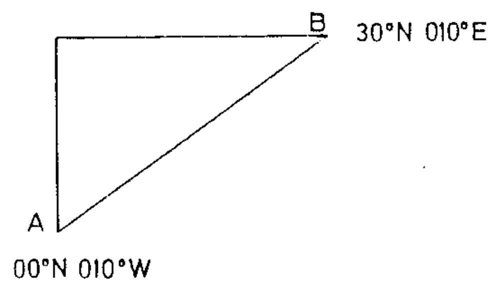
QDR	270°
VAR	- 15°W
QTE	255°

MERIDIONAL PARTS

INTRODUCTION

In essence, meridional parts solves the rhumb line track and distance problem.

Given the following question, from A (00° N 010° W) to B (30° N 010° E), determine the rhumb line track and distance.



Thus far, the suggested method to solve this question has been to convert the dLat into nm's, convert the dLONG into nm's using departure and the cosine of the mid-latitude, and then apply trigonometry to solve the rhumb line track and distance. Unfortunately this method is only accurate for distances up to 600 nm's, mainly due to the fact that the cosine of the mid-latitude is being used to express the dLONG in nm's.

Another possible solution is to physically measure the distance A - B, but due to the continually changing scale on the Mercator chart, this is also not accurate. The solution is to use meridional parts.

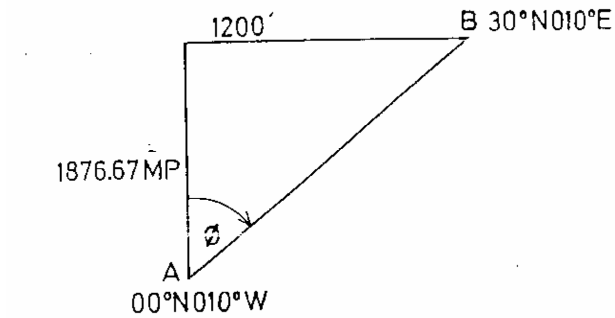
MERIDIONAL PARTS

A meridional part is equal to a minute of longitude. The meridional parts tables indicate “how many times one minute of longitude will fit into a particular change of latitude”.

For example, if you look up 30° (latitude) on the table, you will find 1876.67 meridional parts.

This means that one minute of longitude will fit into the dLat 0° - 30° 1876.67 times.

PRACTICAL APPLICATION



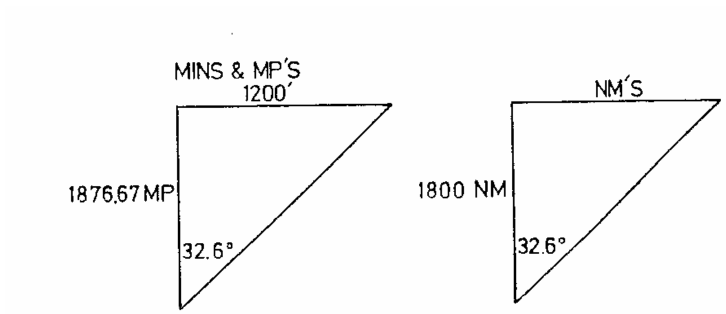
Although expressing the dLONG in nm's by using departure and the COS MID latitude is doubtful in terms of its accuracy, there is absolutely no doubt that the dLONG *is* 1200'. Expressing the dLat in nm's is accurate, but trigonometry can't be applied because the sides of the triangle would have different units.

If we express the LAT in meridional parts, however we can proceed with trigonometry. The sides of the triangle are in the same units because one minute of longitude is equal to one meridional part. (The meridional parts tables do correct for the effect of the earth's compression).

Now, using trigonometry:

$$\text{TAN } \varnothing = \frac{1200}{1876.67 \text{ MP}}$$

$$\varnothing = 32.6° \text{ (track A - B)}$$



Now transfer the track angle to the triangle labelled nm's. Expressing the dLONG in nm's wouldn't be accurate, but expressing the dLat in nm's certainly is.

Now using trigonometry:

$$\text{COS } 32.6° = \frac{1800 \text{ nm}}{x}$$

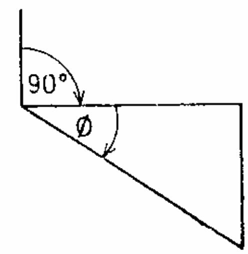
$$x = \frac{1800 \text{ nm}}{\text{COS } 32.6°}$$

$$\mathbf{x = 2137 \text{ nm}} \text{ (rhumb line distance)}$$

RECOMMENDED TECHNIQUES

a) Always draw two sketches, one for MINS LONG/MP's and another for nm's.

b) Sometimes angle ∅ is not the track. In the following sketch, the track is ∅ + 90°.



c) When working from one latitude to another, neither of which is the equator, the latitude side of the triangle will be the difference in meridional parts (DMP), or the sum of meridional parts (SMP) if changing hemispheres.

QUESTIONS

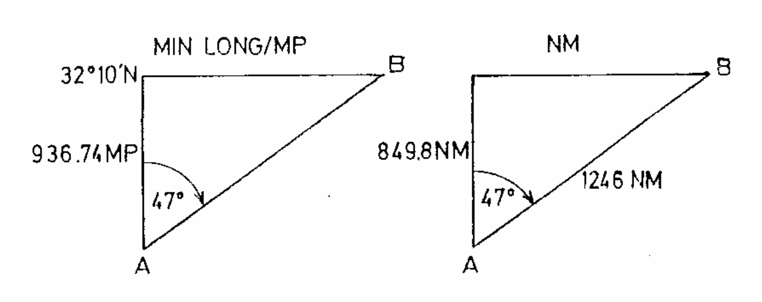
The vast majority of meridional parts questions fall into one of five categories. An example of each follows below, with a heading for each to assist with identification.

a) Determine the rhumb line track and distance flown.

As per previous example.

b) Determine the aircraft's position, given rhumb line track and distance flown.

An aircraft leaves position A (18° N 047° E) on a rhumb line track of 047°. What is its position after flying for 1246 nm's?

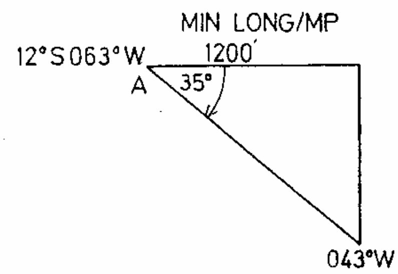


32° 10' N	= 2027.73 MP		COS 47°	=	$\frac{x}{1246 \text{ nm}}$
18° N	= 1090.99 MP		x	=	1246 nm x COS 47
dLAT	= 936.74 DMP		**x**	**=**	**849.8 nm**

TAN 47°	= $\frac{x}{936.74}$		LAT B	= $\frac{849.8}{60}$
x	= 936.74 x TAN 47°			= 14° 10' + 18° N
x	= 1004.53 MIN LONG			**= 32° 10' N**
x	= 16° 45'			
LONG B	= 16° 45' + 047° E			
LONG B	**= 063° 45'E**			

c) At which latitude will an aircraft cross a given meridian?

An aircraft departs A (12° S 063° W) on a track of 125°. At which latitude will the aircraft cross the meridian 043° W?

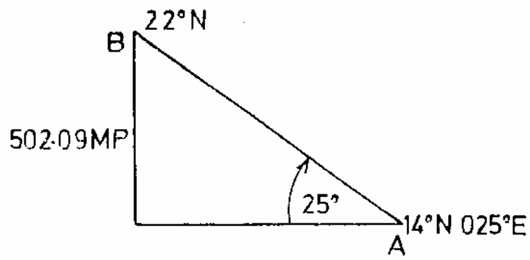


dLONG = 063° W - 043° W

dLONG	=	063° W - 043° W
	=	20° W
	=	**1200'**
TAN 35°	=	$\frac{x}{1200'}$
x	=	1200' x TAN 35°
x	**=**	**840.25 MINS or MP**/s
12° S	=	720.46 MP
DMP	=	<u>840.25 MP</u>
NEW LAT	=	1560.71 MP
NEW LAT	**=**	**25° 19' S**

d) At which meridian will an aircraft cross a given latitude?

An aircraft departs position A (14° N 025° E) on a track of 295°. At which meridian will the aircraft cross the latitude 22° N?



22° N	=	1344.92 MP
14° N	=	842.83 MP
		502.09 DMP

$$\text{TAN } 25° = \frac{502.09}{x}$$

$$x = \frac{502.09}{\text{TAN } 25°}$$

x = 1076.74 MP's or MINS LONG

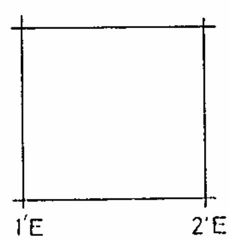
dLONG = 1076.74'

= 17° 57'

NEW LONG = 25° - 17° 57'

NEW LONG = 007° 03'E

e) Meridional parts scale



As can be seen from any Mercator chart, the chart length of one minute of longitude has a constant value chart length, regardless of latitude. The exact value of the chart length of course depends on the scale of the chart at that point. Because the chart length between any two meridians is constant throughout the chart, the scale at any latitude may be used.

If one minute of longitude is equal to one meridional part, then it stands to reason that 1 MP must also have a constant value chart length throughout the chart.

EXAMPLE:

A mercator chart has a scale of $\frac{1}{1000000}$ at the equator. What is the chart length of 1 MP in cm's?

$$SC = \frac{\text{CL (1 MP / 1 MIN LONG)}}{\text{ED}}$$

$$\frac{1}{1000000} = \frac{\text{CL}}{\text{1NM}}$$

$$\frac{1}{1000000} = \frac{\text{CL}}{\text{185300 CM}}$$

$$CL = \frac{185300}{1000000}$$

CL = 0.1853 CM (CL of 1 MP/1 MIN LONG)

As previously stated, because the CL of 1 MIN LONG is constant throughout the chart, the chart length may be calculated at any latitude, provided the scale at that latitude is used.

Calculation of the same question, but at 60°N.

$$\text{SCALE AT } 60° \text{ N} = \frac{1}{1000000} \times \frac{1}{\text{COS } 60°}$$

$$= \frac{1}{500000}$$

Calculate the earth distance of 1 MIN LONG at 60° N.

DEP (nm's) = dLONG' x COS LAT

= 1' x COS 60°

= 0.5 nm's

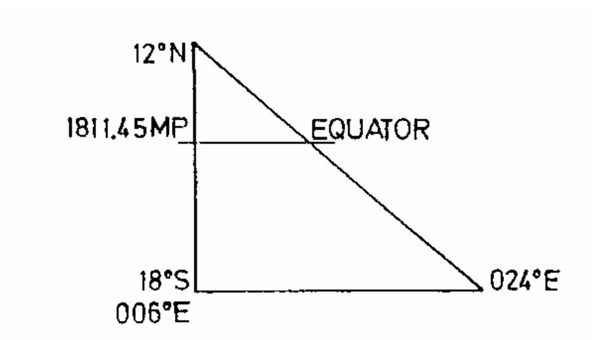
$$SC = \frac{CL\ (1\ MP / 1\ MIN\ LONG)}{ED}$$

$$\frac{1}{500000} = \frac{CL}{0.5\ NM}$$

$$\frac{1}{500000} = \frac{CL}{92650\ CM}$$

$$CL = \frac{92650}{500000}$$

CL = 0.1853 CM (CL of 1 MP/1 MIN LONG)



What is the CL in CM's between A (12° N 006° E) and B (18° S 024° E) if the scale at 52° N is $\frac{1}{400000}$?

Determine the CL of 1 MP/1 MIN LONG

$$SC = \frac{CL}{ED}$$

$$\frac{1}{400000} = \frac{CL}{1' \times COS\ 52° \times 185300}$$

$$\frac{1}{400000} = \frac{CL}{114082\ CM}$$

$$CL = \frac{114082}{400000}$$

CL = 0.2852 CM (CL of 1 MP/1 MIN LONG)

12° N = 720.46 MP

18° S = <u>1090.99 MP</u>

SMP = 1811.45 MP

dLONG = $024° E - 006°E$

dLONG = $18° E$

dLONG = $1080'$

Using Pythagoras:

$$x^2 = 1811.45^2 + 1080^2$$

$$x^2 = 4447751$$

$$x = \sqrt{4447751}$$

$$x = 2109 \text{ MP at } 0.2952 \text{ cm per MP}$$

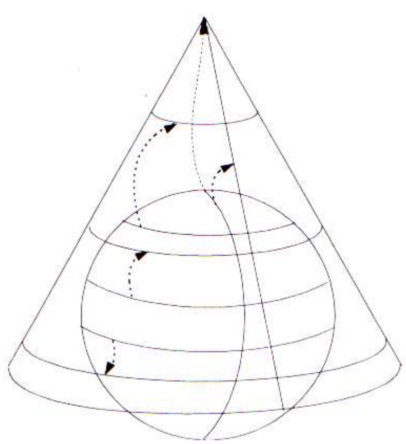
$$x = 601.5 \text{ cm}$$

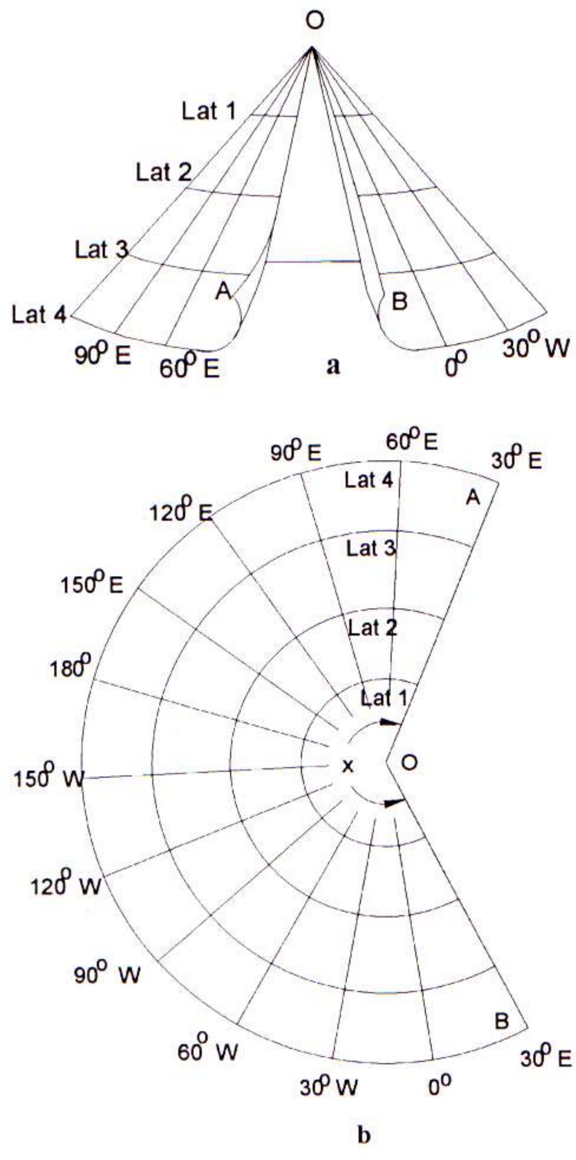
LAMBERT CONFORMAL CONIC CHART

The Lambert's chart was developed from the Simple Conic chart.

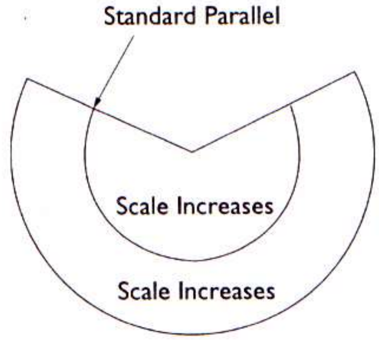
SIMPLE CONIC

A cone is placed over a reduced earth so it is tangential to a selected parallel of latitude. The apex of the cone is above the pole. A light source at the centre of the reduced earth projects details onto the cone. The cone is opened to give a simple conic projection.

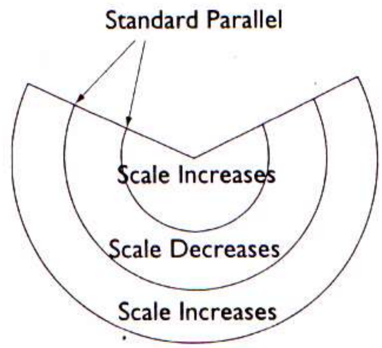




The scale is correct at the parallel of tangency (45N) and expands north and south of 45N. Due to the scale expansion the chart is not suitable for navigation.



a One Standard Parallel



b Two Standard Parallels

The Meridians are straight lines converging on the nearer pole and the value of convergence is constant throughout the chart.

Parallels of Latitude are arcs of circles radius the Pole.

SIMPLE CONIC CONVERGENCE

When the cone is opened, 360° of Longitude is represented by the angular extent of the chart which is 254.5584°. The angular extent of the chart is controlled by the latitude chosen to be the parallel of tangency.

$$\frac{\text{Angular extent of the chart } 254.5584°}{\text{Change of Longitude } 360°} = 0.7071° \text{ Constant of the Cone or 'n' factor}$$

Two Meridians 1° apart have a convergency 0.7071° is called the:

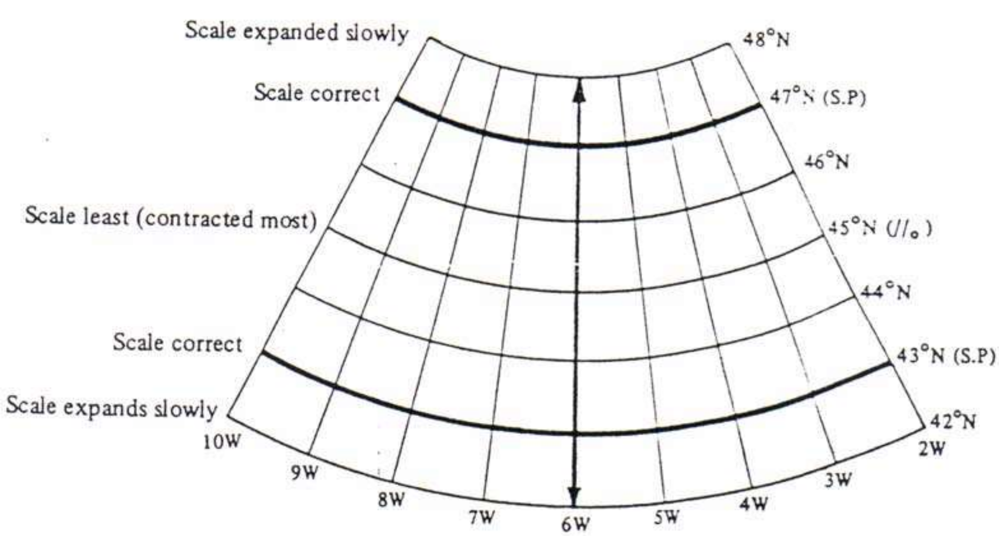
CHART CONVERGENCY FACTOR (CCF)

Parallel of Tangency 45° Sine 45° = 0.7071 = CCF = Constant of the Cone = 'n' factor

LAMBERT CONFORMAL CONIC CHART

The Lambert's chart is based on the simple conic and is produced mathematically from it. Firstly, the scale is reduced throughout the chart. Since scale on the simple conic is correct only on the parallel of tangency and expands either side, the reduction will give two Standard Parallels (SP) on which scale is correct, one on either side of the simple conic parallel of tangency, which is, renamed the **Parallel of Origin.** Further mathematical modification is applied by adjusting the radius of the parallels of latitude to produce an orthomorphic projection.

The above can be shown be lowering the simple conic cone so that it cuts the earth at the two Standard Parallels instead of the original parallel of tangency of the simple conic.



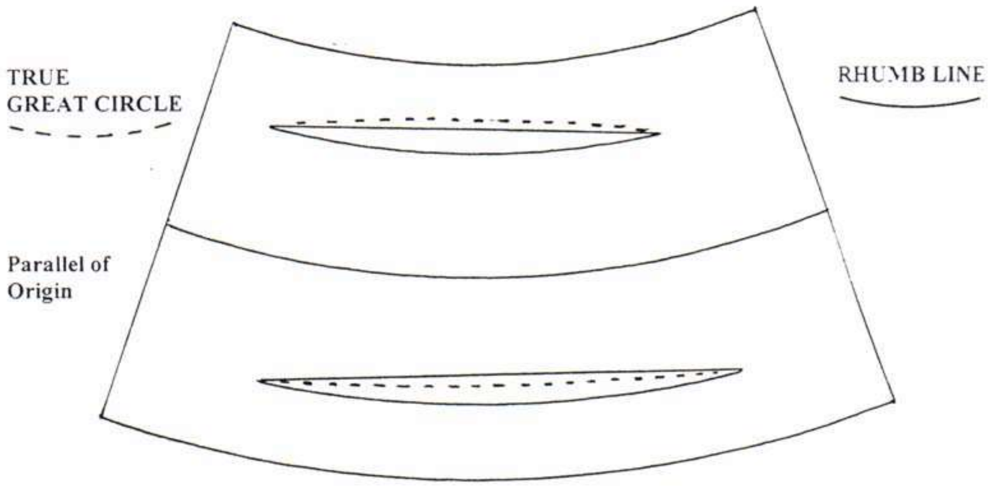
LAMBERT'S CHART PROPERTIES

PARALLELS OF LATITUDE	Arcs of circles, radius the Pole, unequally spaced.
MERIDIANS	Straight lines converging towards the nearer Pole
SCALE	Correct at the two Standard Parallels Expands outside the Standard Parallels Contracts between the Standard Parallels

Scale variation throughout 1:1 000 000 and 1:500 000 charts is negligible and can be considered constant if the band of Latitude projected is small and the Standard Parallels are positioned according to the one sixth rule. That is one sixth of that Latitude band from the top and bottom of the chart. Charts of the North Atlantic with a scale of 1:5 600 000 have a marked scale variation and care must be taken when measuring distances.

RHUMB LINES	Curves concave to the Pole and convex to the Equator
GREAT CIRCLES	A straight line joining two positions on the Parallel of origin, Curves slightly concave to the Parallel of Origin.

For all practical purposes Great Circles can be considered to be a straight line



CONVERGENCY	Constant throughout the chart Correct at the Parallel of Origin
Chart Convergency	Ch. Long x sin Parallel of Origin
Chart Convergency	Ch. Long x CCF (Chart Convergence Factor)
Chart Convergence	CH. Long x 'n'
Chart Convergence	Ch. Long x Constant of the Cone
SHAPES and AREAS	Slight distortion
CHART FIT	Charts of the same scale and Standard Parallels will fit N/S and E/W. Charts with different SP will not fit.

THE ADVANTAGES OF THE CHART

a) Constant scale.

b) Radio bearings are great circles and on this chart, great circles are straight lines, which means that radio bearings can be easily plotted.

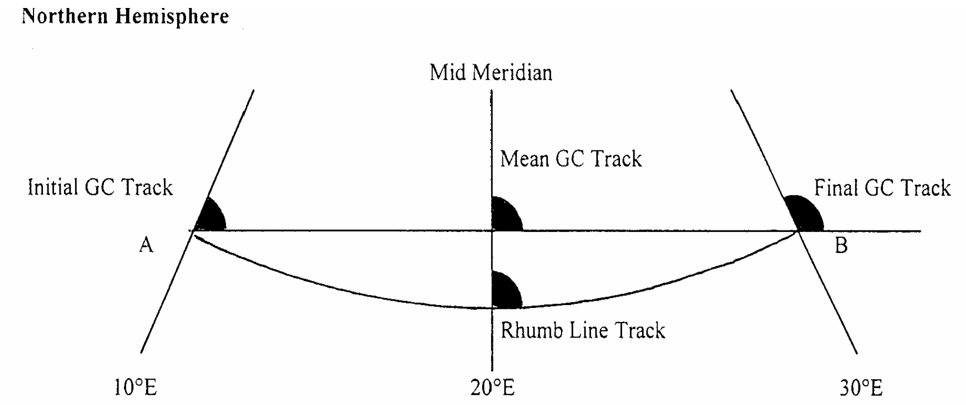
THE DISADVANTAGES OF THE CHART

a) The grid is not rectangular.

b) Light aircraft generally fly rhumb line tracks, but the rhumb line is a curved line on this chart and therefore cannot be accurately plotted.

LAMBERT'S CHART - TRACKS

For all practical purposes the Great Circle is a straight line.



The Rhumb Line track is parallel to the mean Great Circle track at the Mid Meridian between two positions

The difference between the Great Circle and the Rhumb Line is **Chart Conversion Angle (CCA**)

The difference between the Initial Great Circle track and the Final Great Circle track is **Chart Convergency (CC)**

NB: For examination purposes

Unless otherwise stated in a question, the Great Circle is taken to be the straight line and Chart Convergence (CC) is used.

Where a question asks for **'the most accurate value of the Great Circle' or 'the true Great Circle' then Earth Convergency (EC) is used.**

The Parallel of Origin of a Lamberts chart is mid way between the two Standard Parallels

If the Standard Parallels (SP) are 20°S and 40°S - Then the Parallel of Origin (// 0) is 30°S

If one SP is 20°S and the ‖O is 30°S - Then the other SP is 40°S

Chart Convergency (CC) = Change of Longitude x sine Parallel of Origin

Chart Convergency (CC) = Change of Longitude x Chart Convergency Factor

Sine Parallel of Origin = Chart Convergency Factor (CCF)

If a statement regarding convergency is given :-

(e.g. a Lamberts chart has a chart convergency of 5° between the meridians of 10°E and 20°E) then the Parallel of Origin can be calculated (CC 5° = ch. long 10° x sin 30°) and the CCF = 0.5°. As convergency is proportional to the CCF, convergency between any two meridians is easily found.

Q1 On a Northern Hemisphere Lambert's chart (SP 20°N & 40°N) the Initial Great Circle track from A (10°E) to B (42°E) is 065°. The Great Circle track at B is :-

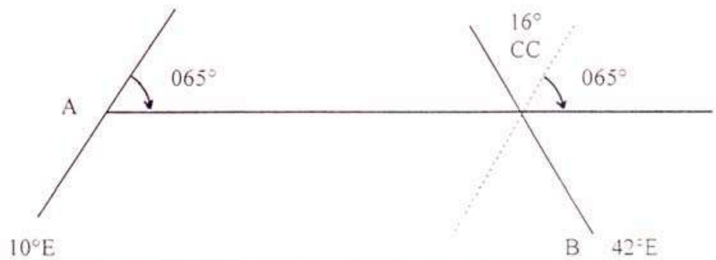
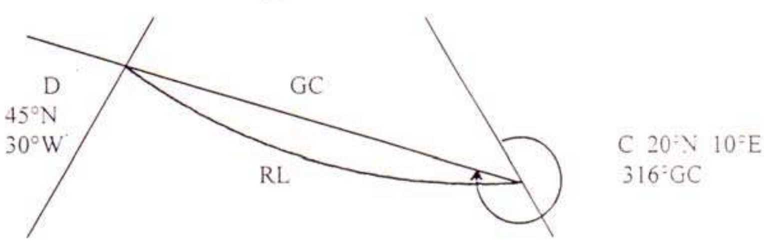


Chart Convergency CC = Ch. Long x sin ∥ O
= 32° x sin 30°
= 16° + 065° = 081° GC track at B

The Meridian at A is paralleled through B to give equal angles of 065° and CC is added to give the GC track at B.

Q2 The Chart Convergency Factor of a Lambert's chart is 0.5
The Great Circle track from C (20°N 10°E) to D (45°N 30°W) measures 316° at C.
The Rhumb Line track from C to D is :-



CC =	Ch. Long x CCF		Track C to D	316°GC
CC =	40° x 0.5		CCA	10°
CC =	20°		Track C to D	306°RL
CCA =	10°			

Q3 The CCF of a Lambert's chart is 0.5
If one Standard Parallel (SP) is 25°S then the Latitude of the other Standard Parallel is :-

The Parallel of Origin (∥O) is midway between the two Standard Parallels

CCF 0.5 = sin∥O = 30°S

SP25°S Parallel of Origin 30°S Other SP35°S

LAMBERT'S CHART PLOTTING RADIO BEARINGS

Radio bearings are Great Circles. Straight Lines on a Lambert's chart are Great Circles and plotting radio bearings is simple.

The final goal when plotting radio bearings on the Lambert's chart is to plot a QTE, and specifically the great circle QTE, because this is a straight line.

STEPS TO PLOTTING ON A LAMBERT'S CHART

a) Always draw a sketch.

b) Orientate the hemisphere correctly.

c) Take the given information and make it true.

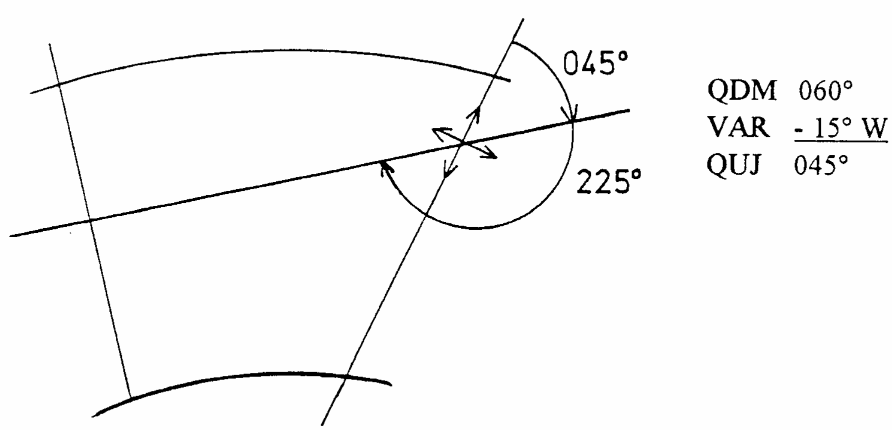
d) Plot this great circle.

e) Measure this bearing where the work was done.

f) Apply convergency if required to obtain the great circle QTE.

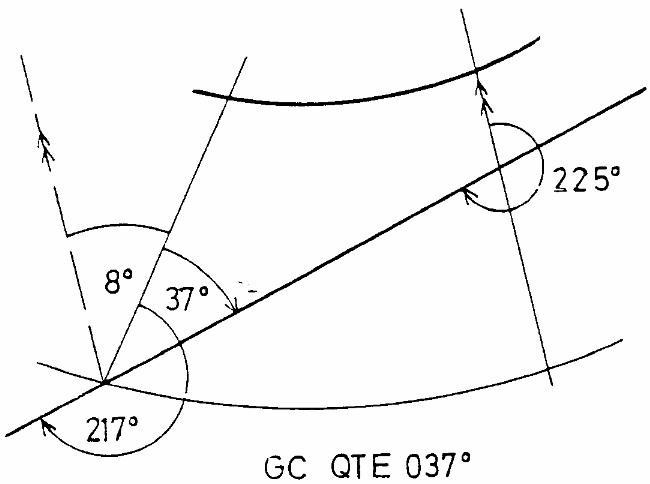
VDF BEARING EXAMPLE

ATC passes an aircraft a QDM of 060°. The variation at the station is 15° W. The variation at the aircraft is 20° W. The deviation is 5° E. The convergency between the aircraft and the station is 10°. Southern hemisphere.



RBI EXAMPLE

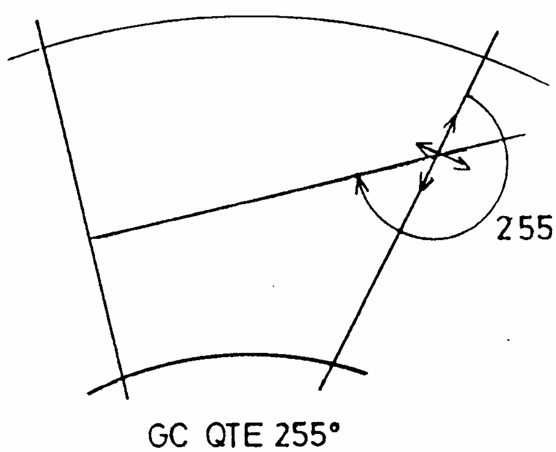
Aircraft compass heading 200°, relative bearing to an NDB station 040°. Aircraft variation 20° W. Station variation 15° W. Deviation 5° E. Convergency between the aircraft and the station is 8°. Northern hemisphere.



HDG	200°
RB	40°
QDM	240°
DEV	+ 5° E
	245°
VAR	- 20° W
QUJ	225°

THE VOR NEEDLE ON THE RMI EXAMPLE

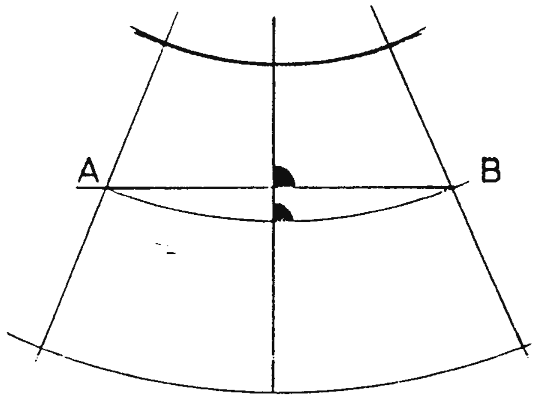
The VOR needle on the RMI indicates a radial of 270° (tail of the needle). The variation at the aircraft is 20° W. The variation at the station is 15° W. The deviation is 5° E. The convergency between the aircraft and the station is 14°. Southern hemisphere.



QDR	270°
VAR	- 15° W
QTE	255°

PLOTTING RHUMB LINE TRACKS

On the Lambert's chart, a rhumb line is a curved line and cannot actually be plotted. Join points A and B on the chart with a straight line (great circle). Measure the track of the great circle at the mid-meridian. If the aircraft departs from position A and maintains this track, it will fly the equivalent rhumb line track. At the mid-meridian, the great circle and the rhumb line are parallel.

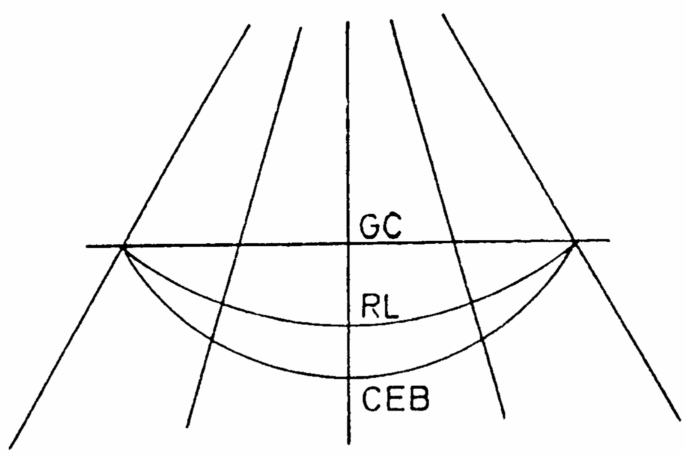


MEASURING RHUMB LINE DISTANCES

The rhumb line (curved) is never actually plotted, thus its distance cannot be measured. Instead, measure the great circle (straight line) distance to obtain the equivalent rhumb line distance. To obtain the greatest degree of accuracy, measure this distance:

a) Along a meridian scale.

b) Across the mid latitude of the track



SCALE PROBLEMS

Lambert's scale 1:2 500 000, SP20° Sand40°S.

The scale is correct at the two Standard Parallels

Scale 20°S = Scale at 40°S

Q1. A Lambert's chart has Standard Parallels of 30°N and 50° N. The Rhumb Line distance from A (50°N 30°E)to B (50°N 10°E) is 13.75 inches.
The scale at 30°N is :-

$$\text{Scale} = \frac{\text{CL}}{\text{ED}} = \frac{13.75 \text{ inches}}{20° \text{ Ch. Long} \times 60 \times \cos 50° \times 6080 \times 12} = \frac{1}{4\ 092\ 898}$$

(Departure in nm)

Q2 On a Lambert's chart the Standard Parallel of 35°S measures 58.4 cms. The other Standard Parallel measures 43.9 cms.

The Latitude of the second Standard Parallel is :-

$$\text{Scale at } 35°\text{S} = \frac{\text{CL} \quad 58.4 \text{ cms}}{\text{ED Ch. Long} \times \cos 35} \qquad \text{Scale at } 2^{nd} \text{ SP} = \frac{\text{CL} \quad 43.9 \text{ cms}}{\text{ED Ch. Long} \times \cos \text{Lat}}$$

The scales are equal.

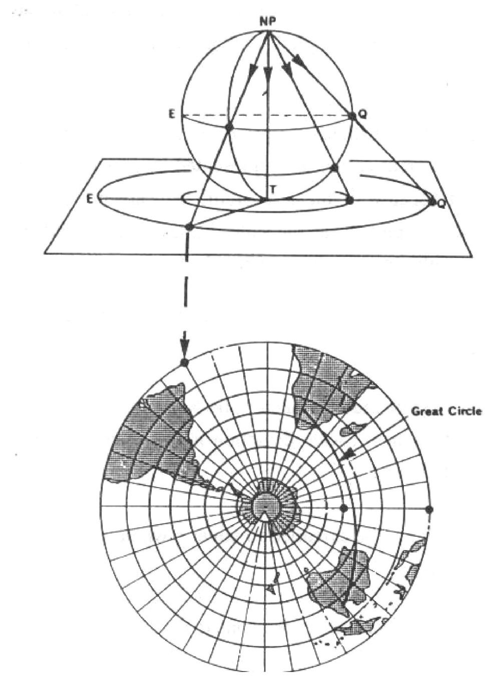
As CH. Long is the same in both equations it disappears

$$\frac{58.4 \text{ cms}}{\cos 35} = \frac{43.9 \text{ cms}}{\cos \text{Lat}} \quad \cos\text{Lat} = 0.6158 \quad = \cos 52°\text{S}$$

THE POLAR STEREOGRAPHIC CHART

THE CONSTRUCTION OF THE CHART

A model earth is constructed in glass with a light source at one of the poles. A flat piece of paper is then placed on top of the pole to be constructed, and opposite to the light source. When the light is switched on, the data is projected onto the flat piece of paper. When the piece of paper is removed, a polar stereographic chart has been created.



SOUTH POLAR STEREOGRAPHIC CHART

THE PROPERTIES OF THE CHART

THE MERIDIANS

The meridians are straight lines radiating from the pole.

THE PARALLELS

The parallels are concentric circles. The spacing between the parallels increases away from the pole.

The formula for determining the chart length of the radius from the pole to a particular parallel of latitude is:

$$r = 2 R \tan \tfrac{1}{2} \text{co-lat}$$

Where R is the radius of the model earth and co-lat is the difference between 90º and the latitude in question.

THE POINT OF TANGENCY

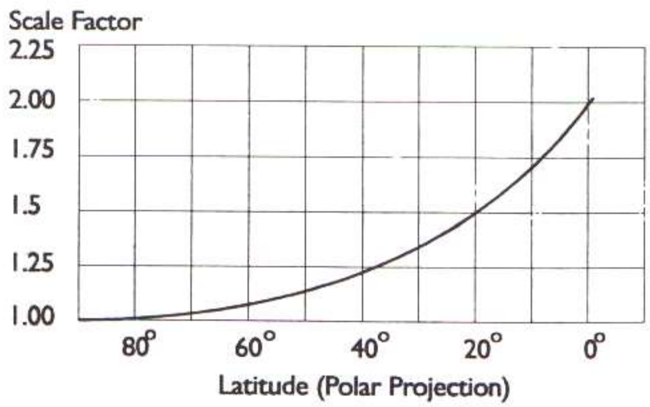
The point of tangency is the north or south pole.

THE POINT OF PROJECTION

The point of projection is a light source at the opposite pole.

SCALE

The scale is correct at the point of tangency (the pole). Elsewhere on the chart, the scale expands with movement away from the pole or contracts with movement towards the pole.



The formula for determining scale expansion away from the pole is:

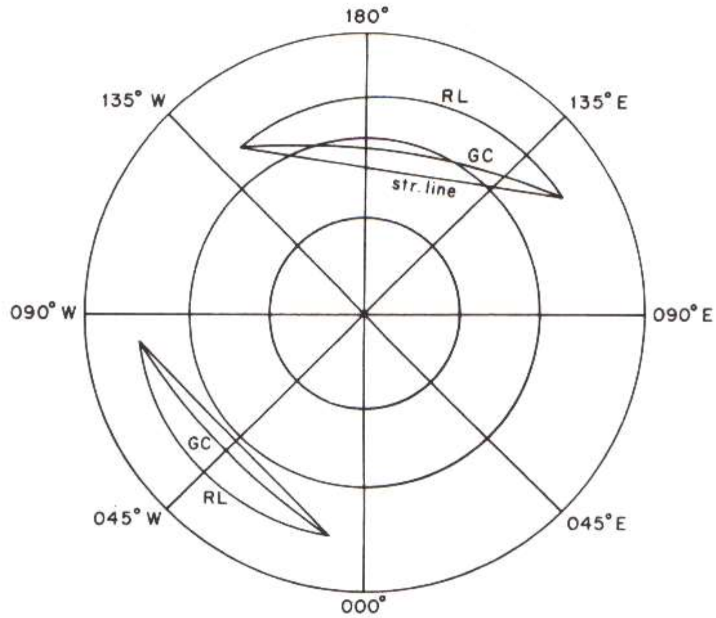
$$\frac{1}{\text{SCALE AT LATITUDE}} = \frac{1}{\text{SCALE AT POLE}} \times \frac{\text{SEC}^2\ \tfrac{1}{2}\ \text{CO-LAT}}{1}$$

RHUMB LINES

Rhumb lines curve towards the equator and cut successive meridians at the same angle.

GREAT CIRCLES

Great circles may be considered to be straight lines and will cut successive meridians at different angles. (In truth the great circle is slightly concave to the pole.)



ORTHOMORPHIC

The chart is orthomorphic because

a) Meridians and parallels cut at 90º.

b) The scale expands at the same rate in all directions over short distances.

CONVERGENCY

On this chart, convergency is correct at the pole.

CONVERGENCYº = dLONGº

However, convergency is constant throughout the chart because the meridians are straight lines. Therefore convergency all over the chart is simply calculated with the formula :

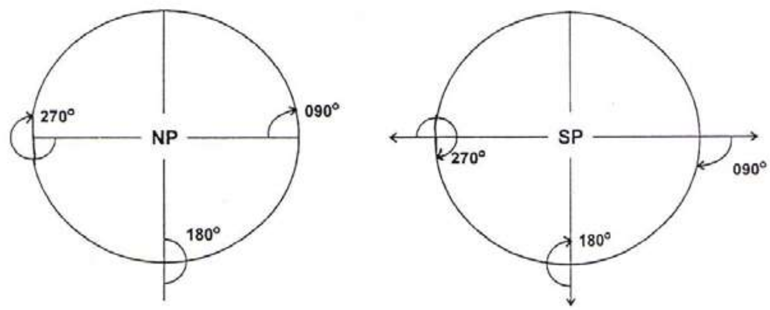
CONVERGENCYº = dLONGº

SHAPES AND AREAS

The nearer the pole, the more accurate the representation of shapes and areas.

Measuring Directions On The Charts

Remember that direction true is always measured clockwise and relative to true north. On a North polar stereographic chart, the North pole is at the centre of the chart. On a South polar stereographic chart, the South pole is at the centre of the chart and true North is 180° away from true South. Remember also that a parallel of latitude runs E/W.



Plotting Radio Bearings On The Polar Stereographic Chart

The final goal when plotting radio bearings on the polar stereographic chart is to plot a QTE, because this is a straight line.

Steps To Plotting On A Polar Stereographic Chart

a) Draw a sketch.

b) Orientate the hemisphere.

c) Take what is given and make it true.

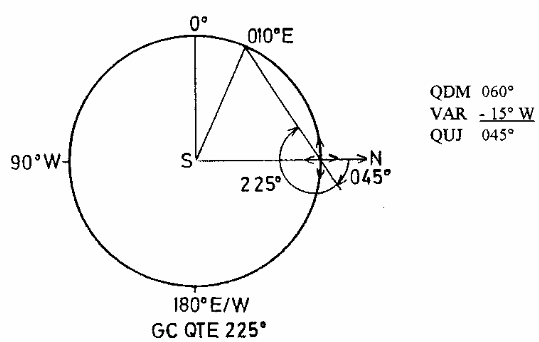
d) Plot this great circle.

e) Measure this bearing where the work was done.

f) Apply convergency if required to obtain the great circle QTE.

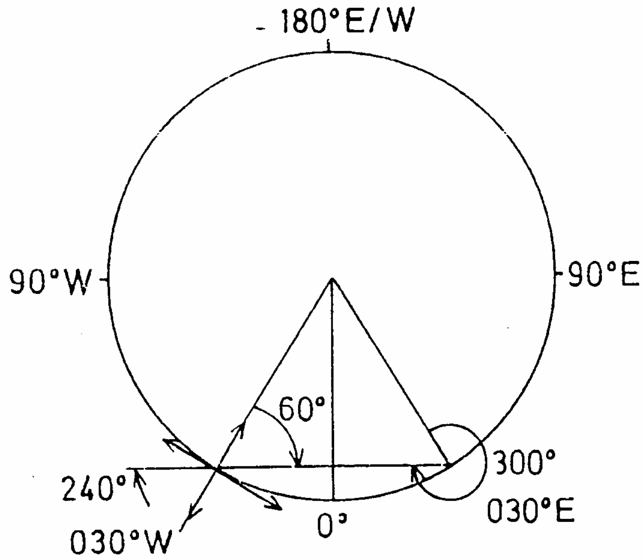
VDF Bearing EXAMPLE

ATC passes the aircraft a QDM of 060°. The variation at the station is 15° W. The variation at the aircraft is 20° W. The deviation is 5° E. The station is at position 70° S 090° E. The aircraft is at position 70°S 010° E. Southern hemisphere. The QTE to plot is?



RBI EXAMPLE

Aircraft compass heading 270°. Relative bearing to an NDB station 040°. Station variation 15° W. Aircraft variation 20° W. Deviation 10° E. The station is at 70° N 030° W. The aircraft is at 70° N 030° E. Northern hemisphere.

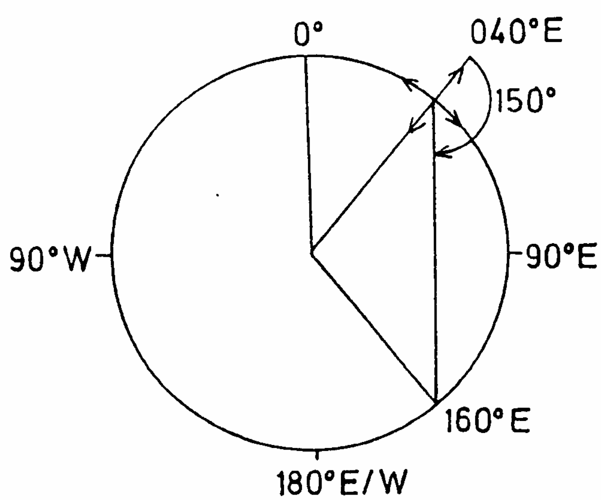


HDG	270°
RB	040°
QDM	310°
DEV	+ 10° E
	320°
VAR	- 20° W
QUJ	300°

GC QTE 060°

VOR Needle On The RMI EXAMPLE

The VOR needle on the RMI indicates a radial of 165°. The variation at the station is 15° W. The variation at the aircraft is 20° W. The deviation is 12° E. The station is at position 70° S 040° E. The aircraft is at position 70° S 160° E. Southern hemisphere.



QDR	165°
VAR	- 15° W
QTE	150°

GC QTE 150°

Determining The Radius Of A Parallel Of Latitude

The scale of the model earth is $\frac{1}{8000000}$. The radius of the real earth is 3438 nm. On a polar stereographic chart of the north pole, calculate the chart length between 70°N and 60° N in cm's.

a. Calculate the radius of the model earth in cm's.

$$SC = \frac{CL}{ED}$$

$$\frac{1}{8000000} = \frac{CL}{3438 \times 185300}$$

$$CL = \frac{637061400}{8000000}$$

CL = 79.6 cm (radius of the model earth)

b. Calculate the radius from 90° N to 60° N.

r = 2 R tan ½ co-lat

= 2 x 79.6 x tan ½ (90° - 60°)

= 2 x 79.6 x tan 15°

= 42.7 cm

c. Calculate the radius from 90° N to 70° N.

r = 2 R tan ½ co-lat

= 2 x 79.6 x tan 10°

= 28.1 cm

The chart length between 70° N and 60° N.

42.7 cm - 28.1 cm = 14.6 cm

Determining Scale On The Polar Stereographic Chart

On a polar stereographic chart of the north pole, the scale at 60° N is $\frac{1}{1000000}$. What is the scale at 70° N.

Take the scale from 60° N to 90° N.

$$\frac{1}{\text{SCALE AT } 90^\circ \text{ N}} = \frac{1}{1000000} \times \frac{\text{COS}^2 \ \frac{1}{2} \text{ co - lat}}{1}$$

$$= \frac{\text{COS}^2 \ \frac{1}{2} \text{ co - lat}}{1000000}$$

$$= \frac{\text{COS}^2 \ 15^\circ}{1000000}$$

$$= \frac{1}{1071797}$$

Take the scale from 90° N to 70° N

$$\frac{1}{\text{SCALE AT } 70^\circ \text{ N}} = \frac{1}{1071797} \times \frac{\text{SEC}^2 \ \frac{1}{2} \text{ co - lat}}{1}$$

$$= \frac{1}{1071797} \times \frac{1}{\text{COS}^2 \ \frac{1}{2} \text{ co - lat}}$$

$$= \frac{1}{1071797} \times \frac{1}{\text{COS}^2 \ 10^\circ}$$

$$= \frac{1}{1039478}$$

By ABBA :

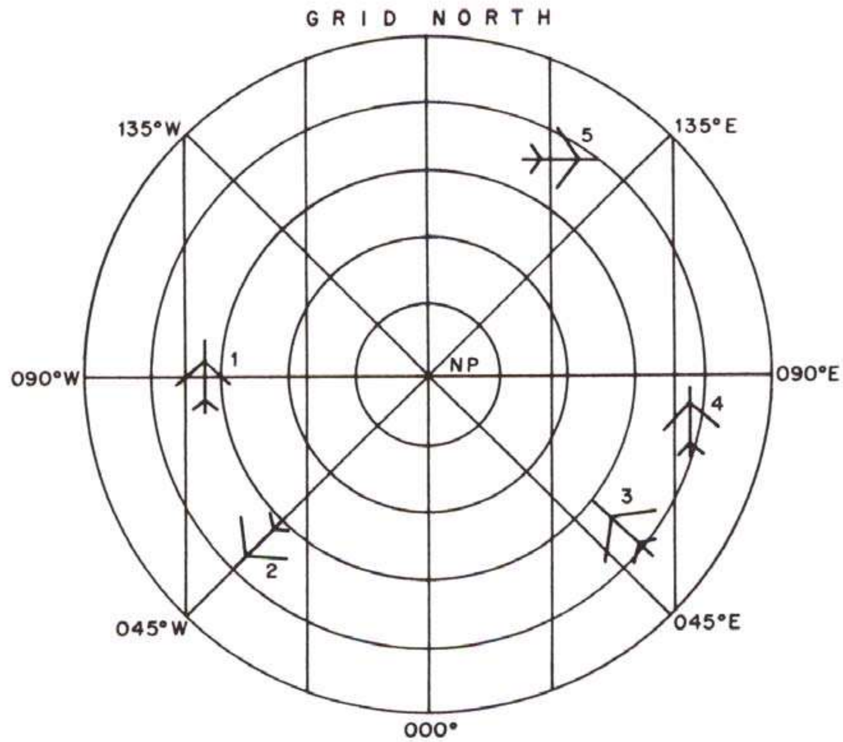
Scale A x {cos (½co-lat)B}² = Scale B x {cos (½co-lat)A}²

Gee that was a short and noisy landing

GRID NAVIGATION

One of the problems associated with the polar stereographic chart is that if you were at the north pole, it would be impossible to plot a course anywhere, because every single direction is south. Similarly, if you were at the south pole, every single direction is north.

Certainly less serious, but also warranting improvement is the Lambert's chart. Flying great circle tracks is ideal, but care must be taken when plotting these tracks, because they cut each meridian at a different angle. The solution to both of these problems is grid navigation. A square grid is placed over the applicable chart, grid north is always at the top of the chart and direction is now referenced to grid north rather than true north. Direction will always be constant relative to grid north because the grid is square.

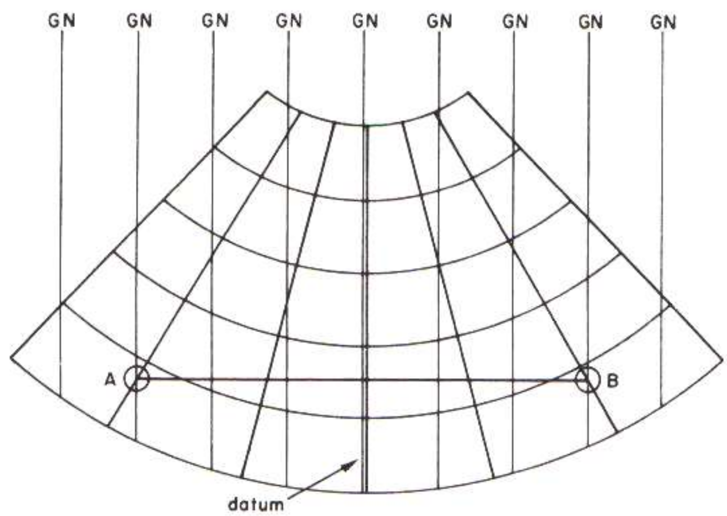


Grid Heading of aircraft 1 is 000
2 is 225
3 is 315
4 is 000
5 is 090

THE POLAR STEREOGRAPHIC GRID

On the polar stereographic grid, the datum meridian (the meridian on the chart with which the grid is lined up) is always the Greenwich meridian / anti-meridian of Greenwich.

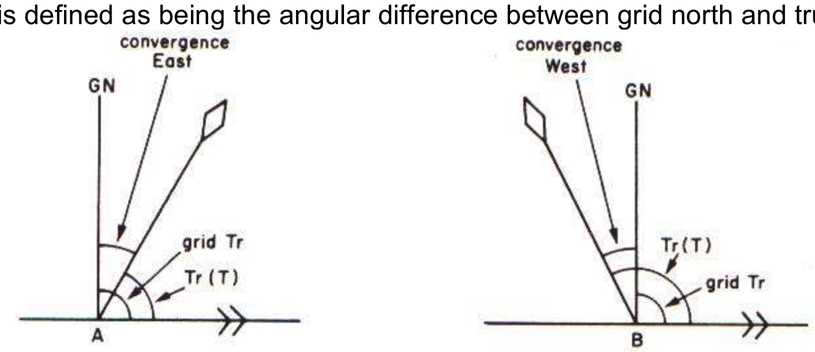
THE LAMBERT'S GRID



On the Lambert's grid, the datum meridian (the meridian on the chart with which the grid is lined up) can vary, and is normally positioned at a meridian closest to where the chart will be used.

CONVERGENCE

Convergence is defined as being the angular difference between grid north and true north.



If convergency is the angular difference between any two meridians, then convergence is the angular difference, not between any two meridians, but between the **datum** meridian and another meridian. Convergence and convergency thus always have the same numerical value.

On the polar stereographic chart :

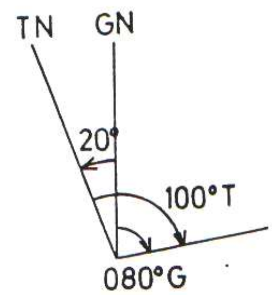
CONVERGENCE° = CONVERGENCY° = dLONG°

On the Lambert's chart

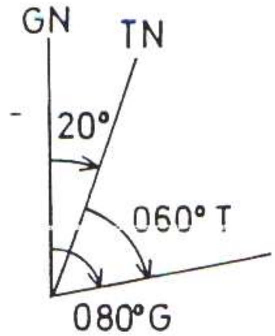
CONVERGENCE° = CONVERGENCY° = dLONG° x SIN LAT//of O

EXAMPLE*:*

Grid heading 080°. Convergence 20° W. What is the true heading?



Grid heading 080°. Convergence 20° E. What is the true heading?



RULE:

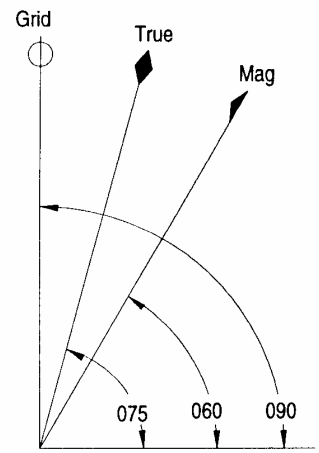
CONVERGENCE WEST - TRUE IS BEST
CONVERGENCE EAST - TRUE IS LEAST

BEWARE:

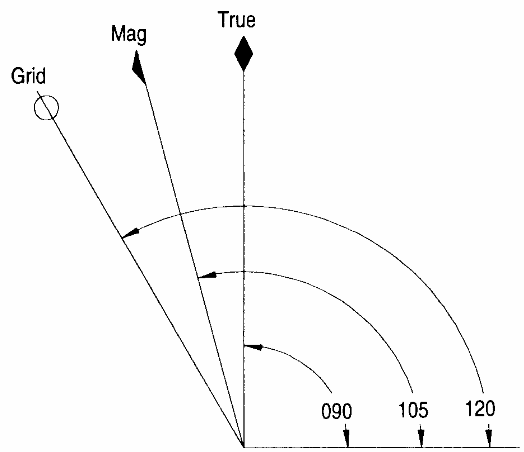
On the polar stereographic grid, although CONVERGENCE° = CONVERGENCY° = dLONG °, an easterly convergence does not necessarily mean that the aircraft is in the eastern hemisphere. Similarly, a westerly convergence does not necessarily mean that the aircraft is in the western hemisphere. This will be the case on a south polar chart, but will *not* be the case on a north polar chart. Always draw a sketch.

GRID VARIATION (GRIVATION)

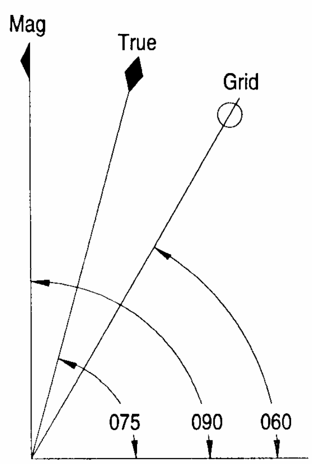
Grivation is defined as the angular difference between grid north and magnetic north. It is thus the algebraic sum of convergence and variation.



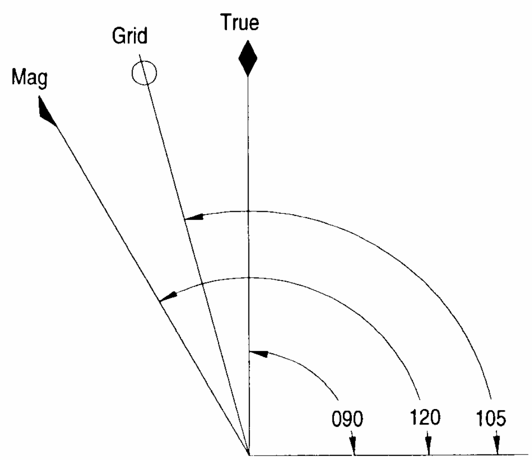
Convergence = 15E
Variation = 15E
Grivation = 30E



Convergence = 30E
Variation = 15W
Grivation = 15E



Convergence = 15W
Variation = 15W
Grivation = 30W



Convergence = 15E
Variation = 30W
Grivation = 15W

ISOGRIVS

Isogrivs are defined as being lines joining places of equal grivation.

QUESTIONS

PART 1

1. On a mercator chart, the scale at 18° N is 1:1000000

   What is the scale at 36° S.

2. On a Mercator chart, a line 21 cm long is drawn along the parallel 36° S.

   What change in longitude does this line represent if the scale of the chart is 1:400 000 at 50° S?

3. What earth distance is represented by a line 18" long drawn along the parallel 27° N if the scale on the Mercator chart is1:250 000 at 60 N?

4. Two lines of equal length are drawn on a Mercator chart, one at the equator and the other at 60° N.

   Which of these two lines represents the greater earth distance?

5. With the needle centralised, the VOR CDI indicates 145° TO. The variation at the aircraft position is 10° W. The variation at the station position is 15° W The deviation is 5° W.

   What bearing should be plotted on a Mercator chart of the northern hemisphere if the convergency between the aircraft and the station is 10°?

6. The ADF bearing on an RMI is 060°. The variation at the aircraft position is 10° W. The variation at the station position is 20° W. the deviation is 5° E the convergency between the aircraft and the station is 12°.

   What bearing should be plotted on a Mercator chart of the southern hemisphere?

7. Using meridional parts, calculate the rhumb line track and distance from A (08°N 016° 30' W) to B (16° 27' S 004° 18' E).

8. An aircraft leaves position A (27˚27' S 014° 28' E) on a rhumb line track of 205° and flies for a distance of 4087 nm.

   What is the aircraft's final position?

9. An aircraft departs from position A (10 18' S 002° 03'E) on a rhumb line track of 040°.

   At which meridian will the aircraft cross the equator?

10. An aircraft departs position A (21° 37' N 012° 12' W) on a rhumb line track of 137°.

    At which latitude will the aircraft cross the Greenwich meridian?

11. On a flight from A (22° N 165° E) to B (37 N 178° W), what is the chart length in cm’s of the rhumb line distance if the scale of the chart is at 60° 1:1 000 000 N?

    If the northernmost latitude of this chart is 60˚N and the north/south length of the chart is 150 cm’s, what is the southernmost latitude?

12. On a flight along the 50th parallel, the measured distance between fixes A and B is 22,5 cm on a Mercator chart of the northern hemisphere. The scale of the chart is 1:2 500 000 at 20°N. What is the aircraft’s groundspeed if the time between fixes was 17 minutes?

13. A Mercator chart has a scale of 1:2 000 000 at latitude 30°N. At what latitude will the scale be 1: 1 500 000?

14. On a Mercator chart, the perpendicular distance between parallels 37°N and 39°N is 4 cms. What is the scale of the chart at 30°N?

Part 2

1. The chart convergency factor on a Lambert's chart is 0.766. On standard parallel is at 40° N.

    What is the latitude of the other standard parallel?

2. The great circle track from A (40° S 015° E) to B (20° S 015° W) cuts the Greenwich meridian at an angle of 45°. The P of O is at 30° S.

    i) What is the great circle track measured at A?

    ii) What is the great circle track measured at B?

    iii) What is the rhumb line track from B - A?

3. The ADF needle on an RMI indicates an QDM of 040°. The variation at the station position is 20° W. The variation at the aircraft position is 15º W. The deviation is 5° E. The dLONG between the aircraft position and station position is 60°. The parallel of Origin is at 30° N.

    What is the bearing to plot on a Lambert's chart of the northern hemisphere?

4. With the needle on the VOR CDI centralised, the indication is 360° TO. The variation at the aircraft position is 15° W. The variation at the station position is 20° W. The deviation is 5° E. The convergency between the aircraft and the station is 10°.

    What is the bearing to plot on a Lambert's chart of the southern hemisphere?

5. On a Lambert's chart of the northern hemisphere, the standard parallel of 30° N has a chart length of 50 cm's. The other standard parallel measures 38 cm's.

    What is the latitude of the other standard parallel?

6. On a Lambert’s chart in the Northern hemisphere, a straight line is drawn from X to Y. The track measured at X is 60°T. If an aircraft leaves X on a constant heading of 60°T in zero wind conditions, will it pass:

a) North of Y.
b) Overhead Y.
c) South of Y.

7. A Lambert's chart has standard parallels 20°N and 60°N. The initial great circle track form a 27°N 061°W to B 47°N 017°W is 52° (T).

The longitude at which the great circle track becomes 084° is ...?

PART 3

1. On a polar stereographic chart, a flight is planned from A (70° N 035° E) to B (70° N 043° W).

   i) What is the great circle track from A - B?

   ii) What is the great circle track from B - A?

2. On a polar stereographic chart, a flight is planned from A - D.

   A - B great circle track 041°.

   B - C great circle track 059°.

   C - D great circle track 064°.

   What is the great circle track from A direct to D if all these positions lie on the parallel 75° N?

3. On a polar stereographic chart, a flight is planned from A (75° S 168° E) to B (75° S x°W).

   The great circle track from A - B is 120°.

   i) What is the longitude of position B?

   ii) What will the great circle track be when crossing the anti-meridian of Greenwich?

4. On a polar stereographic chart, a flight is planned from A (72° N 032° W) to B (72° N 098° W).

   i) What is the great circle heading at A if the drift is 5° right?

   ii) What is the highest latitude which this line will attain?

5. On a polar stereographic chart, a flight is planned from A (75° S 047° E) to B (75° S 063° W).

   i) What is the great circle track from A - B?

   ii) If there was an NDB station at B, what would the QDM be when the aircraft crosses the prime meridian assuming zero deviation and variation 15° W?

6. A Mercator chart and a polar stereographic chart have a rolling fit at 70°N. The scale of the Mercator chart is 1:1 000 000 at the Equator. What is the scale of the polar stereographic chart at 90°N?

Part 4

1. Aircraft heading 231° G. Convergence 15° W.

   What is the aircraft's true heading.

2. A grid is superimposed on a polar stereographic chart of the north pole. An aircraft has a heading of 060° T and 130° G.

   What is the aircraft's longitude?

3. A grid is superimposed on a polar stereographic chart of the south pole. An aircraft has a heading of 210° T and 160° G.

   What is the aircraft's longitude?

4. On a north polar grid chart, an aircraft at position 70° N 040° E, has a heading of 060° G.

   What is the aircraft's heading ° T?

5. On a south polar grid chart, an aircraft at position 75° S 060° W, has a heading of 160° T.

   What is the aircraft's heading ° G?

6. A grid is superimposed on a Lambert's chart of the northern hemisphere with the datum meridian at 030° W. The CCF is 0.5. An aircraft at position 45° N 010° W has a heading of 080° T.

   What is the aircraft's grid heading?

7. A grid is superimposed on a Lambert's chart of the southern hemisphere with the datum meridian at 060° E. The n factor is 0.5. An aircraft at position 20° S 020° E has a grid heading of 160° G.

   If the variation is 15° W, what is the aircraft's magnetic heading?

8. On a Lambert's conformal/Grid chart of the northern hemisphere, an aircraft at position 30° N 050° E has a true heading of 060° T and a grid heading of 100° G. The parallel of origin is at 30° N.

What is the datum meridian used on this chart?

9. On a Lambert's conformal/Grid chart of the southern hemisphere, an aircraft at position 25° S 030° W has a magnetic heading of 120° M. The grid heading is 090°G. The variation is 15° W. The CCF is 0.5.

What is the datum meridian used on this chart?