GENERAL NAVIGATION

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Mathematics - General Reminders

1. Understanding and being able to apply some of the basic principles of mathematics is an essential requirement for anyone studying for a professional pilot's licence or instrument rating. Almost all aspects of aviation can be said to have a mathematical basis or involve mathematics to some degree.

2. The level of mathematical knowledge required by this syllabus is well within the grasp of the average person. However, as it has usually been some time since our students have had to transpose a formula, solve an equation or use trigonometry, this chapter supplies a few reminders of such areas.

3. Please study this chapter before embarking on any other part of the course. There are worksheet questions at the end of the chapter to enable you to test your progress.

4. The first area of mathematics to be considered is simple arithmetic.

Simple Arithmetic

Addition

5. The addition of positive or negative values produces a positive or negative result depending on the sign and magnitude of the two parts being added, for example:

(a) (+2) + (+3) = +5

(b) (-2) + (-3) = -5

(c) (-3) + (+5) = +2

(d) (+3) + (-5) = -2

Subtraction

6. When a number is subtracted from another the sign of the result depends on the sign and magnitude of the two numbers. For example, when both numbers are positive:

(e) (+5) - (+3) = +2

(f) (+3) - (+5) = -2

7. When one or both numbers are negative the result varies depending on which is the larger number as illustrated in the following examples:

(g) (-5) - (-3) = -2

8. In example (g) the double negative becomes positive and so in effect the equation could be rewritten as: -5 +3 = -2.

(h) (-3) - (-5) = +2

9. In example (h) minus, minus 5 becomes plus 5 and so this equation could be rewritten as: -3 +5 and becomes the same as equation (d).

10. Here are two more examples of subtracting negative values:

(i) (+3) - (-5) = +8

(j) (-10) - (-15) = +5

Multiplication

11. When two positive numbers or two negative numbers are multiplied together the result is always positive. However, when only one number is negative the outcome is always negative. The following examples illustrate multiplication:

(k) $(+5) \times (+3) = +15$

(l) $(-5) \times (-3) = +15$

(m) $(-5) \times (+3) = -15$

(n) $(+5) \times (-3) = -15$

Division

12. The rule applicable for division regarding negative numbers is similar to that for multiplication i.e. when both values are positive or both negative the outcome is always positive. If either number is negative, the outcome of the division is negative. The following examples illustrate division:

(o) $(+6) \div (+3) = +2$

(p) $(-6) \div (-3) = +2$

(q) (+6) ÷ (-3) = -2

(r) (-6) ÷ (+3) = -2

Powers, Roots and Indices

13. For the purpose of the examination syllabus it is necessary to know only how to **square** a number (raise it to the power of 2), and how to find the **square root** of a number (raise it to the power of ½). If the reader is familiar with **indices** then this knowledge will stand her/him in good stead in the examinations notably with scale problems and with radio calculations, where 3×10^8 m/sec is easier to deal with than 300,000,000 m/sec. Frankly, however, if you are unfamiliar with indices, you can manage very well without them.

Algebra

14. Within this syllabus there is a limited requirement for simple pure algebra. The major significance of algebra is at the applied level, in as much as it is normally more convenient to manipulate formulae at the pure stage to isolate the unknown factor, and then to add the known values.

15. The secret then lies in the ability to handle simple transposition confidently. Remember the golden rule of transposition: **whatever you do to one side of the equation you must also do to the other.**

EXAMPLE 1-1

EXAMPLE

Given that:
A + B = C + D
Isolate D

SOLUTION

In order to isolate D it is necessary to move C to the other side of the equation. To achieve this, subtract C **from both sides.**

A + B - C = D

Another way of looking at the same problem is to say that, when moving a value from one side of the equal symbol to the other, change the sign (+ or -) of the value.

EXAMPLE 1-2

EXAMPLE

Given that:
W = (X + Y).Z
Isolate Z.
The full stop is by convention used instead of a multiplication symbol to avoid confusing it with the letter X.

SOLUTION

In order to isolate Z it is necessary to move (X + Y) to the other side of the equation. To achieve this divide (X + Y) **into both sides**:

$$W = (X + Y).\,Z$$

$$\frac{W}{(X + Y)} = \frac{\cancel{(X + Y)}.\,Z}{\cancel{(X + Y)}}$$

$$\frac{W}{(X + Y)} = Z$$

EXAMPLE 1-3

EXAMPLE

Given that:

$W = (X + Y).Z$

Isolate X.

SOLUTION

The first step is to remove the brackets by multiplying both X and Y by Z.

$$W = X.Z + Y.Z$$

Now subtract Y.Z from both sides:

$$W - Y.Z = X.Z + \cancel{Y.Z} - \cancel{Y.Z}$$

$$W - Y.Z = X.Z$$

Now divide both sides by Z:

$$\frac{W - Y.Z}{Z} = \frac{X.\cancel{Z}}{\cancel{Z}}$$

$$\frac{W - Y.Z}{Z} = X$$

Note. This equation can be further simplified to:

$$\frac{W}{Z} - Y = X$$

EXAMPLE 1-4

EXAMPLE

Given that:

$X = K\ T$

Isolate T.

SOLUTION

The first step is to divide both sides by K;

$$\frac{X}{K} = \frac{K\ T}{\cancel{K}}$$

In order to remove the square root sign from the T it is necessary to square both sides of the equation:

$$\left(\frac{X}{K}\right)^2 = T$$

EXAMPLE 1-5

EXAMPLE

Given that;

$$D = \frac{S}{\beta} \cdot 4 \cdot K_1 \cdot K_2$$

Isolate S.

SOLUTION

First re-write the formula in its simplistic form;

$$D = \frac{4.S.K_1.K_2}{\beta}$$

now multiply both sides by β**:**

$$D.\beta = \frac{4.S.K_1.K_2.\beta}{\beta}$$

$$D.\beta = 4.S.K_1.K_2$$

Now divide both sides by $4.K_1.K_2$:

$$\frac{D.\beta}{4.K_1.K_2} = \frac{4.S.K_1.K_2.}{4.K_1.K_2}$$

$$\frac{D.\beta}{4.K_1.K_2} = S$$

16. At various points in the notes, worked examples contain simple transposition such as these. Take each transposition one step at a time and remember the golden rule: **always do to one side of the equation exactly what you do to the other.**

Trigonometry

17. As with the preceding consideration of algebra, the required level of knowledge of trigonometry is reasonably basic. There is no requirement for any practical calculations using spherical trigonometry.

18. The sum of the numerical values of the three angles of any triangle is **always** 180 degrees. Since in trigonometry only right-angled triangles are considered, the sum of the two angles other than the right angle **must always add up to 90 degrees.**

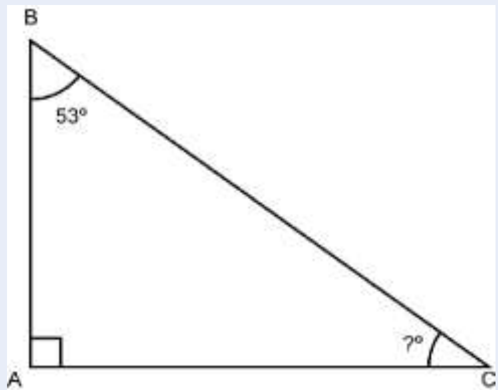
EXAMPLE 1-6

EXAMPLE

Given a right-angled triangle ABC where the angle BAC is 90°, and the angle ABC is 53°, determine the value of the angle ACB.

SOLUTION

FIGURE 1-1



Angle BAC = 90°
Angle ABC = 53°
Angle ACB = (180° - 90°- 53°) = 37°

19. Considering only the right-angled triangle, the length of any side of the triangle may be determined in one of several ways;

(i) by Pythagoras;

(ii) by scale drawing;

(iii) using the 3:4:5 ratio, if applicable;

(iv) Using the basic trigonometric functions of sine, cosine or tangent.

20. Pythagoras' theorem states that the square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides. The hypotenuse is always the side **opposite** the right angle, and is always the longest side.

EXAMPLE 1-7

EXAMPLE

The hypotenuse of a right-angled triangle measures 10 metres. One of the adjacent sides is 8 metres long, determine the length of the remaining side.

SOLUTION

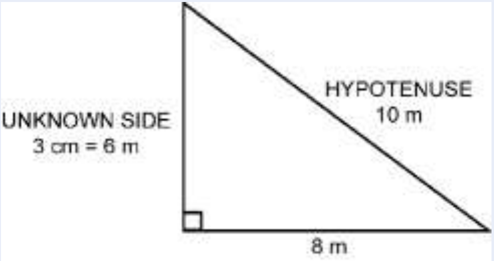
(i) By Pythagoras

Let X represent the unknown side

10^2	=	$8^2 + X^2$
100	=	$64 + X^2$
100-64	=	X^2
36	=	X^2
$\sqrt{36}$	=	X
6 metres	=	X

(ii) By scale drawing (1cm: 2 metre)

FIGURE 1-2



(iii) By 3:4:5 ratio:

Given any right-angled triangle where the ratio of the hypotenuse to one of the adjacent sides is either 5: 4 or 5: 3, then the ratio of the hypotenuse to the remaining side will be either 5: 3 or 5: 4. In other words the length of the sides is in the ratio 5: 4: 3. In the example given the ratio of the hypotenuse to the given side is 5: 4, since the hypotenuse is 10 metres long (5 x 2) and the given side 8 metres long (4 x 2). The length of the remaining side is therefore 6 metres (3 x 2), maintaining the 5: 4: 3 ratio.

(iv) By basic trigonometrical functions. Basic trigonometric functions are discussed shortly

21. A practical example of a Pythagoras type of problem is illustrated in the following example:

EXAMPLE 1-8

EXAMPLE

An airborne distance measuring equipment (DME) gives a **slant** range to the beacon of 15nm when the aircraft is flying at FL 360. Determine the horizontal range of the aircraft from the DME station.

SOLUTION

The question requires the horizontal range, so to this end it is acceptable to convert the altitude (36,000 ft) into nautical miles by dividing by 6000 as opposed to 6080. In this case, the altitude is (approximately) 6nm.

$$15^2 = 6^2 + X^2$$

$$225 = 36 + X^2$$

$$189 = X^2$$

$$\div 189 = X$$

$$13.7 = X$$

The approximate horizontal range is therefore 13.7nm.
The solution by scale drawing is self-explanatory.
The solution by 3:4:5 doesn't apply in this case.

Trigonometrical Functions

22. The candidate should be thoroughly familiar with the three basic trigonometrical functions of sine, cosine and tangent.

The Sine of an Angle

23. The sine of an angle is the ratio of the length of the side **opposite** to the angle divided by the length of the **hypotenuse**. See Figure 1-3. (Remember, the hypotenuse is **always** the side opposite the right angle, the third side is termed the **adjacent)**.

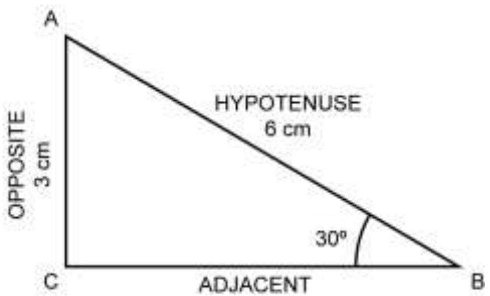
24. At Figure 1-3, the angle ABC is 30°. The construction shows that the hypotenuse is twice as long as the side opposite the 30° angle. Therefore:

$$\text{the sine of } 30° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{6} = 0.5$$

25. Using the same construction technique, it can be shown that:

sine 0°	=	0
sine 30°	=	0.5
sine 45°	=	0.7071
sine 60°	=	0.8660
sine 90°	=	1.0

FIGURE 1-3
Sine of an Angle



The Cosine of an Angle

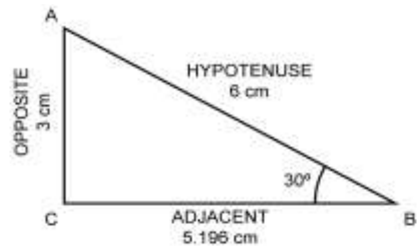
26. The cosine of an angle may be determined by constructing another right-angled triangle such that the angle ABC exists between the hypotenuse and an adjacent side. The cosine of the angle is found by dividing the length of the **adjacent** side by the length of the **hypotenuse**, see Figure 1-4. Therefore:

$$\text{the cosine of } 30° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{5.196}{6}$$

$$= 0.8660$$

FIGURE 1-4
Cosine of an Angle



27. Using the same construction technique, it can be shown that:

$$\cos 0° = 1.0$$

$$\cos 30° = 0.8660$$

$$\cos 45° = 0.7071$$

$$\cos 60° = 0.5$$

$$\cos 90° = 0$$

The Tangent of an Angle

28. Again using a right-angled triangle, the tangent of an angle is determined by dividing the length of the side **opposite** the angle in question by the length of the side **adjacent** to the angle.

The tan of 0° is found to be 0,

The tan of 30° is found to be 0.5773,

The tan of 45° is found to be 1.0,

The tan of 60° is found to be 1.732, and

The tan of 90° is found to be infinity.

29. **Use of electronic calculators.** Examining authorities are permitted to allow candidates to use simple scientific calculators in the JAR-FCL examinations. Such calculators should possess sine, cosine and tangent functions enabling you to process these ratios automatically. Should you not have such a calculator available whilst working on this course, tables are provided at Figure 1-5, Figure 1-6 and Figure 1-7.

FIGURE 1-5

NATURAL COSINE VALUES 0° TO 90°					
(°)	VALUE	(°)	VALUE	(°)	VALUE
1	1.000	31	0.857	61	0.485
2	0.999	32	0.848	62	0.469
3	0.999	33	0.839	63	0.454
4	0.998	34	0.829	64	0,438
5	0.996	35	0.819	65	0.423
6	0.995	36	0.809	66	0.407
7	0.993	37	0.799	67	0.391
8	0.990	38	0.788	68	0.375
9	0.988	39	0.777	69	0.358
10	0.985	40	0.766	70	0.342
11	0.982	41	0.755	71	0.326
12	0.978	42	0.743	72	0.309
13	0.974	43	0.731	73	0.292
14	0.970	44	0.719	74	0.276
15	0.966	45	0.707	75	0.259
16	0.961	46	0.695	76	0.242
17	0.956	47	0.682	77	0.225

18	0.951	48	0.669	78	0.208
19	0.946	49	0.656	79	0.191
20	0.940	50	0.643	80	0.174
21	0.934	51	0.629	81	0.156
22	0.927	52	0.616	82	0.139
23	0.920	53	0.602	83	0.122
24	0.914	54	0.588	84	0.105
25	0.906	55	0.574	85	0.087
26	0.899	56	0.559	86	0.070
27	0.891	57	0.5445	87	0.052
28	0.883	58	0.530	88	0.035
29	0.875	59	0.515	89	0.017
30	0.866	60	0.500	90	0.000

FIGURE 1-6

NATURAL SINE VALUES 0° TO 90°					
(°)	VALUE	(°)	VALUE	(°)	VALUE
1	0.017	31	0.515	61	0.875
2	0.035	32	0.530	62	0.883
3	0.052	33	0.545	63	0.891
4	0.070	34	0.559	64	0.899
5	0.087	35	0.574	65	0.906
6	0.105	36	0.588	66	0.914
7	0.122	37	0.602	67	0.920
8	0.139	38	0.616	68	0.927
9	0.156	39	0.629	69	0.934
10	0.174	40	0.643	70	0.940
11	0.191	41	0.656	71	0.946
12	0.208	42	0.669	72	0.951
13	0.225	43	0.682	73	0.956
14	0.242	44	0.695	74	0.961
15	0.259	45	0.707	75	0.966
16	0.276	46	0.719	76	0.970
17	0.292	47	0.731	77	0.974

18	0.309	48	0.743	78	0.978
19	0.326	49	0.755	79	0.982
20	0.342	50	0.766	80	0.985
21	0.358	51	0.777	81	0.988
22	0.375	52	0.788	82	0.990
23	0.391	53	0.799	83	0.993
24	0.407	54	0.809	84	0.995
25	0.423	55	0.819	85	0.996
26	0.438	56	0.829	86	0.998
27	0.454	57	0.839	87	0.999
28	0.469	58	0.848	88	0.999
29	0.485	59	0.857	89	1.000
30	0.500	60	0.866	90	1.000

FIGURE 1-7

NATURAL TANGENT VALUES 0° TO 90°					
(°)	VALUE	(°)	VALUE	(°)	VALUE
1	0.017	31	0.601	61	1.804
2	0.035	32	0.625	62	1.881
3	0.052	33	0.649	63	1.963
4	0.070	34	0.675	64	2.050
5	0.087	35	0.700	65	2.145
6	0.105	36	0.727	66	2.246
7	0.123	37	0.754	67	2.356
8	0.140	38	0.781	68	2.475
9	0.158	39	0.810	69	2.605
10	0.176	40	0.839	70	2.747
11	0.194	41	0.869	71	2.904
12	0.213	42	0.900	72	3.078
13	0.231	43	0.932	73	3.271
14	0.249	44	0.966	74	3.487
15	0.268	45	1.000	75	3.732
16	0.287	46	1.036	76	4.011
17	0.306	47	1.072	77	4.331

18	0.325	48	1.111	78	4.705
19	0.344	49	1.150	79	5.145
20	0.364	50	1.192	80	5.671
21	0.384	51	1.235	81	6.314
22	0.404	52	1.280	82	7.115
23	0.424	53	1.327	83	8.144
24	0.445	54	1.376	84	9.514
25	0.466	55	1.428	85	11.430
26	0.488	56	1.483	86	14.300
27	0.509	57	1.540	87	19.081
28	0.532	58	1.600	88	28.636
29	0.554	59	1.664	89	57.290
30	0.577	60	1.732	90	-------

Further Trigonometrical Functions

30. Three further trigonometrical functions are achieved by taking the reciprocals of the three basic functions.

31. The reciprocal of the sine of an angle, $\frac{1}{\sin}$ or $\frac{\text{hypotenuse}}{\text{opposite}}$, is the **cosecant**.

32. The reciprocal of the cosine of an angle, $\frac{1}{\cos}$ or $\frac{\text{hypotenuse}}{\text{adjacent}}$, is the **secant**.

33. The reciprocal of the tangent of an angle, $\frac{1}{\tan}$ or $\frac{\text{adjacent}}{\text{opposite}}$, is the **cotangent**.

34. Secant is used in navigation in connection with the properties of a Mercator chart which expands as the secant of the latitude.

Calculation of Sine and Cosine Values in the Second, Third and Fourth Quadrants

35. Certain calculations connected with 'compass swinging' require the calculation of the sine or cosine of angles greater than 90°. The following paragraphs explain how a sine or cosine ratio can be determined for an angle greater than 90°.

36. The quadrants referred to in this calculation are:

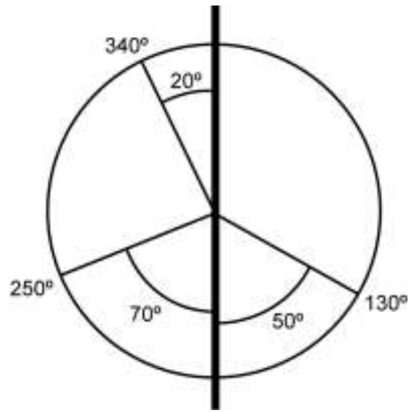
Quadrant 1 - angles < 90°

Quadrant 2 - angles between 90° and 180°

Quadrant 3 - angles between 180° and 270°

Quadrant 4 - angles between 270° and 360°

FIGURE 1-8
Sine / Cosine Angles



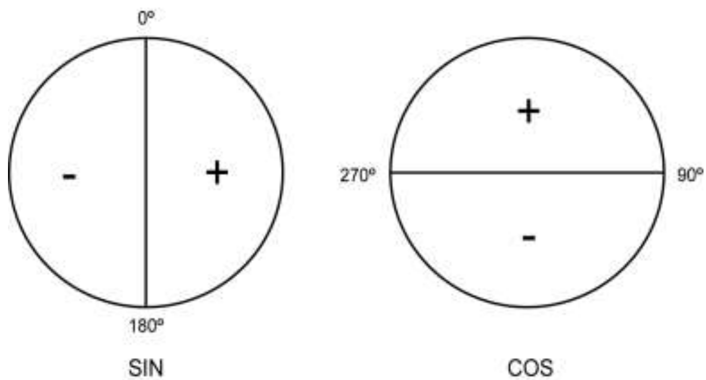
37. Since sine and cosine ratios only apply to angles of less than 90°, a method is required to convert a larger angle to one of less than 90°. The angle required is found by comparing the angle given and the nearest vertical reference line. For example, if the angle given is 130° then the angle whose sine and cosine value has the same numeric value is 50°. See Figure 1-8. If the angle given is 250°, then the sine or cosine value will be numerically the same as 70°. For 340°, the required angle is 20°.

38. The ‘rule of thumb’ which applies is:

Second quadrant	=	180°	- angle
Third quadrant	=	angle	- 180°
Fourth quadrant	=	360°	- angle

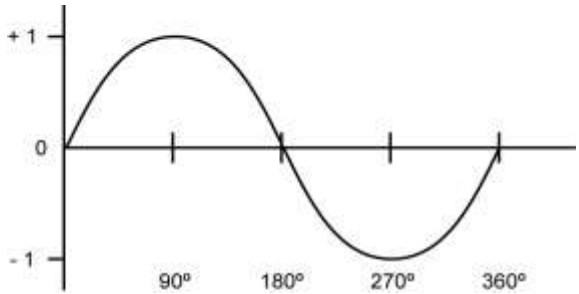
39. Having found the necessary angle which will be used, it is now necessary to establish whether the sine or cosine is positive or negative, and for this it is necessary to refer to Figure 1-9.

FIGURE 1-9
Sine /Cosine Signs



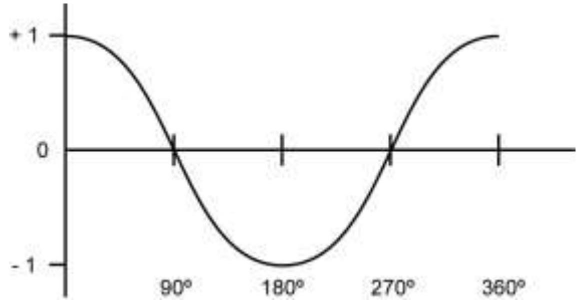
40. At Figure 1-9 we see that the sine values of angles between 0° and 180° are positive, and between 180° and 360° are negative. A graphical plot of sine values produces a 'sine curve' as shown at Figure 1-10.

FIGURE 1-10



41. Figure 1-9 shows that cosine values of angles between 270° and 90° are positive, and between 90° and 270° are negative. A graphical plot of cosine values produces a 'cosine curve' as shown at Figure 1-11.

FIGURE 1-11



42. A scientific calculator should be able to resolve the relevant ratio and its sign automatically. Some examples are given below; check the results with a calculator.

EXAMPLE 1-9

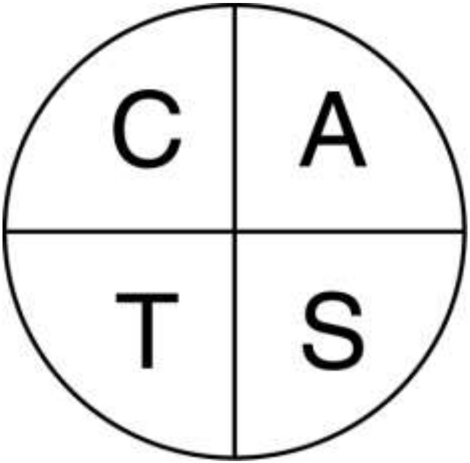
EXAMPLE

Determine the natural sine and cosine values of 157°, 223° and 356°.

SOLUTION

For sine 157° use 23° and the value is positive	=	+0.391
For cosine 157° use 23° and the value is negative	=	-0.920
For sine 223° use 43° and the value is negative	=	-0.682
For cosine 223° use 43° and the value is negative	=	-0.731
For sine 356° use 4° and the value is negative	=	-0.070
For cosine 356° use 4° and the value is positive	=	+0.998

FIGURE 1-12



43. The diagram at Figure 1-12 is useful for remembering which of the sine, cosine and tangent functions are always positive in each quadrant.

44. Just remember CAST, starting at top left and working clockwise. The diagram reminds us that:

In the fourth quadrant only the **C**osine is positive.

In the first quadrant **A**ll values are positive.

In the second quadrant only the **S**ine is positive.

In the third quadrant only the **T**angent is positive.

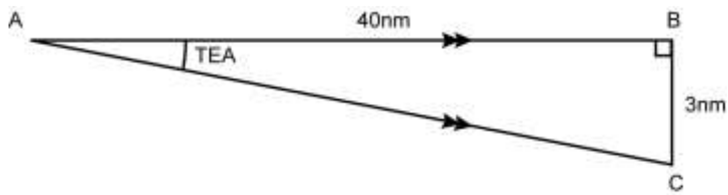
‘1 in 60 Rule’

45. The principle behind the 1 in 60 rule is that the tangent of a small angle is approximately equal to $\frac{1}{60}$th of the angle itself.

46. The 1 in 60 rule has several applications in navigation where it can be used to determine track error angle (the angle between the intended track, sometimes known as the course, and the track acutally flown, also known as the track made good). It can also be used in the vertical plane in aircraft rate of descent calculation.

47. The use of the 1 in 60 rule in the estimation of track error angle (TEA) serves to illustrate the mathematics of a 1 in 60 rule calculation. In Figure 1-13 the right angled triangle represents a navigation problem in which the planned course is represented by side AB and the acheived track by AC. Side BC represents the distance the aircraft is off track at C. Angle BAC represents the track error angle (TEA).

FIGURE 1-13



$$\text{Tan BAC} = \frac{BC}{AB}$$

Therefore,

$$\text{Tan TEA} = \frac{\text{Dist off track (3nm)}}{\text{Dist along track (40nm)}}$$

Since, Tan TEA x 60 = TEA

the formula can be written as,

$$\text{TEA} = \frac{\text{Dist off track x 60}}{\text{Dist along track}}$$

$$= \frac{3 \times 60}{40}$$

$$\text{TEA} = 4\tfrac{1}{2}°$$

Self Assessed Exercise No. 1

QUESTIONS:

QUESTION 1.

(a)	(+14) – (+18) =	(b)	(-10) – (+1) =
(c)	(+15) + (-25) =	(d)	(-18) + (+13) =
(e)	(-30) – (-11) =	(f)	(-8) – (-17) =
(g)	(-2) + (-22) =	(h)	(+6) – (-3) =
(i)	(-7) + (+17) =	(j)	(+9) + (-3) =

QUESTION 2.

(a)	(+88) ÷(-11) =	(b)	(-9) x (+12) =
(c)	(+8) × (+9) =	(d)	(+5) × (-8) =
(e)	(-9) × (-7) =	(f)	(-27) ÷ (-3) =
(g)	(-11) × (+6) =	(h)	(-54) ÷ (+6) =
(i)	(+42) ÷ (-7) =	(j)	(-45) ÷ (+9) =

QUESTION 3.

(a)	$7^{10} \times 7^{2} =$	(b)	$7^{10} \div 7^{2} =$
(c)	$9^{-3} \times 9^{5} =$	(d)	$6^{2} \div 6^{-2} =$

QUESTION 4.

(a)	$\sqrt{(8100)} =$	(b)	$(810) =$
(c)	$\sqrt{(4 \times 49)} =$	(d)	$\sqrt{(81 \div 36)} =$

QUESTION 5.

Given that:

$$x = \frac{15.047 \times (29.37)^2}{4 \sin 78°}$$

Determine the value of X

QUESTION 6.

Given that:

$Y = A + B\,\theta\ + \mathrm{Cos}\,\theta$

and that:

$A = 3$

$B = +1.5$

$C = 22$

$\theta = 42°$

Determine the value of Y

QUESTION 7.

Given that:

$$M = N + P \sin \theta + Q \sin \varphi + \frac{R \tan \theta}{60}$$

and that:

$N = 2.6$

$P = 15$

$Q = + 15$

$R = 480$

$\theta = 480$

$\varphi = 54°$

Determine the value of M

QUESTION 8.

Given that:

$$D = \frac{G \times 4 \times \mathrm{Cos}\theta \times \mathrm{Cos}\varphi}{\lambda}$$

and that:

D = 14,000

$\theta = 65°$

$\varphi = 25°$

$\lambda = 0.03$

Determine the value of G

QUESTION 9.

Given that:

$A = M \times K \times \sqrt{T}$

and that:

$A = 500$

$M = 0.83$

$K = 38.94$

Determine the value of T

QUESTION 10.

Given that:

$$C = \frac{D \times H}{O + H} \text{ and } P = \frac{E \times O \times H}{O + H}$$

and that:

C = P

E = 4

O = 300

Determine the value of D

QUESTION 11.

Find AC. See FIGURE 73 in the Reference Book

QUESTION 12.

Find EF. See FIGURE 74 in the Reference Book

QUESTION 13.

Find J. See FIGURE 75 in the Reference Book

QUESTION 14.

Find k. See FIGURE 76 in the Reference Book

QUESTION 15.

Find PQ. See FIGURE 77 in the Reference Book

QUESTION 16.

Find t. See FIGURE 78 in the Reference Book

QUESTION 17.

Find s. See FIGURE 79 in the Reference Book

QUESTION 18.

Find X. See FIGURE 80 in the Reference Book

QUESTION 19.

Find k. See FIGURE 81 in the Reference Book

QUESTION 20.

Find angle G. See FIGURE 82 in the Reference Book

QUESTION 21.

Find LM. See FIGURE 83 in the Reference Book

QUESTION 22.

Find angle θ. See FIGURE 84 in the Reference Book

QUESTION 23.

secant 25° =

secant 70° =

QUESTION 24.

(a)	Sin 160° =	(b)	Cos 160° =
(c)	Sin 250° =	(d)	Cos 250° =
(e)	Sin 305° =	(f)	Cos 305° =

ANSWERS:

ANSWER 1.

(a)	-4	(b)	-11
(c)	-10	(d)	-5
(e)	-19	(f)	+9
(g)	-24	(h)	+9
(i)	+10	(j)	+6

ANSWER 2.

(a)	-8	(b)	-108
(c)	+72	(d)	-40
(e)	+63	(f)	+9
(g)	-66	(h)	-9
(i)	-6	(j)	-5

ANSWER 3.

(a)	7^{12}	(b)	7^8
(c)	9^2	(d)	6^4

ANSWER 4.

(a)	90	(b)	28.5
(c)	14	(d)	1.5

ANSWER 5.

$x = 3317.4$

ANSWER 6.

$Y = -3.85$

ANSWER 7.

$M = -10.71$

ANSWER 8.

$G = 274.136$

ANSWER 9.

T = 239.327

ANSWER10.

D = 1200

ANSWER 11.

AC = 70

ANSWER 12.

EF = 91.5

ANSWER 13.

j = 85.7

ANSWER 14.

k = 21.5

ANSWER 15.

PQ = 86.4

ANSWER 16.

t = 16

ANSWER 17.

$s = 92.8$

ANSWER 18.

$X = 12.9°$

ANSWER 19.

$k = 39.2$

ANSWER 20.

$G = 48.8°$

ANSWER 21.

$LM = 109.4$

ANSWER 22.

$\theta = 59.3°$

ANSWER 23.

secant $25° = 1.103$

secant $70° = 2.924$

ANSWER 24.

(a)	Sin 160° = +0.342	(b)	Cos 160° = –0.940
(c)	Sin 250° = –0.940	(d)	Cos 250° = –0.342
(e)	Sin 305° = –0.819	(f)	Cos 305° = +0.574

The Earth

Definitions

Position Reference Systems

Direction on the Earth

Distance on the Earth

Conversion Factors

Earth Convergency

Conversion Angle

The Earth

1. Any study of the science of navigation must begin with an appreciation of the properties of the Earth.

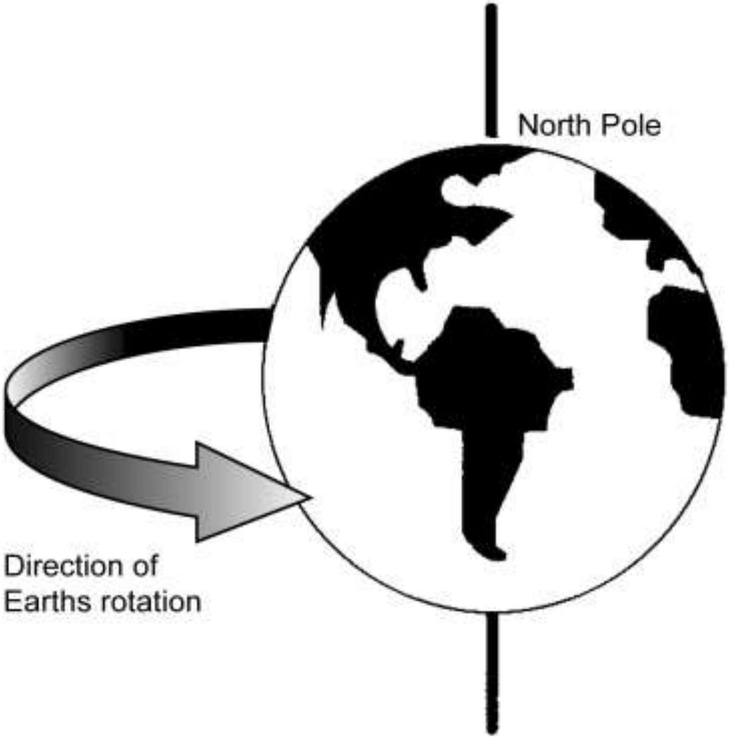
2. The Earth is an almost spherical planet which spins about the axis joining the geographic north and south poles. The Earth is not a true sphere, but is slightly flattened, or compressed, at the poles. In other words, the diameter of the Earth is slightly greater across the equator (6884 nm) than between the poles (6860 nm). The Earth is therefore described as an ellipsoid or **'oblate spheroid'**. This distortion, or 'compression factor', of the perfect sphere is very small, and is normally ignored for the purpose of this syllabus. In addition, when producing maps and charts from a reduced earth model, the compression factor is also small enough to be ignored if required, without incurring significant error.

Definitions

3. The definitions which follow are important. Please refer to Figure 2-1, Figure 2-2 and Figure 2-3.

4. **Direction.** The Earth spins on its own axis whilst travelling on its orbit around the sun; the direction in which the Earth rotates is defined as **east** (see Figure 2-1). Since the Earth is spinning, there must be a spin axis; the geographic, or true, poles are defined as the two points on the Earth's surface through which the spin axis passes.

FIGURE 2-1
The Earth's
Rotation



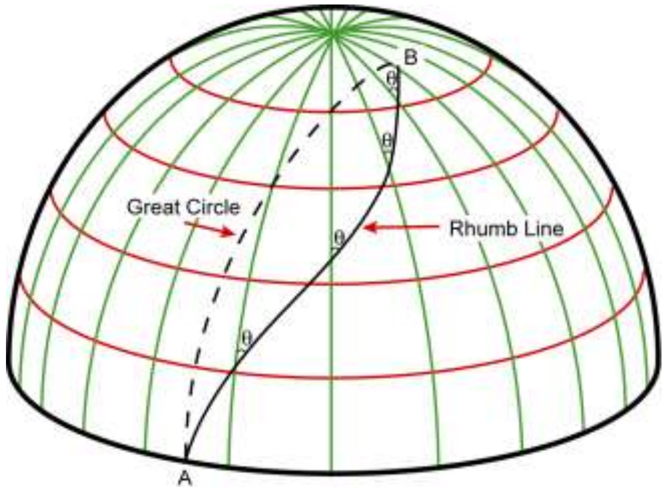
5. The Earth is described as rotating from west to east. If you were looking down on the north pole from space, the Earth would be seen to be rotating anti-clockwise.

6. The direction **north** can then be defined as the direction one would have to travel from any point on the Earth's surface by the shortest route to reach the north pole. Similarly, **south** is the direction one would have to travel to reach the south pole. Finally, **west** is the direction opposite to east.

7. **Great Circle.** A great circle is an imaginary circle on the surface of the Earth whose radius is the same as the Earth and whose plane passes through the centre of the Earth.

8. The shorter arc of a great circle is the shortest distance between any two points on the surface of the Earth; any great circle cuts the Earth into two equal halves or hemispheres. At Figure 2-2 the shorter arc of the great circle between positions A and B is shown as a broken line and represents the shortest distance across the Earth's surface between A and B.

FIGURE 2-2
Great Circle and Rhumb Line Track



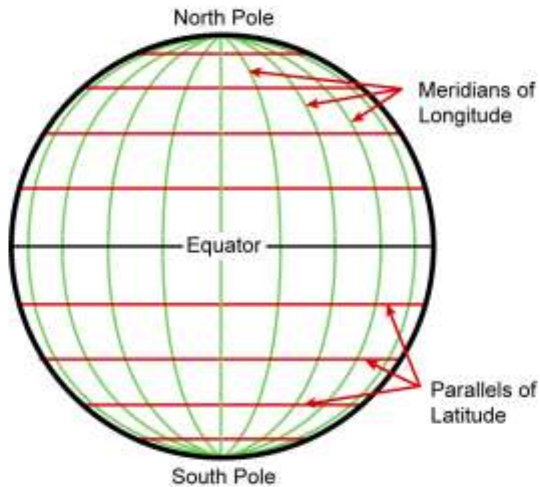
9. **Small Circle.** A small circle is any circle on the surface of the Earth whose radius is **not** that of the Earth, and whose plane does not pass through the centre of the Earth ie any circle other than a great circle. Parallels of latitude are examples of small circles.

10. **Rhumb Line.** A rhumb line is a regularly curved line on the surface of the Earth **which maintains a constant direction with respect to true North.** As will be seen shortly, this definition means that a rhumb line must cross all meridians at the same angle.

11. At Figure 2-2, the rhumb line joining points A and B is shown as a solid line. The distance from A to B along this rhumb line is greater than along the great circle; however, it is sometimes convenient to fly along a track with a constant track direction.

12. **The Equator.** The Equator is the name given to the great circle whose plane lies perpendicular to the Earth's spin axis; therefore, the Equator is the **only** great circle that lies in an east-west direction. The Equator also crosses all meridians at 90°; hence, not only is it a great circle, but it is also a rhumb line. All points on the Equator are equidistant from the geographic poles and the Equator divides the Earth into the northern and southern hemispheres; therefore, the Equator defines 0° North and/or South. See Figure 2-3.

FIGURE 2-3
Meridians and Parallels of Latitude



13. **Parallels of Latitude.** A parallel of latitude is an imaginary line that joins points of equal latitude; it is a small circle that lies parallel to the Equator (the Equator, therefore, is also a parallel of latitude; the only one which is a great circle). Parallels of latitude are used to define position in terms of latitude.

14. Whole degrees of parallels of latitude are numbered from 1° to 89° North or South of the Equator. Since parallels of latitude are parallel to the Equator and run East/West they cross all meridians at right angles and are therefore rhumb lines. See Figure 2-3.

15. **Meridians.** A meridian is a semi-great circle between the North and South poles (see Figure 2-3). It is a line joining points of equal longitude and crosses the Equator at right angles. All meridians define the directions of True North and South.

16. **The Prime Meridian.** The Prime Meridian is the semi-great circle passing through the poles and also passing through Greenwich, in London; it is also known, therefore, as the Greenwich Meridian and defines a longitude of 0° East and/or West. The Greenwich anti-meridian lies at 180° East/West.

17. Meridians are used to define the angular difference of a position from the Prime Meridian which is referred to as longitude. Whole degrees of meridians are numbered from 0° to 180° and are labelled east or west of the 0° meridian of longitude (the Prime Meridian). For every meridian, there is an anti-meridian that lies geometrically opposite (in other words 180° of longitude removed). A meridian together with its anti-meridian form a complete great circle.

18. The longitudes of a meridian and its anti-meridian will always add up to 180°; they will also be in opposite hemispheres. Hence the anti-meridian of 030°W is 150°E.

Position Reference Systems

19. The purpose of a Position Reference System is to be able to define uniquely any point on the surface of the Earth.

20. There are a number of different ways of defining position. You will probably be familiar with the method of using x-axis and y-axis co-ordinates (where the x-axis and y-axis are at right angles to each other) to plot a graph; these are called **Cartesian Co-ordinates**. However, the Cartesian Co-ordinate system is not very convenient when applied to navigation on the curved surface of the Earth.

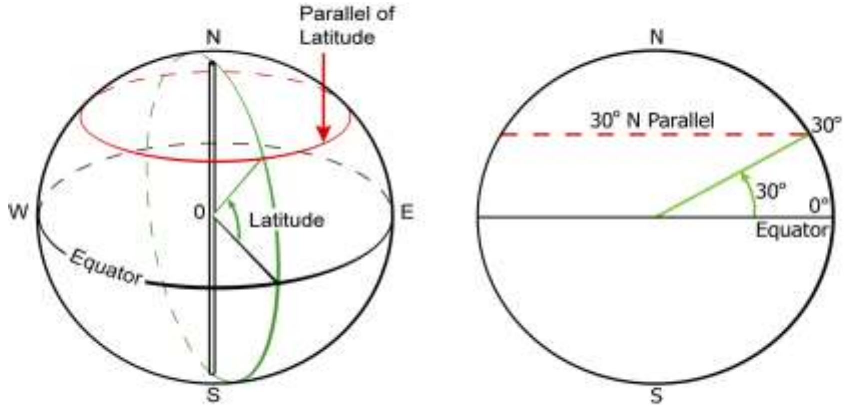
21. An alternative system is the **Polar Co-ordinates** system. Using Polar Co-ordinates it is possible to define **any** point on the surface of a sphere in terms of the radius of the sphere, (r), and 2 angles, represented by the Greek letters θ and ϕ.

22. When applied to navigation, the angles θ and ϕ are called latitude and longitude. Since we treat the Earth as a true sphere, r, the radius of the Earth is constant and is not therefore required in order to define position.

Latitude

23. The **latitude** of any point on the Earth's surface is defined as the arc of the meridian intercepted between the equator and the parallel of latitude passing through the point (see Figure 2-4). The latitude is given as an angle subtended at the centre of the Earth and expressed in degrees and minutes North or South of the Equator. (Note that each degree can be divided into 60 minutes of arc and each minute subdivided into 60 seconds of arc.)

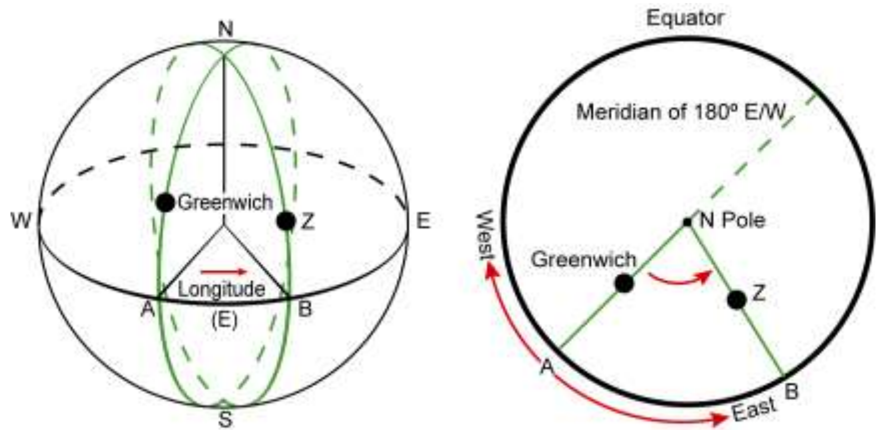
FIGURE 2-4
Latitude



Longitude

24. The **longitude** of any point on the Earth is defined as the smaller arc along the Equator intercepted between the Prime Meridian and the meridian passing through the point (see Figure 2-5). The longitude is given as an angle subtended at the centre of the Earth and expressed in degrees and minutes of arc, East or West of the Prime Meridian.

FIGURE 2-5
Longitude



Position

25. We can now uniquely define any point on the Earth's surface in terms of latitude and longitude. Latitude and longitude may be written as whole degrees only or as degrees and minutes (and sometimes seconds) as illustrated below:

- Latitude 14 degrees north could be expressed as: 14N or 14°N or 1400N or 14°00'N or 14°00'00"N
- Longitude 15 degrees east could be expressed as: 015E or 015°E or 01500E or 015°00'E or 015°00'00"E

Geodetic and Geocentric Latitude

26. Our definition of latitude is based on the assumption that the Earth is a true sphere. This definition should properly be called the **Geocentric latitude** ie the angle subtended between the line joining a point on the surface of the Earth to the centre of the Earth and the plane of the Equator.

27. There is an alternative definition of latitude, known as **Geodetic** (or **Geographic**) **latitude**. Geodetic latitude is defined as the angle between the normal to the meridian at a given point (ie the perpendicular to the Earth's surface at that point) and the plane of the Equator. When we look, later on, at chart properties, we will see that the first stage in map projection is to produce a physical model of the Earth which is called a **Reduced Earth**. This model can either be spherical (ignoring the irregularities in the shape of the Earth) or spheroidal, taking the irregularities into account.

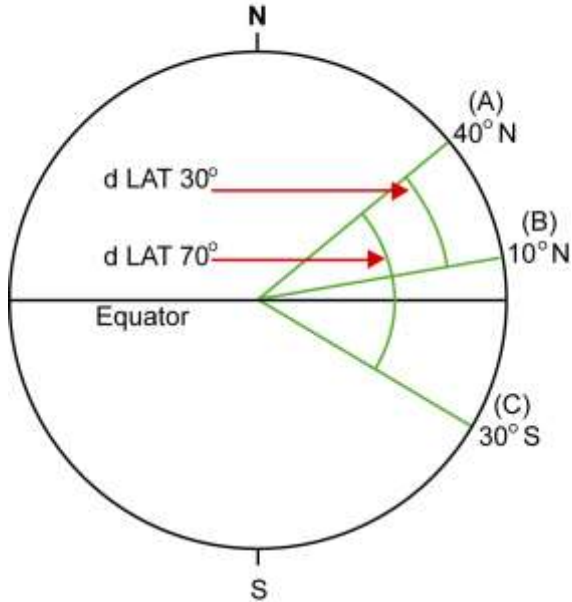
28. If the model is spherical then in fact there will be no difference between the Geodetic and Geocentric latitudes because the angle is measured in both cases at the centre of the Earth. If however, the model does take account of the irregular shape of the Earth, then there will be a difference between the two latitudes. This difference is known as **Reduction of Latitude** and is at a maximum at a latitude of 45° when its value is approximately 11.6' of arc. It should be noted that the latitude plotted on modern aeronautical charts is, in fact, the Geodetic (or Geographic) latitude.

29. When it comes to very accurate navigation systems such as Global Positioning System (GPS), the difference between Geocentric and Geodetic latitude becomes significant. GPS requires a very accurate mathematical 'model' of the Earth in order to function properly. This model is provided by a Geodetic Survey; the one used for GPS is the **World Geodetic System 1984** (WGS84).

Difference or Change of Latitude

30. Difference of latitude (d lat), which is also known as change of latitude (ch lat), is the difference between two latitude angles normally expressed in degrees and minutes of arc. Figure 2-6 shows three positions at different latitudes but at the same longitude (that is to say, on the same meridian). Point A is at 40°N, point B is at 10°N and point C is at 30°S. In this example the d lat between A and B is 30°, whereas the d lat between A and C is 70°.

FIGURE 2-6
Difference in Latitude



31. The following examples illustrate the calculation of d lat:

EXAMPLE 2-1

EXAMPLE

Point X is at 26°43'N 100°00'W, point Y is at 34°22'N 100°00'W. What is the difference in latitude

SOLUTION

	Lat Y	=	34°22'N
-	Lat X	=	26°43'N
therefore,	d lat	=	7°39'

EXAMPLE 2-2

EXAMPLE

Point X is at 26°43’N 100°00’W as before, point Z is at 34°22’S 100°00’W. What is the difference in latitude?

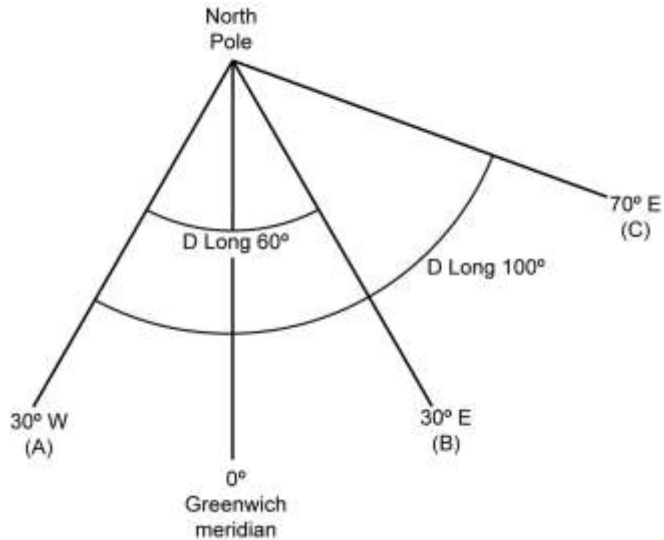
SOLUTION

		Lat X	= 26°43'N
	+	Lat Z	= 34°22’S
therefore,		d lat	= 61°05'

Difference or Change of Longitude

32. Difference of longitude (d long), which is also known as change of longitude (ch long), is the angular difference between two longitude angles normally expressed in degrees and minutes of arc. Figure 2-7 represents three positions at different longitudes but at the same latitude. Point A is at 030°W, point B is at 030°E and point C is at 070°E. In this example the d long between A and B is 60°, whereas the d long between A and C is 100°.

FIGURE 2-7
Difference in Longitude



33. Assuming the shorter arc of longitude difference is required in each case, the following examples illustrate the calculation of d long:

EXAMPLE 2-3

EXAMPLE

Point P is at 40°00’N 007°42’W, point Q is at 40°00’N 023°26’W. What is the difference in longitude?

SOLUTION

	Long Q	=	023°26'W
-	Long P	=	007°42’W
therefore,	d long	=	15°44'

EXAMPLE 2-4

EXAMPLE

Point P is at 40°00’N 007°42’W, point R is at 40°00’N 023°26’E. What is the difference in longitude?

SOLUTION

		Long P	= 007°42'W
	+	Long R	= 023°26’E
therefore,		d long	= 31°08'

34. Because it is normal to calculate the shorter arc between 2 points, a problem can arise when calculating d lat between points at high latitudes on opposite meridians or when calculating d long either side of the Greenwich anti-meridian. The following examples illustrate such situations.

EXAMPLE 2-5

EXAMPLE

Point X is at 79°42’N 100°00’W, point Y is at 84°49’N 080°00’E. What is the difference in latitude.

SOLUTION

The angular difference between these points is not a simple subtraction of one from the other because they are on opposite sides of the North pole. (Notice that the two longitudes add up to 180°.) The 2 points are on meridian and anti-meridian and therefore we must calculate the difference in latitude via the North pole. See Figure 2-8.

Step 1 d lat from 79°42’N to the north pole (90°00’N)

= 90°00'N

- 79°42'N

= 10°18'

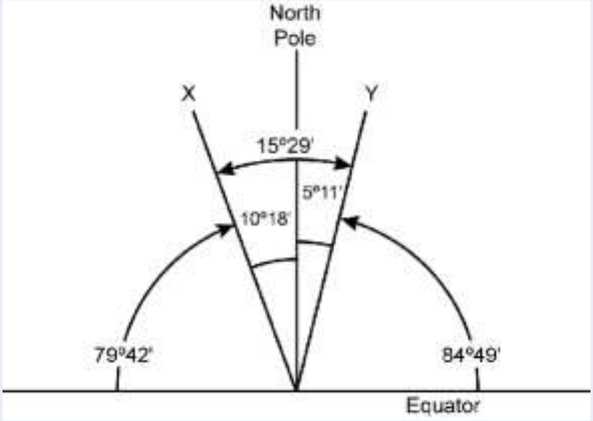
Step 2 d lat from the north pole to 84°49'

$$
\begin{array}{ll}
= & 90°00'N \\
 & -\ 84°49'N \\
= & 05°11'
\end{array}
$$

Step 3 d lat between point x and point y is the sum of the 2 values found:

$$
\begin{array}{ll}
= & 10°18' \\
+ & 05°11' \\
= & 15°29'
\end{array}
$$

FIGURE 2-8



EXAMPLE 2-6

EXAMPLE

Point L is at 60°N 160°W, point M is at 60°N 170°E. Calculate the shorter arc of d long in minutes between the two points.

SOLUTION

Step 1 Point A has a d long from the Greenwich anti- meridian of:

180°E/W - 160°W

= 20°

Step 2 Point B has a d long from the Greenwich anti- meridian of:

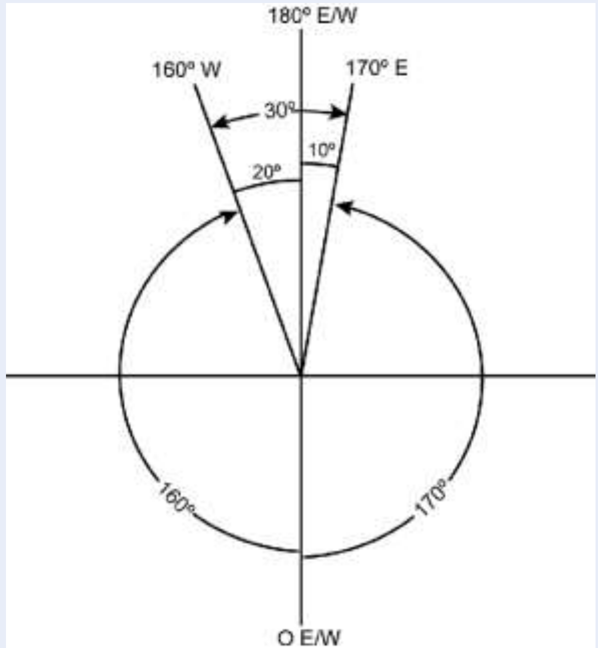
180°00'E/W - 170°E

= 10°

Step 3 Therefore, the total d long between points A and B and in minutes of arc is 30 x 60'

= 1800 minutes

FIGURE 2-9



Direction on the Earth

35. We have used the concept of direction in establishing our position reference system. When flying an aircraft, we steer it in a particular direction, the aircraft heading, and it is essential to know which datum or reference that heading is measured against.

True Direction

36. Directions in navigation are normally measured clockwise from a given datum. True North has been defined as the direction in which one would have to travel from any point on the Earth's surface to reach the North Pole.

37. True direction is expressed as an angle measured clockwise from **True North.**

38. As already stated, a meridian defines the directions of the True North datum (000°(T)) and South (180°(T)) at any given position. Furthermore, since all meridians and parallels cross at 90°, all parallels of latitude define the directions of East (090°(T)) and West (270°(T)).

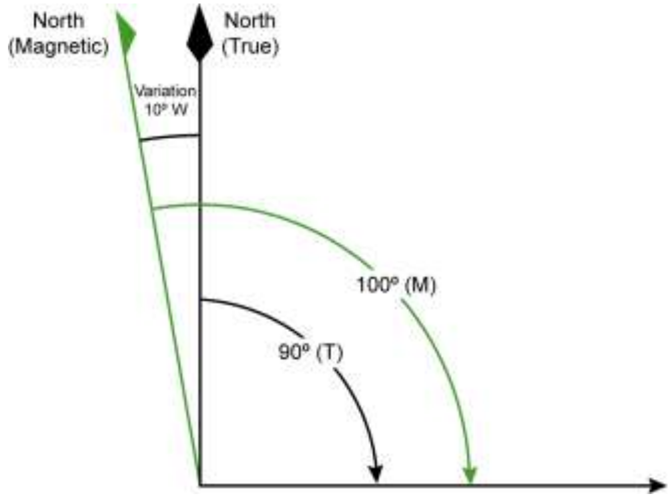
Magnetic Direction

39. Because meridians and parallels of latitude are only imaginary lines on the Earth, the actual measurement of direction in its simplest form is achieved using a magnetic compass. In a compass, small magnets or a magnetised pointer seek to align with the direction of the Earth's magnetic field (they 'seek' **Magnetic North**).

40. The direction of Magnetic North at any point defines the 'magnetic meridian' at that point i.e. local Magnetic North. Unfortunately, because the North magnetic pole is not coincident with the true pole, a difference between the true and magnetic meridians occurs. This angular difference may vary between 0° and 180° and is called **variation**. Knowledge of the local value of variation enables magnetic directions to be converted to true.

41. If Magnetic North lies to the West of True North, variation is said to be westerly; if Magnetic North lies to the east of True North, the variation is said to be easterly; hence variation is measured either West or East from True North. See Figure 2-10.

FIGURE 2-10
Comparing True North and Magnetic North (Variation West)



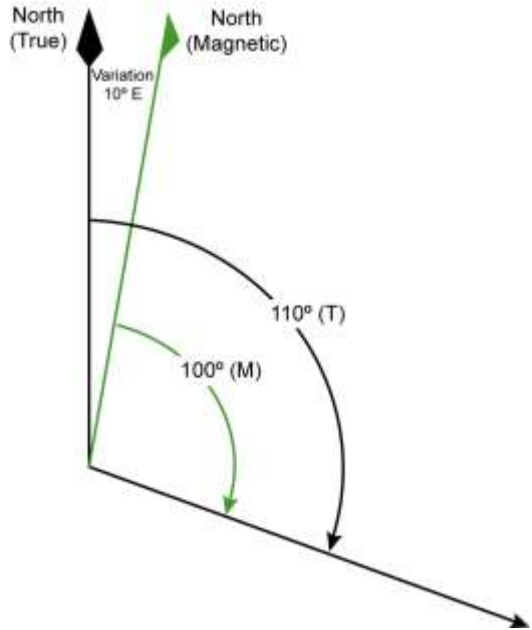
42. In the example in Figure 2-10, an aircraft is heading 090°(T). Variation is West of True North and has a value of 10°. From the diagram, we can see that the magnetic heading is a larger angle and must be larger by 10°; therefore, the magnetic heading is 100°(M).

43. This gives rise to a saying to help you remember which way to apply variation:

VARIATION WEST, MAGNETIC BEST

VARIATION EAST, MAGNETIC LEAST

FIGURE 2-11
Comparing True North and Magnetic North (Variation East)



44. Similarly in Figure 2-11, an aircraft is heading 110°(T). Variation is East of True North and has a value of 10°. From the diagram, it can be seen that the magnetic heading is a smaller angle and must be smaller by 10°; therefore, the magnetic heading is 100°(M).

45. Lines joining points of equal variation are known as **isogonals** and are normally printed on navigation charts. See Figure 2-12

FIGURE 2-12
Isogonals as Shown on a Jeppesen 1:500,000 Chart

Compass Direction

46. Magnetic fields within the aircraft may deflect the compass magnets from their intended alignment. The direction in which the compass needle actually indicates North is called **Compass North.** The error angle between Magnetic North and Compass North is called **deviation**. Deviation may also be caused by a misalignment of the reference line on the compass itself. Deviation is covered in more detail later.

47. If Compass North lies to the West of Magnetic North the deviation is westerly (or **negative**) as shown at Figure 2-13. If Compass North lies to the East of Magnetic North the deviation is easterly (or **positive**) as shown at Figure 2-14.

FIGURE 2-13
Deviation West

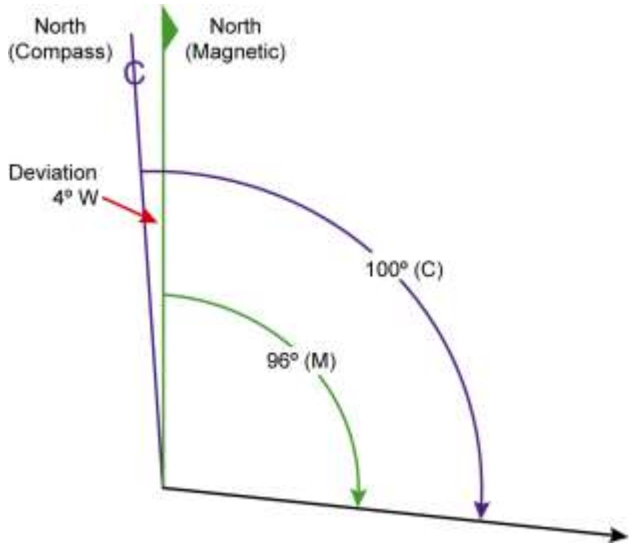
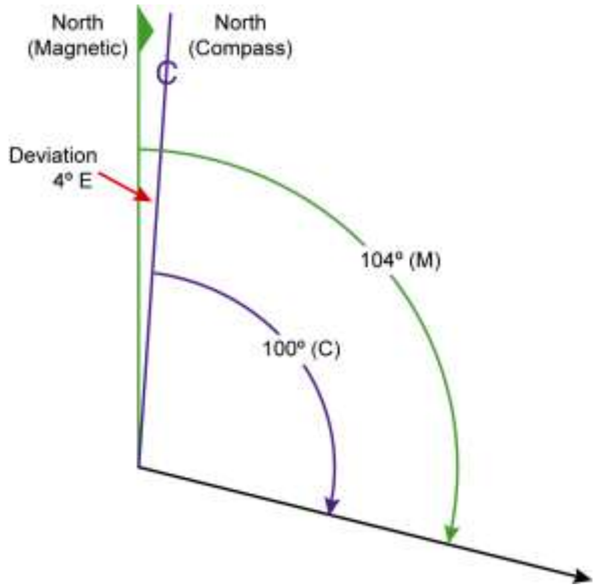


FIGURE 2-14
Deviation East



48. Note that, unlike variation, deviation changes with change of aircraft heading. Deviation in an aircraft compass is reduced to a minimum by conducting a compass swing. Residual deviations are then recorded on a compass deviation card, which is mounted in the aircraft. An illustration of one type of cockpit deviation card is shown at Figure 2-15.

FIGURE 2-15
A Compass Deviation Card

For Magnetic Heading		Steer by Compass
N	000°	002°
NE	045°	046°
E	090°	087°
SE	135°	136°
S	180°	182°
SW	225°	225°
W	270°	267°
NW	315°	315°

49. The relationship between compass, magnetic and true direction is summarised as shown at Figure 2-16.

FIGURE 2-16
Relationship between Compass, Magnetic and True Directions

Compass direction °(C)	+ Deviation East ------------> - Deviation West	Magnetic direction °(M)	+ Variation East ------------> - Variation West	True direction °(T)

Definitions:

- Deviation is east when compass north lies to the east of magnetic north
- Deviation is west when compass north lies to the west of magnetic north
- Variation is east when magnetic north lies to the east of true north
- Variation is west when magnetic north lies to the west of true north

50. Here are some examples to check your understanding of the relationships between compass, magnetic and true directions.

EXAMPLE 2-7

EXAMPLE

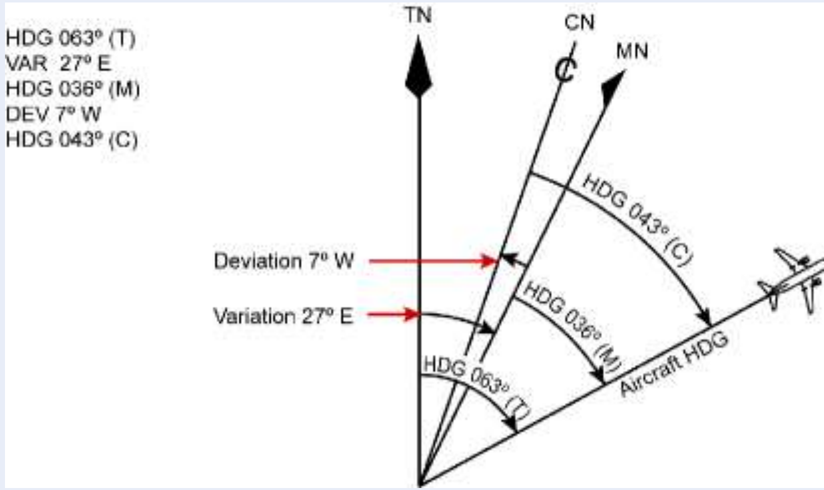
Draw a diagram to illustrate an aircraft on a heading of 063°(T), variation 27°E, deviation -7° (west) to show compass and magnetic north in relation to true north.

SOLUTION

See Figure 2-17.

FIGURE 2-17

Comparison of True, Magnetic and Compass Heading



EXAMPLE 2-8

EXAMPLE

An aircraft is steering 247° on the compass. The variation is 13°E and there is -1° (west) compass deviation. What is the aircraft's true heading?

SOLUTION

Deviation west, compass best which means the heading °(M) is less than compass:

therefore, Heading °(M) = 247° - 1°

= 246°(M)

Variation east, magnetic least, which means the heading °(T) is greater.

therefore, Heading °(T) = 246° + 13°

= 259°(T)

EXAMPLE 2-9

EXAMPLE

An aircraft is to fly a heading of 090°(T). If the variation is 6°W and the compass deviation is -3° (west), what heading is required on the compass to achieve the desired true heading?

SOLUTION

Variation west, magnetic best, which means the heading °(M) is greater.

therefore,	Heading °(M)	=	090°	+	6°
		=	096°(M)		

Deviation west, compass best, which means the heading °(C) is greater.

therefore,	Heading °(C)	=	096°	+	3°
		=	099°(C)		

Grid Direction

51. Grid navigation involves the use of a grid overprinted on a normal navigation chart to simplify navigation in polar regions. Grid direction is covered in a later chapter and also in the Operational Procedures notes.

Distance on the Earth

52. The measurement or calculation of distance is an essential element in navigation. On a chart, distance is normally measured using a scale bar or using dividers, but a knowledge of the means of calculating distance is also required and is covered in this section.

53. There are three main systems of measurement:

(a) Imperial measurement – statute mile, yards, feet and inches

(b) Systeme Internationale (SI) – The Metric System: kilometres, metres, centimetres and millimetres

(c) Angular Measurement – The nautical mile (nm)

Imperial Measurement

54. The Imperial system of measurements is based on:

12 inches	=	1 foot
3 feet	=	1 yard
1, 760 yards	=	1 statute mile

55. For most of the calculations required in this syllabus, it is necessary only to remember that 1 statute mile equates to 5,280 feet.

Systeme Internationale

56. The Systeme Internationale is more commonly known as the metric system. The basic unit of measurement for distance in the SI is the metre (m). There is an exact and scientific definition of the metre available, but for the purposes of this syllabus, we use the definition of the kilometre (km) provided by Napoleon: one kilometre is equal to one ten-thousandth of the average distance from the Equator to either pole. Hence the distance from the Equator to the north or south pole is 10,000 km.

1 kilometre	=	1000 metres
1 metre	=	100 centimetres

Nautical Mile

57. The unit of distance most frequently used in navigation is the nautical mile (nm). The relationship between the nautical mile and latitude/longitude is fundamental to navigation.

58. **Definition.** The nautical mile is defined as the distance on the surface of the Earth along a great circle which subtends an angle of 1’ of arc measured at the centre of curvature of the Earth. However because the radius of curvature varies (the Earth is an oblate spheroid) so does the actual length of a nautical mile. For most practical purposes, the angle of 1’ is assumed to be subtended at the centre of the Earth and the variability in length of the nautical mile is ignored. For calculations required in this syllabus, it is necessary to remember the conversion rate:

1 nautical mile = 6,080 feet

(Note: This distance equates to 1854m; however, for reference purposes the ICAO nautical mile is actually defined as a distance of 1852 metres)

59. A meridian is a semi-great circle; therefore, 1’ along a meridian can be assumed to represent 1nm. Therefore, 1° of change of latitude along a meridian is assumed to represent a distance of 60nm.

Calculating Distances

60. The syllabus requires you to be able to **calculate** the following:

(a) Great circle distance. The great circle distance may be along a meridian or along the Equator.

(b) Rhumb line distance. The rhumb line distance is the east/west distance along a parallel of latitude. Rhumb line distance is also called ‘departure’.

61. The calculation of the distance between two points that are neither on the same meridian or anti-meridan, nor on the same parallel of latitude is not required in this syllabus.

rcle Distance

62. Some examples of distance calculation are given below. Assume the shorter arc of distance is required in each case:

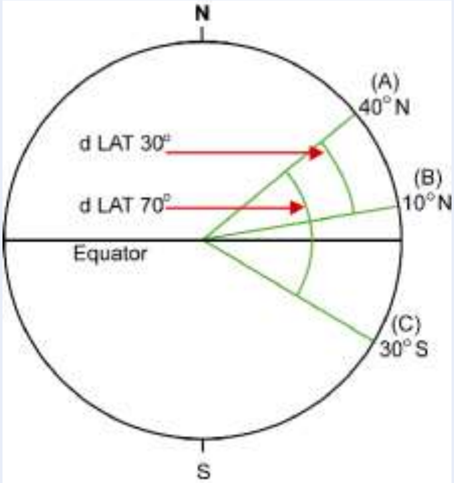
EXAMPLE 2-10

EXAMPLE

Determine the great circle distance in nautical miles between points A and B, and between points A and C in Figure 2-18

SOLUTION

FIGURE 2-18



First check that A and B are on the same Meridian.

Point A is at 40°N; point B is at 10°N.

The difference in latitude (d lat) is 30° which we must convert to minutes (multiply by 60).

Therefore the distance between A and B	=	30 x 60'
	=	1800'
	=	1800 nm

Point A is at 40°N; point C is at 30°S

The d lat is 70° and A and C lie on the same meridian

Therefore the distance between A and C	=	70 x 60'
	=	4200'
	=	4200 nm

EXAMPLE 2-11

EXAMPLE

Determine the great circle distance in nautical miles between point D (47°15'N 027°30'W) and point E (32°20'N 027°30'W).

SOLUTION

Point D is at 47°15’N; point E is at 032°20’N; both are on the same meridian

The d lat is 14°55’

Therefore the great circle distance is (14 x 60’) + 55’

= 895’

= 895 nm

EXAMPLE 2-12

EXAMPLE

Determine the great circle distance in nautical miles between point F (80°N 030°W) and point G (85°N 150°E).

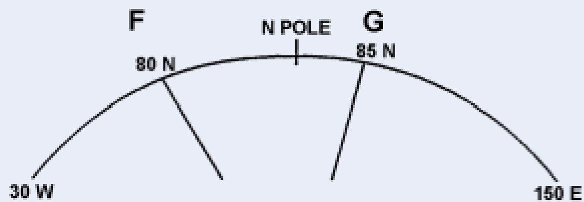
SOLUTION

At first sight these two points appear not to be on the same meridian, but remember, a meridian and its anti-meridian will always add up to 180° and they are always in opposite hemispheres.

In this case point F lies on 030°W and point G lies on 150°E; they add up to 180° and are in opposite hemispheres. Therefore, G is on the anti-meridian of F and both points lie on the same great circle We can proceed by calculating the distance via the North Pole. See Figure 2-19.

Calculating great circle distance over the pole

FIGURE 2-19



Point F is at 80°N 30°W; Point G is at 85°N 150°E

From 80°N to the North Pole (90°N) = 10°

From the North Pole to 85°N = 5°

Therefore the d lat is 15° and we have established that F & G lie on a meridian and its anti meridian

Therefore the distance between F and G = 15 x 60’

= 900’

= 900 nm

EXAMPLE 2-13

EXAMPLE

Determine the great circle distance in nautical miles between point J (00°N/S 075°W) and point K (00°N/S 150°W).

SOLUTION

Points J and K are not on the same meridian, but they are both on the Equator. The Equator is a great circle; hence we can calculate the distance that J and K are apart.

Point J is at 075°W and point K is at 150°W; therefore, the d long is 75°. That difference in longitude is along a great circle (the Equator) where 1’ of arc equates to 1nm.

Therefore the distance between J to K	=	75 x 60’
	=	4500’
	=	4500nm

Calculating Departure (Rhumb Line) Distance

63. One degree change of latitude along any meridian always represents a distance of 60 nm. One degree of change of longitude will, however, only measure 60 nm at the Equator; this is because **the Equator is the only parallel of latitude which is a great circle**.

64. It follows, therefore, that a change of 1’ of arc of longitude along the Equator will equate to 1nm whereas a change of 1’ of arc of longitude at either pole is not a distance at all, simply a very small rotation. So the distance represented by 1’ of longitude decreases as latitude increases. This means that (except at the Equator) we **cannot** convert a difference in longitude to distance simply by converting d long to minutes.

65. However, there is a trigonometrical function which varies in the same way as longitudinal distance ie is a maximum of +1 at 0°, (the Equator) and a minimum of 0 at 90° (the pole). That function is the cosine and it is the cosine of the latitude that is used to convert d long in minutes to departure in nautical miles. It is essential to note that this formula will only produce an exact answer if both positions have the same latitude.

66. The formula relating change of longitude to distance is called the departure formula. The formula states that:

Departure Distance (nm) = Difference in long (min) x cos lat

or, in other words,

Departure Distance (nm) = d long’ x cos lat

67. Some examples of departure calculation are given below. Once again, unless the question states otherwise, assume the short arc of distance is required.

EXAMPLE 2-14

EXAMPLE

Determine the shorter arc of rhumb line distance between point A (35°15'N 027°40'W) and point B (35°15'N 046°25'E).

SOLUTION

Distance (nm) = d long' x cos lat

Add the longitude values to give d long, thus

= (46°25' + 27°40') x cos 35°15'

= 74°05' x cos 35°15'

= ((74 x 60) + 05) x 0.817 (rounded)

= 4445' x 0.817

= 3632nm

EXAMPLE 2-15

EXAMPLE

Determine the shorter arc of rhumb line distance between point C (48°00'N 027°00'E) and point D (48°00'N 014°35'E).

SOLUTION

Departure Distance (nm) = d long' x cos lat

= (27°00' - 14°35') x cos 48°

= 12°25' x cos 48°

= ((12 x 60) + 25)' x 0.669

= 745' x 0.669

= 498.4nm

EXAMPLE 2-16

EXAMPLE

Determine the shorter arc of rhumb line distance between point E (65°00’N 162°17’E) and point F (65°00’N 157°24’W).

SOLUTION

In this example, the points are both at 65°00’N, but they are either side of the Greenwich anti-meridian. It is therefore necessary to work out the d long from 162°17E to 180°00’ E/W and from 180°00’ E/W to 157°24’W and add them together, thus:

180°00'	- 162°17'	=	17°43'
180°00’	- 157°24'	=	22°36'
17°43'	+ 22°36'	=	40°19'
therefore,	d long	=	40°19'
Departure Distance (nm)		=	d long’ x cos lat
		=	40°19' x cos 65°
		=	((40 x 60) + 19)’ x cos 65°
		=	2419’ x 0.423
		=	1023nm

Comparison of Rhumb Line and Great Circle Distances

EXAMPLE 2-17

EXAMPLE

FIGURE 2-20

Comparison of Rhumb Line and Great Circle Distance

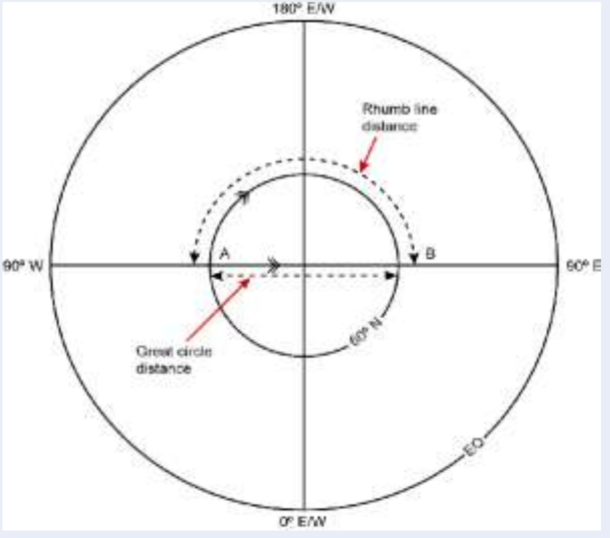


Figure 2-20 illustrates an extreme case of the advantage of flying a great circle track as opposed to a rhumb line. In this example, point B is on the anti-meridian of point A, hence the great circle track between them passes directly over the north pole.

In addition, since points A and B are both at 60°N, we can calculate the east/west departure distance between them.

SOLUTION

From Point A (at 60°N) to the north pole (90°N)	=	30° d lat
From the north pole to point B (also at 60°N)	=	30° d lat
Therefore, the total d lat from A to B	=	60°
Great circle distance from A to B	=	60 x 60’
	=	3600’
	=	3600nm

On the other hand, the E/W Rhumb Line or Departure Distance is calculated as follows:

Departure distance (nm)	=	d long'	x cos lat
	=	(180 x 60)'	x cos 60
	=	10800'	x 0.5
	= 5400nm		

In this example the rhumb line distance is 1800nm, or 50% more than the great circle distance, indicating why airlines prefer great circle routes (to save time, fuel and money).

Conversion Factors

68. Check your computer and ensure that it makes provision for converting from one unit of distance to another. If your computer does not provide this facility, you will need to memorise the following conversion factors:

1 nm (6080ft)	=	1.854km	=	1854 metres
1 nm	=	1.152 statute miles	=	6080 ft
1 statute mile	=	1.610 km	=	1610 metres
1 inch	=	2.54 cm		
1 metre	=	3.28 ft		

69. Speed given in knots represents the number of nautical miles flown in one hour; therefore:

1 knot = 1 nm/hour

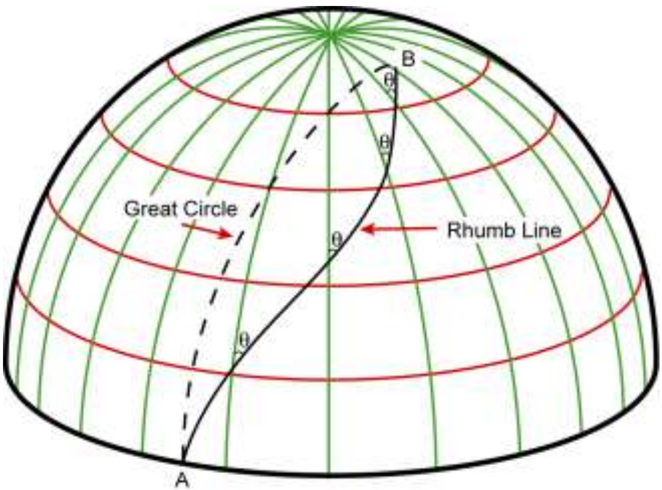
70. To convert speed given in knots to metres/second, divide the speed in knots by 1.95, ie. 1.95kts = 1m/s

71. Note that when expressing altitude or height, some countries choose to use feet whilst others choose to use metres.

Earth Convergency

72. When discussing the form of the Earth, we observed that all meridians lie north/south and join the two true poles. What this means is that the meridians **converge** at the poles and the distance between adjacent meridians reduces with increase in latitude.

FIGURE 2-21



73. The effect of this convergency of meridians is that the direction of a great circle track or bearing constantly changes (unlike a rhumb line which remains the same). The magnitude of the change can be determined by calculating the **convergency** between the start point and the finish point (or destination) on the track.

Definition

74. **Earth Convergency.** Earth convergency can be defined as the angle that one meridian forms with another.

75. Expressed more technically, earth convergency (EC) is the angle of inclination between any two meridians at a given latitude.

76. At the Equator earth convergency is zero, since all meridians are parallel with one another. However, at the poles the value of Earth convergency between any two meridians is equal to the value of the change of longitude between them.

77. If we consider two meridians 1° of longitude apart at the Equator, the angle they form with each other is 0°. However, if we consider the same two meridians at the north pole (or south pole), they will form an angle of 1° which is the same as the difference (or change) in longitude. Hence earth convergency between these two meridians is 0° at the Equator and 1° at the poles.

78. There is a trigonometrical function which varies in the same way as earth convergency ie is a minimum of 0 at 0°, (the Equator) and a maximum of +1 at 90° (the pole). This is the sine function and it is the sine of the latitude that is applied to d long to calculate earth convergency.

79. To calculate earth convergency the formula is:

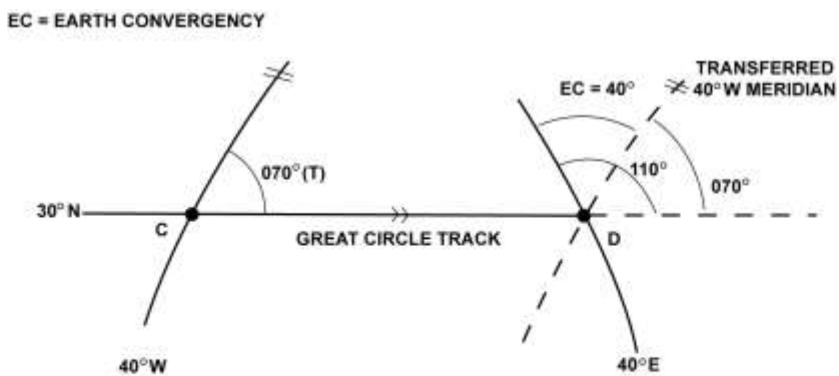
Earth Convergency° = Difference in Longitude (°) x Sine of the Latitude

80. Unlike the formula for departure distance, which is accurate only when the two points are at the same latitude, in the case of earth convergency the two points may be at different latitudes, and the formula for calculating earth convergency then becomes:

Earth Convergency° between 2 points = Difference in Longitude (°) x Sine of the Mean Latitude

Consider Figure 2-22:

FIGURE 2-22
Change in Great Circle Track Due to Earth Convergency



81. Earth convergency between the two meridians shown in Figure 2-22 is 40° at latitude 30°N (80° x sine 30).

82. The great circle track measured at C is 070°(T); measured at D, it is 110°(T). The great circle track has changed by 40°; due to the convergency between the 040°W meridian and the 040°E.

83. An example of an Earth convergency calculation is given below:

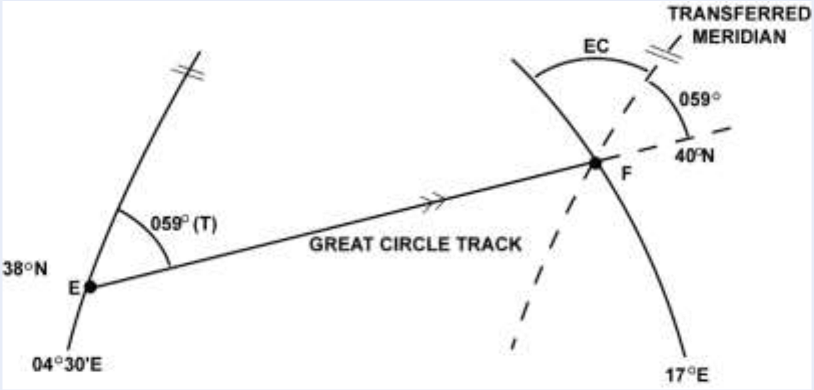
EXAMPLE 2-18

EXAMPLE

The great circle track between point E (38°N 04°30'E) and point F (43°N 17°E) measures 059°(T) at E. Determine the direction of this great circle track at F. See Figure 2-23.

SOLUTION

FIGURE 2-23



Earth Convergency° between 2 points	**=**	**Difference in Longitude (°)**	**x**	**Sine of the Mean Latitude**
Earth convergency	=	d long°	x	sin mean lat
	=	(17°00’ - 04°30’) x sin $\frac{(38 + 43)}{2}$		
	=	12.5°	x	sin 40°30’
	=	12.5°	x	0.649
	=	8°		

By observation, the track angle at F must be larger than the track angle at E and therefore we **add** the earth convergency (in this case) to the initial great circle track at E to obtain the new great circle track at F:

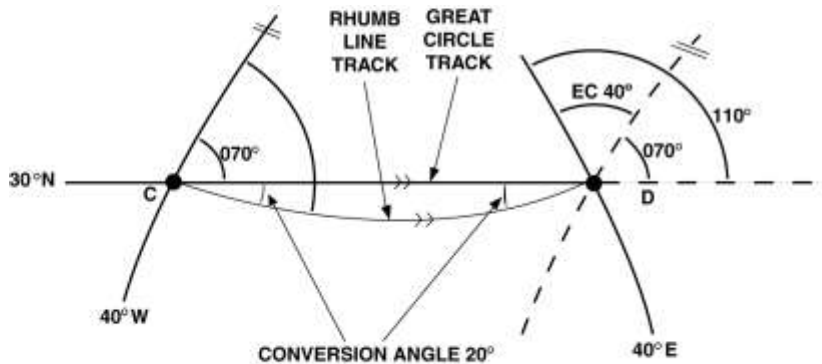
great circle track at F	=	great circle track at E	+ earth convergency
	=	059°	+ 8°
	=	067°(T)	

84. The practical application of convergency on a chart is covered later in 'plotting'.

Conversion Angle

85. When flying between one point and another it is usual to use either a great circle track (the shortest distance) or a rhumb line track (with a constant track angle).

FIGURE 2-24
Rhumb Line Track



86. Figure 2-24 is the situation repeated from Figure 2-22, but this time also shows the rhumb line track between C and D. We have already seen that the great circle track **increases** from 070°(T) to 110°(T); implying that at some point the great circle track must be 090°(T).

87. The rhumb line track between C and D follows the 30°N parallel of latitude and therefore the rhumb line track must be a constant 090°(T).

88. Therefore, at some point, the two tracks are parallel ie they are **both** 090°(T).

89. The great circle track changes (because of earth convergency) at a constant rate from 070° to 110°; therefore, to be parallel with the rhumb line track, it must have changed by 20°. The total track change between C and D is 40°. Therefore, the point at which the great circle track and rhumb line track are equal and parallel is at the **mid-longitude** of the track ie at 0°E/W.

90. The difference in track angle between the initial great circle track and the rhumb line track is the angular difference subtended between these two tracks at the departure point.

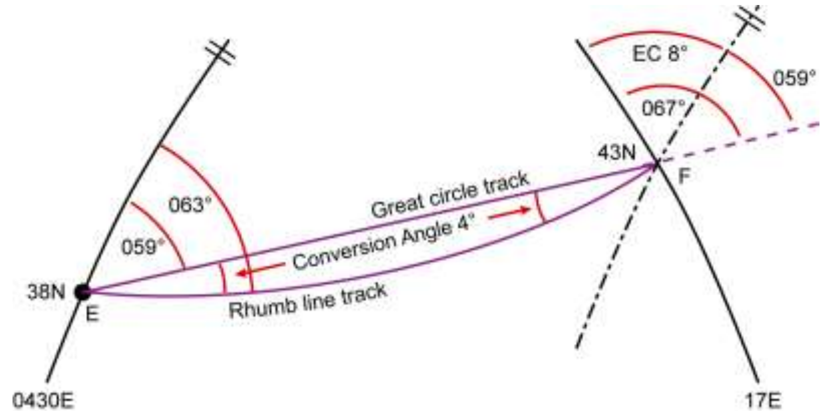
91. The initial great circle track is 070°(T); the rhumb line track is 090°(T); therefore, the difference is 20°. The angular difference between the tracks at D is **also** 20°.

92. This angle is called the **conversion angle**. That angle of 20° equals half of the earth convergency between points C and D. The formula for calculating conversion angle is:

$$\textbf{Conversion angle} = \textbf{0.5} \times \textbf{d long}^{\circ} \times \textbf{sin mean lat}$$

Figure 2-25 is as Figure 2-23, but once again the rhumb line track is included.

FIGURE 2-25
Rhumb Line Track



93. Check that the logic outlined in the previous example holds true for Figure 2-25 and the rhumb line track is 063°(T).

94. To sum up, the changes in great circle track over the Earth are due to earth convergency.

earth convergency = d long° x sin mean lat

95. The difference between the great circle track and rhumb line track between two points is called conversion angle.

conversion angle = half of earth convergency

= 0.5 x d long° x sin mean lat

Try the following example:

EXAMPLE 2-19

EXAMPLE

Point A is at 65°00'N 05°00'E; point B is at 60°00'N 15°00'W. If the initial great circle track at the departure point A is 252°(T), find:

(a) the great circle track B to A measured at B

(b) the rhumb line track A to B?

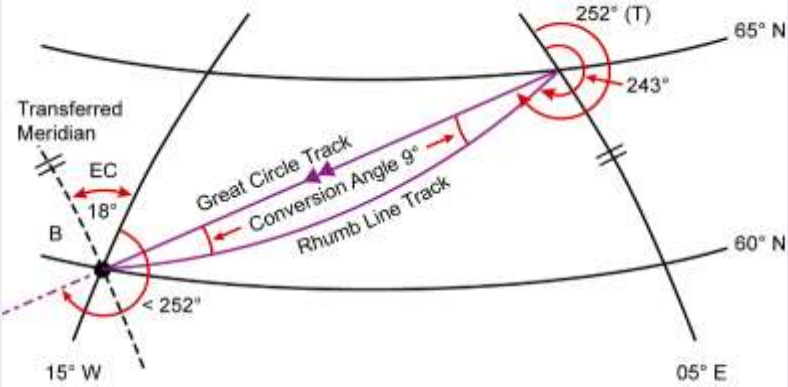
SOLUTION

The recommended technique here is to determine the track direction from A to B measured at B and **then** take the reciprocal. A thumb-nail sketch is usually sufficient to show which **way** to apply convergency, but be sure to give the correct relative positions according to the latitude and longitude of the two points.

EXAMPLE 2-20

EXAMPLE

FIGURE 2-26



Solution to (a)

Earth Convergency	=	d long°	x sin (mean lat)
	=	20°	x sin 62.5°
	=	20°	x 0.887
	=	17.74°	= 18° (approx)

From Figure 2-26 it can be seen that the track angle at B will be smaller than 252°; therefore, we have to **subtract** the convergency:

great circle track at B	=	great circle track at A	-	convergency
	=	252° - 18°		
	=	234°		

But this is the track A to B measured at B; we now take the reciprocal to determine the great circle track B to A

$$234° - 180° = 054°$$

Therefore, the great circle track B to A measured at B is **054°(T)**

Solution to (b)

The rhumb line track A to B is determined by applying the conversion angle to the given great circle track at A.

Figure 2-26 confirms that the conversion angle must be subtracted from the great circle track at A.

$$\text{Conversion Angle} = \tfrac{1}{2} \times \text{Earth Convergency}$$

$$= \tfrac{1}{2} \times 18°$$

$$= 9°$$

$$\text{rhumb line track A to B} = \text{great circle track A to B} - \text{conversion angle}$$

$$= 252° - 9°$$

$$= 243°$$

Note: By definition the rhumb line track does not change (it crosses **all** meridians at the same angle) so, although it is not required in this question, the reciprocal will also be the same all the way from B to A (063°).

Self Assessed Exercise No. 2

QUESTIONS:

QUESTION 1.

State the definition of the direction East.

QUESTION 2.

State the definition of the direction North.

QUESTION 3.

State the definition of a great circle.

QUESTION 4.

State the definition of a small circle.

QUESTION 5.

State the definition of the Equator.

QUESTION 6.

State the definition of a parallel of latitude.

QUESTION 7.

State the definition of a meridian.

QUESTION 8.

What is the anti-meridian of the 060°E meridian.

QUESTION 9.

Define geocentric latitude.

QUESTION 10.

Define reduction of latitude.

QUESTION 11.

Where is reduction of latitude at a maximum.

QUESTION 12.

Point A is at 47°25’N 02°30’W; point B is at 11°57’N 02°30’W

Calculate d lat between points A and B

Calculate the great circle distance between A and B

Determine the bearing (°T) of B from A.

QUESTION 13.

Define Earth convergency.

QUESTION 14.

Point C is at 45°00'N 12°25'E; point D is at 45°00'N 10°46'W.

Calculate d long between points C and D

Calculate the rhumb line (departure) distance between C and D

Determine the rhumb line track (°T) C to D

Determine the great circle track (°T) D to C measured at D

QUESTION 15.

Direction – fill in the blanks.

Hdg°(C)	Deviation	Hdg°(M)	Variation	Hdg°(T)
120	10W		15E	
345	-7		27E	
	4E	132	24W	
	12W		12W	220
	+10		7E	270
	16W	300	16E	
030	10E		4E	
000	10W		17W	
	3E	215	6W	
220	-4		5W	

QUESTION 16.

Great Circle Distance – fill in the blanks.

A		B		D Lat	Distance (nm)
Latitude	Longitude	Latitude	Longitude		
Great Circle Distances					
49°00'S	012°25'E	45°00'S	012°25'E		
37°27'S	049°21'E	43°21'S	049°21'E		
10°21'S	113°47'W	05°17'N	113°47'W		
53°25'N	108°00'E	47°17'N	108°00'E		
78°47'N	035°45'W	84°35'N	144°15'E		

QUESTION 17.

Rhumb Line Distance – fill in the blanks.

A		B		D Long	Distance (nm)
Latitude	**Longitude**	**Latitude**	**Longitude**		
Rhumb Line (departure) Distances					
52°00’N	21°31’W	52°00’N	010°17’E		
13°00’S	175°17’E	13°00’S	168°31’W		
49°00’S	012°25’E	49°00’S	025°45’E		
21°00’N	010°43’E	21°00’N	016°17E		
47°30’N	025°00’W	47°30’N	044°28’W		

QUESTION 18.

Great Circle and Rhumb Line Tracks fill in the blanks.

Point A		**Point B**		**GC Tk @ A**	**Conv**	**GC Tk @ B**	**CA**	**RL Track**
Latitude	Longitude	Latitude	Longitude					
59°00'N	010°00'W	65°00'N	010°00'E	050				
47°00'S	125°00'W	41°00'S	135°00'W	300				
55°00'N	110°00'W	75°00'N	010°00'E	022				
75°00'N	010°00'E	60°00'N	075°00'W					245
75°00'S	160°00'W	60°00'S	165°00'E					312
75°00'N	100°00'W	50°00'N	025°00'W					128
73°00'S	120°00'E	55°00'S	150°00'E			023		
50°00'S	170°00'W	73°00'S	135°00'W			129		
55°00'S	120°00'E	73°00'S	170°00'E			107		

ANSWERS:

ANSWER 1.

East is defined as the direction in which the Earth rotates.

ANSWER 2.

Since the Earth is spinning, it has a spin axis. The extremities of the spin axis on the Earth's surface are the True Poles. Then facing in the direction of the Earth's rotation, ie. East, the True North Pole is on the left. The direction North is then defined as the direction in which one would have to travel from any point on the Earth's surface by the shortest route to reach the True North Pole.

ANSWER 3.

A great circle is an imaginary circle on the surface of the Earth whose radius is the same as the Earth and whose plane passes through the centre of the Earth.

ANSWER 4.

A small circle is an imaginary line on the surface of the earth whose radius is not the same as the Earth and whose plane does not pass through the centre of the Earth, ie. any circle that is <u>not</u> a great circle.

ANSWER 5.

The Equator is the name given to the great circle whose plane lies perpendicular to the Earth's spin axis; it is the only great circle that lies in an east/west direction.

ANSWER 6.

A parallel of latitude is an imaginary line that joins points of equal latitude; it is a small circle that lies parallel to the Equator and hence lies in an east/west direction. Parallels of latitude are used to define position in terms of latitude, north or south of the equator.

ANSWER 7.

A meridian is a semi-great circle between the north and south poles. It is a line joining points of equal longitude and crosses the Equator at right angles. Meridians define the directions of True North and South and are used to define position in terms of longitude east or west of the Greenwich (Prime) Meridian.

ANSWER 8.

120°W

ANSWER 9.

Geocentric latitude is the angle subtended between a line joining a point on the surface of the Earth to the centre of the Earth and the plane of the Equator; it assumes the Earth is a true sphere.

ANSWER 10.

Reduction of latitude is the difference between geocentric and geodetic latitude. It is the difference between treating the Earth as a true sphere and taking the irregularities into account.

ANSWER 11.

Reduction of latitude is at a maximum at a latitude of 45° where its value is approximately 11.6’ of arc

ANSWER 12.

I. D.lat = 47° 25’ – 11° 57’ = 35° 28’

II. Great circle distance = (35 x 60) + 28 = 2128 nm

III. True bearing of B from A = 180°T (on the same meridian)

ANSWER 13.

Earth convergency is caused by the meridians converging at the poles. It is the angle that the meridian at one point on the Earth’s surface form with the meridian at another point. Earth convergency = d long° x sin (mean lat).

ANSWER 14.

a) difference in longitude = 10°46’ + 12°25’

= 23°11’

b) Rhumb line (departure) distance = d long’ x cos lat

= [(23 x 60) +1]’ x cos 45

= 1391 x 0.707

= 983.4 nm

c) Rhumb line track $C \rightarrow D$ follows 45°N parallel of latitude that is parallel to the Equator which lies east/west. Therefore, 45°N parallel lies east/west; therefore, track $C \rightarrow D$ is 270°T

d) Rhumb line track $C \rightarrow D$ = 270°T

$\therefore$ Rhumb line track $D \rightarrow C$ = 090°T (can take reciprocal of rhumb line anywhere)

Great circle track $D \rightarrow C$ is smaller angle than rhumb line.

$\therefore$ Great circle track $D \rightarrow C$ = rhumb line track $D \rightarrow C$ – conversion angle

conversion angle = ½ x d long° x sin (mean lat)

= ½ x 23.2° x 0.707

= Great circle track $D \rightarrow C$ = 090° – 8.2° = 081.8°T

See FIGURE 85 and FIGURE 86 in the Reference Book

ANSWER 15.

Hdg°(C)	Deviation	Hdg°(M)	Variation	Hdg°(T)
120	10W	110	15E	125
345	-7	338	27E	005
128	4E	132	24W	108
244	12W	232	12W	220
253	+10	263	7E	270
316	16W	300	16E	316
030	10E	040	4E	044
000	10W	350	17W	333
212	3E	215	6W	209
220	-4	216	5W	211

ANSWER16.

A		B		D Lat	Distance (nm)
Latitude	**Longitude**	**Latitude**	**Longitude**		
Great Circle Distances					
49°00'S	012°25'E	45°00'S	012°25'E	4°00'	240
37°27'S	049°21'E	43°21'S	049°21'E	5°54'	354
10°21'S	113°47'W	05°17'N	113°47'W	15°38'	938
53°25'N	108°00'E	47°17'N	108°00'E	6°08'	368
78°47'N	035°45'W	84°35'N	144°15'E	16°38'	998

ANSWER17.

A		B		D Long	Distance (nm)
Latitude	**Longitude**	**Latitude**	**Longitude**		
Rhumb Line (departure) Distances					
52°00'N	21°31'W	52°00'N	010°17'E	1908'	1174.7
13°00'S	175°17'E	13°00'S	168°31'W	972'	947
49°00'S	012°25'E	49°00'S	025°45'E	800'	524.8
21°00'N	010°43'E	21°00'N	016°17E	334'	311.8
47°30'N	025°00'W	47°30'N	044°28'W	1168'	789.1

ANSWER 18.

Point A		Point B		GC Tk @ A	Conv'y	GC Tk @ B	CA	RL Track
Latitude	**Longitude**	**Latitude**	**Longitude**					
59°00'N	010°00'W	65°00'N	010°00'E	050	17.6°	067.6	8.8°	058.8
47°00'S	125°00'W	41°00'S	135°00'W	300	6.9°	306.9	3.5°	303.5
55°00'N	110°00'W	75°00'N	010°00'E	022	108.8°	130.8	54.4°	076.4
75°00'N	010°00'E	60°00'N	075°00'W	284.3	78.5°	205.7	39.3°	245
75°00'S	160°00'W	60°00'S	165°00'E	295.8	32.3°	328.1	16.2°	312
75°00'N	100°00'W	50°00'N	025°00'W	094.7	66.5°	161.2	33.3°	128
73°00'S	120°00'E	55°00'S	150°00'E	050	27°	023	13.5°	036.5
50°00'S	170°00'W	73°00'S	135°00'W	159.8	30.8°	129	15.4°	144.4
55°00'S	120°00'E	73°00'S	170°00'E	151.9	44.9°	107	22.5°	129.4

The Triangle of Velocities and Navigation Computer

- Triangle of Velocities
- Navigation Computer
- Speed, Distance and Time Calculations
- Calibrated (Rectified) Airspeed to True Airspeed Calculations
- Mach Number Calculations
- Conversion Factors

The Triangle of Velocities and Navigation Computer

Triangle of Velocities

1. The triangle of velocities is a vector triangle which is used to establish the relationship between the elements governing an aircraft's speed and direction through the air and its direction and rate of progress over the surface of the earth. The difference between these two values is caused by the wind.

Definitions Used in the Triangle of Velocities

2. **Velocity**. A velocity comprises two elements, a direction and a speed. In navigation the direction is measured clockwise from a given datum which is usually true north. Speed is the rate of movement per unit of time which may be specified in a number of different ways. Normally the speeds used in navigation are nautical miles per hour which is abbreviated to knots or kt.

3. **True Airspeed (TAS) and Groundspeed**. The speed of movement of an aeroplane relative to the undisturbed air is referred to as the true airspeed. If the speed is measured relative to the path over the surface of the Earth, it is referred to as the groundspeed of the aircraft (G/S).

4. **Heading.** The heading of an aircraft is the direction the aircraft is moving in the undisturbed air. In other words, the direction in which the nose is pointing. However, rarely is the air completely still; it is moved by the wind and as a consequence will move an aircraft flying in that air downwind.

5. **Wind Velocity**. Wind velocity is the direction from which the wind is blowing and its speed of movement over the Earth.

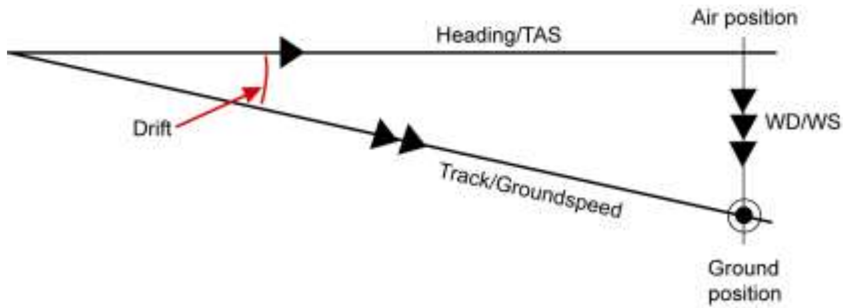
Wind Effect

6. If an aeroplane is heading 090(T) at a TAS of 500kt, after 1 hour it will be 500nm east of its start position in still air. This is referred to as its air position. However, if during that hour a wind had been blowing from the north at 50kts, the aeroplane would have been blown downwind by 50nm. In other words, its ground position would be 50nm south of its air position.

7. What has occurred is that although the aeroplane was heading 090(T) at a TAS of 500kts, it was being continuously blown downwind for the whole hour. Its path over the ground can be determined by joining the start ground position to the end ground position and is called its **track made good** (TMG). The distance between these two positions in the hour is a measure of its groundspeed.

8. This principle is depicted in Figure 3-1 by vectors. (A vector is a straight line in the direction of the velocity having a length representing the distance travelled). The heading and TAS vector by convention is denoted by one arrow on the line, the track and groundspeed vector by two arrows and the wind vector and speed by three arrows. This is known as the triangle of velocities.

FIGURE 3-1
Triangle of Velocities



9. **Drift**. The angle subtended between the heading and the track is referred to as the drift. It is defined as starboard if the aeroplane is blown to the right of its heading, (as in Figure 3-1), or port if it is blown to the left of its heading. Provided all three velocities remain constant then the drift angle will be the same no matter what the period of time. For instance, in the example, after 30 minutes, the aeroplane will have only travelled half of the distance in the air and over the ground but the drift will remain the same.

(Note. Drift angle is also known as the wind correction angle).

10. **Solving the Triangle of Velocities**. Of the six variables that make up the triangle of velocities, if four of them are known it is always possible to determine the other two. When plotting these velocities it is essential that they all be drawn to the same scale.

11. The following two examples require the construction of the triangle of velocities in each case. If you are in doubt as to how to proceed, read the solution first.

EXAMPLE 3-1

EXAMPLE

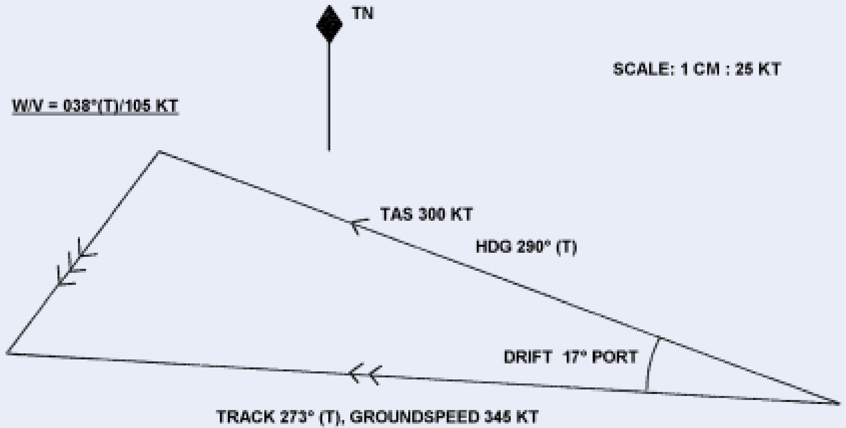
An aircraft has a TAS of 300 kt and is heading 290°(T). The drift is 17° port and the groundspeed as 345 kt. Determine by vector construction the wind velocity affecting the aircraft at this time.

SOLUTION

Remember that drift is the angle between the aircraft's heading and track measured left or right (port or starboard) of the **heading.**

1. Construct true north datum.
2. Select a suitable scale ie. 1cm - 25nm
3. Plot the true heading 290(T). Mark with one arrow.
4. Measure the distance to scale for 1 hour at TAS (300nm = 12cm). This point is the air position.
5. Calculate the true track (290-17) = 273°(T)
6. Plot the track 273(T). Mark with two arrows.
7. Along the track to the same scale, measure 1 hour at groundspeed (345nm = 13.8cm). This point is the ground position.
8. Join the ends of the two vectors.
9. Remember that wind blows from air position to ground position. Mark with three arrows.
10. Measure the length of the wind vector (approx. 4.2cm).
11. Convert to a wind speed using the scale selected.

FIGURE 3-2


TN
SCALE: 1 CM : 25 KT
W/V = 038°(T)/105 KT
TAS 300 KT
HDG 290° (T)
DRIFT 17° PORT
TRACK 273° (T), GROUNDSPEED 345 KT

EXAMPLE 3-2

EXAMPLE

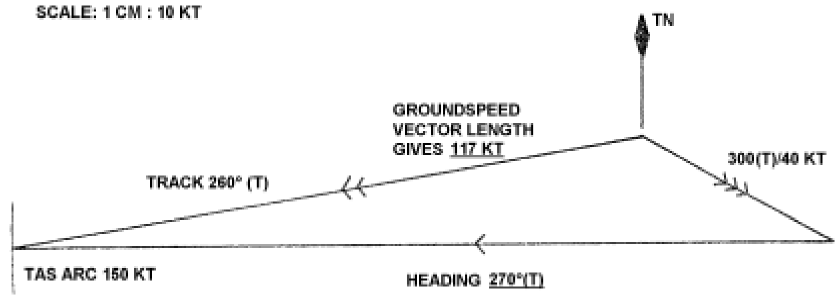
An aircraft is required to make good a track of 260°(T). The TAS is 150 kt and the W/V is 300°(T)/40 kt. Determine by vector construction the required heading and the groundspeed.

SOLUTION

The construction is as follows:

1. Choose a suitable scale. At Figure 3-3, 1 cm is taken to represent 10 kt.
2. Construct a True North Reference.
3. Construct a track line bearing 260°(T).
4. Draw the W/V vector. From 300°(T) for 4cm. (40kts).
5. From the end of the W/V vector, arc a curve 15 cm (TAS 150 kt) across the track line.
6. Join the end of the W/V vector to the point where the TAS arc crosses the track line, this line is the heading/TAS vector.
7. Measure the angle between True North and the Hdg/TAS vector.
8. Hdg = 270°(T).
9. Measure the length of the track/groundspeed vector, and using the chosen scale convert this distance to a speed (approx 11.7 cm).
10. G/S = 117 kt.

FIGURE 3-3



12. The plotting solution to the vector triangle is long and laborious. The solution is more easily obtained by using a navigation computer.

Navigation Computer

13. All candidates must be equipped with a suitable navigation computer. The Pooleys International CRP 5 computer is recommended because it performs all of the necessary functions. Candidates already possessing a computer should check that the plastic slide is calibrated up to at least 800 kt and that the reverse (circular sliderule) side includes the facility to compute Mach No. and to apply **compressibility corrections** to TAS calculations. Ideally, the sliderule side of the computer should include conversion markings to enable the candidate to convert units of distance (nautical miles, statute miles, kilometres), volume (Imperial gallons, US gallons, litres), and weight (pounds, kilograms). Specific gravity scales are also a great help when converting volumes of fuel to weight of fuel or vice versa.

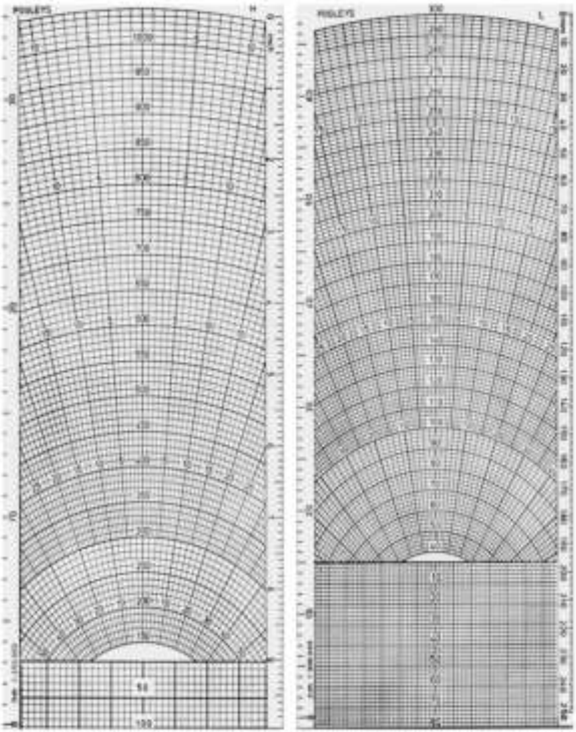
Description of the Navigation Computer (CRP 5)

14. **The sliding Speed Scale**. There are two scales, one on each side of the slider (see Figure 3-4). On one side is the low speed scale ranging from 40kts to 300kts with speed arcs at 2kt intervals and labelled every 10kts. The other side has a high speed scale which ranges from 150kts to 1050kts with speed arcs at 10kt intervals and labelled every 50kts. On both scales the centre of radius of the speed arcs is at 0kt. From this point, the straight line through the middle of the slider parallel to the sides represents the heading of the aeroplane. Drift lines radiate from the 0kt point. On the low speed scale they are at 2° intervals from 40kts to 100kts and at 1° intervals from 100kts to 300kts. On the high speed scale, they are at 2° intervals from 150kts to 300kts and at 1° intervals from 300kts to 1050kts.

15. Along the sides of the slider are distance scales, two on each side. On the left of the low speed side of the slider is a 1:500,000 nautical mile scale for measuring distance on that scale chart. A mm ruler is provided on the right of the low speed side. The scales on the high speed side are 1:250,000 for measuring distance on that scale chart, and an inch ruler on the right of that side.

16. By selecting the scale appropriate to the TAS and positioning the exact TAS beneath the centre dot on the face of the computer, commonly called the TAS dot, it is possible to reproduce the wind vector end of the triangle of velocities. The TAS dot is effectively the air position after one hour. The ground position can be found when the wind vector is added as will be shown in the following examples. At the bottom end of both sliding speed scales is a squared portion which is used to solve wind component problems.

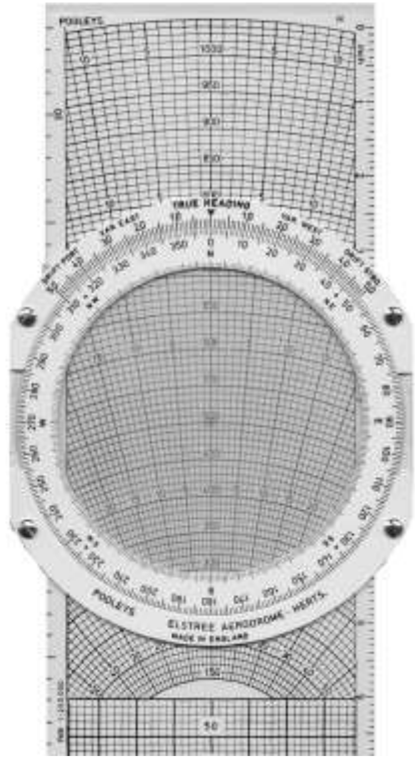
FIGURE 3-4
The Sliding Speed Scale



17. **The Computer Face.** The front of the computer consists of a rotating perspex face and scale divided into 360° known as the bezel (see Figure 3-5). The bezel is kept in position by two locking brackets and four screws. On the upper bracket is the true heading datum with a scale marked either side of the datum at 1° intervals. The left scale is used to indicate port drift or easterly variation and the right scale for starboard drift or westerly variation.

18. By rotating the bezel the centre line of the sliding scale is correctly aligned with the direction indicated against the heading datum. **Heading must always end up against the heading datum to complete the solution to any vector triangle problem.**

FIGURE 3-5
The Computer
Face



Using the Navigation Computer

19. There are two ways to solve the vector triangle known as the dot down' and the 'dot up' methods. The dot down method is the preferred method used on this course. There are three main triangle of velocities problems that are solved on the computer and three additional types of velocity problems, each of them is dealt with in this chapter. They are:

(a) Given: heading, TAS and wind velocity
Find: track and groundspeed.

(b) Given: heading, TAS, track and groundspeed
Find: wind velocity

(c) Given: track, TAS and wind velocity
Find: heading and groundspeed

(d) Given: wind velocity and runway direction
Find: along and across track wind component

(e) Given: wind direction, runway direction, maximum crosswind limitation and minimum acceptable headwind
Find: maximum and minimum wind speeds suitable for take-off

(f) Given: two or more headings and the drift on each and the TAS
Find: wind velocity

Worked Examples

20. The following examples and explanations cover these six computer solutions; please work slowly through them and repeat the exercise until the procedures are learned.

EXAMPLE 3-3
Given Heading TAS and Wind Velocity, Find Track and Groundspeed

EXAMPLE

An aircraft is heading 100°(T) with a TAS of 150 kt. The wind velocity is given as 360°(T)/25 kt. Determine by computer the expected track and groundspeed.

SOLUTION

Initially select the low speed sliding scale face up.

FIGURE 3-6

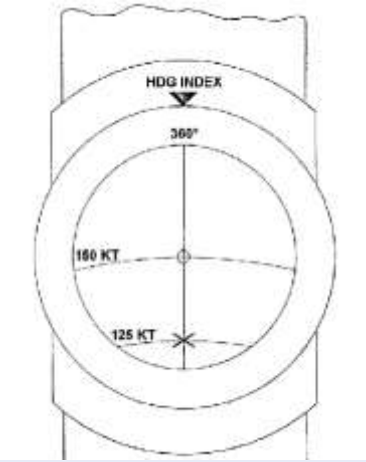
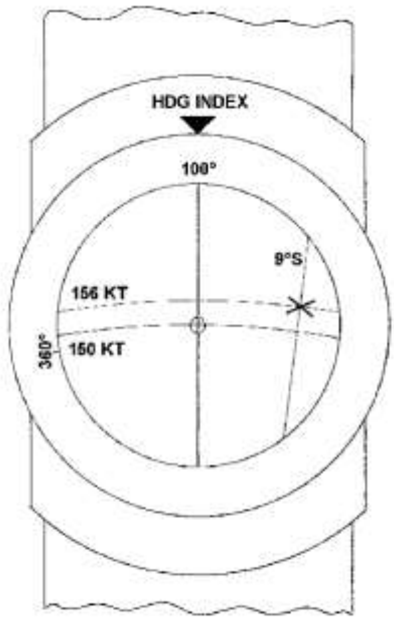


Figure 3-3 shows the first step in the solution. The compass rose is rotated until the direction **from** which the wind is blowing is positioned next to the heading index. A cross is then placed 25 kt **down** the perspex screen from the centre ring to represent the wind velocity.

The next step is illustrated at Figure 3-7. The compass rose is rotated until the heading, in this case 100°(T), is next to the heading index. The centre ring is aligned with the appropriate TAS on the speed scale, in this case 150 kt. The drift (9° starboard) is read off at the cross relative to the drift lines, and the groundspeed (156 kt), also at the cross, relative to the curved speed lines.

FIGURE 3-7



By reading 9° starboard drift on the upper outer scale, the track can be read against it on the inner scale as 109(T).

The solution in this case gives a track of 109°(T), and a groundspeed of 156 kt. Figure 3-8 relates back to the vector triangle to show just what has been achieved. The lines with the appropriate arrows (one for Hdg/TAS, two for track/groundspeed, three for W/V) show the wind vector end of the triangle of velocities which has effectively been reproduced on the computer. The Hdg/TAS and track/groundspeed vectors would, of course, meet at 0 kt on the speed scale.

FIGURE 3-8

EXAMPLE 3-4

Given Heading, TAS, Drift and Groundspeed, Find Wind Velocity

EXAMPLE

An aircraft has a TAS of 300 kt and is heading 290°(T). The drift is known to be 17° port and the groundspeed as 345 kt. Determine by computer the wind velocity affecting the aircraft at this time.

SOLUTION

This is example 3-2 reworked by computer. Initially select the high speed scale face up. Figure 3-9 shows the first steps in the solution. The compass rose is rotated until the heading, in this case 290°(T), appears under the heading index. The centre ring is placed over the appropriate TAS, in this case 300 kt. The cross is placed on the perspex to coincide with, in this case, 17° port drift and a groundspeed of 345 kt.

FIGURE 3-9

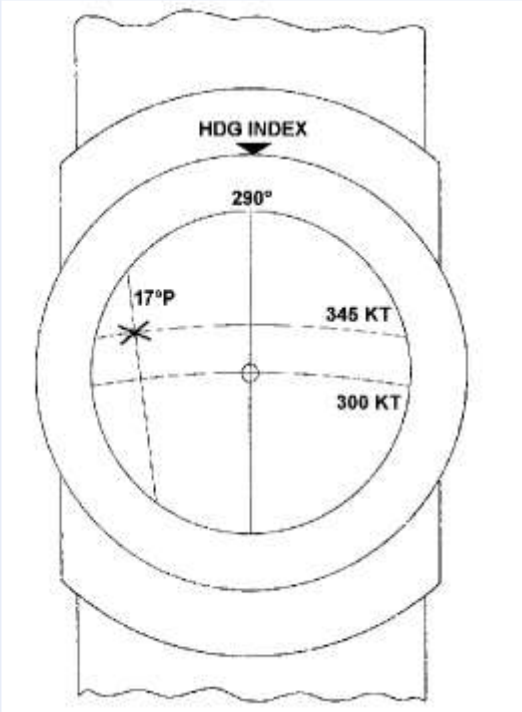


Figure 3-10 is identical to Figure 3-9, except that again the wind vector end of the triangle of velocities is shown.

FIGURE 3-10

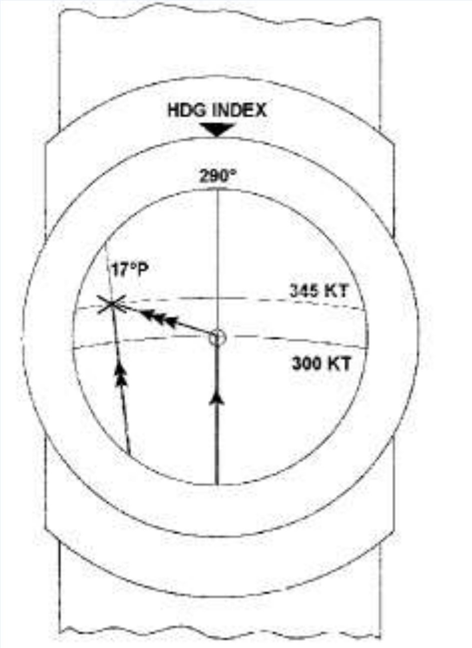
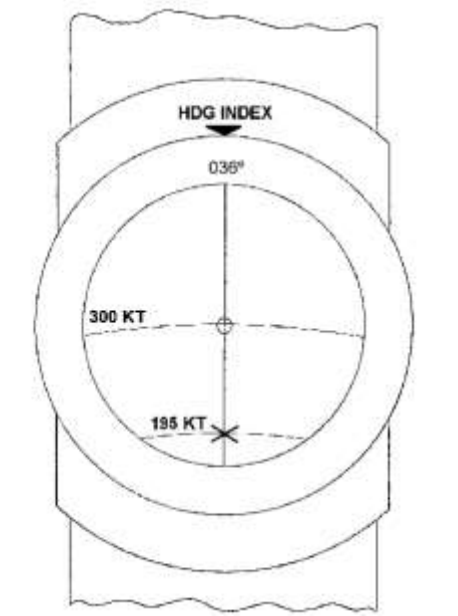


Figure 3-11 shows the final step in the solution. The compass rose is rotated until the cross is directly beneath the circle (on the zero degree drift line). The direction from which the wind is blowing 036°(T)] now appears against the heading index. The wind speed (105 kt) is extracted against the speed scale. In this example the W/V is therefore 036°(T)/105 kt.

FIGURE 3-11



EXAMPLE 3-5
Given required Track, TAS and Wind Velocity, Find Heading required and Groundspeed

EXAMPLE

An aircraft is required to make good a track of 260°(T). The TAS is 150 kt and the W/V is 300°(T)/40 kt. Determine by computer the required heading and the groundspeed.

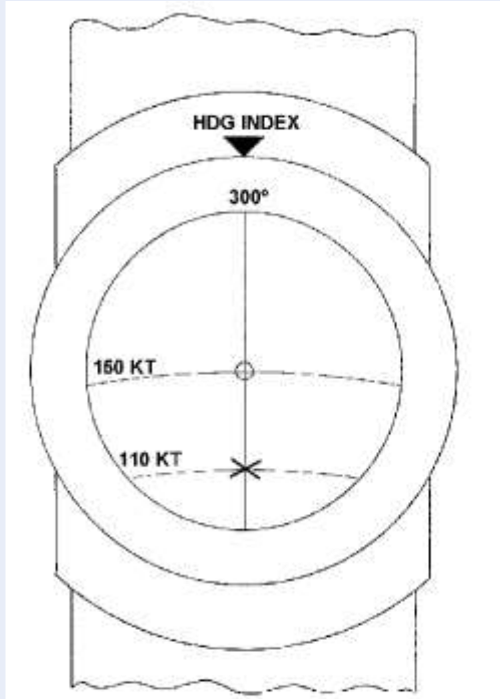
SOLUTION

See Figure 3-12, Figure 3-13, Figure 3-14 and Figure 3-15.

The computer solution of this problem requires a little more manipulation than in the previous two cases, and it is this solution which is most commonly completed incorrectly.

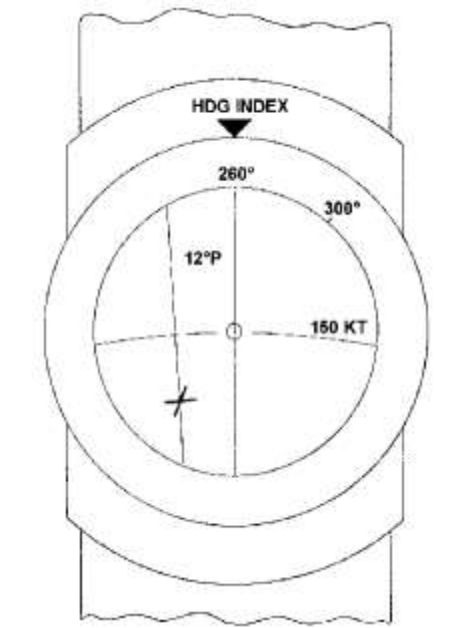
The first step is to put on the wind velocity as shown at Figure 3-12, in this case the W/V is 300°(T)/40 kt.

FIGURE 3-12



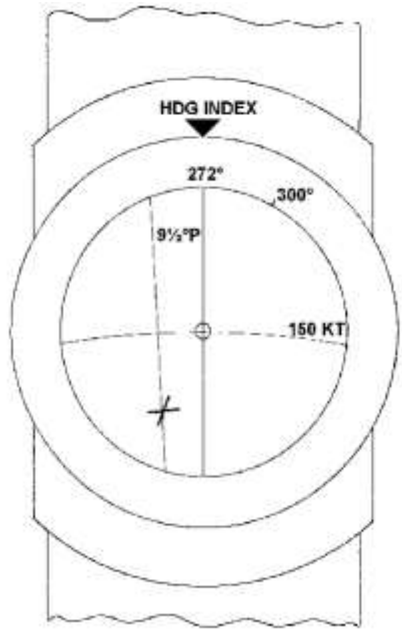
The next step is to place the ring over the TAS, in this case 150 kt. It is impossible to put the heading under the index mark, and so the track, in this case 260°(T), is placed under the index mark instead, see Figure 3-13.

FIGURE 3-13



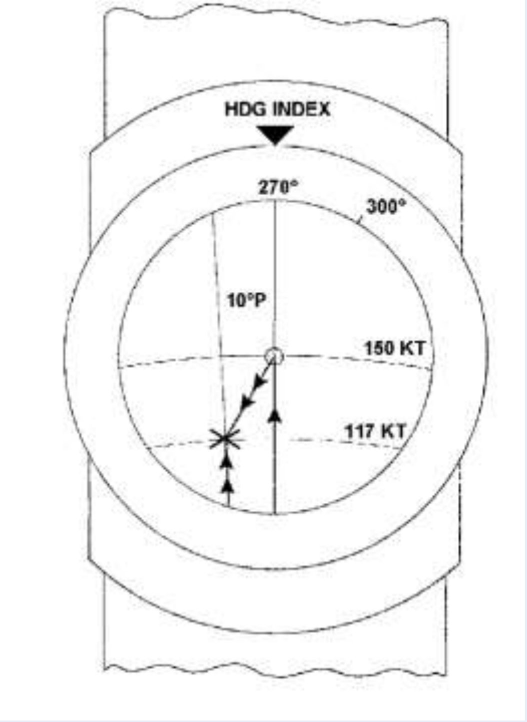
Were the heading of 260°(T) to be steered, the drift experienced would be 12° port (Figure 3-13). Using this assessment of drift the heading required to make good a track of 260°(T) would be (260° + 12° port) 272°(T), so set 272°(T) under the heading index, as shown at Figure 3-14.

FIGURE 3-14



With 272°(T) under the heading index the drift is now 9.5° port (Figure 3-14). In order to make good the required track of 260°(T) with this drift it is now necessary to head 269.5°(T), so put this value under the heading index. On this heading the drift now shows 10° port and so to make good a track of 260°(T) the heading required is 270°(T) and this value is set under the heading index arrow, the drift remains at 10° port, as shown at Figure 3-15.

FIGURE 3-15



Now take stock of the situation shown at Figure 3-15. The TAS is given as 150 kt and this value appears under the ring. The drift is shown as 10°port, the required track is 260°(T) and the value under the heading index is 270°(T), these three values therefore equate correctly.

Having completed the check outlined above, it is now possible to note the correct heading, in this case 270°(T), and the groundspeed of 117 kt.

It might be useful now to go through this example again before attempting the next example.

EXAMPLE 3-6
Given required Track, TAS and Wind Velocity, Find required Heading and Groundspeed

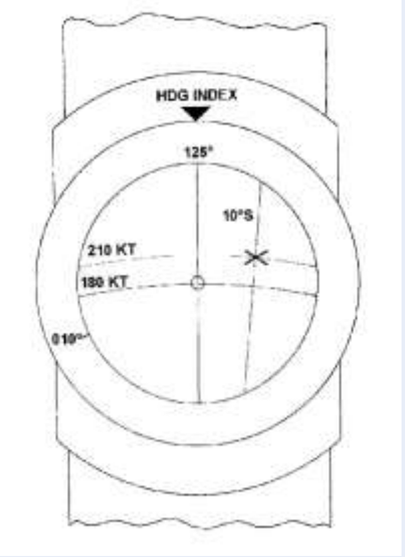
EXAMPLE

An aircraft is to make good a track of 135°(T) at a TAS of 190 kt with a forecast W/V of 010°(T)/40 kt. Determine by computer the required heading and the expected groundspeed.

SOLUTION

First of all mark the wind velocity on the computer. Then set the TAS under the ring and set the track under the heading index. The drift then shows 9° starboard.

FIGURE 3-16



To make good a track of 135°(T) with 9° starboard drift it is necessary to head 126°(T), so set this under the heading index. Drift now shows 10° starboard, so set 125°(T) under the heading index.

Check that the TAS is under the ring, and that heading 125°(T) plus or minus drift (in this case plus 10° starboard drift) equals the required track 135°(T).

Having completed the check note the heading and read off the groundspeed of, in this case, 210 kt, as shown at Figure 3-16.

The required heading is therefore 125°(T), and the groundspeed expected is 210 kt.

In all of the previous examples the north datum used has been true north. It is equally possible to solve any triangle of velocities problem using a magnetic north or grid north datum. Appreciate however that all three vector directions (heading, track and wind direction) **must use the same north datum.**

Wind Component Calculations

21. It is probable that the slide on your computer is printed with a square grid on one side. If this is so, you possess a quick and easy way of determining, for example, the head or tail component and the crosswind component for take-off and landing.

EXAMPLE 3-7

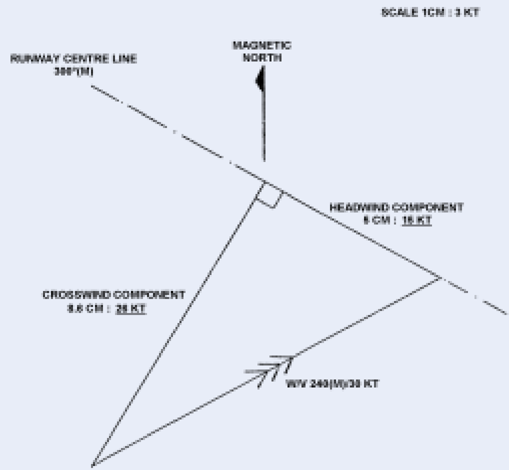
EXAMPLE

Given that the surface W/V is 240°(M)/30 kt, determine the wind components along and across runway 30.

SOLUTION

By vector diagram

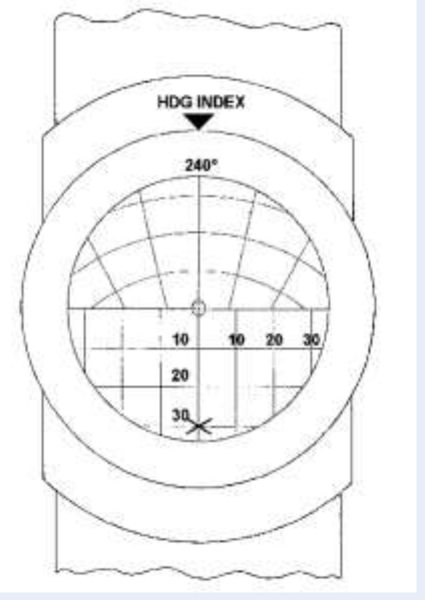
FIGURE 3-17



By computer:

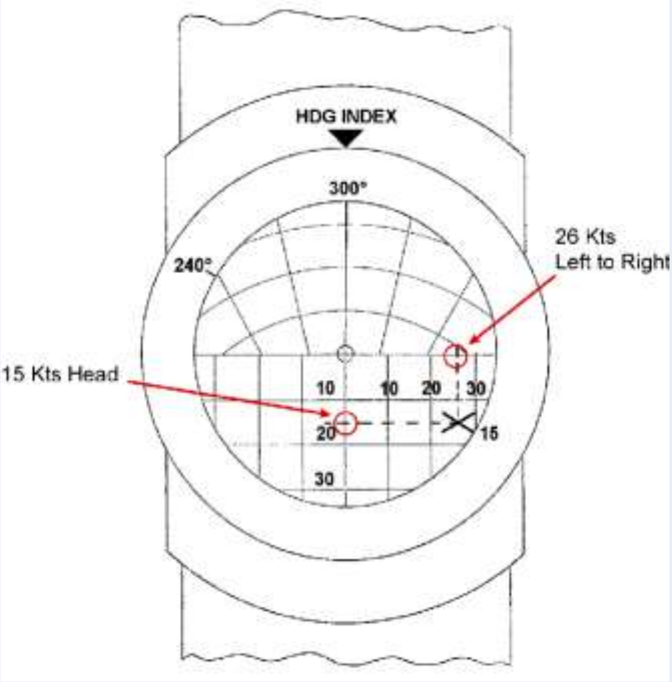
1. Select the low speed scale
2. Position the speed slide to zero on the squared portion
3. Mark on the wind velocity in the usual way down from the TAS dot.

FIGURE 3-18



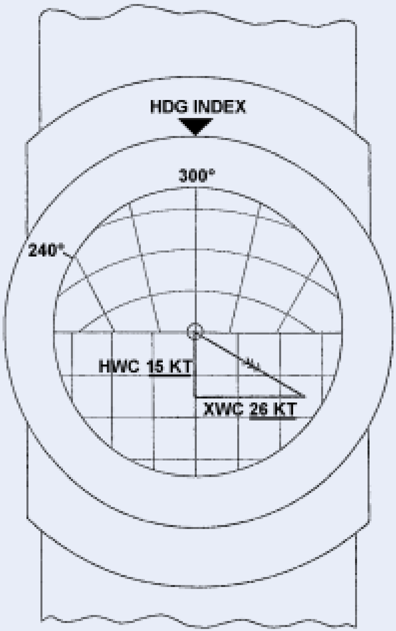
Next, rotate the compass rose until the runway direction (based on the same referenced direction as the wind) lies beneath the heading index, as shown at Figure 3-19 . It is now possible to read off the headwind and crosswind components using the chosen scale. From the wind dot make a horizontal line to intersect the central vertical scale. Read the headwind component. A vertical from the wind dot to intersect the horizontal scale. The crosswind component is measured from the TAS dot and must be labelled left to right or right to left. It is **not** a drift, it is a wind component

FIGURE 3-19



The following diagram shows the vector diagram as an aid to comprehension.

FIGURE 3-20



Note that the same procedure can be used where the wind component along track is required. In which case, the wind velocity is plotted in the same way and then the compass roise is rotated to put track under the heading index.

EXAMPLE 3-8
Given the Runway Direction and the Limiting Components, Find the Maximum and Minimum Wind Speeds

EXAMPLE

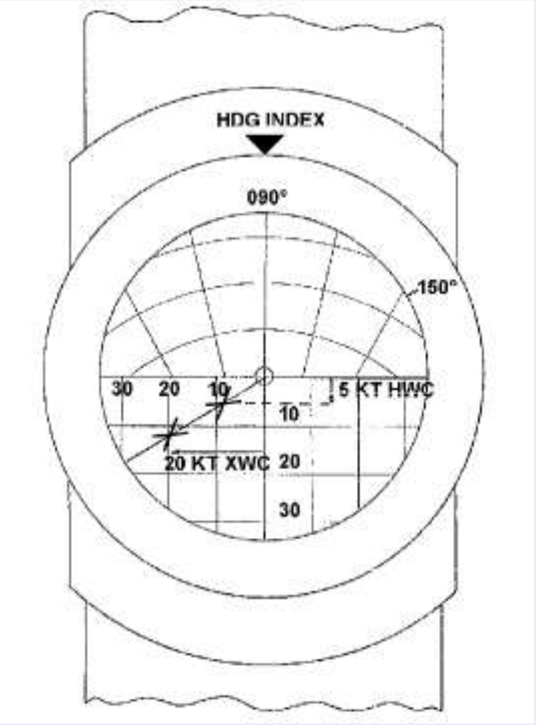
In order to take off on runway 09 an aircraft needs a headwind component of at least 5 kt. The maximum permissible crosswind component for this aircraft is 20 kt. The wind direction is given as 150°(M).

Determine the minimum and maximum wind speeds acceptable for take-off.

SOLUTION

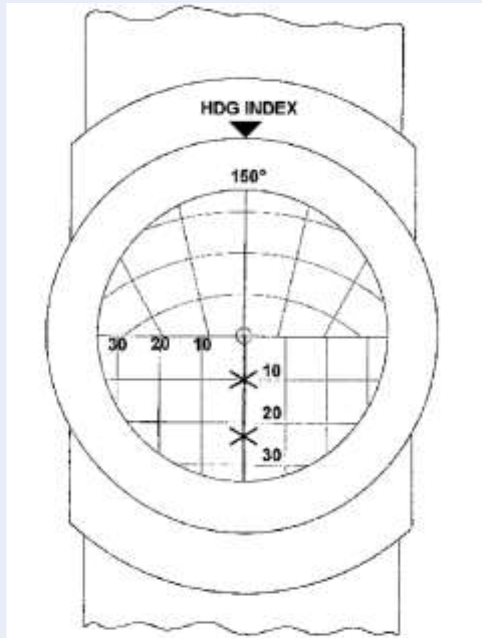
See Figure 3-21, and Figure 3-22. The first step is to set the wind direction against the heading index and to draw a line vertically all the way down the centre of the computer from the TAS ring. With the TAS ring over the top of the squared grid set the runway direction against the heading index. Move down the vertical axis to 5kt. Take a horizontal line to intersect the line already drawn. Mark this point. Travel horizontally from the TAS dot to 20kts crosswind. Now take a vertical to intersect the wind line. Mark this point.

FIGURE 3-21



Rotate the bezel until the wind direction line is again vertical and, using the chosen scale, read off the minimum and maximum permissible wind speeds of, in this case, 10 kt minimum and 23 kt maximum.

FIGURE 3-22



Multi Drift Wind Finding

22. A wind velocity can be found using a minimum of two headings and drifts provided the TAS is known. The following example uses 3 headings and drifts.

23. Determine the TAS of the aircraft and set this value under the ring on the computer.

24. Determine the drift on a given heading, for example, at a TAS of 150 kt let us assume that the drift is measured as 5° left on a heading of 030°(T). With 150 kt under the ring and 030° under the heading index draw a line all the way down the 5° left drift line.

25. Now turn the aircraft through 120° and determine the drift on the new heading, let's say that the drift on 150°(T) is determined as 9° right. Place the heading of 150° under the heading index and draw a line down the 9° right drift line.

26. Turn the aircraft through a further 120° and determine the drift. In this example the heading is 270° and the drift is determined as 4° left. Again place the heading under the index and draw a line down the 4° left drift line.

27. This process will result in three drift lines crossing at a common point. Rotate the compass rose until the intersection of the three drift lines is vertically beneath the TAS ring and read off the W/V of, in this case, 058°(T)/24 kt.

Note: Although a 'three-drift wind velocity' is the most accurate method, W/V can still be found from just two drifts and two headings.

Speed, Distance and Time Calculations

28. Speed is expressed in units of distance per unit time, for example nautical miles per hour, or knots.

$$\text{Thus: Speed (kt)} = \frac{\text{distance (nm)}}{\text{time (hours)}}$$

29. For navigational purposes it is usually more convenient to work in minutes rather than decimals of an hour. For example, use 75 minutes rather than 1.25 hours. Remember that an hour is 60 minutes, which is why the 60 unit on the inner scale of your computer is outlined or otherwise highlighted in some way.

30. To utilise the reverse side of the navigation computer for these purposes, the rotating inner scale represents time in minutes and the outer scale nautical miles.

EXAMPLE 3-9

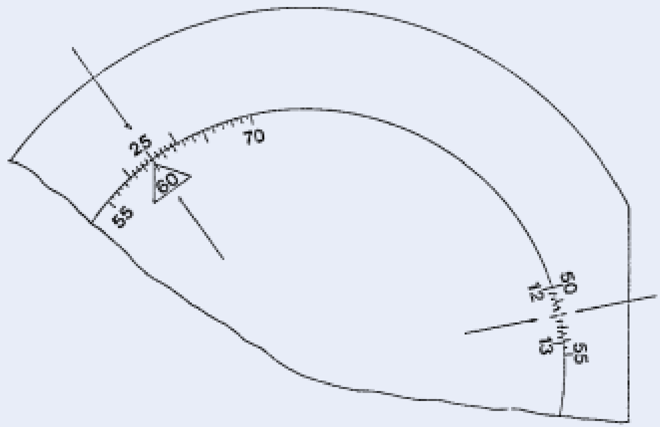
EXAMPLE

An aircraft travelled 52 nm over the ground in 12.5 minutes. Compute the groundspeed of this aircraft.

SOLUTION

Place 52 (nm) on the outer ring over 12.5 (mins) on the inner ring. The number which appears on the outer ring opposite 60 minutes (1 hour) on the inner ring which in this example is 25.0.

FIGURE 3-23



Mental arithmetic is required to place the decimal point. In this example, since the aircraft has covered approximately 50 nm in just over one fifth of an hour the groundspeed is obviously 250 kt.

Note. In the examinations, a calculator will be provided. Thus by using the original formula:

$$\text{Speed} = \frac{\text{distance}}{\text{time in minutes}} \times 60$$

By calculator:

$$\frac{52\ (\text{nm})}{12.5\ (\text{min})} \times 60 = 249.6\ \text{kt}$$

EXAMPLE 3-10

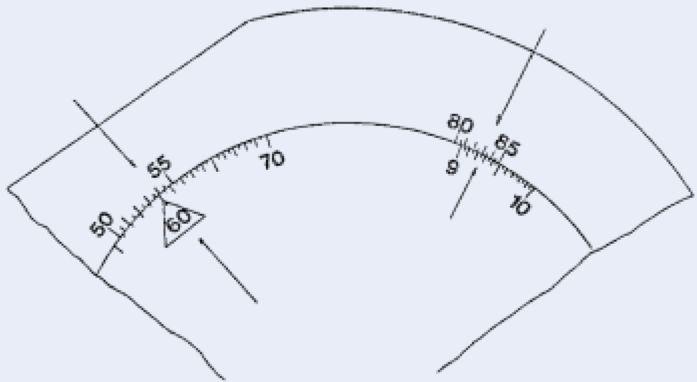
EXAMPLE

Determine how long will it take an aircraft with a TAS of 625 kt to cover a distance of 830 nm over the ground with a headwind component of 87 kt.

SOLUTION

The groundspeed is (625 - 87) 538 kt and this figure is placed on the outer ring over 60 on the inner ring. Find the distance (830 nm) on the outer ring, and the number which appears opposite it on the inner scale is the time, expressed in minutes. In this case the time required is 92.5 mins, or 1 hour 32.5 mins.

FIGURE 3-24



Transposing the original formula then:

$$\text{Time in minutes} = \frac{\text{dis tan ce}}{\text{groundspeed}} \times 60$$

By calculator:

$$= \frac{830}{538} \times 60$$

$$= 92.5\text{min}$$

$$= 1\text{hr } 32.5\text{min}$$

EXAMPLE 3-11

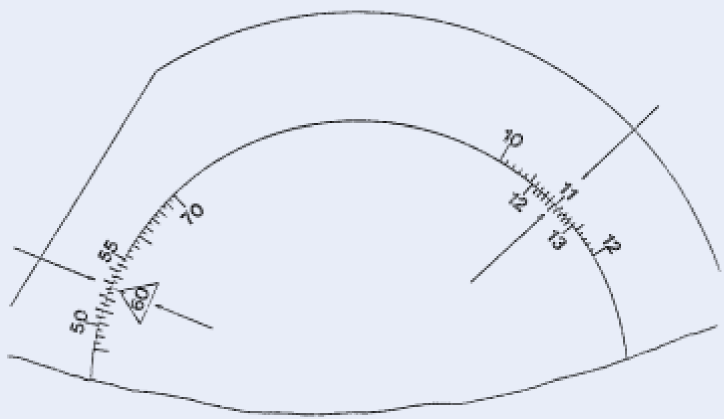
EXAMPLE

Determine how far an aircraft will travel over the ground in 2 hours and 5 minutes given a TAS of 465 kt and a tailwind component of 62 kt.

SOLUTION

By computer.

FIGURE 3-25



The groundspeed is (465 + 62) 527 kt and this figure is placed on the outer ring over 60 on the inner ring. Find the time on the inner ring (125 minutes). The number which appears over this figure is the distance, expressed in nautical miles. In this case the distance travelled is 1098 nm.

Transposing the formula:

$$\text{Distance} = \frac{\text{groundspeed} \times \text{time in minutes}}{60}$$

By calculator

$$= \frac{527(\text{kt}) \times 125\ (\text{min})}{60}$$

$$= 1098\ \text{nm}$$

Calibrated (Rectified) Airspeed to True Airspeed Calculations

31. Before making airspeed conversions, it is essential to understand the following definitions:

(a) **ASIR - Airspeed Indication Reading**. That which is indicated by an ASI without correction.

(b) **IAS - Indicated Airspeed.** The speed shown by the pitot/static airspeed indicator calibrated to reflect standard atmosphere adiabatic compressible flow at MSL and uncorrected for airspeed system errors.

(c) **CAS - Calibrated Airspeed**. The IAS corrected for position and instrument error. It is equal to TAS at MSL in a standard atmosphere. (Note. In the UK, CAS is often referred to as Rectified Airspeed (RAS) and is labelled as such on the CRP 5 Computer).

(d) **EAS - Equivalent Airspeed.** The CAS corrected for compressibility at the particular pressure altitude under consideration. It is equal to CAS in a standard atmosphere.

(Note that when applying the effect of compressibility using the CRP 5, the raw TAS is found first; if TAS is >300kt, this value is then corrected (reduced) by the amount necessary for compressibility).

(e) **TAS - True Airspeed**. The EAS corrected for density error and is the true speed of the aeroplane relative to the undisturbed air.

32. The following paragraphs deal with the method of converting CAS to TAS, given pressure altitude and temperature. (The reason for this conversion is dealt with in 022 - Instruments & Electronics).

EXAMPLE 3-12

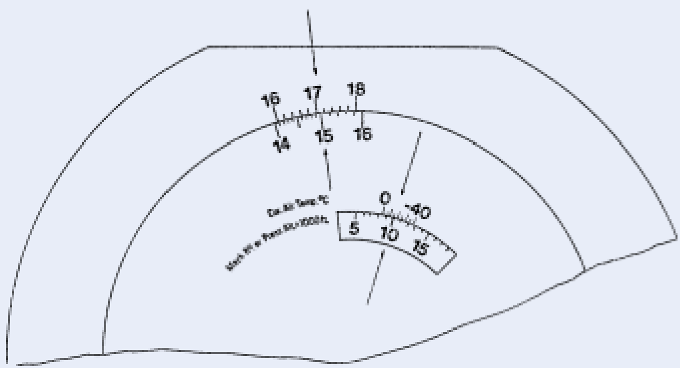
EXAMPLE

An aircraft is flying at Flight Level 100. The correct outside air temperature at this altitude is -15°C. Given that the aircraft is flying at a CAS of 150 kt, determine the TAS.

SOLUTION

First find the window on the rotating disc labelled **AIR SPEED**. FL 100 is 10,000 ft pressure altitude, so position 10 in the window against -15°C on the temperature scale. Check the reading on the outside scale against 150 kt CAS (RAS) on the inside scale. In this case the answer is 171 kt, and since the TAS is below 300 kt there is no need to apply a correction for compressibility.

FIGURE 3-26



Correction for Compressibility

33. The following example covers the use of the compressibility correction window. Again, the need for this correction (practically speaking at true airspeeds in excess of 300 kt) is explained fully in 022 - Instruments & Electronics.

EXAMPLE 3-13

EXAMPLE

An aircraft is flying at FL 350. The correct outside air temperature is -43°C. Given that the aircraft is flying at a CAS of 266 kt, determine the TAS.

SOLUTION

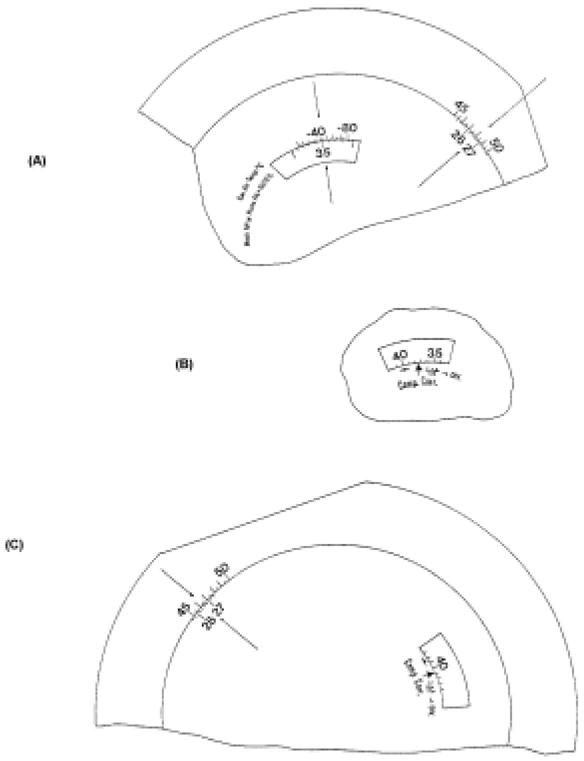
Set the pressure altitude (35,000 ft) against the correct outside air temperature(- 43°C) and read the TAS on the outside scale against the CAS (RAS) (266 kt) on the inside scale. In this case the initial reading of TAS (the raw TAS) is 480 kt, as shown at Figure 3-27.

Without moving the inner ring, turn the whole computer upside down in order to bring the compressibility correction window to the top, as shown at Figure 3-27. Now follow the instructions printed under the correction window. These instructions tell us to take the initial (raw) TAS, to divide this figure by 100, and to then subtract 3. In this example the initial TAS is 480 kt, which when divided by 100 gives 4.8. By subtracting 3 from this figure we finish up with 1.8.

In this case then, it is necessary to move the compressibility arrow 1.8 divisions in the direction shown by the small arrow (anti-clockwise), see Figure 3-27. The corrected TAS is now read on the outside scale against the CAS (266 kt) on the inner scale. The TAS, corrected for compressibility, is now 463 kt.

34. Different types and makes of computer will give slightly different answers to TAS corrected for compressibility, and in any event the process of correction is not a particularly accurate one.

FIGURE 3-27



35. In the event that your computer is not able to correct for compressibility, it is necessary to use the table shown at Figure 3-28. You are permitted to take this table into the examination with you. The procedure for using the table is outlined at the top of the table.

FIGURE 3-28

COMPRESSIBILITY CORRECTION TABLE

To use: Enter with CAS/RAS and altitude to extract the correction factor. Multiply the CAS/RAS by this factor and determine TAS from the computer using the CORRECTED CAS/RAS.

	Rectified Airspeed in Knots							
Pressure Altitude in ft	200	250	300	350	400	450	500	550
10,000	1.0	1.0	0.99	0.99	0.98	0.98	0.97	0.97
20,000	0.99	0.98	0.97	0.97	0.96	0.95	0.94	0.93
30,000	0.97	0.96	0.95	0.94	0.94	0.91	0.90	0.89
40,000	0.96	0.94	0.92	0.90	0.90	0.87	0.87	0.86
50,000	0.93	0.90	0.87	0.86	0.86	0.84	0.84	0.84

Mach Number Calculations

36. Again Machmeters are dealt with fully in Instruments. One quick and easy way of converting Mach number to TAS, and of determining the local speed of sound (LSS), is on the computer, and therefore these solutions are covered in the following paragraphs.

37. When working with Mach number always remember the following important points:

(a) The local speed of sound (Mach 1.0) depends **only** on the ambient temperature.

(b) Mach number is the TAS divided by the local speed of sound, or: $\text{Mach no.} = \dfrac{\text{TAS}}{\text{LSS}}$

(c) The Machmeter does not suffer from compressibility errors and so it is not necessary to apply a compressibility correction when converting Mach number to TAS.

EXAMPLE 3-14

EXAMPLE

An aircraft is flying at Mach 0.86 and the true ambient air temperature is -42°C. Determine:

(a) The local speed of sound.

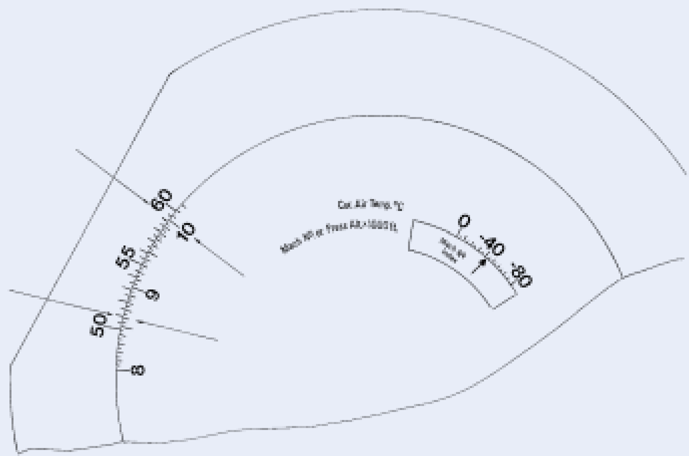
(b) The TAS.

SOLUTION

(a) The first step is to align the **Mach number index arrow** in the window marked **Mach No. or Pressure Altitude x 1000 ft** with the ambient temperature. The figure on the outer ring which appears opposite 10 (Mach 1.0) on the inner ring is the local speed of sound in knots. In this case the LSS is 592 kt.

(b) In this example the aircraft is flying at Mach 0.86 (592 x 0.86 kt), and the appropriate TAS is found on the outside ring opposite 8.6 (Mach 0.86) on the inner ring. In this case the TAS is 509 kt.

FIGURE 3-29



38. It is also necessary to understand the relationship between Mach No and CAS/RAS. The Machmeter indicates the ratio between the aircraft's TAS and the local speed of sound but it does this by comparing airspeed (CAS or RAS) with the static pressure. Thus, the output of the Machmeter is directly proportional to the CAS (RAS), but inversely proportional to static pressure.

$$\text{Mach No} \propto \frac{\text{CAS (RAS)}}{\text{Static Pressure}}$$

39. From this relationship it can be deduced that if an aircraft climbs at a constant CAS, the static pressure reduces with altitude, so Mach No increases. A constant CAS in a descent would have the opposite effect.

40. This principle can be demonstrated on the navigation computer and is illustrated in the following example.

EXAMPLE 3-15

EXAMPLE

An aircraft is climbing at a constant CAS in the International Standard Atmosphere. Determine the effect on:

(a) TAS

(b) Mach No

SOLUTION

(a) As the aircraft climbs, air density decreases. Select a CAS of 100 kts, say. In the International Standard Atmosphere the temperature is +15°C at MSL. Set these readings on the CRP-5 and read off the TAS on the outer ring against the CAS, 100 kts, on the inner ring.

Now repeat the process for FL100. Since temperature is assumed to decrease at 1.98°/1000ft the temperature at FL100 should be approximately -5°C. Set these conditions on the CRP-5 Airspeed window and read off the TAS on the outer ring against the CAS, constant at 100kts, on the inner ring.

TAS at MSL = 100kts
TAS at FL100 = 116kts
Therefore TAS increases

(b) Mach No $= \frac{TAS}{LSS}$ From a) above we know that TAS is increasing when an aircraft climbs at a constant CAS.

The local speed of sound varies only with the ambient temperature.

As the aircraft climbs, the temperature reduces; as temperature reduces, the local speed of sound reduces.

So, TAS is increasing, LSS is decreasing.

$\therefore$ the ratio $\frac{TAS}{LSS}$ MUST be increasing.

$\therefore$ Mach No will increase in a constant CAS Climb.

Conversion Factors

41. On the outer scale of the circular slide rule are datum arrows to be used for conversions of weight, volume and distance. In the event that your computer does not have any or all of the necessary conversion marks it is important that you memorise the conversion factors given at Figure 3-30. The diagram at Figure 3-32 may help with this task although the answers may differ slightly because of the abbreviated conversion factors. Using the diagram permits the use of the calculator to complete the conversion of volume to mass and vice versa.

Note. Conversion factors must be learned and such aide memoires are not permitted in the examination.

FIGURE 3-30

1 kilogram	=	2.2 pounds
1 Imperial gallon	=	1.2 US gallons
1 Imperial gallon	=	4.55 litres
1 US gallon	=	3.8 litres
1 nautical mile (6080ft)	=	1.854 kilometres
1 nautical mile	=	1.152 statute miles
1 statute mile	=	1.610 kilometres
1 metre	=	3.28 feet
1 inch	=	2.54 centimetres
1 metre/second	=	1.95 knots

EXAMPLE 3-16

EXAMPLE

Convert 3615 kilograms into pounds.

Using the conversion datum arrows on the computer.

Place the weight in kilograms (3615) on the inner scale under the **kg** mark on the periphery of the sliderule. The number which appears on the inner scale under the **lbs** datum arrow on the periphery of the sliderule read the corresponding weight in pounds, in this case 7970 lb.

FIGURE 3-31

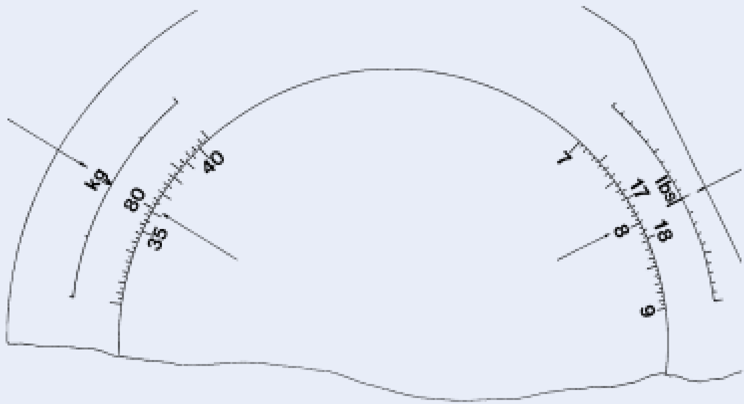
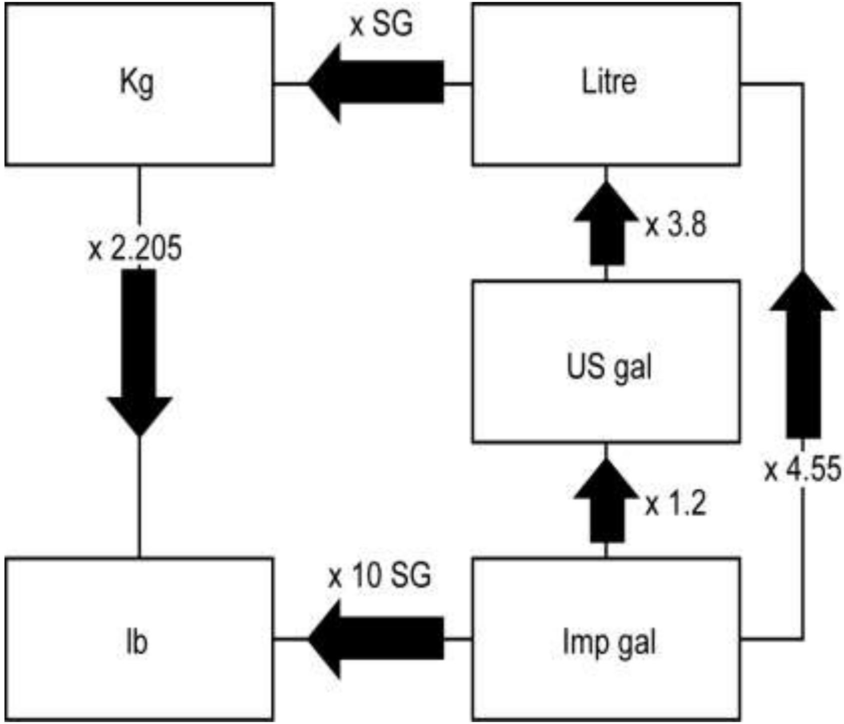


FIGURE 3-32



42. When travelling in the direction of the arrows multiply, when travelling in the opposite direction, divide.

43. When converting litres of any liquid to kilograms the volume must be multiplied by the specific gravity, or when converting kilograms to litres the weight must be divided by the specific gravity. Similarly, when converting Imperial gallons to pounds the volume must be multiplied by 10 x the specific gravity, or to convert pounds to Imperial gallons the volume must be divided by 10 x specific gravity of the liquid.

44. Aviation fuels and oils are lighter than pure water, therefore their specific gravities will be less than 1.0.

EXAMPLE 3-17

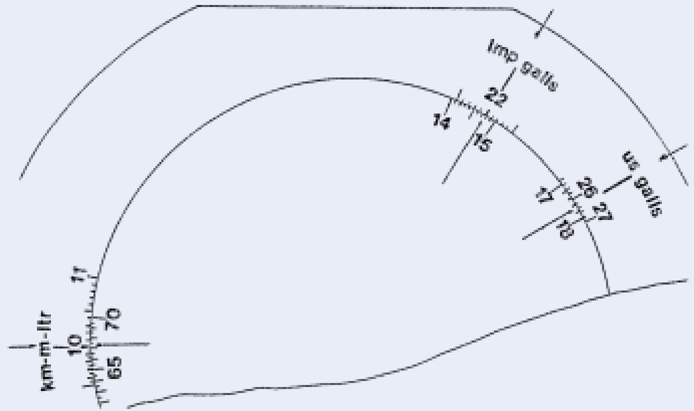
EXAMPLE

Convert 6720 litres into both US and Imperial gallons.

SOLUTION

Using the conversion marks on the computer.

FIGURE 3-33



Place the value in litres (6720) on the inner scale under the **km-m-ltr** mark on the periphery of the sliderule. The volume in Imperial gallons is given on the inner scale under the **imp gals** mark on the periphery, in this case 1477 gallons. Likewise, the volume in US gallons is found under the **US gals** mark, in this case 1768 gallons.

By calculator:

Litres	÷	3.8	=	US gals
US gals	÷	1.2	=	Imp gals
or				
Litres	÷	4.55	=	Imp gals
6720	÷	3.8	=	1768 US gals
1768	÷	1.2	=	1474 Imp gals
6720	÷	4.55	=	1470 Imp gals

45. Exactly the same procedures are adopted when converting units of distance, taking full advantage of conversion marks when available, but otherwise using the conversion factors and multiplying or dividing as required.

46. Of course the conversion marks or conversion factors used for converting units of distance can equally well be employed when converting units of speed, for example knots into statute miles per hour or kilometres per hour.

47. For the radio syllabus it is often necessary to convert speed in knots into metres per second or vice versa. The conversion factor in this case is 1.95, such that:

(a) When converting speed in knots to speed in metres per second, divide the speed in knots by 1.95.

(b) When converting the speed in metres per second to speed in knots, multiply by 1.95.

48. Try the following examples.

EXAMPLE 3-18

EXAMPLE

The weight of fuel required for a flight is calculated as 2660 kg. If the specific gravity of the fuel is 0.78, give the volume of fuel required in:

(a) Litres.

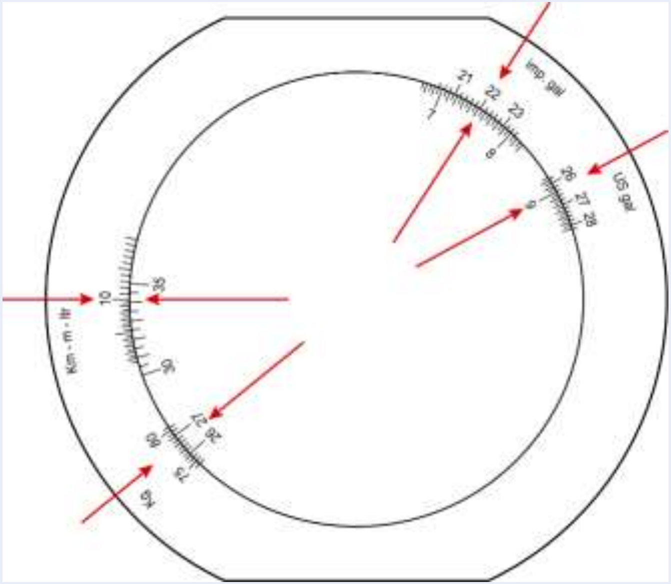
(b) Imperial gallons.

(c) US gallons.

SOLUTION

By computer.

FIGURE 3-34



By calculator:

(a)	Kg	÷	SG	= Litre	
	2660	÷	0.78	= 3410 Litres	
(b)	Litre	÷	4.55	= Imp gals	
	3410	÷	4.55	= 749.5 Imp gals	
(c)	Litre	÷	3.8	= US gals	
	3410	÷	3.8	= 897 US gals	
	OR				
	Kg	x	2.205	÷ 10SG	= Imp gals
	2660	x	2.205	÷ 7.8	= 752 Imp gals
	Imp gals	x	1.2		= US gals
	752	x	1.2		= 902 US gals

Although the answers are slightly different, examination choices of answer should accommodate such differences.

EXAMPLE 3-19

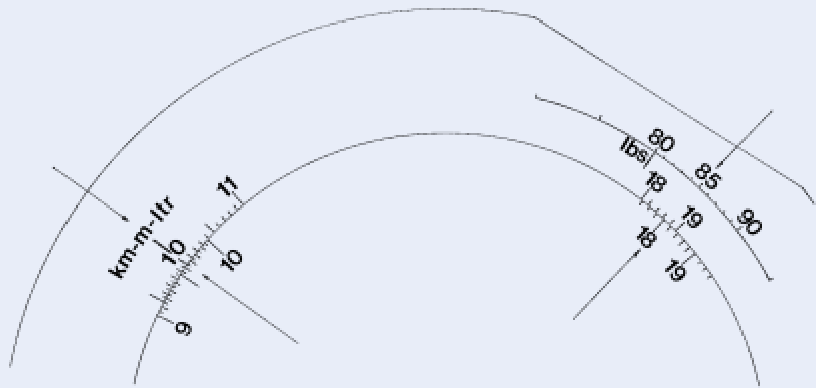
EXAMPLE

Determine the weight in pounds of 9650 litres of fuel having a specific gravity of 0.85.

SOLUTION

By computer.

FIGURE 3-35



By calculator.

9650 litres x 0.85	=	8203 kg
8203 kg x 2.2	=	18,045 lb

By calculator.

Litres	x	SG	x	2.205	= lbs
9650	x	.85	x	2.205	= 18087 lbs
OR					
Litres	÷	4.55	x	10 SG	= lbs
9650	÷	4.55	x	8.5	= 18027 lbs

Altitude Calculations

49. Before any calculations can be attempted, it is essential to understand the following definitions:

(a) **Height** - the vertical distance of a point or object considered as a point measured from a specified datum.

(b) **Elevation** - the vertical distance of a fixed point above mean sea level.

(c) **Altitude** - the vertical distance of a level, a point or an object considered as a point measured from mean sea level.

(d) **QFE** - the atmospheric pressure at the aerodrome pressure datum point; when set in the sub scale of an altimeter of an aircraft the altimeter will indicate height above the QFE reference datum.

(e) **QNH** - the atmospheric pressure at the aerodrome pressure datum point converted to mean sea level using the standard atmospheric lapse rate of temperature and pressure. When set in the sub-scale of the altimeter it will cause the altimeter to indicate the altitude above msl.

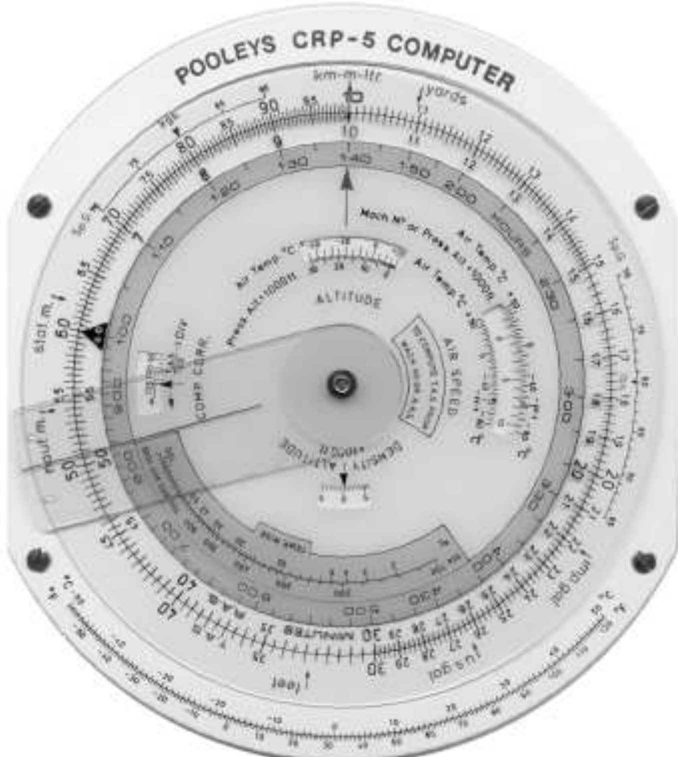
(f) **Pressure Altitude** - the altitude above the standard atmosphere pressure datum of 1013.25mbs. When set on the altimeter sub scale the altimeter will indicate pressure altitude. At the surface level of an aerodrome this is known as QNE.

(g) **Density Altitude** - is the altitude in the standard atmosphere at which the prevailing density occurs.

The Calculation of True and Indicated Altitude

50. The navigation computer uses the standard atmosphere temperatures and pressure altitude to enable indicated altitude to be converted to true altitude. This can be seen if the inner and outer circular scales are aligned at the same value. Line up 10 on the outer scale with 10 on the inner scale. In the **ALTITUDE** window against 0 feet pressure altitude is a temperature of +15°C ie the standard atmosphere temperature at MSL. The standard atmosphere temperature can be determined from this window for any pressure altitude up to 30,000 feet. See Figure 3-36.

FIGURE 3-36



POOLEYS CRP-5 COMPUTER
km-m-ltr
yards
stat m.
naut m.
feet
Imp gal
US gal
MINUTES
HOURS
ALTITUDE
AIR SPEED
COMP CORR
DENSITY ALTITUDE
Air Temp °C
Press Alt ×1000ft.
Mach No or Press Alt ×1000ft.
R.A.S.

51. By setting the ambient temperature against the pressure altitude, in the altitude window, the altitude is corrected for the effect of the difference of temperature from standard.

52. It can be seen that the standard temperature at 25,000 feet pressure altitude is -35°C. If, however, the ambient temperature is actually -20°C, it is warmer than standard by 15°C. Therefore, the atmosphere is less dense than standard and the true altitude will be higher.

53. Set pressure altitude 25,000 feet against -20°C in the altitude window. Read on the inner of the two outer circular scales the indicated pressure altitude 25 (representing 25,000ft), the true altitude is opposite to this on the outer scale and reads 26,600 feet.

EXAMPLE 3-20

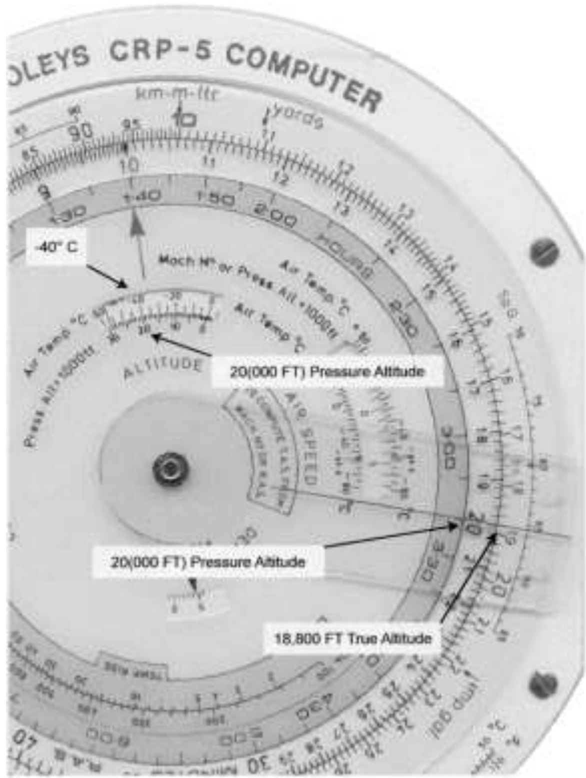
EXAMPLE

Pressure altitude 20,000 feet; Ambient temperature -40°C. Determine the true altitude

SOLUTION

Set pressure altitude 20,000 feet against temperature -40°C. Against 20 on the inner scale, the true altitude is 18,800 feet.

FIGURE 3-37


CRP-5 COMPUTER
-40° C
20(000 FT) Pressure Altitude
20(000 FT) Pressure Altitude
18,800 FT True Altitude

54. Note. A useful rule of thumb solution for this type of problem is:

Altitude difference = 4 x temp deviation x altitude in thousands.

55. In this example, the temperature deviation is calculated by comparing ambient temperature with standard. Thus, at 20,000ft, the standard is +15-40 = -25°C. Ambient temperature is -40°C, ie. 15° colder. Therefore, correction to indicated altitude is:

4 x -15 x 20 = -1200'.

Therefore, true altitude is 18,800'.

EXAMPLE 3-21

EXAMPLE

A mountain is 9000 feet elevation. Determine the pressure altitude that will clear the obstacle by 1000 feet if the ambient temperature at 9,0000 feet pressure altitude is -20°C.

SOLUTION

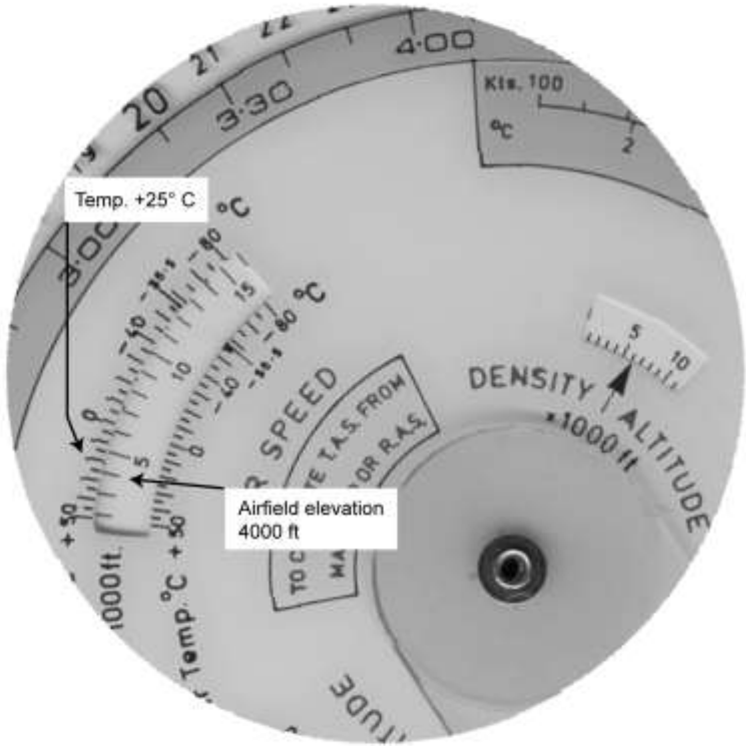
The true altitude required is 9000 feet + 1000 feet = 10,000 feet. Set -20°C against pressure altitude 10,000 feet. Against 10,000 feet on the outer scale read pressure altitude 10,580 feet.

56. From these examples it can be deduced that if an aircraft is flying at a constant indicated altitude from a cold air mass to a warm air mass, the true altitude increases due to the temperature change and when flying from a warm to a cold air mass, the altitude reduces.

The Calculation of Density Altitude

57. The density of the air directly affects the performance of an aeroplane. In a dense atmosphere, it will perform well but in a low density atmosphere the performance will be reduced. Most aeroplane flight manuals schedule the performance against pressure altitude but some smaller types do not. The performance of these aircraft is scheduled against density altitude which for any given aerodrome elevation or pressure altitude and ambient temperature, the density altitude can be determined from the navigation computer. The aerodrome elevation on the pressure altitude scale is set against the ambient temperature in the AIRSPEED window. The density altitude is then displayed against the datum arrow in the density altitude window.

FIGURE 3-38



Temp. +25° C
Airfield elevation
4000 ft
DENSITY ALTITUDE
×1000 ft
Kts. 100
°C
4·00
3·30
3·00
°C
Temp. °C
1000ft.

EXAMPLE 3-22

EXAMPLE

Aerodrome elevation 4000ft. Ambient temperature +25°C

SOLUTION

Density altitude 6100ft.

58. An alternative 'Rule of Thumb' method to determine the density altitude is to use the formula:

Density altitude = Aerodrome elevation + (120 x ISA deviation)

59. In the above example then:

Density altitude = 4000 + (120 x 18) = 6160 ft.

Note. When temperature is warmer than standard, the correction to altitude is added and, when colder than standard, is subtracted.

EXAMPLE 3-23

EXAMPLE
Aerodrome elevation 6000ft. Ambient temperature -10°C
SOLUTION
By computer Density altitude 4500ft. By calculator = 4440ft [(6000 + (120 x -13))]

Temperature Calculations

60. Temperature is the measure of the intensity of heat in a body. The ambient air temperature is therefore a measure of the heat in the free air at that time and is referred to as the static air temperature (SAT). An aircraft travelling through the air compresses the air ahead of it and suffers the full effect of that compression at the aircraft leading edges of the wing etc. where the air is brought almost to a halt momentarily. The compression heats the air adiabatically, also in and around the area of the temperature sensing probe. The temperature sensed and indicated is therefore a falsely high temperature. The increase in temperature due to this effect is known as 'ram-rise' and the indicated temperature is referred to as the total air temperature (TAT). Depending on the type of temperature probe used to determine the temperature, 'ram-rise' can be calculated with a varying degree of accuracy and the TAT corrected by that amount. This is called the 'recovery factor' or 'recovery coefficient'.

61. The recovery factor can be calculated from the appropriate formula or from a graph.

62. TAT cannot be used for airspeed conversions; it must first be corrected to become the corrected outside air temperature (COAT).

63. The CRP 5 possesses a temperature rise scale calibrated against indicated airspeed values. The values enable TAT to be corrected to give an approximate SAT if required.

Self Assessed Exercise No. 3

QUESTIONS:

QUESTION 1.

Find Track and Groundspeed (G/S)

Heading	TAS	W/V	Drift	Track	G/S
355	400	210/60			
170	200	090/50			
090	150	360/30			
160	310	040/70			
302	240	270/50			
025	200	340/40			
062	260	140/50			
115	230	100/60			
145	365	270/35			
170	410	100/110			

QUESTION 2.

Find wind velocity

Heading	TAS	Track	G/S	Drift	W/V
270	250	267	213		
155	300	159	330		
040	350	035	380		
210	620	205	538		
226	260	205	169		
310	400	300	450		
260	355	268	282		
225	150	217	187		
047	173	058	197		
083	197	070	228		

QUESTION 3.

Find heading and groundspeed

Track	TAS	W/V	Drift	Heading	G/S
120	180	340/50			
260	300	140/75			
020	240	120/40			
219	200	310/74			
163	120	075/23			
010	220	350/65			
350	420	010/100			
290	510	260/90			
230	240	250/50			
120	120	210/20			

QUESTION 4.

Find headwind and crosswind components

Runway(M)	W/V(M)	Headwind	Crosswind
090	120/40		
310	270/30		
230	270/30		
030	010/50		
140	310/30		
270	060/20		
210	180/40		
160	060/20		
110	350/30		
060	030/25		

QUESTION 5.

Find minimum and maximum wind speeds

Runway (M)	Wind Direction (M)	Min H/ Wind	Max C/Wind	Min Wind Speed	Max Wind Speed
030	070	10 kt	20 kt		
075	040	20 kt	20 kt		
250	290	15 kt	25 kt		
290	250	20 kt	30 kt		
345	010	20 kt	20 kt		
010	330	15 kt	15 kt		
330	010	20 kt	20 kt		
270	240	15 kt	20 kt		
240	270	5 kt	12 kt		
210	265	10 kt	17 kt		

QUESTION 6.

Convert Rectified Air Speed (RAS) to True Air Speed (TAS) or vice versa.

Flt Level	COAT (C)	RAS	TAS
250	-30	180	
200	-10	250	
300	-30	250	
120	-30	150	
250	-35	340	
420	-60	230	
350		165	300
270	-25	300	
310	-40	250	
210	-10		250

QUESTION 7.

Convert Mach No to True Air Speed (TAS) or vice versa

Ambient Temperature	Mach No	TAS
-30	0.75	
-40		542
	0.85	510
-70	0.90	
-20		510
	0.76	460
	0.85	515
-20	0.75	
-25		500
-35		421

QUESTION 8.

Speed/Distance/Time computations

Groundspeed	Distance	Time (mins)
210	198	
	324	42
278		34
352		27
420	450	
180	69	
	150	32
	60	10
450	55	
310		17

QUESTION 9.

Convert the distances

Kilometres	Statute Miles	Nautical Miles
		91
	215	
200		
	168	
		280
420		
	420	
	77	
		31
2,457		

QUESTION 10.

Convert the volumes

Imperial Gallons	US Gallons	Litres
1,420		
	10,900	
		26,010
	2,320	
622		
		14,500
47		
	7,530	
685		
		2,870

QUESTION 11.

Convert the weights

Pounds (lbs)	Kilograms (kg)
723	
	19,400
	6,120
22,320	
	520
980	
250	
	7,530
685	
	93,000

QUESTION 12.

Convert the weight to volume/volume to weight

Pounds (lbs)	Kilograms (kg)	SG	US Gallons	Imp. Gallons	Litres
15,400		0.80			
		0.79			51,000
10,000			1,600		
		0.78		750	
153,500				18,000	
	1,400	0.80			
71,000					40,000
	2,450		818		
	1,980			560	
	490				610

ANSWERS:

ANSWER 1.

Heading	TAS	W/V	Drift	Track	G/S
355	400	210/60	4S	359	450
170	200	090/50	15S	185	198
090	150	360/30	11S	101	153
160	310	040/70	10S	170	350
302	240	270/50	8S	310	199
025	200	340/40	9S	034	174
062	260	140/50	11P	051	253
115	230	100/60	5S	120	172
145	365	270/35	4P	141	383
170	410	100/110	15S	185	387

ANSWER 2.

Heading	TAS	Track	G/S	Drift	W/V
270	250	267	213	3P	286/39
155	300	159	330	4S	014/38
040	350	035	380	5P	173/45
210	620	205	538	5P	239/96
226	260	205	169	21P	257/120
310	400	300	450	10P	070/88
260	355	268	282	8S	233/85
225	150	217	187	8P	009/44
047	173	058	197	11S	288/43
083	197	070	228	13P	199/57

ANSWER 3.

Track	TAS	W/V	Drift	Heading	G/S
120	180	340/50	10S	110	216
260	300	140/75	12S	248	330
020	240	120/40	10P	030	243
219	200	310/74	22P	241	188
163	120	075/23	11S	152	117
010	220	350/65	6S	004	158
350	420	010/100	5P	355	325
290	510	260/90	5S	285	429
230	240	250/50	4P	234	192
120	120	210/20	10P	130	118

ANSWER 4.

Runway(M)	**W/V(M)**	**Headwind**	**Crosswind**
090	120/40	35 H/W	20 R-L
310	270/30	23 H/W	19 L-R
230	270/30	23 H/W	19 R-L
030	010/50	47 H/W	17 L-R
140	310/30	30 T/W	5 R-L
270	060/20	17 T/W	10- R-L
210	180/40	35 H/W	20 L-R
160	060/20	4 T/W	20 L-R
110	350/30	15 T/W	26 L-R
060	030/25	22 H/W	12 L-R

ANSWER 5.

Runway (M)	Wind Direction (M)	Min H/ Wind	Max C/ Wind	Min Wind Speed	Max Wind Speed
030	070	10 kt	20 kt	13 kt	31 kt
075	040	20 kt	20 kt	25 kt	34 kt
250	290	15 kt	25 kt	19 kt	39 kt
290	250	20 kt	30 kt	26 kt	47 kt
345	010	20 kt	20 kt	22 kt	47 kt
010	330	15 kt	15 kt	20 kt	23 kt
330	010	20 kt	20 kt	27 kt	31 kt
270	240	15 kt	20 kt	17 kt	40 kt
240	270	5 kt	12 kt	5 kt	24 kt
210	265	10 kt	17 kt	18 kt	21 kt

ANSWER 6.

Flt Level	COAT (C)	RAS	TAS
250	-30	180	268
200	-10	250	348
300	-30	250	406
120	-30	150	173
250	-35	340	485
420	-60	230	455
350	-40	165	300
270	-25	300	456
310	-40	250	406
210	-10	175	250

ANSWER 7.

Ambient Temperature	Mach No	TAS
-30	0.75	456
-40	0.91	542
-36	0.85	510
-70	0.90	500
-20	0.82	510
-32	0.76	460
-31	0.85	515
-20	0.75	465
-25	0.815	500
-35	0.70	421

ANSWER 8.

Groundspeed	Distance	Time (mins)
210	198	56.6
465	324	42
278	158	34
352	158	27
420	450	64.4
180	69	22.6
282	150	32
360	60	10
450	55	7.3
310	88	17

ANSWER 9.

Kilometres	Statute Miles	Nautical Miles
169	105	91
345	215	186
200	124	108
269	168	145
519	323	280
420	260	226
675	420	365
124	77	67
57.5	35.7	31
2,457	1,560	1,328

ANSWER 10.

Imperial Gallons	US Gallons	Litres
1,420	1,704	6,475
9,083	10,900	41,420
5,704	6,845	26,010
1,933	2,320	8,816
622	746	2,836
3,187	3,830	14,500
47	56.2	214
6,300	7,530	28,600
685	822	3,115
630	755	2,870

ANSWER 11.

Pounds (lbs)	Kilograms (kg)
723	329
42,680	19,400
13,464	6,120
22,320	10,145
1,144	520
980	445
250	114
16,600	7,530
685	312
204,600	93,000

ANSWER 12.

Pounds (lbs)	Kilograms (kg)	SG	US Gallons	Imp. Gallons	Litres
15,400	6,980	0.80	2,310	1,922	8,750
89,000	40,400	0.79	13,500	11,200	51,000
10,000	4,550	0.75	1,600	1,330	6,050
5,869	2,668	0.78	900	750	3,420
153,500	69,500	0.85	21,600	18,000	82,080
3,080	1,400	0.80	461	386	1,746
71,000	32,400	0.81	10,520	8,780	40,000
5,400	2,450	0.79	818	683	3,110
4,380	1,980	0.78	671	560	2,540
1,075	490	0.80	161.5	134.5	610

Terrestrial Magnetism

Magnetism – General Principles

Terrestrial Magnetism

Terrestrial Magnetism

Magnetism – General Principles

1. Magnetism is a property which may be possessed by iron, steel and a few other chemical elements such as nickel, cobalt and certain special alloys. The main properties of a magnet are:

(a) the ability to attract material made of iron and other magnetic materials;

(b) when freely suspended it will tend to align in an approximately North-South direction depending on its position on the earth;

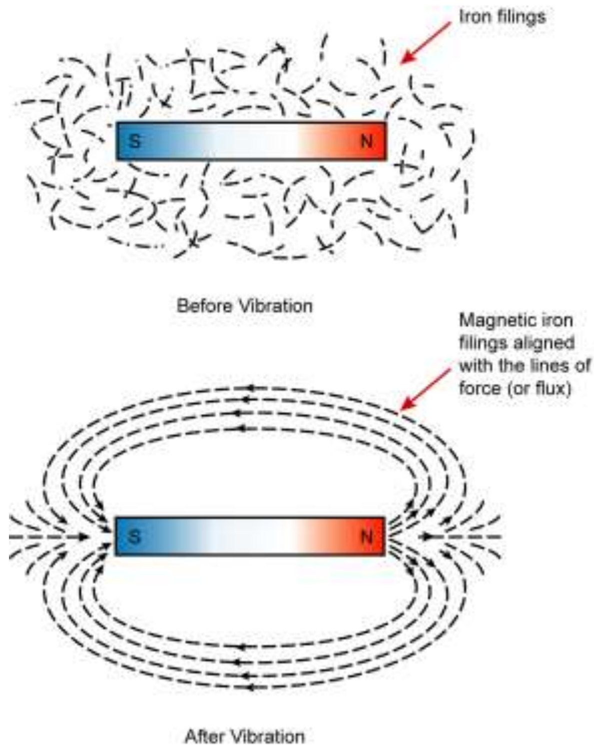
(c) the attracting force tends to be concentrated at each end of the magnet. The end which is north-seeking is called the North (or red) pole and the end which is south-seeking the South (or blue) pole;

(d) like poles repel, unlike poles attract;

(e) the strength of the attracting or repelling force between two magnets decreases as the square of the distance between them.

2. Magnetic force can be shown to exist around a magnet by, for example, sprinkling some easily magnetised light material such as iron filings around it. The small iron particles will themselves become magnetised by the magnetic force but less so with increased distance from the magnet. If the area is then vibrated slightly to permit movement the iron filings tend to align themselves with the lines of magnetic force emanating from the magnet. This phenomenon is illustrated at Figure 4-1.

FIGURE 4-1
Diagram of the Magnetic Field Around a Bar Magnet



3. Each iron filing itself becomes magnetised and aligns with the flow of the magnetic force or flux. The direction of the lines of force in the magnetic field are, by convention, assumed to emanate from the North-seeking pole of the magnet and re-enter at the South-seeking pole. The strength of this force is measurable and the SI units used are ‘tesla’. Material can be magnetised only up to a certain point and is then said to be saturated.

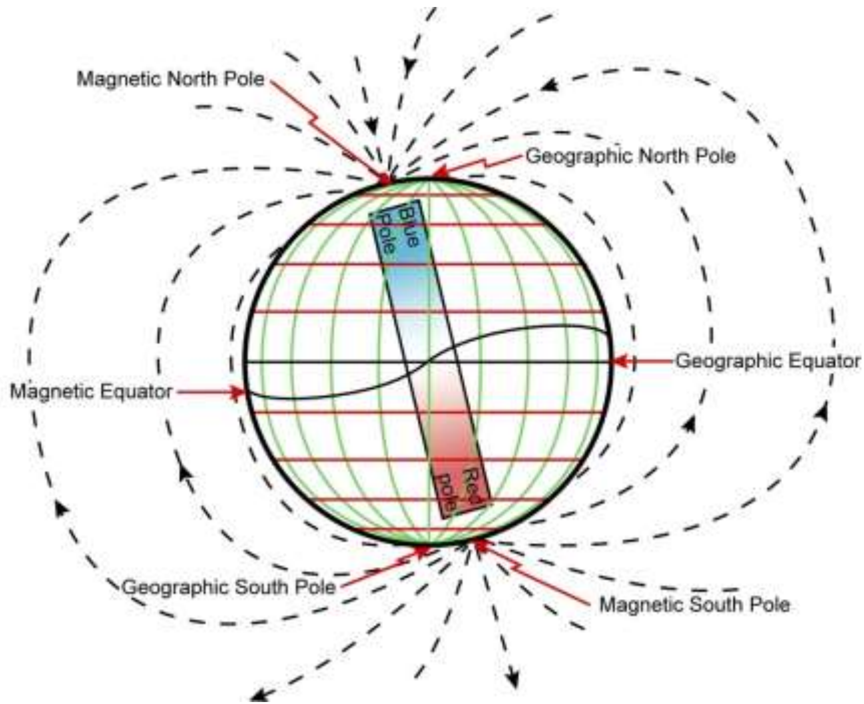
4. **Magnetic moment.** The magnetic moment of a magnet is the product of its pole strength and its length and is a measure of its tendency to turn or be turned by another magnetic field. The greater the pole strength and/or the longer the magnet the greater will be the magnetic moment and the stronger will be its tendency to align with the magnetic field in which it is located.

Terrestrial Magnetism

5. The Earth possesses a magnetic field similar to that of a magnet. The cause of this magnetism has not been fully explained, but the presence of large quantities of iron in its core and the rotation of the earth itself may play a part in the formation of the magnetic field. Suffice to say that a freely suspended magnetic needle will align itself in a direction reasonably consistent with this field and will therefore point to the North Magnetic Pole.

6. The Earth’s magnetic field behaves as if there were a very powerful bar magnetic located at its centre with, by convention, a blue (south-seeking pole) near the North geographic pole. In practice the North Magnetic Pole is located some 750 nm from the true pole in northern Canada. An equivalent South Magnetic Pole is located in Antarctica. The maximum strength of a magnet is normally found at each of its poles (hence the term ‘pole strength’) however, in the case of the Earth the maximum strength has been found to be at two locations near to each pole. In addition the magnetic poles are not single points but large areas where it is assumed the lines of magnetic flux emanate from or enter the Earth vertically. Figure 4-2 illustrates diagrammatically the bar magnet concept and the Earth’s magnetic field.

FIGURE 4-2
The Earth's Magnetic Field



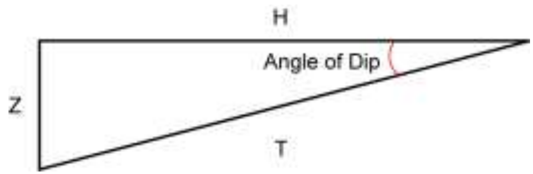
Components of Terrestrial Magnetism

7. **Magnetic meridian.** A freely suspended magnet will align itself with the local lines of force that are part of the Earth's magnetic field and a line drawn through the plane of the magnet and the centre of the Earth would define the local magnetic meridian. This meridian therefore defines the direction of Magnetic North and South at any location. It is not always consistent because there are known anomalies in the Earth's magnetic field where local deposits of iron ore for example will deflect the magnet from its expected alignment.

8. **Directive force.** The force that causes the alignment of the magnet with the Earth's field is known as the directive force and it is given, by convention, the representative letter H. It is the **horizontal component** of the total force present (represented by the letter T) at any location.

9. **Vertical component.** The total magnetic force T may consist of both horizontal and vertical components depending on location, the vertical component is given the representative letter Z. Figure 4-3 illustrates the relationship between T, H and Z.

FIGURE 4-3
Components of Earth Magnetism



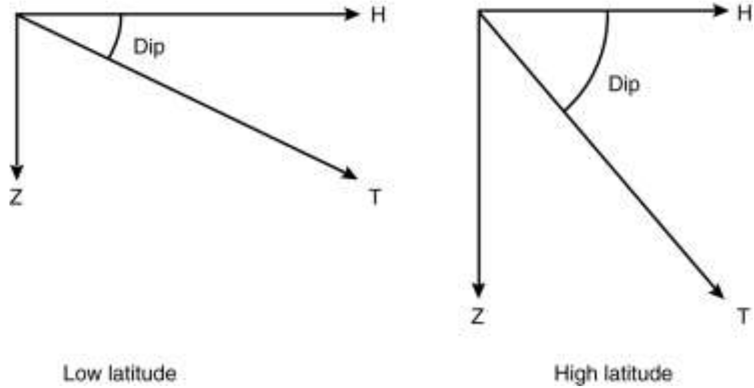
10. **Magnetic Equator.** At every point midway between the North and South Magnetic Poles on the Earth the lines of total force are horizontal to the Earth's surface; the line joining all these points is known as the magnetic equator. At this point, H represents the total force of the Earth's magnetic field and is at its greatest strength. Z is non-existent at the magnetic equator.

11. **Dip.** Dip is the name given to the angle below the local horizontal, taken up by a freely suspended magnet when aligning itself with the Earth's total magnetic field. Since this field tends to follow the contours illustrated in Figure 4-2, it can be seen that the angle of dip increases with increase in 'magnetic latitude' until, at the North and South Magnetic Poles, it is 90° (with the magnet vertical). It can be concluded therefore that the angle of dip is zero at the magnetic equator increasing to a maximum of 90° at the magnetic poles. (Note, the term magnetic inclination may also be used to describe dip).

12. A line joining points with the same value of dip on the Earth's surface is known as an **isoclinal** line.

13. The line joining points with zero dip angle is known as an **aclinal** line and is a description of the magnetic equator The change in dip angle with latitude is illustrated in Figure 4-4.

FIGURE 4-4
Variation of H, Z and Dip T at Two Different Latitudes

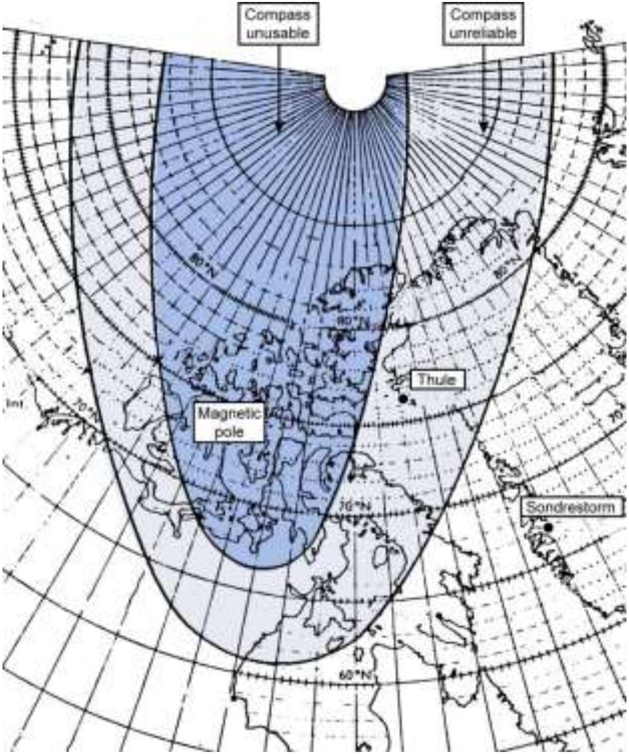


14. **Directionality.** The freely suspended magnet has been used since very early times for providing directional guidance and the principles involved are still used in magnetic compass systems. The directionality relies on the magnet (compass needle in a simple compass) aligning with Magnetic North and for this to be achieved the directive force H must be of sufficient strength for the sensitivity of the compass magnet(s).

15. As can be seen from Figure 4-4, the proportion of the total force which is H must reduce with increase in magnetic latitude until it is zero at the poles (where only Z is present). It can be concluded, therefore, that magnetic compass systems give poor indications near to the poles and are completely useless very close to the poles.

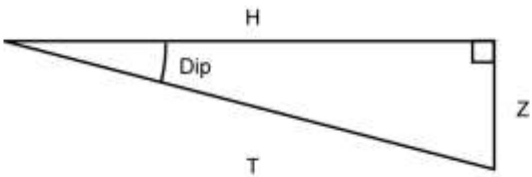
16. Polar navigation charts usually show areas where magnetic compass systems are unreliable and unusable. (The minimum strength of the directive force for operation of a magnetic compass system near to the North pole is considered to be 6 micro-tesla). Figure 4-5 illustrates the type of warning given on a polar chart.

FIGURE 4-5
Depiction of Compass Unreliability Warning Areas



17. **Calculations of H and Z from total force T at any given latitude.** It is possible to make an approximate calculation of the comparative values of Z and H for a given magnetic latitude with a given value of the total magnetic force (T) and the angle of dip. The simplified relationship between components is that based on a the right angled triangle in which trigonometry and/or Pythagoras' formula may be used according to the information given.

FIGURE 4-6
H, T, Z Relationship



18. From Figure 4-6 it can be seen that,

$$\text{Tan dip angle} = \frac{Z}{H} \text{ and therefore}$$

(a) $Z = H \times \text{Tan dip}$: and

(b) $H = \frac{Z}{\text{Tan dip}}$;

also, from Figure 4-6, Cos dip angle = $\frac{H}{T}$ and therefore,

(a) $H = T \times \text{Cos dip}$; and

(b) $$T = \frac{H}{\text{Cos dip}}$$

in addition, $T^2 = Z^2 + H^2$.

19. The following examples illustrate the use of these formula.

EXAMPLE 4-1

EXAMPLE

Given that T = 50 micro-tesla (mT), dip angle = 60°, calculate H and Z.

SOLUTION

	H = T x cos dip,
therefore, in this case	H = 50 x cos 60,
	H = 25 mT;
and,	Z = H x tan dip,
therefore, in this case	Z = 25 x tan 60,
	Z = 25 x 1.732,
	Z = 43.3 mT

EXAMPLE 4-2

EXAMPLE

Given that T = 60mT, dip angle 30 (deg), calculate H and Z.

SOLUTION

H = T x cos dip,

therefore, H = 60 x .866,

H = 51.96 mT

Z = H x tan dip,

therefore, Z = 51.96 x .577

Z = 30 mT.

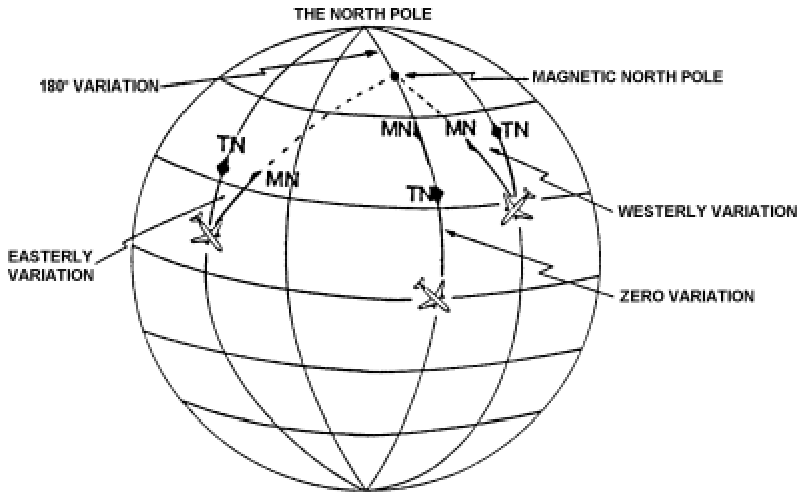
Summary of Changes in H and Z with Latitude

20. It can be seen that the strength of H decreases with increase in dip angle and magnetic latitude, whereas Z increases with increase in magnetic latitude.

Variation

21. Because the magnetic and geographic poles are not in the same positions, magnetic meridians and true (geographic) meridians do not always coincide and there will exist in most places on the Earth an angular difference between the directions of magnetic and true north. The difference between the direction of True North and Magnetic North at any point on the Earth's surface is commonly known as **variation**. (Strictly speaking, variation is the angle between the direction taken up by component H and True North). The minimum value of variation is zero when the poles and the observer's position are in alignment, the maximum value is 180° in any position that is exactly between the true and magnetic poles. Figure 4-7 is a reminder of the principle of variation.

FIGURE 4-7
Calculation of Magnetic Variation



22. **Application of variation.** The application of variation to convert magnetic direction to true and vice versa has already been covered in Chapter 2. Variation is described as East and positive when Magnetic North is on the eastern side of True North, and West and negative when Magnetic North is on the western side of True North. For example 'var 10W' annotated on a chart means that to convert magnetic direction to true in that position 10° must be subtracted from it.

(Note. The sign convention is arranged to allow a conversion from magnetic to true, when converting back the other way the signs must be reversed.)

23. **Changes in variation**. There are several factors which can alter the value of variation at any given position. Local magnetic anomalies have already been mentioned. Other changes are:

(a) **Secular changes.** The magnetic poles are moving in relation to the true poles following a roughly clockwise movement around the true pole every 960 years. (The magnetic pole has moved in a north-westerly direction in last hundred years). The value of variation at any given place will change with time according to this movement, this change may be referred to as a secular change and the annual rate is normally depicted on navigation charts.

(b) **Annual changes.** An annual change in the value of variation takes place in a yearly cycle with maxima occurring in conjunction with the spring and autumn equinoxes (March 21 and September 21). The change is small, varying from 2'E to 2'W in the northern hemisphere. The change is in the opposite sense in the southern hemisphere.

(c) **Daily changes.** Daily changes in variation can occur due to changes in the ionosphere associated with changes from night to day. The earth's magnetic field is modified by changes in the ionosphere and alter the directionality of the magnetic field. The changes are relatively small, around 4' with maxima occurring about sunrise and noon.

(d) **Periodic changes.** The 11 year sunspot cycle has significant effect on the ionosphere and on the Earth's magnetic field. Changes of 12' on a daily basis are possible. Magnetic storms associated with increased sunspot activity cause errors in terms of several degrees.

24. **Variation on charts.** Navigation charts normally show lines of equal variation known as **isogonals.** (Note variation values change rapidly over short distances near to the poles which is an additional problem for navigation in polar areas.)

25. The annual change in variation is normally shown somewhere on the chart along with the date of the chart to enable isogonal values to be amended for current date. It is inadvisable to use an old chart without this amendment. The information might be depicted as shown in Figure 4-8.

FIGURE 4-8
Representation of Variation Change on a Navigation Chart

LINES OF MAGNETIC VARIATION FOR 1999

(Annual Rate of Change 6’ decrease)

(Note. There are two ‘lines’ on the Earth’s surface where zero variation exists: One approximates to a portion of the meridian running through the Magnetic North Pole to the True South Pole, the other approximates to its anti-meridian between the Magnetic South Pole and the True North Pole. These lines, joining points of zero variation are called agonic lines.)

Aircraft Magnetism

General Principles

Analysis of Hard and Soft Iron Magnetism

Aircraft Magnetism

General Principles

1. The ability of an aircraft magnetic compass system to align with or to ‘sense’ the local magnetic meridian accurately not only depends on the strength of the horizontal directive force H at that point but also on the magnetic environment within the aircraft. Other magnetic forces will exist within an aircraft and especially in the cockpit of an aircraft and these forces can give rise to what is called deviation. Deviation is the angular difference between the compass indicated direction and the direction of the local magnetic meridian (Magnetic North). The purpose of a compass swing is to analyse and correct for as much of this deviation as possible.

Causes of Deviation

2. Deviation can occur as a result of hard iron and soft iron magnetism within an aircraft and also as a result of misalignment of reference lines from which readings are taken or of the detector element in a remote sensing compass system.

Permeability

3. Permeability is the name given to the capacity of a material to become magnetised. In scientific terms it is expressed as the ratio between the flux density (magnetic field strength) within the material compared with the magnetising force. Materials possessing high permeability are easily magnetised whereas materials such as rubber, brass, copper, aluminium, plastic, and carbon fibre have very low or zero permeability and cannot be magnetised. Material such as Permalloy has a high permeability but once the magnetising force is removed it decays very quickly. In the case of iron however, once magnetised the magnetism is likely to decay at a much slower rate. The measure of whether iron is hard or soft is this capacity to retain magnetism. The magnetisation process is called magnetic induction.

Hard Iron Magnetism

4. Hard iron magnetism is the name given to magnetism that is virtually permanent or decays very slowly. It is caused by the presence of iron and steel components within the aircraft structure that acquire magnetism slowly from the Earth's magnetic field during manufacture (hammering, riveting, vibration assist the process). Also it can form when an aircraft remains on the ground on one heading for a long period of time (weeks). A very powerful electrical shock such as that received in a lightning strike can also establish hard iron magnetism in an aircraft. Hard iron magnetism is very difficult to remove.

Soft Iron Magnetism

5. Soft iron magnetism is the name given to temporary magnetism. Such magnetism is usually induced within highly permeable ‘soft’ iron or steel components by the presence of magnetic fields, but decays rapidly when the magnetising force is removed. Such magnetism is frequently induced by the magnetic fields surrounding electrical components. For example, the operation of an electrical motor (such as a windscreen wiper) in the cockpit of an aircraft can therefore induce temporary soft iron magnetism that disappears when the component is switched off.

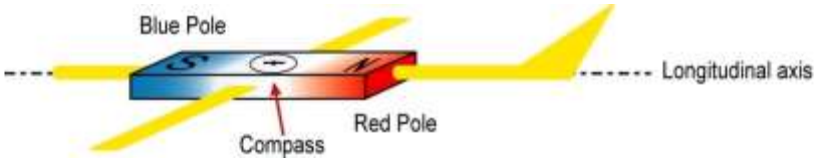
Analysis of Hard and Soft Iron Magnetism

Components of Hard Iron Magnetism

6. In order to analyse and correct for the effect of magnetism within an aircraft it is necessary first to analyse its effect in detail. To achieve this the magnetism is assessed to act in each of three planes within the aircraft structure and thus to have three identifiable components. The three components are labelled with the capital letters P, Q and R depending on whether they act in the fore/aft, athwartships (lateral axis) or vertical axis of the aircraft respectively.

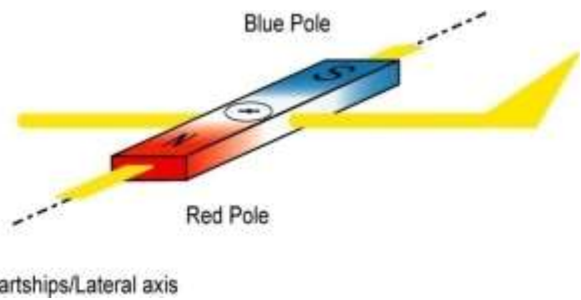
7. **Magnetism component P.** The conventional description of the effect of magnetism component P draws on the analogy of a ‘bar magnet’ positioned in the fore/aft axis with its poles on either side of the aircraft compass system. Component P is, by convention, positive when the south-seeking or blue pole is ahead of the compass system and negative when behind it. Figure 5-1 illustrates diagrammatically the principle and shows a component +P.

FIGURE 5-1
Hard Iron Magnetism (Component +P)



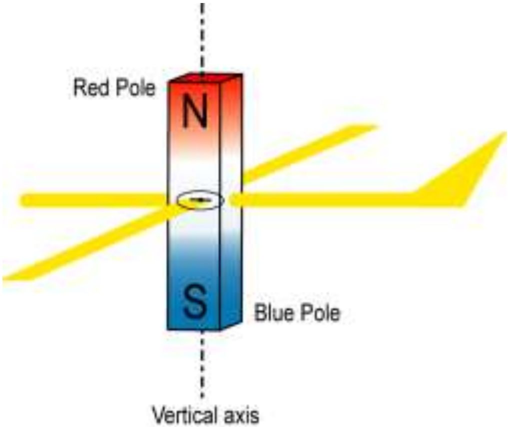
8. **Magnetism component Q.** A similar analogy is used to identify component Q, but this time the 'bar magnet' is positioned so that its poles are ether side of the compass system. By convention, a south-seeking (blue) pole on the starboard side is positive and when on the port side is negative. Figure 5-2 illustrates +Q.

FIGURE 5-2
Hard Iron Magnetism (Component +Q)



9. **Magnetism component R.** This component is assumed to act in the vertical plane of the aircraft above and below the compass system. A south-seeking (blue) pole below the compass system is positive and a red pole negative. Figure 5-3 illustrates +R.

FIGURE 5-3
Hard Iron
Magnetism
(Component +R)



Effects of Components P, Q and R

10. In an aircraft compass system the compass magnets are mounted horizontally to enable them to align with the horizontal component of the Earth's magnetic field. The effect of component R on the compass magnets should not interfere with this alignment providing the aircraft remains in level flight. Were the aircraft to pitch up or down the R would no-longer remain vertical and would then cause some deviation. In practice the effect on a compass system is negligible compared to other errors likely to be present and is generally ignored.

11. In normal operation the magnets in a direct reading compass remain nearly horizontal and aligned with North. However, the position of components P and Q relative to the compass magnets will change with change of aircraft heading and whilst the strength of the components does not change with heading their deviating effect does. This aspect is covered in detail in the chapter on compass swinging.

12. In principle, the effect of each of the components P and Q is to produce a proportion of the total deviation called a 'coefficient of deviation'. Each coefficient is also given an identifying letter. Component +P produces coefficient +B, component +Q produces coefficient +C, (similarly -P produces –B etc.). The purpose of the compass swing is to identify these coefficients to enable corrections to be made to cancel their effect.

Soft Iron Magnetism

13. Soft iron magnetism is temporary, induced magnetism, it is present only when an inducing magnetic field is present. One of the main causes of soft iron magnetism is the Earth's magnetic field in which the horizontal and vertical components may induce horizontal and vertical soft iron magnetism respectively in the aircraft structure near to the compass magnets. The effect of horizontal soft iron is rather complex, but as it is strongest when the H component of Earth magnetism is strongest, the deviation caused is relatively small. Vertical soft iron magnetism is more important because it is strongest in high latitudes where Z is stronger, whereas H is weaker and the magnets of the compass are more easily deflected.

14. The bar magnet analogy is continued with respect to vertical soft iron magnetism. In this case the reader must imagine vertical bar magnets positioned ahead and behind the compass magnets on the longitudinal axis giving soft iron component 'cZ'. Also, vertical bar magnets positioned on the lateral (or athwartships) axis causing component 'fZ'.

15. From the positioning of cZ and fZ it can be seen that these components contribute to coefficients of deviation B and C respectively.

Summary of Effects of Hard and Vertical Soft Iron Magnetism

16. Hard iron magnetism is permanent and does not vary with magnetic latitude but, the deviation caused by hard iron magnetism increases with latitude because in high latitudes where H is weaker, the compass magnets are more easily deflected. The deviating effect of hard iron magnetism varies with the heading of an aircraft.

17. Vertical soft iron magnetism increases when an aircraft moves to a higher latitude because Z increases. The deviation caused by vertical soft iron magnetism in higher latitudes is doubly increased because H is reduced. The deviating effect of the vertical soft iron component of coefficients B and C varies with the heading of an aircraft.

Direct Reading Compasses

Direct Reading\Standby Compasses

Horizontality

Sensitivity

Aperiodicity

Serviceability Tests

Compass Safe Distances

E Type Compass

Acceleration Errors

Inertia

The Z Field Effect

Turning Errors

Inertia

The Z Field Effect

Liquid Swirl

Direct Reading Compasses

Direct Reading\Standby Compasses

1. A direct reading compass enables the pilot to read the aircraft heading directly in relation to a magnetic assembly.

2. This type of compass basically consists of two or more pivoted magnets, which are free to align themselves with the horizontal component of the Earth's magnetic field. It is desirable that any direct reading compass should possess three elementary properties:

(a) Horizontality

(b) Sensitivity

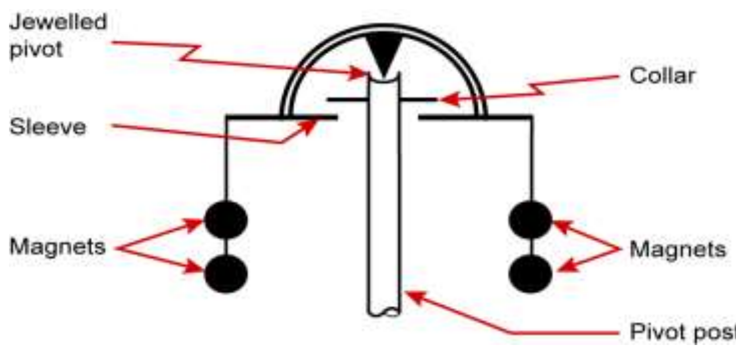
(c) Aperiodicity

Horizontality

3. As already discussed, it is the horizontal component of the Earth's magnetic field which enables the compass magnets to align themselves with north. It is therefore essential that the compass magnets should lie as close as possible to the horizontal plane.

4. If a magnet were pivoted at its centre on a pin, it would dip to lie in the plane of the Earth's total field. Even in mid latitudes, the dip angle would be unacceptably high. To overcome this problem, a system of pendulous suspension is employed which is schematically outlined at Figure 6-1. The success of the system lies in the fact that the centre of gravity of the magnets lies below the pivot point. Thus the dipping effect due to the vertical (Z) component of the Earth's magnetic field is opposed by the weight of the magnets.

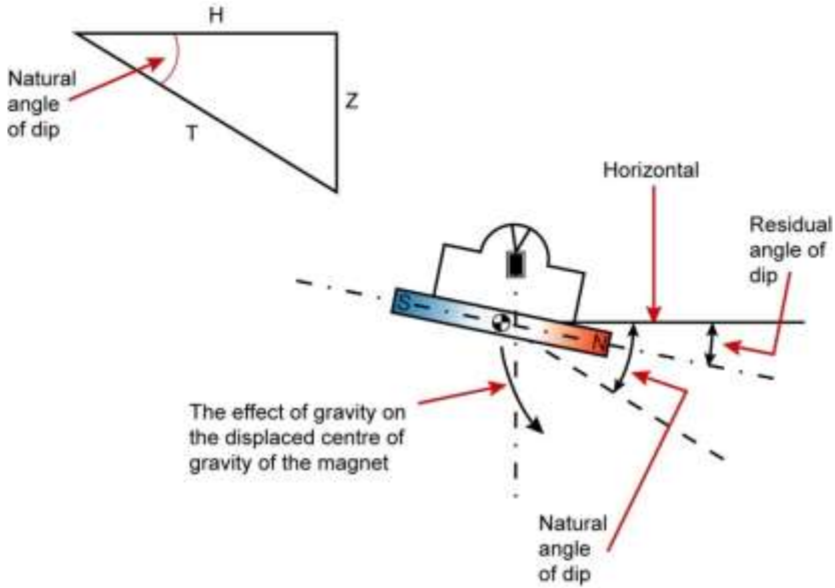
FIGURE 6-1
DRC Magnet Assembly Suspension System



5. The collar and sleeve assembly Figure 6-1 prevents the magnet assembly from parting company with the pivot during inverted flight.

6. The result of pendulous suspension is that the magnets will lie close to the horizontal. Except at the magnetic equator there will however be a small residual dip, see Figure 6-2. In mid latitudes the residual angle of dip should be less than three degrees.

FIGURE 6-2
Natural angle of a dip



Sensitivity

7. Sensitivity is a measure of the ability of the compass magnetic assembly to point accurately towards north.

8. Nothing can be done to increase the strength of the weak terrestrial magnetic field and so it is necessary to use several magnets with high pole strengths. The magnetic assembly is made as light as possible to reduce friction at the pivot.

9. The pivot itself normally incorporates a jewelled bearing which is lubricated by the viscous fluid which fills the bowl. Being fairly dense the fluid effectively lightens the magnet assembly still further, once again reducing friction at the pivot.

Aperiodicity

10. The aperiodic quality of a compass may be defined as the ability of the magnet assembly to settle quickly, pointing towards the magnetic north pole, following displacement during manoeuvres or turbulence.

11. If a compass is not entirely aperiodic, the effect is that the magnets oscillate, or hunt, around magnetic north, coming to rest only slowly.

12. Aperiodicity is achieved by using short length magnets, thereby keeping the mass near to the centre of rotation and reducing the moment of inertia. This is also aided by the use of light materials in the magnetic assembly. The fluid within which the magnets are immersed will tend to dampen any oscillation of the magnet assembly.

13. The compass fluid discussed above must obviously be transparent. Additionally, the liquid must also completely fill the compass bowl in order to prevent liquid swirl during turns, as this would deflect the magnet assembly. To ensure that the bowl is always completely full despite change in temperature, an expansion bellows is fitted which acts as a fluid reservoir.

Serviceability Tests

14. The following tests would normally be carried out after compass installation, following a compass swing, or whenever the accuracy of the compass is in doubt.

(a) Check that the compass liquid is free from discolouration, bubbles and sediment. The movement of bubbles could deflect the magnetic assembly and the presence of sediment could prevent its free movement.

(b) Carry out a damping test. Using a small magnet deflect the compass by 90° and hold for at least 20 seconds to allow the liquid to stabilise. Now remove the magnet. The time taken for the compass reading to return to within 5° of the original reading should be 2 to 3 seconds for an E type compass.

(c) Carry out a pivot friction test. Turn the aircraft so that it is on one of the cardinal headings (N, E, S or W). Again using a small magnet, deflect the compass by 10° and hold for 10 seconds. Remove the magnet and note the reading when the compass settles. Now repeat the procedure but deflect the compass in the opposite direction. For an E type compass the readings after deflection should be within 2½° of the readings prior to deflection. Now repeat the process on each of the remaining cardinal headings.

Compass Safe Distances

15. From experience, you know that one of the major problems with a direct reading compass is that it must necessarily be mounted in the cockpit or on the flight deck. It is therefore surrounded by equipment which is capable of causing deviation, and the compass must be carefully sited.

16. The compass should be sited such that no single item of non electrical equipment causes a compass deviation of more than 1°. The sum of the deviations caused by all such equipment must not exceed 2°. Similarly, no single item of electrical equipment, or its associated wiring, may cause a compass deviation of more than 1°, and again the sum of the deviations caused by all such equipment and wiring may not exceed 2°. The operation of the aircraft controls is not permitted to change the deviation suffered by the compass by more than 1°.

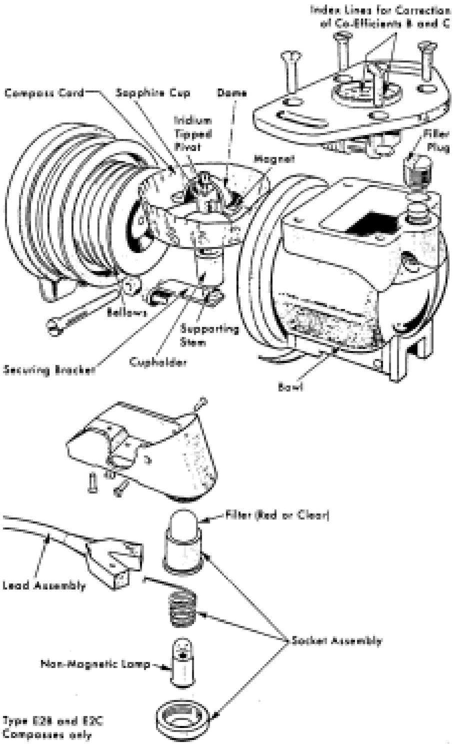
17. Where the compass concerned is the primary heading reference, the maximum permissible deviation on any heading is 3° and the maximum permissible value of coefficients B and C is 15°. When a direct reading compass is installed as a standby heading reference, the maximum permissible deviation on any heading is 10°.

E Type Compass

18. The E Type compass, which is illustrated at Figure 6-3, is otherwise known as the vertical card compass. It is in very common use, as the main magnetic heading reference in modern light aircraft, and as a standby compass in larger aircraft.

FIGURE 6-3

E-type compass



19. Notice that with the E Type compass, the compass card is attached to the magnet assembly. Appreciate that (ignoring all errors) as the aircraft turns the magnets and compass card remain stationary whilst the instrument case and lubber line move around them:

(a) A displacement of the magnet assembly away from north in an **anti-clockwise** direction will result in an erroneous increase in indicated heading, ie. **the compass will overread**.

(b) Conversely, a displacement of the magnet assembly away from north in a **clockwise** direction will result in an erroneous decrease in indicated heading, ie. **the compass will underread**.

Acceleration Errors

20. We know from basic flight training that direct reading compasses will give incorrect readings during aircraft acceleration or deceleration. There are two reasons for this: inertia is one, and the effect of the vertical (Z) component on the displaced magnets is the other.

Inertia

21. Figure 6-4 shows a pendulously suspended magnet (with residual dip) in the northern hemisphere. Notice that the vertical line through the pivot point lies closer to the nearer (North) magnetic pole than does the centre of gravity of the magnet.

22. Figure 6-5 shows the same magnet viewed from above (the diagram is rotated through 90° to put North conventionally at the top of the diagram).

Figure 6-6 shows the effect on the magnet of an acceleration on a heading of 090°(M). The acceleration is felt through the pivot but, due to inertia, the magnet wishes to maintain its state of uniform motion and so a reaction is evident which acts through the centre of gravity which will cause the magnet assembly to rotate.

FIGURE 6-4
Effect of Residual Angle of Dip - Northern Hemisphere

23.

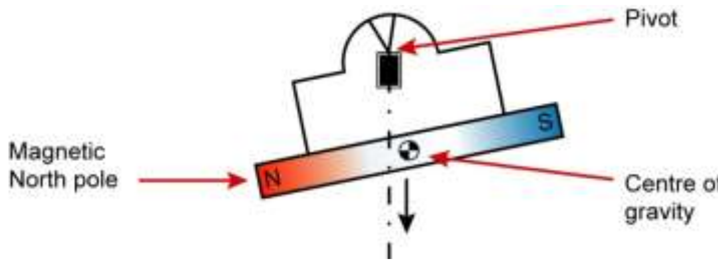


FIGURE 6-5
Relative Positions of Pivot and Centre of Gravity (Northern Hemisphere)

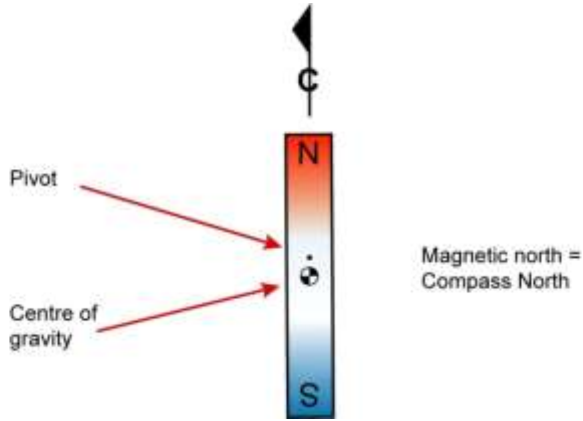
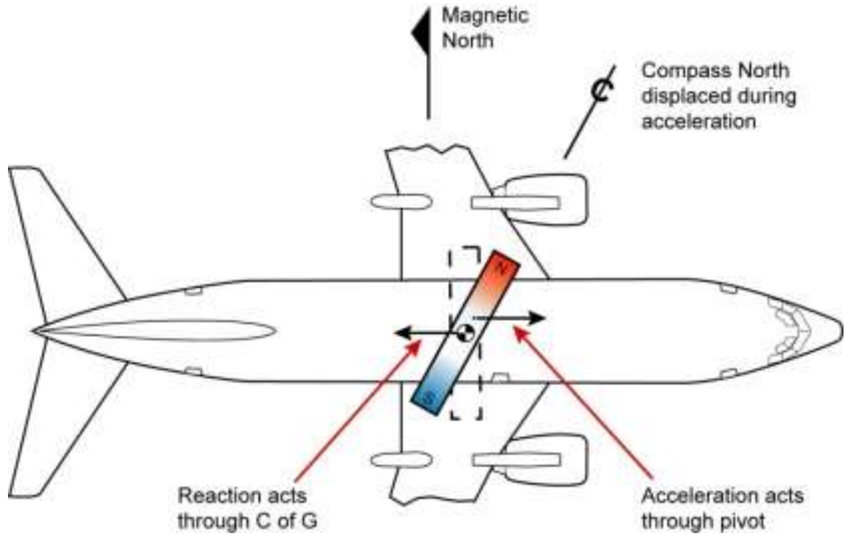


FIGURE 6-6
Inertia Effect - Aircraft Accelerating East in the Northern Hemisphere



24. The needle has swung clockwise when viewed from above. The compass will therefore read less than 090° during the acceleration. A deceleration on a heading of 270°(M) will also cause the magnet to swing clockwise when viewed from above, causing the compass to read less than 270° during the deceleration.

Figure 6-7 shows a pendulously suspended magnet in the southern hemisphere. Again the pivot point is nearer than the centre of gravity to the nearer (South) pole, as illustrated at Figure 6-8. An acceleration on a heading of 090°(M) will cause the magnet to swing anti-clockwise when viewed from above, see Figure 6-9. This will cause the compass reading to increase during the acceleration. A deceleration on a heading of 270°(M) will cause the needle to rotate in the same direction as an acceleration on 090°M, again causing an increase in the compass reading.

FIGURE 6-7
Effect of Residual Angle of Dip - Southern Hemisphere

25.

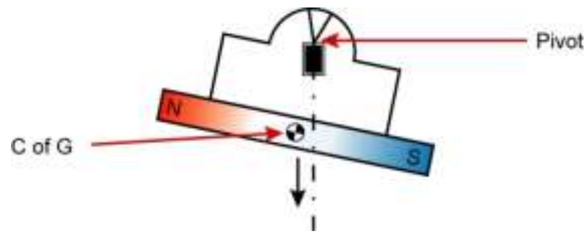


FIGURE 6-8
Relative Positions of Pivot and Centre of Gravity (Southern Hemisphere)

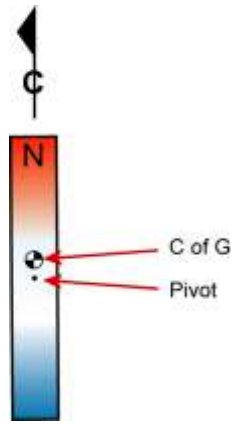
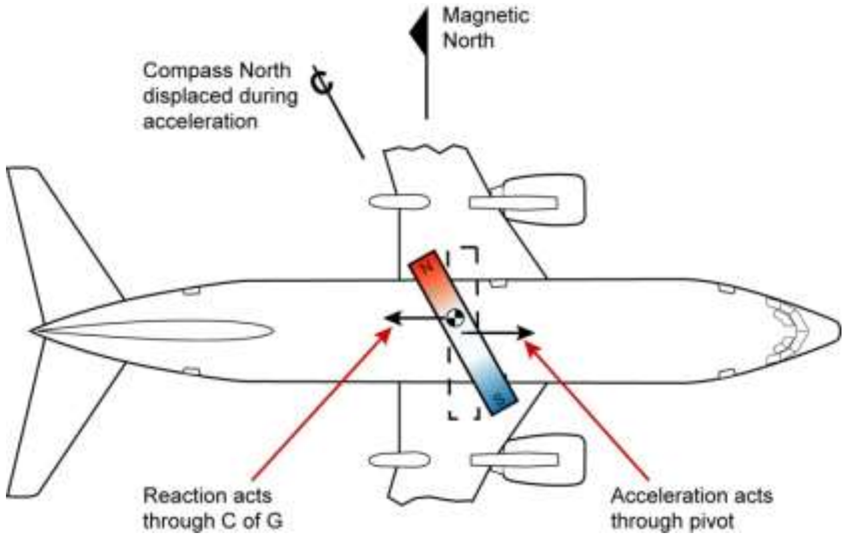


FIGURE 6-9
Inertia Effect - Aircraft Accelerating East in the Southern Hemisphere



26. From Figure 6-4, Figure 6-5, Figure 6-6, Figure 6-7, Figure 6-8, and Figure 6-9 it is apparent that:

(a) An acceleration **always** produces an **apparent** turn towards the magnetic pole which is physically closest to the aircraft;

(b) A deceleration **always** produces an **apparent** turn towards the magnetic equator.

27. At the magnetic equator there will be no acceleration error. This is because the magnet lies in the horizontal plane (no residual dip) and therefore the pivot point and the centre of gravity are vertically co-incident.

28. On headings of north and south there will again be no acceleration error, since both the centre of gravity and the pivot lie along the aircraft's fore and aft axis. In this situation acceleration or deceleration causes the magnet to move in the vertical plane (causing a change in the residual dip angle) rather than the horizontal plane (which would cause an erroneous heading indication).

The Z Field Effect

29. The effect of the Earth's vertical field component during change of speed is not as easy to explain as the effect of inertia. Remember, however, that the error caused by the Z field effect **always** acts in the same direction as the error caused by inertia.

30. Figure 6-10 shows a pendulously suspended magnet in the northern hemisphere. Since the magnet lies beneath the pivot, the Z field can have no turning effect. Figure 6-11 shows the effect of an acceleration to the east. The magnet is left behind (because of inertia) and no longer lies vertically beneath the pivot. The Z field can now exert a turning force on the dipped end of the magnet. The magnet can only turn by rotating about the pivot, causing the already dipped north-seeking end to move downwards under the influence of the Z field. The magnet therefore moves in a clockwise direction when viewed from above, as shown at Figure 6-12.

FIGURE 6-10
Z Field Effect

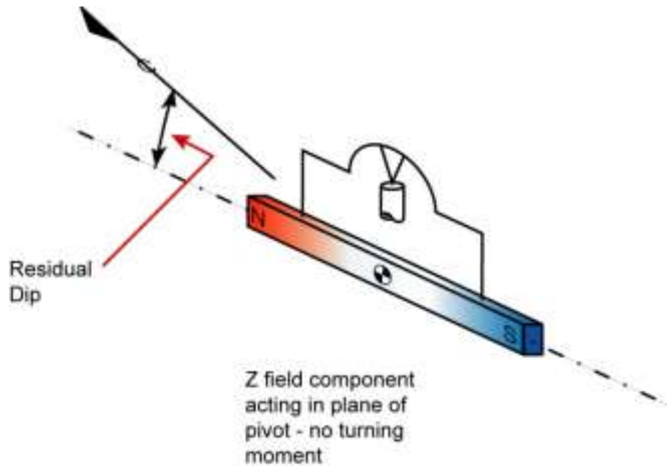


FIGURE 6-11
Z Field Effect
(Continued)

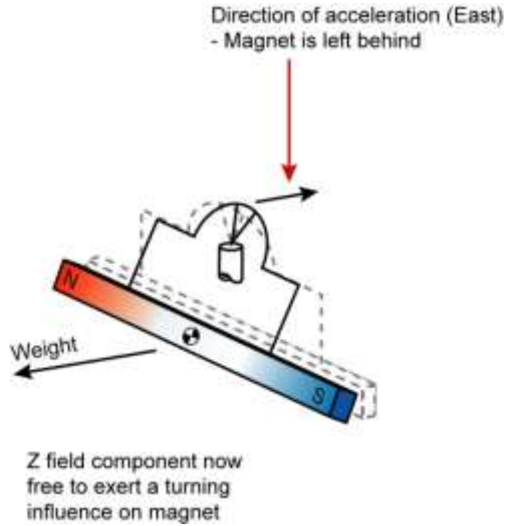
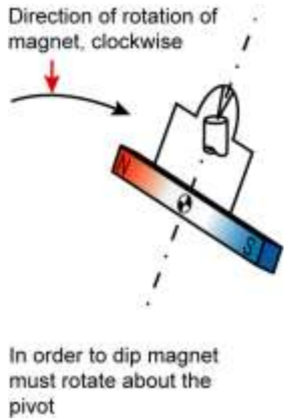


FIGURE 6-12
Z Field Effect
(Continued)



31. Refer back to Figure 6-6 and check that the inertia induced error and the Z field error are in fact complementary. They are, and this is always so.

Turning Errors

32. Turning errors should be easy to understand if it is appreciated that any turn is effectively an acceleration towards the centre of curvature of the turn. Again it is the effect of inertia and of the Z field which are primarily responsible for errors in a turn.

33. Once again, appreciate that the errors induced by inertia and by the vertical field component are **always** complementary.

Inertia

34. Figure 6-13 shows a pendulously suspended magnet (with residual dip) in the northern hemisphere. The aircraft within which the compass is fitted is presently heading 315°(M) and compass deviation is assumed to be zero. Notice that, as always, the pivot point lies closer to the nearer magnetic pole than does the centre of gravity of the magnet. If you are in doubt as to why this is so refer back to Figure 6-4 Figure 6-5.

FIGURE 6-13
Turning Errors - Northern Hemisphere

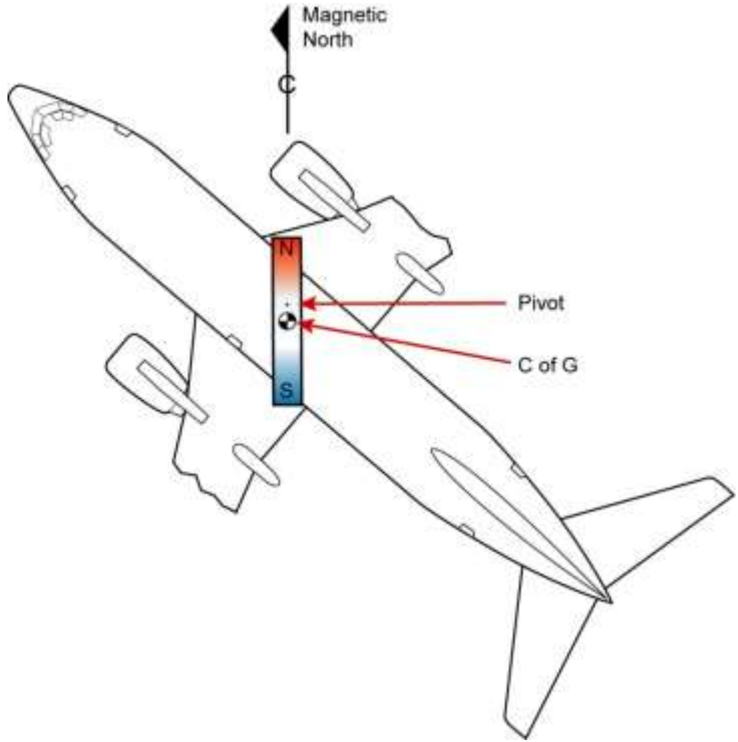
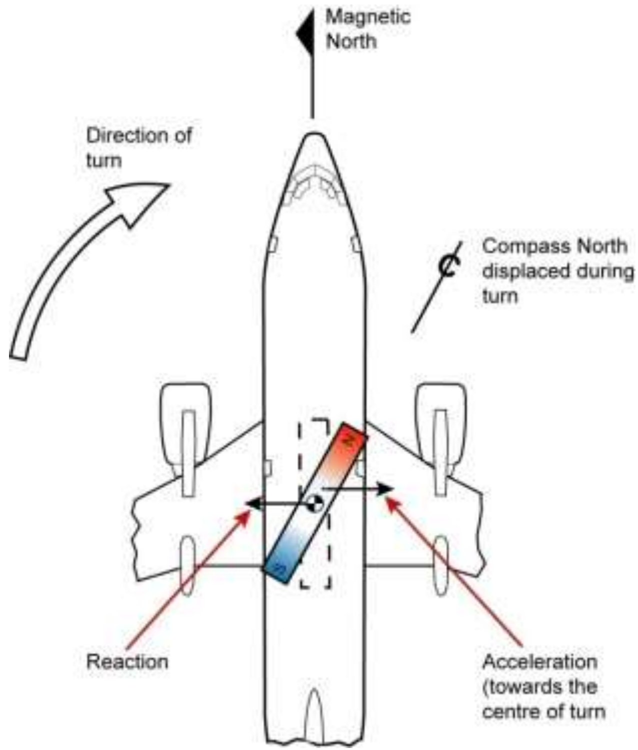


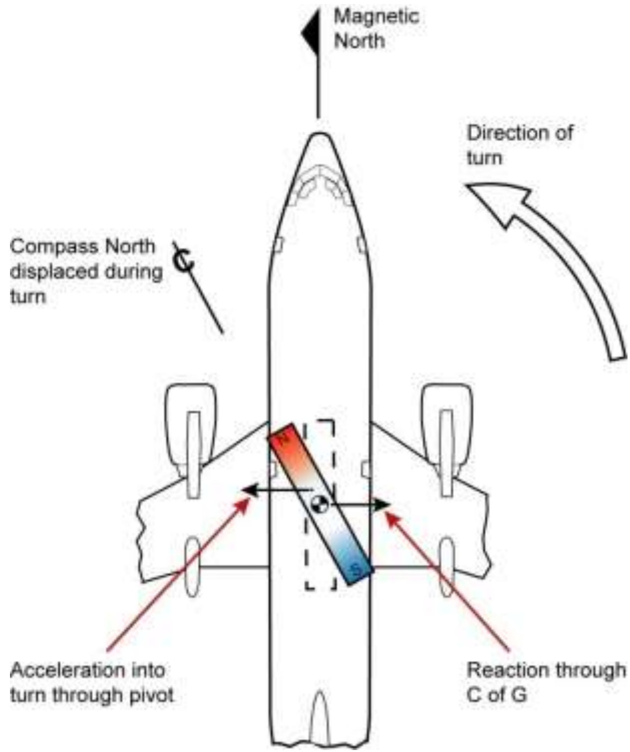
FIGURE 6-14
Aircraft Turning From 315°(M) onto 045°(M) - Northern Hemisphere



35. Figure 6-14 shows the same aircraft during a turn from 315°(M) through north onto 045°(M). The aircraft is presently passing through 360°(M). The acceleration force (towards the centre of curvature of the turn) is acting through the pivot and the reaction to this force is acting through the centre of gravity of the magnet. The result is that, during the turn, the magnet will swing clockwise when viewed from above, and consequently the compass will under-read.

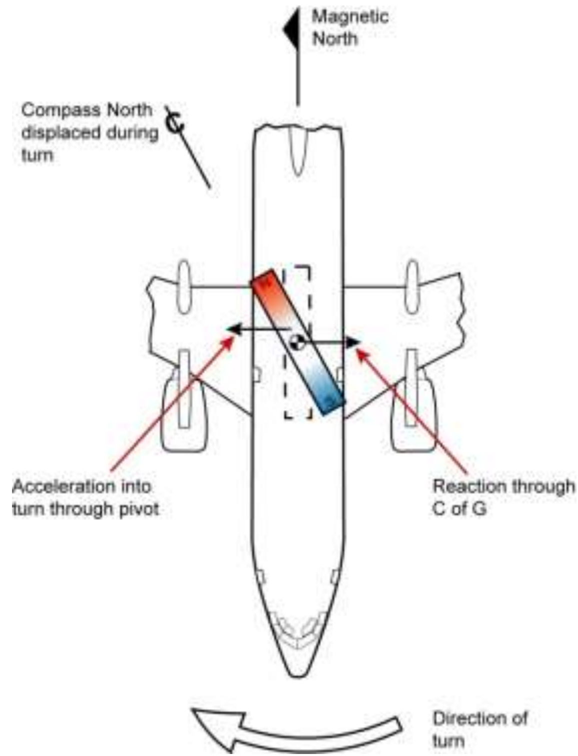
36. Figure 6-15 shows an aircraft in the northern hemisphere turning from 045°(M) through north onto 315°(M). Again, for simplicity, deviation is assumed to be zero during straight and level flight at a constant airspeed. As the aircraft passes through 360°(M), the acceleration force and the reaction to it have caused the magnet to swing anti-clockwise when viewed from above, and the compass will over-read during the turn.

FIGURE 6-15
Aircraft Turning From 045°(M) onto 315°(M) - Northern Hemisphere



37. Figure 6-16 shows an aircraft in the northern hemisphere turning from 135°(M) through south onto 225°(M). As the aircraft passes through 180°(M), the magnet has swung anti-clockwise when viewed from above and is causing the compass to over-read during the turn.

FIGURE 6-16
Aircraft Turning From 135°(M) onto 225°(M) - Northern Hemisphere



38. Three of the eight possible conditions where turning errors will exist have now been illustrated and discussed. The table at Figure 6-17 summarises the turning errors in all eight cases. The turning errors listed at Figure 6-17 are for reference only and you shouldn't try to memorise them. It is far better to apply the logic previously discussed to resolve the effect of turning errors.

39. The term ‘sluggish’ which appears in the right-hand column of the table denotes that the compass heading is lagging behind the aircraft heading. Conversely, when the term ‘lively’ is used, the compass is leading the aircraft around the turn.

FIGURE 6-17
Turning errors - Summary

Direction of Turn	Hemi-sphere	Displacement of Magnet (viewed from above)	Compass Reading Error	Compass Condition
315°(M) through N to 045°(M)	Northern	Clockwise	Under-read	Sluggish
045°(M) through N to 315°(M)	Northern	Anti-clockwise	Over-read	Sluggish
135°(M) through S to 225°(M)	Northern	Anti-clockwise	Over-read	Lively
225°(M) through S to 135°(M)	Northern	Clockwise	Under-read	Lively
315°(M) through N to 045°(M)	Southern	Anti-clockwise	Over-read	Lively
045°(M) through N to 315°(M)	Southern	Clockwise	Under-read	Lively
135°(M) through S to 225°(M)	Southern	Clockwise	Under-read	Sluggish
225°(M) through S to 135°(M)	Southern	Anti-clockwise	Over-read	Sluggish

From Figure 6-17, the following rules of thumb can be formulated:

(a) During a turn through the pole which is physically nearer to the aircraft, the compass will be sluggish. It is therefore necessary to roll out early when using the direct reading compass.

(b) During a turn through the pole which is physically further from the aircraft the compass will be lively. It is therefore necessary to roll out late when using the direct reading compass.

40. From the two statements above it can be seen that, at the magnetic equator, there is no turning error (since there is no residual dip). Furthermore, when rolling out on headings near to 090°(M) and 270°(M), the turning error will be minimal as the acceleration force and the reaction to it will lie close to a north-south direction and will result only in a change of the residual angle of dip.

41. Remember that it is a displacement of the magnet in a clockwise direction when viewed from above which causes the compass to under-read, and a displacement in an anti-clockwise direction which causes the compass to over-read.

The Z Field Effect

42. The effect of the Earth's vertical field component during a turn is not easy to illustrate with two-dimensional diagrams. Remember, however, that the error caused by the Z field effect always acts in the same direction as the inertia induced error.

43. Consider an aircraft in the northern hemisphere turning from 315°(M) through north onto 045°(M). During the turn, the magnet will be thrown outwards (towards the high wing) and will no longer be vertically beneath the pivot. The Z field component will now cause the dipped (in this case the north-seeking end) of the magnet to swing downwards and clockwise about the pivot. This situation is as illustrated at Figure 6-12, remembering that the acceleration is into the turn. Check Figure 6-14 and you will see that the clockwise movement of the magnet under the influence of the Z field complements the displacement of the magnet due to inertia.

Liquid Swirl

44. A third factor involved in turning errors is liquid swirl. Ideally the compass bowl will have a smooth internal surface and be full of a low viscosity fluid. If these conditions are not fully met, there will be a tendency for the liquid to be dragged around the bowl during turns. Once in motion, the liquid will continue to swirl under its own momentum. Should it occur, this fluid swirl will carry with it the magnetic assembly, thereby displacing it from its correct orientation. The displacement will therefore be in the direction of the turn and a clockwise turn (135° through south to 225°) with liquid swirl will result in a clockwise turn of the magnets and an under-reading compass.

45. Depending on the hemisphere and the direction of turn, the effect of liquid swirl will either increase or decrease the turning error. At the magnetic equator liquid swirl would be the only source of any error in a turn.

The Slaved Gyro\Remote Reading Compass

The Detector Unit

The Transmission System

The Slaving System

Annunciator Indicators

The Gyro Self Levelling System

Remote-Reading Compass System Errors

Advantages of the Slaved Gyro Compass

The Slaved Gyro\Remote Reading Compass

1. The direct-reading compass (DRC) which forms the primary heading reference in most light aircraft is relegated to the role of a standby system in larger aircraft. Direct reading compasses suffer from significant turning and acceleration errors and therefore have serious limitations. Furthermore, by virtue of the design of a DRC, the sensing elements (the magnets), must necessarily be housed on the flight deck; an area rich in deviating materials and components. In other words, whilst the long term accuracy of the DRC is reasonable, its overall performance will be degraded by short term inaccuracies.

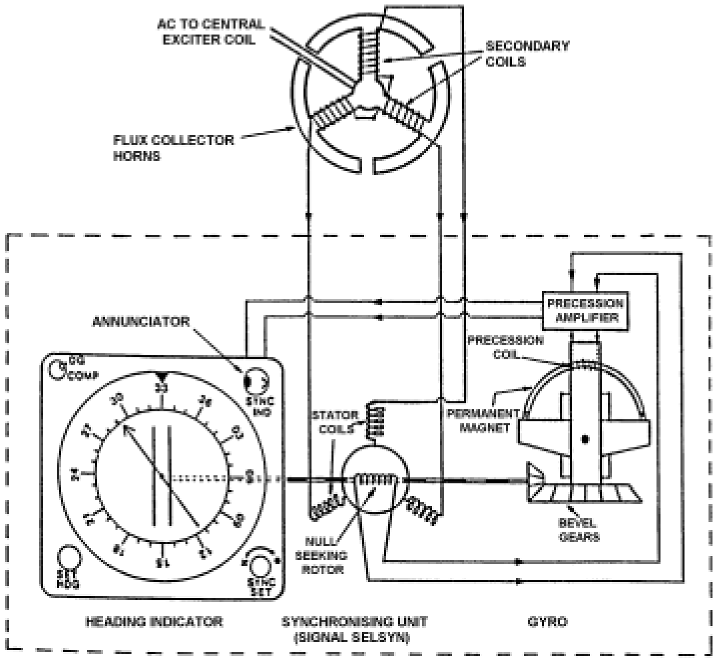
2. The directional gyro indicator (DGI) goes some way to solving the problems discussed above. If the DGI is set with reference to the DRC heading corrected for deviation when the aircraft is in straight and level flight and at a constant airspeed, then the DGI will read the correct value of magnetic heading. Unfortunately the DGI will subsequently drift because of both real and apparent errors, and so the problem of producing an accurate heading reference, whilst reduced, is still evident. The advantage of the short term accuracy of the directional gyro is degraded therefore by long term errors due to drifting.

3. The slaved gyro compass (otherwise known as the gyro magnetic compass or remote-reading compass) essentially solves the problem by automatically and continuously comparing the output of a magnetic sensing element with the indicated heading of the gyro indicator, and by resetting the gyro whenever a discrepancy exists. The gyro output is therefore slaved to magnetic north and combines the advantages of the long term accuracy of the sensing element with the short term accuracy of a directional gyro.

4. The pilot is no longer required to reset the gyro indicator periodically; or, putting it another way, significant errors of indicated heading do not occur if he fails to do so.

5. The major components of a slaved gyro compass system are shown at Figure 7-1.

FIGURE 7-1
Slaved Gyro Compass System - Block Schematic



The Detector Unit

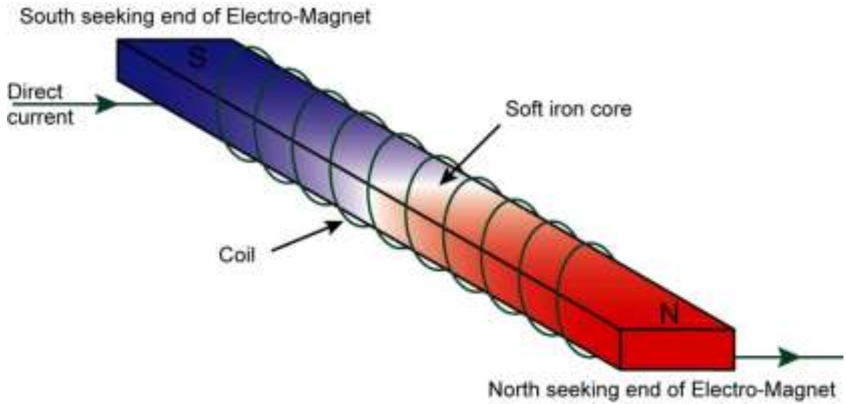
6. The element which senses the direction of magnetic north, the detector unit, is normally mounted in a wing tip or at the top of the fin, in an area where the deviating influence of the aircraft is at an absolute minimum.

7. The output of the detector is a series of electrical currents which represent magnetic heading in a manner which will be discussed shortly. The detector unit itself suffers from turning and acceleration errors, however the electrical output currents can be interrupted whenever the aircraft accelerates or turns. When this happens the gyro unit will function as a pure DGI for the duration of the manoeuvre. Once the aircraft returns to constant velocity flight the gyro heading is automatically updated with reference to the detector output.

8. The heart of any detector unit is the flux valve. The principle of operation of such a valve is now discussed.

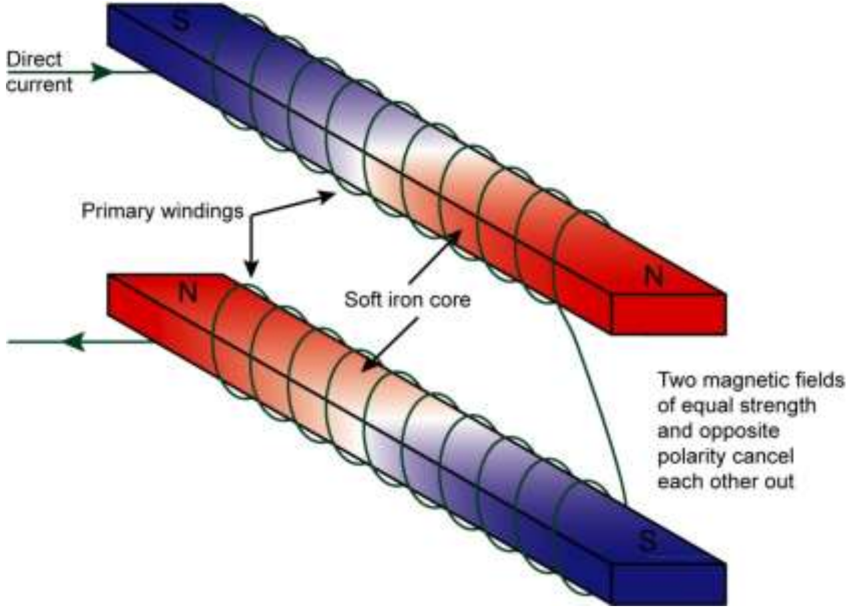
9. If a direct current is passed through a coil wound around a soft iron core, the core will become magnetized as shown at Figure 7-2.

FIGURE 7-2
Induced Magnetism in a Soft Iron Core



10. If the soft iron core is now split at the middle and the two halves are laid side by side without disturbing the coil, two magnets of equal strength and opposite polarity are produced as shown at Figure 7-3.

FIGURE 7-3
Two Soft Iron Cores Resulting in Equal and Opposite Magnetic Fields



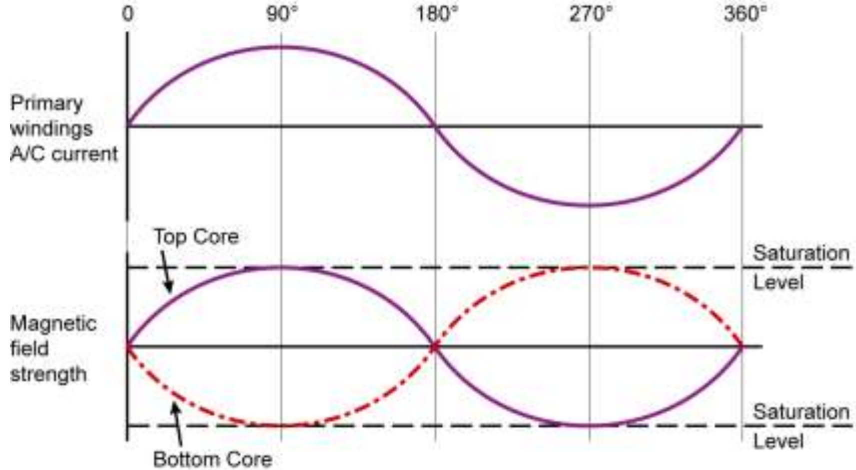
11. Were the value of the current passed through the coil to be steadily increased a stage would be reached when the soft iron cores would become saturated. That is to say that any further increase in coil current would not result in a corresponding increase in the strength of the magnetic fields produced.

12. In fact, in a flux valve, it is an alternating current rather than a direct current which is fed to the primary windings shown at Figure 7-3.

13. The effect of the alternating current is to completely reverse the magnetic polarity of both soft iron cores each time that the direction of current flow changes.

14. The peak value of the alternating current fed to the primary coil is just sufficient to saturate the soft iron cores. In other words, ignoring any external magnetic influences, the cores would just saturate at the 90° and 270° phase points of the primary winding alternating current flow, see Figure 7-4.

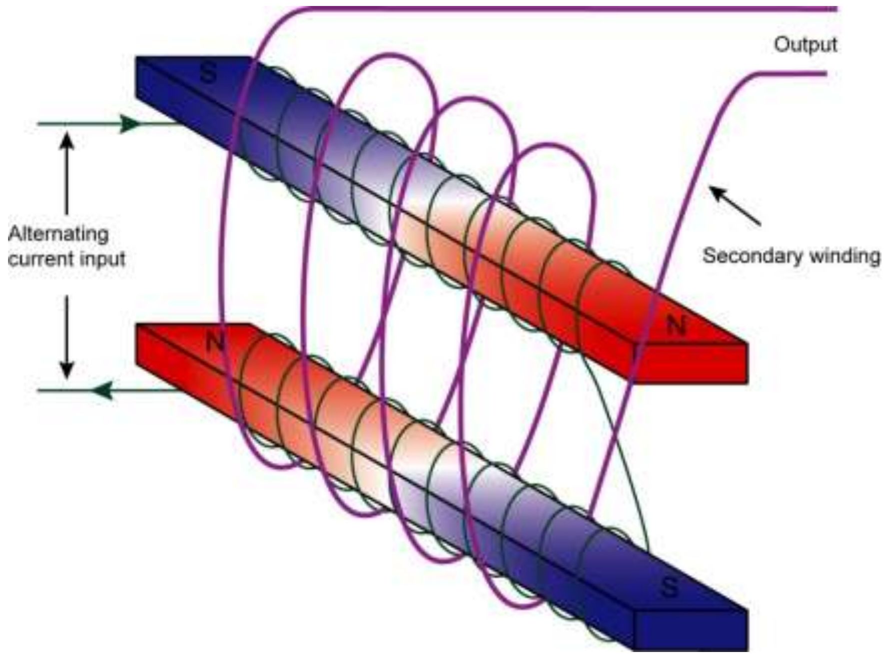
FIGURE 7-4
Variation in Fluxvalve Field Strength with Phase of AC.



15. Thus far we have established that an electric current may be used to produce a magnetic field. Of course the reverse is also true. If a magnetic field, of changing field strength and/or polarity, cuts a conducting element an alternating electric current will be induced to flow through the conductor.

16. A secondary coil is wound around the flux valve as shown at Figure 7-5.

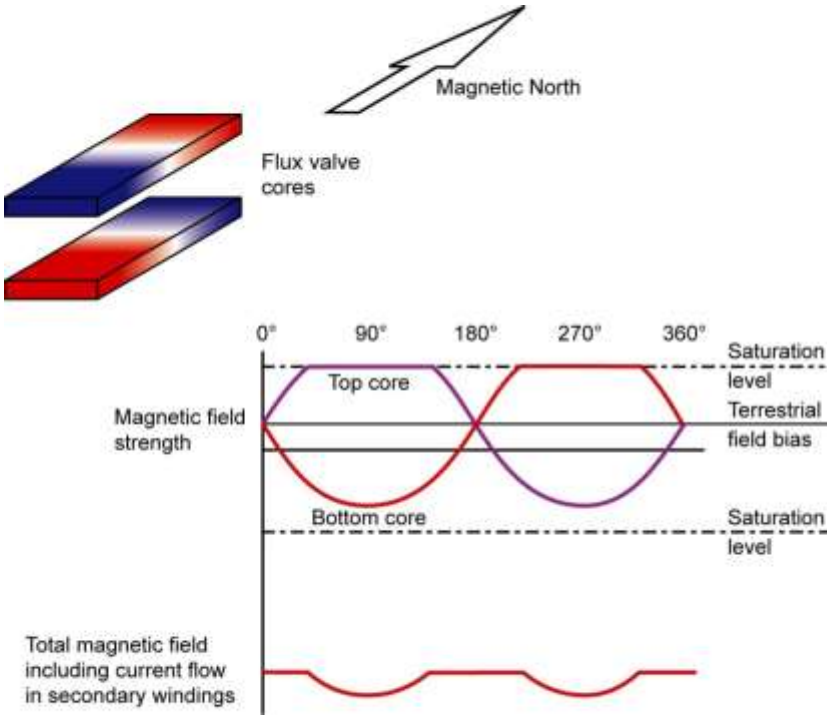
FIGURE 7-5
Fluxvalve Construction Showing Input and Output Coils



17. If the flux valve were to be placed in a totally screened container such that there were no external magnetic influences acting upon it, there would be no current induced into the secondary coil. This is because the two soft iron cores are producing equal but opposite magnetic fields which effectively cancel each other.

18. When the flux valve is placed within the terrestrial magnetic field the equality of the magnetic fields produced by the soft iron cores is disturbed; consequently, a current will be induced to flow in the secondary (output) winding, as shown at Figure 7-6.

FIGURE 7-6
Effect of Terrestrial Bias on Fluxvalve Operation



19. Comparing Figure 7-4 and Figure 7-6 should give some idea of what is happening. At Figure 7-4 no external magnetic influence was evident at the fluxvalve; consequently, both cores just achieved saturation twice during each 360° phase cycle of the alternating current fed to the primary coil. At Figure 7-6 the Earth's own magnetic field is acting upon the fluxvalve and its effect is to apply a **magnetic bias** to the system. The result is that one core will become totally saturated at the 90° phase point and the other at the 270° phase point. Differing magnetic field strengths now result in a current being induced to flow in the secondary winding.

20. It should be said at this point that the explanation given above does not represent the complete picture, however the syllabus does not require a study of the hysteresis characteristics of the soft iron cores. Without such a study it is necessary to accept that the explanation offered above is factually correct, if incomplete.

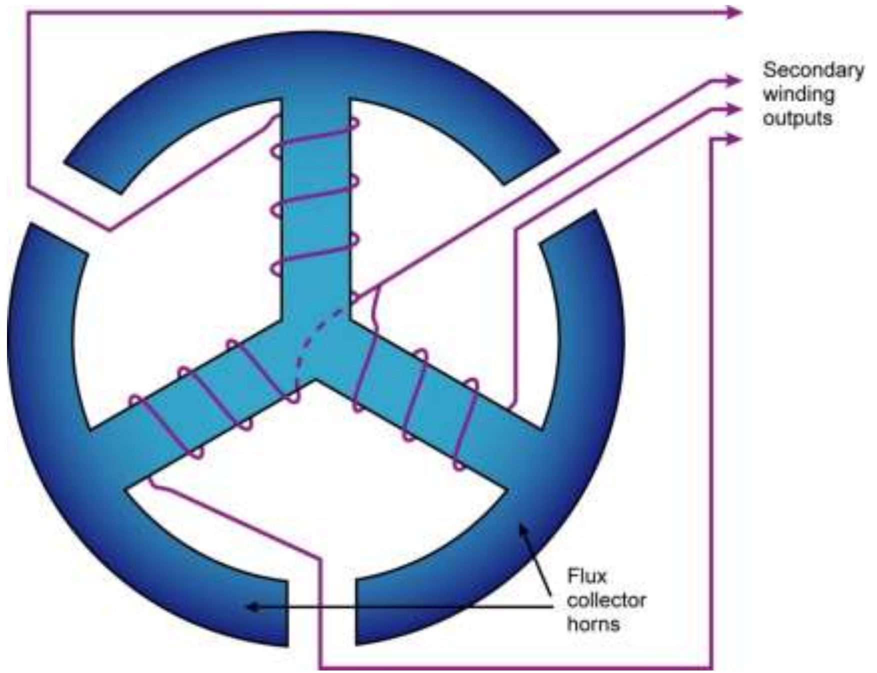
21. Were the flux valve to be turned through 180° the bias effect of the terrestrial magnetic field would be reversed, as would be the flow of current induced into the secondary coil.

22. Were the flux valve to be placed at 90° to the Earth's own magnetic field there would be no current induced into the secondary coil.

23. A detector unit employs three flux valves, positioned 120° apart, as shown at Figure 7-7. Appreciate that the detector unit is fixed in azimuth with respect to the aircraft. In other words, if the aircraft turns through 90°, so does the detector unit. Therefore the orientation of the detector unit to the Earth's magnetic field (and the currents generated within the secondary windings) vary with aircraft heading.

24. At Figure 7-7 only the secondary windings are shown. The **flux collector horns** are simply extensions of the soft iron cores and are employed to concentrate the terrestrial magnetic field.

FIGURE 7-7
Detector Unit/
Flux Valve System



25. The whole detector unit is required to lie in the Earth's horizontal plane, so that it is the H component of the terrestrial magnetic field which is sensed rather than the Z component. In order that the detector can remain horizontal when the aircraft is pitching or rolling the unit is suspended by a universal joint knows as a **Hooke's Joint.** This arrangement allows, typically, 25° of freedom in pitch and roll.

26. When the freedom of movement limits imposed by the Hooke's joint are exceeded the electrical outputs of the fluxvalve are isolated from the gyro unit. Since the detector unit in this condition is no longer in the horizontal plane an element of the Earth's Z component would necessarily be sensed at the detector and the resultant turning/acceleration errors would cause an eventual misalignment of the gyro.

27. Even if the freedom of movement limits of the Hooke's joint were not exceeded during a turning or acceleration manoeuvre the detector unit would still depart from the Earth's horizontal plane as its own mass reacted under the effect of inertia. Again a part of the Earth's Z component would be sensed and the gyro would eventually become misaligned. In order to minimise this error the gyro unit is precessed to align itself to the detector output at a slow rate (only 2° per minute typically) during normal operation. During any manoeuvre of limited duration the heading indicated by the gyro magnetic compass will not therefore be significantly in error.

28. The next step is to consider the technique by which the gyro is maintained in alignment with the detector unit output.

The Transmission System

29. The signals generated in the detector must somehow be transmitted to the gyro unit in order to keep the gyro slaved to (or synchronised with) magnetic north as determined by the detector. This is achieved within the synchronising unit by means of a self synchronous control unit (or **selsyn**).

30. The selsyn is effectively a detector unit in reverse. The electrical currents which are produced in the secondary windings of the fluxvalves within the detector unit, and which synthesize the Earth's magnetic field direction, are fed to the selsyn, as shown at Figure 7-1.

31. The **stator coils** within the synchronising unit are positioned mutually at 120° to each other, as are the flux valves in the detector. The flux valve currents flowing through these stator coils will produce a magnetic field which represents the Earth's magnetic field sensed at the detector. The **null seeking rotor coil** of the selsyn will have a current induced into it whenever it lies at any angle other than 90° to the magnetic lines of flux produced by the stator coils.

The Slaving System

32. The gyro unit itself is mechanically coupled to the null seeking rotor within the selsyn, as is the compass rose of the heading indicator, see Figure 7-1.

33. In the event that the gyro is misaligned with Magnetic North as defined by the detector unit, then necessarily the null seeking rotor must be at other than 90° to the magnetic field manufactured within the selsyn, and consequently an error signal will be produced by the rotor. This error signal is amplified, rectified and fed to the **precession coil** on the gyro itself, again as shown at Figure 7-1. The precession coil now generates its own magnetic field which acts upon the semi-circular shaped permanent magnet which is attached to the inner gimbal of the gyro. This exerts a downward force on the inner gimbal, the consequence of which is that the gyro will precess in the aircraft's yawing plane and, because of the mechanical linkage, both the null seeking rotor and the compass rose on the face of the instrument will also rotate. This will continue until the error signal at the null seeking rotor ceases, at which time the rotor will again lie at 90° to the selsyn's magnetic field and the gyro will now be aligned with magnetic north as sensed at the detector unit.

34. An alternative to the precession coil and permanent magnet arrangement described above is a torque motor, which is more commonly found in modern systems.

Annunciator Indicators

35. It is obviously desirable that the pilot should know when an error current is flowing from the null seeking rotor, since at such times the heading indication will be erroneous. The annunciator circuit is located between the precession amplifier and the precession coil. An indicator shows when precession currents are flowing and therefore indicates that the compass rose is not aligned with magnetic north as sensed by the detector unit. Two types of annunciator indicator are shown at Figure 7-8.

36. It has already been mentioned that a typical rate for the gyro to automatically align with the detector unit output is 2° per minute. Remember that this is deliberately kept to a low value so that the system will not become grossly misaligned whenever the detector output is affected by the Z component during turning or acceleration manoeuvres.

37. If the operator notices a gross misalignment, as may well occur when the equipment is first switched on, then the system can be rapidly re-aligned using the manual synchronisation control. Figure 7-9 shows the indicator of a G4F compass, the manual resetting control is positioned at bottom right and the annunciator indicator (dot/cross type) is positioned at top right. The set heading knob is used to position the heading 'bug', and the DG/COMP switch gives the operator the option of operating in the pure DGI mode whenever the output of the detector unit is suspect, following a lightning strike, or at very high magnetic latitudes.

FIGURE 7-8
Types of Annunciator Indicator

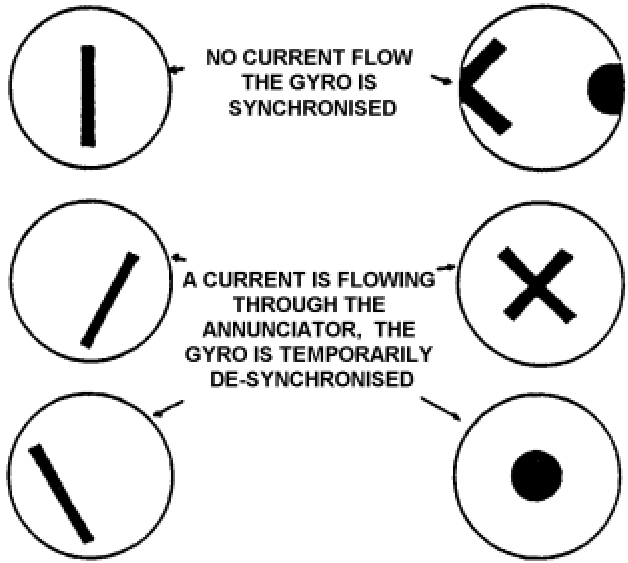
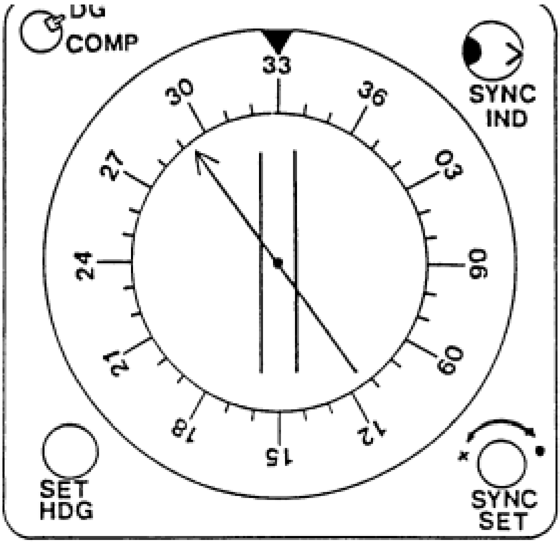


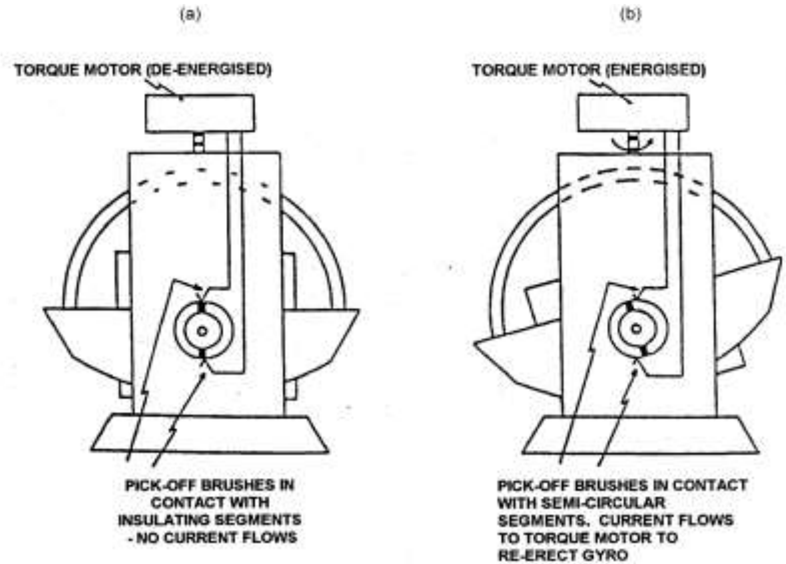
FIGURE 7-9
G4F Compass indicator



The Gyro Self Levelling System

38. Finally it is necessary to consider the technique employed to maintain the gyro spin axis in the yawing plane of the aircraft. Figure 7-10 shows schematically how this is achieved.

FIGURE 7-10
The gyro self-levelling system



39. The **commutator switch** consists of two semi-circular contacts insulated from each other and attached to the pivot joining the inner and outer gimbals. With the gyro spin axis lying in the yawing plane (Figure 7-10(a)) the pick-off brushes are in contact with the insulating segments and no current flows to the torque motor. With the gyro toppled (Figure 7-10(b)) the commutator contacts have rotated under the pick-off brushes and an electrical current flows to the torque motor. The precessing force from the torque motor is applied around the aircraft's normal (vertical) axis and the resultant movement of the spin axis is back towards the yawing plane. When the spin axis again lies in the correct plane the torque motor is de-energised as the pick-off brushes contact the insulating segments.

Remote-Reading Compass System Errors

40. The remote reading compass system suffers from the following errors:

Fluxvalve Tilt Errors

41. Any horizontal accelerations which cause fluxvalve tilt can cause heading errors in a simple uncompensated remote-reading compass system. Accelerations are caused by coriolis, vehicle movement (rhumb line), aircraft turns, linear changes of velocity and flux valve vibrations. Fluxvalve induced heading errors will not manifest themselves immediately and the rate of any heading error introduction depends on the limiting precession rate and the response time of the system (time constant).

(a) **Turning Error.** Although a high rate of turn in a fast aircraft would show the greatest flux valve heading error, little of the error is displayed since the time spent in the turn is minimal. Slow prolonged turns at high speeds generate the greatest errors. The errors decay after level flight is resumed. Flux valve induced errors due to tilt can be limited by switching the system to an unslaved directional gyro mode whenever turns are sensed by suitable detection devices.

(b) **Coriolis Error.** An aircraft flies a curved path in space and in consequence there will be a force acting to displace the pendulously suspended flux valve. The error is calculable, depending on groundspeed, latitude, dip and track, and can be compensated automatically.

(c) **Vehicle Movement Error.** Whenever flying a true or magnetic rhumb line the aircraft must turn to maintain a constant track with reference to converging meridians. As with coriolis error, the acceleration displaces the detector from the local horizontal plane. A correction can be applied in a similar manner to the coriolis error.

(d) **Fluxvalve Vibration.** Fluxvalve vibration results in a heading oscillation, the mean of which is not the actual mean heading. Since the gyro slaving loop tends to average fluxvalve headings over a period of time, the gyro would eventually be precessed to the erroneous fluxvalve mean heading. The effect can be limited to small values by careful design of the pendulous detector damping mechanism and through consideration of the location of the detector in the aircraft.

Northerly Instability

42. Northerly instability or weaving is a heading oscillation experienced in high speed aircraft attempting to fly straight and level at or near a heading of magnetic North. Starboard bank of the aircraft induces starboard tilt, and this causes an under reading of the heading. Thus, if an aircraft on North banks to starboard to correct a small error, the magnetic meridian appears to rotate in the same direction. The aircraft continues to turn and eventually reaches the false meridian. On levelling out, the fluxvalve senses the correct meridian and starts to precess the gyro towards it. The indicated heading changes and the aircraft is banked to port to regain a northerly indicated heading. This tilts the fluxvalve which has the effect of appearing to rotate the meridian to port. The new false meridian is chased until, upon resuming level flight, the sensor detects the correct meridian again and precesses the gyro to starboard. The pattern is repeated and the amplitude can be as great as 6°. The amplitude of the weave tends to increase with an increase in dip and aircraft velocity.

Hang-Off Error

43. Gyroscopic drift is a constant source of error signal in a gyro-magnetic compass system, and although it will be compensated for by the precession loop, at any given time there must be an element of error present. This is known as hang-off error. Gyro drift may be due to:

(a) **Real Drift.** Real drift can only be reduced by the incorporation of a high quality azimuth gyro having a low real drift rate.

(b) **Earth Rate.** Apparent azimuth gyro drift due to Earth rotation can be countered by correcting the gyro at a rate of 15 sin lat °/hr. The correction can be supplied through a manually set latitude correction mechanism or through a constantly biased gyro.

(c) **Transport Wander.** To compensate for transport wander due to the convergence of geographic meridians the gyro must be corrected at a rate equal to:

$$\frac{G/S}{60} \tan \text{lat} \ °/\text{hr} (\text{where } G/S = \text{east-west groundspeed})$$

44. The correction can be applied manually or through a computer using inputs of groundspeed, heading and latitude. However, although the gyro can be compensated in this way for the apparent change in the direction of **geographic** North, the output from the fluxvalve is in terms of **magnetic** North. Therefore, as the aircraft moves over the Earth there will be a difference between fluxvalve and gyro since the variation is changing (unless the aircraft is flying along an isogonal). To remove this error, variation must be applied to the output of the detector unit before the gyro error loop so that both the gyro and fluxvalve give directional information relative to true North. The value of variation can be inserted manually or by means of an automatic variation setting control unit. Failure to update the variation value will result in small hang-off errors.

Gimbal Error

45. When a 2 degree of freedom gyroscope with a horizontal spin axis is both banked and rolled, the outer gimbal must rotate to maintain orientation of the rotor axis, thereby inducing a heading error at the outer gimbal pick-off. The magnitude of this error depends upon the angle of bank and the angular difference between the spin axis and the longitudinal axis. In most systems the spin axis direction is arbitrary, relative to North, so the error is not easily predicted.

Transmission Errors

46. Overall system accuracy is worsened by the errors in the synchro system. Typically, each synchro might be expected to have an error in the order of 0.1°, with an overall system error of perhaps 0.5°. This shows in a compass swing as a coefficient D or E error (not part of this syllabus).

Compass Swinging Errors

47. It is not possible to obtain absolute accuracy in compass swinging, and even refined methods are considered to be only accurate to 0.2°.

Variation and Deviation Errors

48. Charted values of variation may be considered to vary between 0.1° and 2°. Over the UK the uncertainty at height is considered to be within 1° but the value varies both with height and locality. Setting of variation and deviation is likely to be accurate to 0.25°.

Advantages of the Slaved Gyro Compass

49. Some of the advantages of this type of system over a direct reading system have already been mentioned, the main advantages are summarised below:

(a) The detector unit is located in an area which is low in aircraft magnetism.

(b) The compass rose is mechanically driven, giving a stable compass heading, notably in turbulence.

(c) The compass rose is flat faced and located centrally in front of each pilot on the flight instrument panel, removing the parallax error which occurs with direct reading compasses.

(d) The electro/mechanical output of the compass system can be used to feed other instruments and systems, for example RMIs or a Flight Management Computer.

(e) The compass rose can be electrically or mechanically corrected with variation to enable true headings to be flown.

(f) Turning and acceleration errors are much reduced.

(g) The system can be operated as a DGI in high latitudes or in the vicinity of thunderstorms, where magnetic compasses are unreliable.

(h) The detector unit senses rather than seeks the magnetic meridian, giving increased sensitivity.

Pre-flight Checks

50. When power is available to the system, operate the synchronising control in the direction indicated by the annunciator. Now check the heading readouts of the primary display and the RMI against the other gyro slaved compass system (if two are fitted), the direct reading compass and the known heading of the aircraft (with reference to the stand centre line markings and the aerodrome manoeuvring area chart. Be aware that if the aircraft is on a stand and is surrounded by ground power units, fuel bowsers and so on, the readings of each of the gyro compasses and the direct reading compass may disagree. Whilst the aircraft is taxiing, compare the compass readouts against each other, especially during the turns. If any doubt still exists, a final check can be conducted with the aircraft aligned with the runway centre line.

51. In an aeroplane with two slaved gyro compasses, it is normal for the captain's primary display (horizontal situation indicator, HSI) and the first officer's RMI to be driven by one gyro compass. The first officer's HSI and the captain's RMI are naturally driven by the second gyro compass. It is normal to include a compass comparator, which will warn the pilot's whenever a discrepancy (of, for example, 5° or more exists for 5 seconds or more) is sensed between one compass system and the other. Finally, it is normally possible for the pilots to drive all of the HSIs and RMIs from a single gyro compass in the event that the other system fails.

Self Assessed Exercise No. 4

QUESTIONS:

QUESTION 1.

Describe the 5 main properties of a magnet.

QUESTION 2.

External lines of magnetic flux are said to have direction; from which pole do they emanate?

QUESTION 3.

State the units of measurement of magnetic force.

QUESTION 4.

Explain the term 'saturated' when applied to magnetic material.

QUESTION 5.

Define magnetic moment.

QUESTION 6.

State the convention for assigning colours to the Magnetic Poles.

QUESTION 7.

Define directive force.

QUESTION 8.

Define angle of dip.

QUESTION 9.

Define the Magnetic Equator.

QUESTION 10.

Define the term 'Magnetic Meridian'.

QUESTION 11.

Define an isoclinal line.

QUESTION 12.

Given that $T=50mT$ and angle of dip$=30°$, calculate H and Z.

QUESTION 13.

Where on the Earth's surface is the vertical component of the Earth's magnetic field (Z) at a maximum and a minimum?

QUESTION 14.

State the minimum strength of the directive force for operation of a magnetic compass system.

QUESTION 15.

Define variation.

QUESTION 16.

The maximum value of variation is 180°; state where on the Earth's surface this occurs.

QUESTION 17.

Explain why the value of variation for any given point on the Earth's surface changes with time.

QUESTION 18.

Define an agonic line.

QUESTION 19.

Define an isogonal.

QUESTION 20.

Define the term 'deviation'.

QUESTION 21.

Explain the terms 'hard iron magnetism' and 'soft iron magnetism'.

QUESTION 22.

Explain the effects that change in latitude and change in heading have on the deviating effect of hard iron magnetism.

QUESTION 23.

Explain the effects that change in latitude and change in heading have on the deviating effect of vertical soft iron magnetism.

QUESTION 24.

State the components of hard iron magnetism in an aircraft and with which aircraft axis each is associated.

QUESTION 25.

State which components of hard iron magnetism are associated with which Coefficients of Deviation.

QUESTION 26.

State the three basic properties a direct reading compass should have.

QUESTION 27.

State the system of suspension employed to improve horizontality.

QUESTION 28.

State the methods of improving compass sensitivity.

QUESTION 29.

Define the aperiodicity of a compass.

QUESTION 30.

State the conditions in which the indications on a DRC may be unreliable or in error.

QUESTION 31.

State the causes of turning and acceleration errors.

QUESTION 32.

State the effect on a direct reading compass of an acceleration to the East in the Northern Hemisphere.

QUESTION 33.

State the effect on a direct reading compass of an acceleration to the East in the Southern Hemisphere.

QUESTION 34.

State the effect on a DRC's magnets (when viewed from above) when an aircraft at 45°N turns from 315°M through North to 045°M.

QUESTION 35.

State the general rule for the effect of liquid swirl on the magnets of a DRC.

QUESTION 36.

State the major components of a gyro magnetic compass.

QUESTION 37.

Explain the operation of the detector unit.

QUESTION 38.

Explain the function of the Hooke's Joint mounting for the detector unit.

QUESTION 39.

Explain the function of the Signal Selsyn in a gyro magnetic compass.

QUESTION 40.

Explain the operation of the Annunciator Indicators in a Gyro Magnetic Compass.

QUESTION 41.

State the advantages of a gyro magnetic compass over a direct reading compass.

ANSWERS:

ANSWER 1.

1. The ability to attract other magnetic materials.

2. The tendency, when freely suspended, to align in an approximately North/South direction.

3. The attracting force tends to be concentrated at each end of the magnet.

4. Like poles repel; unlike poles attract.

5. The strength of the attracting or repelling force between two magnets decreases as the square of the distance between them.

ANSWER 2.

Lines of magnetic flux are, by convention, assumed to emanate from the North-seeking pole of the magnet and re-enter at the South-seeking pole. See figure 4-1.

ANSWER 3.

Tesla

ANSWER 4.

The strength of a magnetic field is measurable; the units are 'tesla'. The strength of a magnetic field induced in a magnetic material can be increased only up to a certain point and no more, at which point the material is said to be saturated.

061-4-3

ANSWER 5.

The magnetic moment of a magnet is the product of its pole strength multiplied by its length; it is a measure of its tendency to turn or to be turned by another magnetic field.

ANSWER 6.

The north-seeking end is red; the south-seeking is blue.

ANSWER 7.

The force that causes the alignment of a magnet with the Earth's magnetic field. It is the horizontal component, H, of the total magnetic field.

ANSWER 8.

Angle of dip is the angle a freely suspended magnet takes up below the local horizontal when aligning itself with the Earth's total magnetic field.

ANSWER 9.

The Magnetic Equator is a line joining all the points where the lines of total force are horizontal to the Earth's surface.

ANSWER 10.

A freely suspended magnet will align with the local lines of flux of the Earth's magnetic field. A great circle line drawn from the magnet to the North Magnetic Pole would then define the local magnetic meridian.

ANSWER 11.

An isoclinal line is a line joining points on the Earth's surface that have the same value of dip.

ANSWER 12.

$H = T \times \cos dip$ $Z = H \times \tan dip$

$= 50 \times 0.866$ $= 34.64 \times 0.577$

$= 34.64\ \mu T$ $= 19.99\ \mu T$

ANSWER 13.

Vertical component (Z) is at a maximum at the Magnetic Poles and at a minimum, zero, at the Magnetic Equator.

ANSWER 14.

6 micro-tesla.

ANSWER 15.

The true and magnetic North Poles are not co-located; variation is the angular difference between True North and Magnetic North at any point on the Earth's surface.

ANSWER 16.

The maximum value of variation occurs on the meridian between the True North Pole and the Magnetic North Pole.

ANSWER 17.

Secular change. The Magnetic North Pole moves clockwise around the True North Pole every 960 years.

Annual change. A yearly cycle; maxima occurring at Spring and Autumn Equinoxes.

Daily changes associated with diurnal changes in the ionosphere.

Periodic changes due to 11 year sunspot cycle. Magnetic storms associated with sunspot activity cause changes in variation.

ANSWER 18.

An agonic line is a line joining points of zero variation; it approximates the meridian passing through the True North Pole and the Magnetic North Pole (except the portion actually between the poles) and its anti-meridian.

ANSWER 19.

An isogonal is a line joining points of equal variation.

ANSWER 20.

Deviation is the angular difference between the compass indicated direction of North and the direction of the local magnetic meridian (ie. the difference between Compass North and Magnetic North).

ANSWER 21.

Hard iron magnetism is the name given to magnetism that is virtually permanent or decays only very slowly. Soft iron magnetism is the name given to temporary magnetism.

ANSWER 22.

Although hard iron magnetism is permanent and does not change with magnetic latitude, the deviating effect it produces will vary; H is weaker at higher latitudes and therefore the compass magnets are more easily deflected. The deviation effect of hard iron magnetism varies with aircraft heading.

ANSWER 23.

Vertical soft iron magnetism increases with increase in magnetic latitude because Z increases; therefore, the deviation effect increases. The increase in deviating effect is compounded by the fact that the H component is reducing and the compass magnet is more easily deflected. The deviating effect of vertical soft iron magnetism will vary with aircraft heading.

ANSWER 24.

Component P is associated with the fore/aft axis (positive when blue pole is ahead of the compass system); component Q is associated with the athwartships axis (positive when the blue pole is on the starboard side); component R is associated with the vertical axis (positive when the blue pole is below the compass system). See Figures 5-1 to 5-3.

ANSWER 25.

Component +P produces coefficient +B; component +Q produces coefficient +C.

ANSWER 26.

1. Horizontality
2. Sensitivity
3. Aperiodicity

ANSWER 27.

Pendulous suspension.

ANSWER 28.

1. Use several small magnets with high pole strengths.
2. Assembly is made as light as possible to reduce friction at the pivot.
3. Pivot itself incorporates a jewelled bearing that is lubricated by the viscous fluid that fills the bowl.
4. Viscous fluid effectively lightens the magnet system.

ANSWER 29.

The ability of the magnetic assembly to settle quickly, pointing towards the Magnetic North Pole, following displacement during manoeuvres or turbulence.

ANSWER 30.

Incorrect readings can occur when the aircraft accelerates, decelerates and when the aircraft turns.

ANSWER 31.

1. Inertia

2. Z-field effect: the effect of the vertical component of the Earth's magnetic field on the displaced magnets

ANSWER 32.

Inertia and Z-field effect are complementary, both causing the compass to under-read (ie. less than 090°). The compass appears to indicate a turn towards the North (nearer) Magnetic Pole.

ANSWER 33.

Inertia and Z-field effect are complementary, both causing the compass to over-read (ie. more than 090°). The compass appears to indicate a turn towards the South (nearer) Magnetic Pole.

ANSWER 34.

The magnets are displaced clockwise when viewed from above resulting in the compass under-reading and being 'sluggish'. See figure 6-14

ANSWER 35.

When turning through the nearer pole, liquid swirl will increase any movement of the magnets; when turning through the further pole, liquid swirl will decrease the movement of the magnets.

ANSWER 36.

The detector unit (in wingtip); the gyro unit; the precession amplifier; the synchronising unit (signal selsyn); the annunciator circuit; the heading indicator (display). See Figure 7-1.

ANSWER 37.

See paragraph 6 to paragraph 24 061-7

Figure 7-2, Figure 7-3, Figure 7-4, Figure 7-5, Figure 7-6 and Figure 7-7.

ANSWER 38.

See paragraph 25 to paragraph 27 061-7

ANSWER 39.

See paragraph 29 to paragraph 31 061-7

ANSWER 40.

See paragraph 35 to paragraph 37 061-7

Figure 7-8 and Figure 7-9

ANSWER 41.

See paragraph 47 061-7

Properties of a Chart

The Ideal Chart Projection

Orthomorphism

Conformality

Chart Scale

Projections

Properties of a Chart

1. Now that the properties of the Earth have been studied, it is logical to consider the maps or charts which are used to portray the Earth in two dimensions.

2. To produce or project a chart, it is necessary to produce a spherical model of the Earth which is called the **reduced Earth**. The actual size of the reduced Earth used for any projection determines the scale of the resulting chart.

3. The terms map and chart are normally considered to be inter-changeable. To be totally correct, a chart shows the parallels of latitude and meridians together with the minimum of topographical features, and would normally be used for plotting purposes. A map normally shows the graticule of latitude and longitude together with topographical detail in far greater depth, for example the Ordinance Survey maps of Great Britain.

The Ideal Chart Projection

4. An ideal projection (which can not exist) should have the following properties:

    (a) The scale should be both correct and constant at all points on the chart. Scale will be discussed in depth later in the syllabus.

    (b) Great circles and rhumb lines should be represented on the chart as straight lines (clearly this is impossible).

(c) Bearings on the chart should be identical to the corresponding bearing on the surface of the Earth.

(d) Parallels of latitude and meridians should cross each other at right-angles on the chart, as they do on the Earth. This is an essential property of any chart used for navigational purposes.

(e) World-wide coverage should be possible.

(f) Adjacent sheets should fit together, with the graticule of latitude and longitude aligned from one sheet to the next.

(g) Shapes (land masses, towns etc.) should be correctly represented.

(h) Areas should be correctly represented.

5. It is impossible to produce the perfect chart. Any chart which is to be used for navigation must, however, be **orthomorphic.**

Orthomorphism

To be orthomorphic a chart must possess two properties:

(a) It must have **equal scale expansion.** This is achieved if the chart has a constant scale, that is to say that there is no scale expansion. On a chart which has a varying scale, orthomorphism still exists provided that the scale expansion at any point on the chart is the same in all directions, over a relatively short distance.

(b) The parallels of latitude must cross the meridians at right-angles on the chart, as they do on the Earth. From a practical point of view this means that the bearing between any two points on the Earth will be correctly represented on an orthomorphic chart between the same two points.

Conformality

6. An orthomorphic chart which also has the property that a straight line approximates a great circle is referred to as conformal eg. The Lamberts Conical orthormorphic chart is also referred to as the Lamberts Conformal Chart.

Chart Scale

7. The scale of a map or chart is the ratio of a given distance on the map or chart to the actual distance it represents on the Earth. The scale of a chart at any point may be expressed in several ways. The four examples of scale descriptions given below refer to the same chart, and all four are correct:

(a) $\dfrac{1}{63{,}360}$

(b) 1 to 63,360

(c) 1: 63,360

(d) 1 inch to 1 statute mile (one statute mile being 63,360 inches).

8. Of the four examples given above, the first, the **representative fraction**, lends itself best to mathematical manipulation.

9. The scale of this chart is such that 1 unit of distance on the chart (chart distance) represents 63,360 of the **same** units of distance on the surface of the Earth (Earth distance).

10. The scale, **expressed as a representative fraction**, is equal to the chart distance (CD) divided by the Earth distance (ED) it represents providing that **both CD and ED are expressed in the same units of distance**.

$$\text{Thus: } \frac{1}{F} = \frac{CD}{ED}$$

$$\text{or by transposition } \frac{ED}{F} = \mathbf{CD}$$

$$\text{and } \mathbf{ED = CD \times F}$$

11. The smaller the denominator of the representative fraction, the larger the scale of the chart described.

EXAMPLE 8-1

EXAMPLE

On chart A one inch represents 15.78 statute miles. Express the scale of this chart as a representative fraction. The scale of chart B is given as 1: 850,000.
How many kilometres on the Earth are represented by a line 5 cm long on the chart?
Which of the two charts (A or B) has the larger scale ?

SOLUTION

$$\frac{1}{F} = \frac{CD}{ED}$$

$$\frac{1}{F} = \frac{1''}{15.78 \times 63360''}$$

$$\frac{1}{F} = \frac{1}{1{,}000{,}000}$$

$$\frac{1}{F} = \frac{CD}{ED}$$

$$\frac{1}{850{,}000} = \frac{5\ \text{cm}}{ED}$$

$$ED = 5 \times 850{,}000\ \text{cm}$$

$$ED = \frac{5 \cdot 850{,}000}{100{,}000}$$

$$ED = 5 \times 8.5\ \text{km}$$

$$ED = 42.5\ \text{km}$$

Chart B has the larger scale

EXAMPLE 8-2

EXAMPLE

The scale of a chart is given as 1: 750,000. Determine the length in centimetres on this chart of a straight line joining two points which are 75.5 nm apart.

SOLUTION

$$\frac{1}{F} = \frac{CD}{ED}$$

$$\frac{1}{750{,}000} = \frac{CD}{75.5 \text{ nm}}$$

$$CD = \frac{75.5 \times 6080 \times 12 \text{ (inches)}}{750{,}000}$$

$$CD = \frac{75.5 \times 6080 \times 12 \times 2.54 \text{ (cm)}}{750{,}000}$$

$$CD = 18.66 \text{ cm}$$

EXAMPLE 8-3

EXAMPLE

Given a constant scale chart with a published scale of 1: 500,000, determine the length in centimetres of a straight line joining point A at 50°N 2°E with point B at 48°N 2°E.

SOLUTION

The two points lie on the same meridian (great circle) and are therefore 120 nm (120 minutes of arc of latitude) apart.

$$\frac{1}{F} = \frac{CD}{ED}$$

$$\frac{1}{500{,}000} = \frac{CD}{120 \text{ nm}}$$

$$CD = \frac{120 \times 6080 \times 12 \text{ (inches)}}{500{,}000}$$

$$CD = \frac{120 \times 6080 \times 12 \times 2.54 \text{ (cm)}}{500{,}000}$$

CD = 44.48 cm

Projections

12. Different chart projections have different properties and are therefore suitable for different purposes. Chart selection is dependent on the purpose for which it is to be used and the area involved. Projections may be grouped under four main headings:

(a) Mathematically constructed graticules not based on a geometric projection.

(b) Conical projections and their modifications. i.e. a cone placed on either pole of the reduced earth.

(c) Cylindrical projections and their modifications i.e. a cylinder enclosing the reduced earth.

(d) Azimuthal projections. i.e. a flat plane tangential to the reduced earths surface.

13. There are thirteen projections contained in these groups. However, only the following are required to be studied in detail for this syllabus. They are:

(a) The Mercator projections: normal, transverse and oblique.

(b) The Lamberts conformal.

(c) The Polar stereographic.

The Mercator Projection

Mercator Scale Expansion

Measurement of Distance on a Mercator Chart

The Appearance of Rhumb Lines and Great Circles

Plotting Radio Bearings on a Mercator Chart

Uses of the Mercator Chart

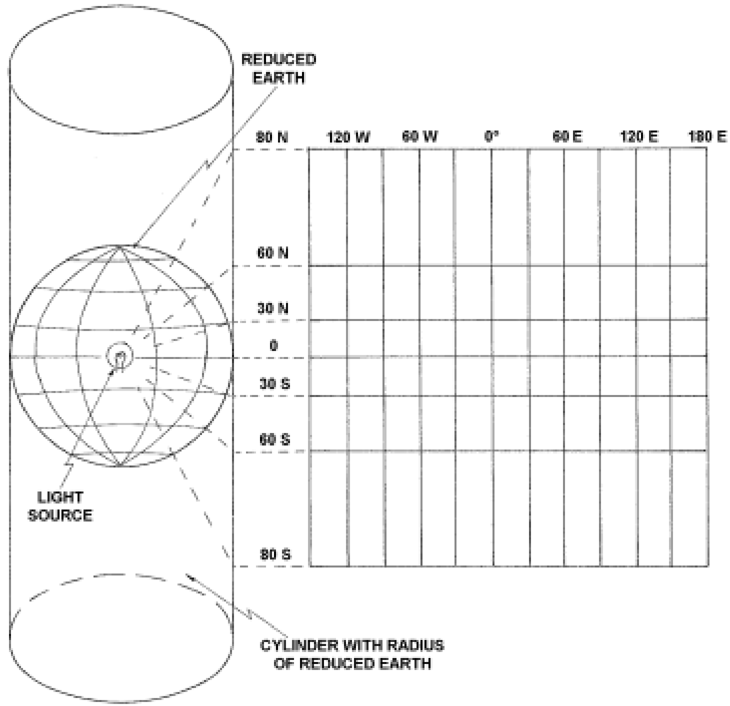
The Mercator Projection

1. The Mercator chart is a **cylindrical** projection. A cylinder having a radius identical to that of the reduced Earth is placed over the reduced Earth such that the cylinder and the globe are touching only at the equator.

2. The graticule of latitude and longitude, which is etched on to the reduced Earth, is projected onto the cylinder using a light source at the centre of the globe.

3. Having fixed the graticule on to the cylinder, the cylinder is now laid flat and appears as shown at Figure 9-1.

FIGURE 9-1
Mercator Cylindrical Projection



4. Refer to Figure 9-1 and note the following points:

(a) All meridians appear as parallel, vertical, straight lines, and they are equally spaced.

(b) The parallels of latitude also appear as straight parallel lines, but the distance between them increases with increase of latitude.

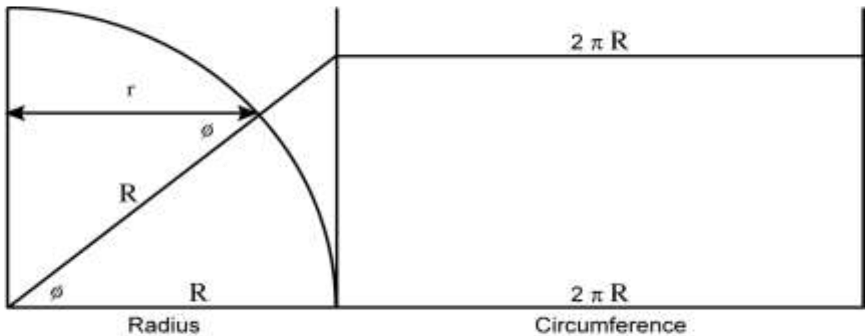
(c) It is impossible to project the poles using this projection.

(d) The parallels of latitude cross the meridians at right-angles.

Mercator Scale Expansion

5. On a Mercator chart the distance between adjacent parallels of latitude increases as the latitude increases. The Earth distance between adjacent parallels of latitude is constant (one degree change of latitude equals 60 nm at all latitudes). Thus, as latitude increases, the chart distance representing a constant Earth distance is increased, and consequently the chart scale becomes larger.

FIGURE 9-2
Mercator Scale Expansion



6. If R is the equatorial radius of the earth, then the width of the chart representing 360° of longitude is $2\pi R$. This width is constant over the whole chart. At latitude ∅° the earth radius is r and the earth distance for 360° of longitude at that latitude is $2\pi r$. However, r = R cos ∅° ; therefore, the earth distance is $2\pi R \cos \varnothing$. This is represented on the chart by $2\pi R$.

From the scale formula:

$$\text{Scale} = \frac{\text{Chart distance}}{\text{Earth distance}}$$

Then $$\frac{1}{F} = \frac{2\pi R}{2\pi R \cos \varnothing} = \frac{1}{\cos \varnothing} = \sec \varnothing$$

Thus the scale expands as the secant of the latitude.

7. Based on this principle, it can be demonstrated using a scale drawing that scale also expands north/south at the same rate. Hence we have equal scale expansion.

8. Since parallels of latitude cross meridians at right angles, bearings on the Earth are correctly represented on the chart; therefore, the chart can be described as orthomorphic.

EXAMPLE 9-1

EXAMPLE

The scale of a Mercator chart is given as 1: 500,000 at the equator. Determine the scale of this chart at 60°S.

The scale at 60°S = The scale at the equator x the secant of 60°

$$\sec 60° = \frac{1}{\cos 60°} = \frac{1}{0.5} = 2$$

$$\text{The scale at } 60°S = \frac{2}{500{,}000} = \frac{1}{250{,}000}$$

Remember that the secant of any angle is the reciprocal of the cosine of the same angle. Example 1 could therefore equally well have been written:

$$\text{The scale at } 60° = \frac{1}{500{,}000 \times \cos 60°} = \frac{1}{500{,}000 \times 0.5} = \frac{1}{250{,}000}$$

9. Of course the answer is the same in both cases whichever technique is used. Always apply a common-sense check to ensure that no gross mathematical errors have occurred. The scale at a latitude other than 0° must be larger than that at the equator, and the denominator of the representative fraction must therefore be smaller. The denominator decreases with increase of latitude.

EXAMPLE 9-2

EXAMPLE

The scale of a Mercator chart is given as 1: 380,000 at the equator. Determine the scale of this chart at 42°N.

SOLUTION

The scale at 42° = The scale at the equator x secant 42°

The secant of 42° $= \dfrac{1}{\cos 42^\circ} = \dfrac{1}{0.743} = 1.346$

The scale at 42° N $= \dfrac{1}{380{,}000} \times 1.346$

$= \dfrac{1.346}{380{,}000}$

$= \dfrac{1}{282{,}300}$

or

The scale at 42° N

$$= \frac{1}{380{,}000 \times \cos 42°}$$

$$= \frac{1}{380{,}000 \times 0.743}$$

$$= \frac{1}{282{,}300}$$

EXAMPLE 9-3

EXAMPLE

The scale of a Mercator chart is given as 1:675,000 at 30°S. Determine the scale of this chart at the equator.

SOLUTION

Obviously, if the scale of a Mercator chart expands as the secant of latitude as the latitude is increased away from the equator, then the scale must contract as the reciprocal of the secant (the cosine) of latitude as the latitude is decreased towards the equator.

Therefore:

The scale at the equator = The scale at a given latitude x the cosine of that latitude

The scale at the equator

$$= \frac{1}{675{,}000} \times \cos 30°$$

$$= \frac{1}{675{,}000} \times 0.866$$

$$= \frac{0.866}{675{,}000}$$

$$= \frac{1}{779{,}400}$$

or

The scale at the equator

$$= \frac{1}{675{,}000 \cdot \sec 30^\circ}$$

$$= \frac{1}{675{,}000 \times 1.155}$$

$$= \frac{1}{779{,}400}$$

EXAMPLE 9-4

EXAMPLE

Given that the scale of a Mercator chart is 1: 240,000 at 20°N, determine the scale of the same chart at 40°S.

SOLUTION

The scale at 20° N $= \dfrac{1}{240{,}000}$

The scale at the equator $= \dfrac{1}{240{,}000} \div \sec 20$

The scale at 40° $= \dfrac{1}{240{,}000} \div \sec 20 \text{ x } \sec 40$

$$= \frac{1}{240{,}000 \times \sec 20° \times \cos 40°}$$

$$= \frac{1}{240{,}000 \times 1.064 \times 0.766}$$

$$= \frac{1}{195{,}600}$$

The final latitude is greater than the initial latitude and therefore the scale should be greater and the denominator of the representative fraction smaller.

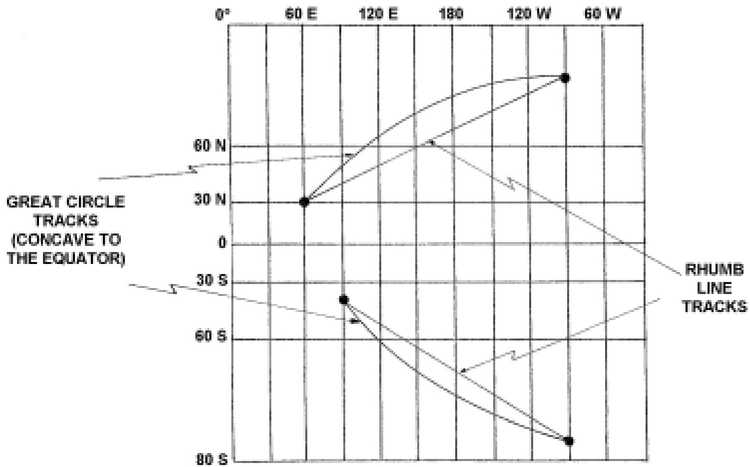
Measurement of Distance on a Mercator Chart

10. Since the scale of a Mercator chart is expanding with increase of latitude, measurement of distance poses a problem. In order to overcome this problem, it is necessary to use the meridian distance scale (1° of latitude = 60 nm) at a **point on the chart which is adjacent to the distance to be measured.**

The Appearance of Rhumb Lines and Great Circles

11. Figure 9-3 shows two rhumb line tracks and two great circle tracks drawn on a Mercator projection.

FIGURE 9-3
Rhumb Line and Great Circle Tracks on a Mercator Chart



12. Note that all **rhumb lines** on this projection are shown as **straight lines.** All **great circle tracks** (apart from meridians and the equator) are shown as **curved lines which are concave to the equator**, ie. on the polar side of the rhumb line.

13. Earth convergency is only correctly shown on this projection at **the equator.** At zero degrees latitude Earth convergency is zero (the meridians are parallel) and this is correctly portrayed on the chart. At all other latitudes Earth meridians converge, but on the Mercator chart they are shown as parallel straight lines, regardless of latitude.

Plotting Radio Bearings on a Mercator Chart

14. The concept of convergency on the Earth was discussed earlier in this section.

Remember that:

(a) Earth convergency = ch long x sin **mean** lat

(b) Conversion angle = 0.5 x ch long x sin **mean** lat

(c) The conversion angle is the angle between the rhumb line and great circle tracks joining the same two points, measured at either of the two points.

EXAMPLE 9-5

EXAMPLE

Given that point A is at 30°N 40°E and point B is at 30°N 70°E determine:
The direction of the great circle track from A to B at A.
The direction of the great circle track from B to A at B.

SOLUTION

See Figure 9-4 and Figure 9-5.

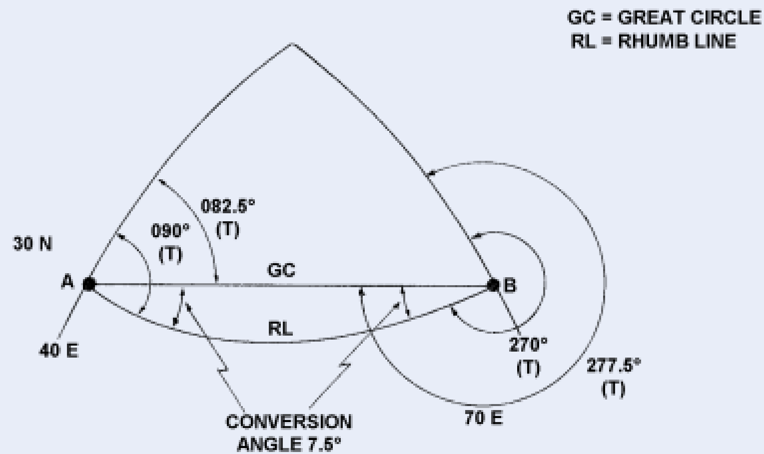
$$\text{Conversion angle between A and B} = 0.5 \times \text{ch long} \times \sin \text{mean lat}$$

$$= 0.5 \times 30° \times \sin 30°$$

$$= 0.5 \times 30° \times 0.5$$

$$= 7.5°$$

FIGURE 9-4

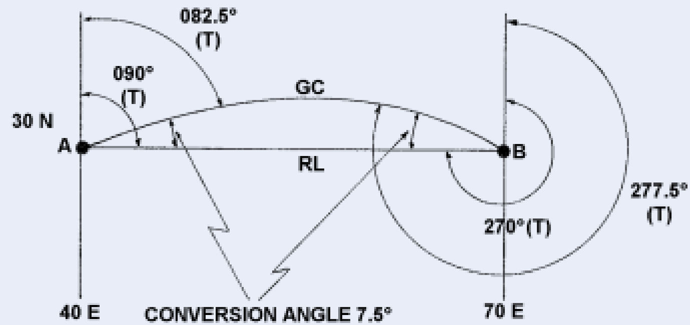


By inspection:

(i)	The great circle track A to B at A	=	082.5°(T)
(ii)	The great circle track B to A at B	=	277.5°(T)

Now, suppose that the whole problem is presented on a Mercator chart rather than on the Earth. The diagram would appear as at Figure 9-5, **but the answers are still the same.**

FIGURE 9-5



15. The problem now arises of plotting radio bearings on a Mercator chart. Radio waves travel along great circle paths. Unfortunately, great circles appear as curves on the Mercator chart, and are therefore impossible to plot. Consequently, if the change of longitude between beacon and aircraft is sufficiently great, the conversion angle must be determined and applied to the great circle bearing in order to convert it to a rhumb line bearing for plotting.

16. Additional complications arise in that:

(a) With an NDB/ADF bearing, the bearing is **measured** at the aircraft.

(b) With VDF bearings, the bearing is **measured** at the station, whilst with VOR bearings, the bearing is **manufactured** at the station.

17. Some significant facts emerge from the above statements, and they are very important.

(a) When converting an NDB bearing from magnetic to true, **apply the variation at the aircraft.**

(b) When converting a VOR or VDF bearing from magnetic to true, **apply the variation at the station.**

(c) When converting NDB great circle bearings to equivalent rhumb line bearings, **apply the conversion angle at the aircraft.**

(d) When converting VOR or VDF great circle bearing to equivalent rhumb line bearings, **apply the conversion angle at the station.**

18. The best way to clarify the above statements is by worked examples.

EXAMPLE 9-6

EXAMPLE

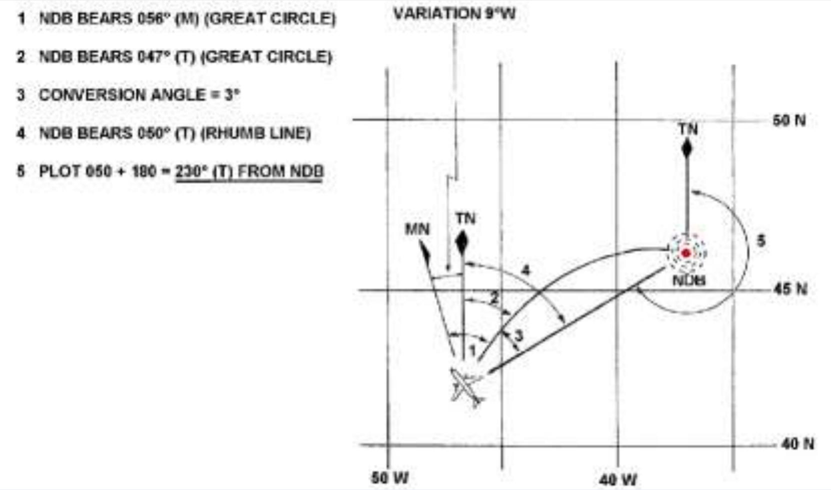
An aircraft's position is estimated as 42°N 46°W. An NDB situated at 46°N 37°W bears 056°(M) from the aircraft. Variation at the aircraft is 9°W. Determine the true bearing that you would plot from the meridian passing through the NDB on a Mercator chart.

SOLUTION

Conversion angle $= 0.5 \times 9° \times \sin 44°$

$= 3°$

FIGURE 9-6


1 NDB BEARS 056° (M) (GREAT CIRCLE)
2 NDB BEARS 047° (T) (GREAT CIRCLE)
3 CONVERSION ANGLE = 3°
4 NDB BEARS 050° (T) (RHUMB LINE)
5 PLOT 050 + 180 = 230° (T) FROM NDB
VARIATION 9°W
TN
MN
TN
NDB
50 N
45 N
40 N
50 W
40 W
1
2
3
4
5

EXAMPLE 9-7

EXAMPLE

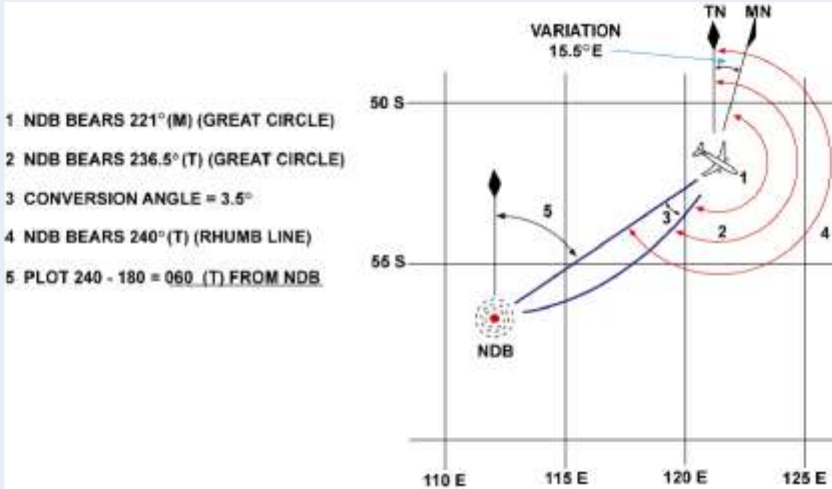
An aircraft's position is estimated as 52°S 121°E. An NDB situated at 56°S 112°E bears 221°(M) from the aircraft. Variation at the aircraft is 15.5°E. Determine the true bearing to plot from the NDB on a Mercator chart.

SOLUTION

Conversion angle $= 0.5 \times 9° \times \sin 54°$

$= 3.5°$

FIGURE 9-7



1 NDB BEARS 221°(M) (GREAT CIRCLE)
2 NDB BEARS 236.5°(T) (GREAT CIRCLE)
3 CONVERSION ANGLE = 3.5°
4 NDB BEARS 240°(T) (RHUMB LINE)
5 PLOT 240 - 180 = 060 (T) FROM NDB
VARIATION
15.5°E
TN
MN
50 S
55 S
NDB
1
2
3
4
5
110 E
115 E
120 E
125 E

EXAMPLE 9-8

EXAMPLE

An aircraft's position is estimated as 60°N 11°W. A VOR station at 64°N 3°W gives a bearing indication of 052°(M) **TO** the station. The variation at the VOR station is 8°W. Determine the bearing to plot from the meridian passing through the VOR station on a Mercator chart.

SOLUTION

Conversion angle	= 0.5 x 8° x sin 62°
The sin of 62°	= 0.883
	= 3.5°

Remember that, since the VOR bearing is manufactured at the ground station, the conversion angle is applied at the ground station rather than at the aircraft.

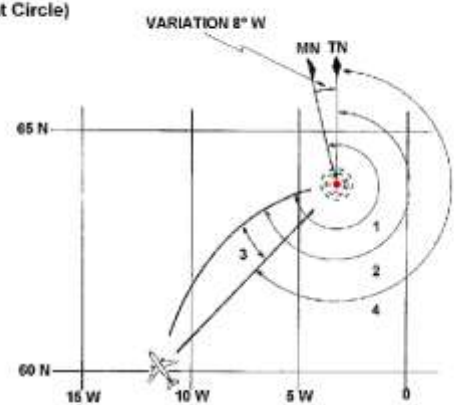
FIGURE 9-8

1 Aircraft bears 052 + 180 = 232° (M) (Great Circle)

2 Aircraft bears 224° (T) (Great Circle)

3 Conversion angle = 3.5°

4 Plot 224 - 3.5 = <u>220.5° (T) from VOR</u>



EXAMPLE 9-9

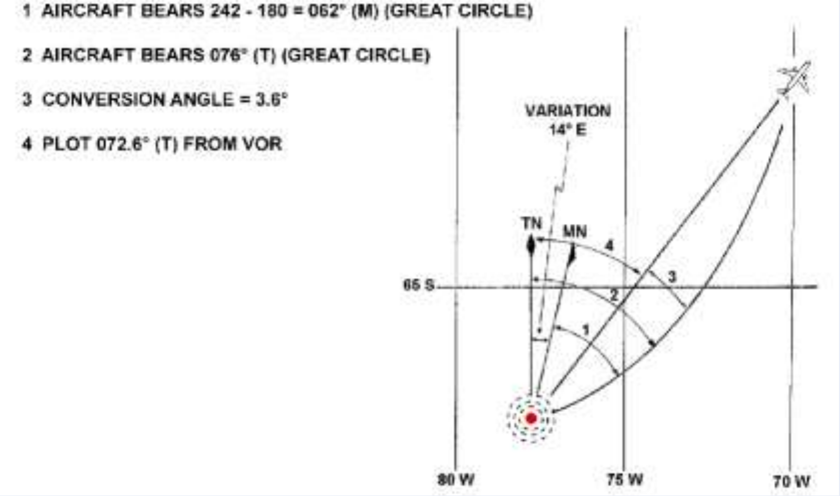
EXAMPLE

An aircraft's position is estimated as 63°S 70°W. A VOR station at 66°S 78°W gives a bearing indication of 242°(M) **TO** the station. The variation at the VOR station is 14°E. Determine the true bearing to plot from the VOR station on a Mercator chart.

SOLUTION

Conversion angle	= 0.5 x 8° x sin 64°30’
The sin of 64°30’	= 0.903
Conversion angle	= 3.6°

FIGURE 9-9


1 AIRCRAFT BEARS 242 - 180 = 062° (M) (GREAT CIRCLE)
2 AIRCRAFT BEARS 076° (T) (GREAT CIRCLE)
3 CONVERSION ANGLE = 3.6°
4 PLOT 072.6° (T) FROM VOR
VARIATION
14° E
TN
MN
65 S
80 W
75 W
70 W
1
2
3
4

Uses of the Mercator Chart

19. Mercator charts are used mainly for the following:

(a) Plotting charts for latitudes 70°N to 70°S where the requirement is to fly rhumb line tracks rather than great circle tracks.

(b) Topographical maps of equatorial regions. In low latitudes the chart is almost constant scale, since the value of the secant of small angles changes only slowly. Consequently ground features are displayed with little distortion in shape in area.

Self Assessed Exercise No. 5

QUESTIONS

QUESTION 1.

Using FIGURE 87 in the Reference Book, the chart distance from Boston (4220N 07105W) to Detroit (4220N 08305W) is 32.9 cm along the rhumb line. What is the scale of the chart?

QUESTION 2.

The scale of the chart is 1:5 million. If the chart distance between two places is 19.7 cm; what is the earth distance between them in statute miles?

QUESTION 3.

The distance between Cardiff (5130N 00310W) and Edinburgh (5555N 00310W) is 305 statute miles. The chart distance is 7.7 inches; what is the chart scale?

QUESTION 4.

The chart distance between Ascension (0800S 01400W) and Freetown (0800N 01400W) is 5.84 inches; what is the scale of the chart?

QUESTION 5.

The rhumb line distance from Perth (3153S 11553E) to Broken Hill (3153S 14125E) is measured as 24.1 cm; what is the scale of the chart?

QUESTION 6.

Using the details given, fill in the Mercator scale table.

	Lat	F Number	Lat	F Number	Lat	F Number
1	0°		35°N	1,000,000	45°N	
2	0°		20°N	250,000	35°N	
3	0°		56°N		25°N	810,371
4	0°		75°N		30°N	300,000
5	0°	500,000	30°N		60°N	
6	0°		60°N	250,000	30°N	
7	0°	100,000	25°N		55°N	
8	0°		52°N	10,000	56°N	
9	0°		56°N		60°N	5,000
10	0°		52°N		56°N	100,000

QUESTION 7.

What is the scale of the chart at FIGURE 87 in the Reference Book at 60N?

QUESTION 8.

If the scale of the chart at FIGURE 87 in the Reference Book at 60N is 1:20 million, what is the scale at 45N?

QUESTION 9.

Using the chart at FIGURE 87 in the Reference Book, what is the great circle track from PWK to THT, measured at PWK?

QUESTION 10.

Using the chart at FIGURE 87 in the Reference Book, at DR position 60N 015W (variation 15W) a bearing is obtained from YQX NDB (variation 24.5W) of 276.5 (M) on the RMI. What is the bearing to plot from the beacon?

QUESTION 11.

Using the chart at FIGURE 87 in the Reference Book, what is the great circle track from YYQ to SF measured at YYQ?

QUESTION 12.

From the chart at FIGURE 87 in the Reference Book, what is the great circle track from YYQ to SF measured at SF?

QUESTION 13.

Use the chart at FIGURE 87 in the Reference Book. On a route from 60N 120W to 60N 010W at what longitude does the great circle track reach its most northerly point?

QUESTION 14.

Use the chart at FIGURE 87 in the Reference Book. On a route from 60N 120W to 60N 010W what is the great circle track direction at its most northerly point?

QUESTION 15.

Use the chart at FIGURE 87 in the Reference Book. At DR position 61N 030W (variation 16W) a bearing is obtained from YG NDB (variation 8W) of 090 (RMI). What is the bearing to plot from the beacon?

QUESTION 16.

Use the chart at FIGURE 87 in the Reference Book. At DR position 61N 030W (variation 16W) a bearing is obtained from OZN NDB (variation 20W) of 285.5 (RMI). What is the bearing to plot from the beacon?

QUESTION 17.

Use the chart at FIGURE 87 in the Reference Book. At DR position 55N 045W (variation 22W) a bearing is obtained from YQX VOR (variation 24W) of 253 (RMI). What is the bearing to plot from the beacon?

QUESTION 18.

Use the chart at FIGURE 87 in the Reference Book. At DR position 45N 105W (variation 20E) a bearing is obtained from YWG VOR (variation 15E) of 039 (RMI). What is the bearing to plot from the beacon?

QUESTION 19.

Using the chart at FIGURE 87 in the Reference Book, what is the great circle track from YWR to YQX, measured at YQX?

QUESTION 20.

Using the chart at FIGURE 87 in the Reference Book, at what longitude will the rhumb line track and the great circle track be the same between YWR and YQX?

ANSWERS:

ANSWER 1.

Rhumb line distance = 12 x 60 x cos 42.33 = 532.3 nm

532.3 nm = 532.3 x 1.854 x 100,000

= 98,666,647 cm

$$\frac{1}{F} = \frac{CD}{ED} = \frac{32.9}{98,666,647} = \frac{1}{2,998,986F}$$

scale is 1 : 3 million

ANSWER 2.

$$\frac{1}{F} = \frac{CD}{ED}$$

$$\frac{1}{5,000,000} = \frac{19.7}{ED}$$

ED = 19.7 x 5,000,000 cm

= 19.7 x 50 km

= 985 km

$$= \frac{985}{1.610} \text{ st miles}$$

= 611.8 statute miles

ANSWER 3.

$$\frac{1}{F} = \frac{CD}{ED}$$

$$\frac{7.7}{305 \times 5280 \times 12} = \frac{1}{2,509,714}$$

$$= 1: 2.5 \text{ million}$$

ANSWER 4.

ED = 16 x 60 x 6080 x 12

= 70,041,600 inches

$$\frac{1}{F} = \frac{CD}{ED}$$

$$= \frac{5.84}{70,041,600}$$

$$= \frac{1}{11,993,425}$$

$$= 1.\ 12 \text{ million}$$

ANSWER 5.

$$ED = 15326 \text{ x } \cos(31.88) \text{ x } 6080$$

$$= 7{,}909{,}515 \text{ ft}$$

$$= \frac{7{,}909{,}515 \text{ ft}}{3.28} \times 100 \text{ cm}$$

$$= 241{,}143{,}760 \text{ cm}$$

$$\frac{1}{F} = \frac{CD}{ED}$$

$$= \frac{24.1}{241{,}143{,}760}$$

$$= \frac{1}{10{,}005{,}965}$$

$$= 1: 10 \text{ million}$$

ANSWER 6.

	Lat	F Number	Lat	F Number	Lat	F Number
1	0°	1,220,775	35°N	1,000,000	45°N	863,218
2	0°	266,044	20°N	250,000	35°N	217,931
3	0°	894,146	56°N	500,000	25°N	810,371
4	0°	346,410	75°N	89,658	30°N	300,000
5	0°	500,000	30°N	433,013	60°N	250,000
6	0°	500,000	60°N	250,000	30°N	433,013
7	0°	1,000,000	25°N	906,308	55°N	573,576
8	0°	16,243	52°N	10,000	56°N	9,083
9	0°	10,000	56°N	5,592	60°N	5,000
10	0°	178,829	52°N	110,098	56°N	100,000

ANSWER 7.

$$\text{Scale} = \frac{\text{Chart Distance}}{\text{Earth Distance}}$$

CD for 120° longitude = 23.7cm
ED for 120° longitude = 120 x 60 x cos 60
= 3600nm
CRP 5 3600nm = 6660Km
= 666,000,000cm

$$\text{Scale} = \frac{23.7}{666{,}000{,}000}$$

$$= \frac{1}{28.1 \text{ million}}$$

= 1 : 28 million

ANSWER 8.

$$\text{Scale at B} = \frac{1}{F \div \cos A \times \cos B}$$

$$= \frac{1}{20{,}000{,}000 \div 0.5 \times 0.71}$$

= 1: 28.4 million

ANSWER 9.

Measured RL track = 311(T)

CA = ½ x ch. long x sin mean lat
= ½ x 64 x 0.91
= 29°
GC track = 311 + 29 = 340(T)

ANSWER 10.

BRG = 276.5(m) - 15W
= 261.5(T)
CA = ½ x 40 x 0.82
= 16.5
RL BRG = 261.5 - 16/5
= 245(T)

Plot 065(T)

ANSWER 11.

Measured RL track = 066½(T)

CA = ½ x 43 x 0.89
= 19°
GC track = 066½ - 19
= 047½(T)

ANSWER 12.

RL at SG = 066½(T)

GC at SG = 066½ + 19

= 085½(T)

ANSWER 13.

At mid-longitude = 65W

ANSWER 14.

090°(T)

ANSWER 15.

GC BRG VG = 090(M) - 16W
= 074(T)
CA = ½ x 23 x 0.88
= 10°
RL BRG VG = 074 + 10
= 084(T)
Plot 264(T) from VG

ANSWER 16.

GC BRG OZN = 285½(M) - 16W
= 269½(T)
CA = ½ x 13 x 0.87
= 5½°
RL BRG OZN = 269½(T) - 5½
= 264(T)
Plot 084(T) from OZN

ANSWER 17.

CA = ½ x 10 x 0.79

= 4°

GC BRG = 073(M) + 24W

= 049(T)

RL BRG = 249(T) + 4°

= 053(T)

ANSWER 18.

CA = ½ x 8 x 0.74

= 3°

GC BRG = 219(M) + 15E

= 234(T)

RL BRG = 234(T) - 3°

= 231(T)

ANSWER 19.

RL Track = 090(T)

CA = ½ x 68½ x 0.75

= 25.7°

GC at YQX = 090 + 25.7

= 115.7(T)

ANSWER 20.

088°45'W ie. mid-longitude

The Lambert Projection

The Appearance of Rhumb Lines and Great Circles

Chart Convergency

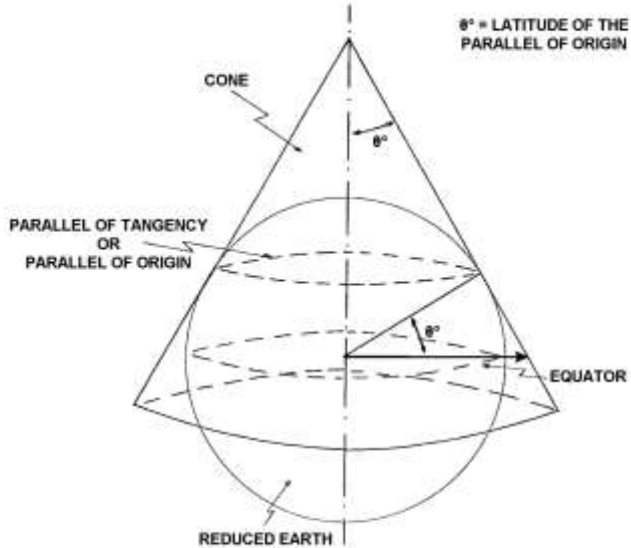
Calculations Involving the Lambert Projection

The Lambert Projection

1. The Lambert Projection, otherwise known as the Lambert's Conformal, is an improvement on the Mercator projection in that it produces a very nearly constant-scale chart.

2. The Lambert chart is a **conical** projection. Figure 10-1 shows a reduced Earth with a cone placed over it, such that the apex of the cone lies directly above the pole. This ensures that the cone touches the reduced Earth at only one parallel of latitude. This latitude is known as the latitude of **the parallel of tangency** or **parallel of origin.** The graticule of latitude/longitude is projected onto the cone, using a light source at the centre of the reduced Earth.

FIGURE 10-1
Simple Conical Map Projection



3. Figure 10-1 shows a **simple** conical projection, and the problem of scale expansion away from the parallel of origin remains, as shown at Figure 10-2.

4. The Lambert projection overcomes this scale expansion problem as illustrated at Figure 10-3. The cone is now **inset** into the reduced Earth. The angle θ (half the value of the angle at the apex of the cone) is **still the latitude of the parallel of origin.** The cone now cuts the reduced Earth at two latitudes, which are approximately equidistant about the parallel of origin, and **it is at these two latitudes that the scale of the chart is correct**. These two latitudes are described as being the **standard parallels.**

FIGURE 10-2
Simple Conical Projection Showing Scale Expansion Problem

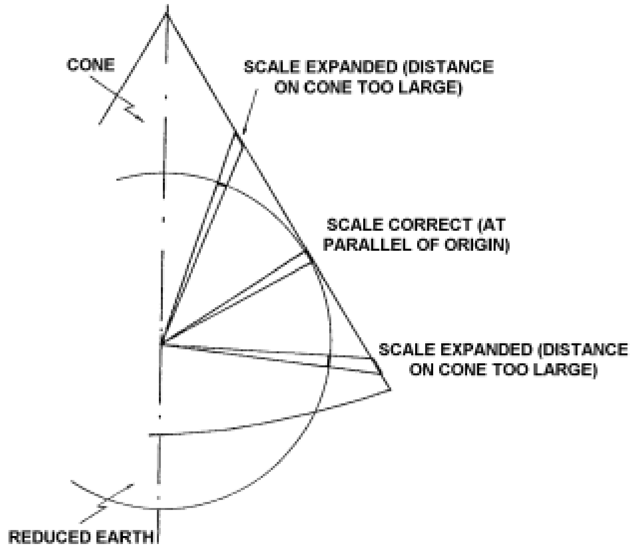
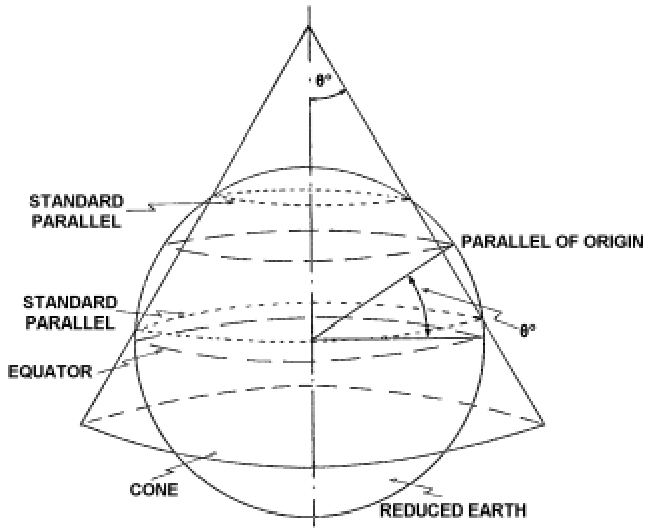
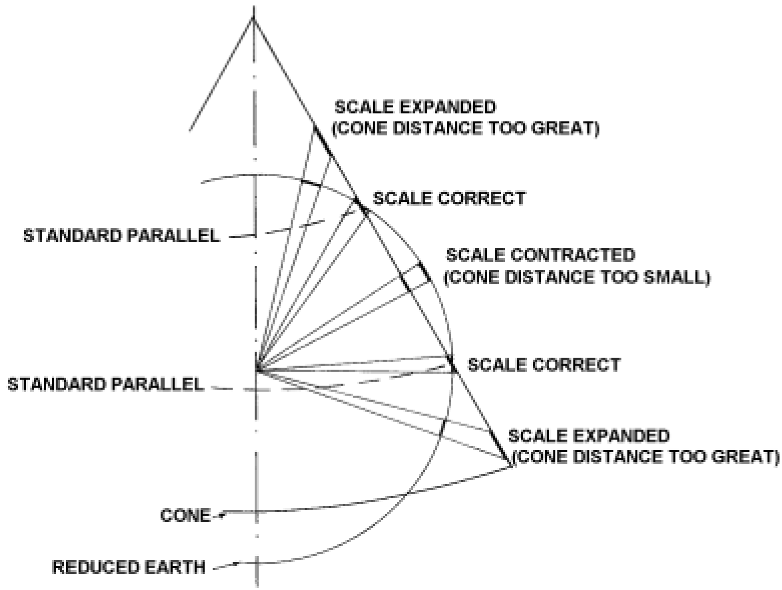


FIGURE 10-3
Lambert Conical Projection



5. Figure 10-4 shows how the problem of scale distortion is affected by the insetting procedure. Now the scale is correct at the standard parallels, is contracted between the standard parallels, and expands outside the standard parallels. Although the rate of scale distortion is very much reduced, it cannot be considered to be constant scale.

FIGURE 10-4
Lambert Projection Showing Reduced Scale Distortion



6. With careful spacing of the standard parallels, the scale distortion may however be reduced to one percent or less and the resulting chart may be **considered** to be constant in scale.

7. The rule of thumb which governs the spacing of the standard parallels is known as the Rule of Two Thirds and takes account of the fact that scale expands at a greater rate outside the standard parallels than it contracts between them. It states that scale distortion will be reduced to within acceptable limits if:

(a) Two-thirds of the chart is contained within the standard parallels and one-sixth of the chart lies above the upper standard parallel, and one-sixth of the chart lies below the lower standard parallel.

(b) The spacing between the standard parallels should be limited to no more than 14° of latitude.

8. Having fixed the graticule of latitude/longitude on to the cone, it will appear as shown at Figure 10-5 when laid flat. Note that:

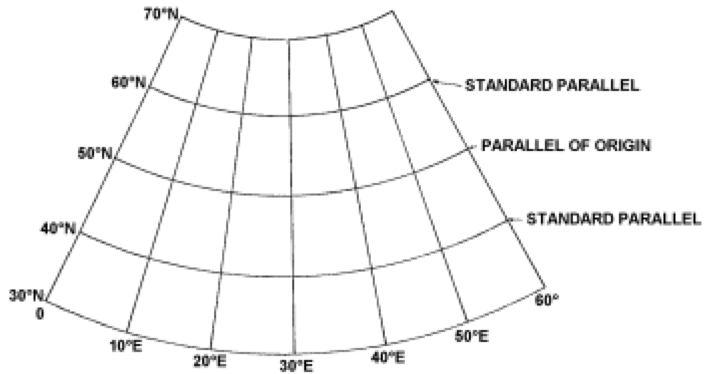
(a) The meridians appear as straight lines which converge towards the apex of the cone.

(b) The parallels of latitude appear as concentric arcs of circles centered on the apex of the cone. The distance between them is (almost) constant.

(c) The parallels of latitude cross the meridians at right-angles.

The chart is therefore orthomorphic.

FIGURE 10-5
Conical Projection Laid Flat



The Appearance of Rhumb Lines and Great Circles

9. Rhumb lines, for example the parallels of latitude, appear as regular curves which are concave to the nearer pole.

10. Great circles appear as very shallow curves which are concave to the parallel of origin. For plotting purposes great circles are assumed to be straight lines.

11. Figure 10-6 and Figure 10-8 summarise the above paragraphs both for northern and southern hemisphere charts.

FIGURE 10-6
Rhumb Line and Great Circle Tracks on a Flat Conical Projection (Northern Hemisphere)

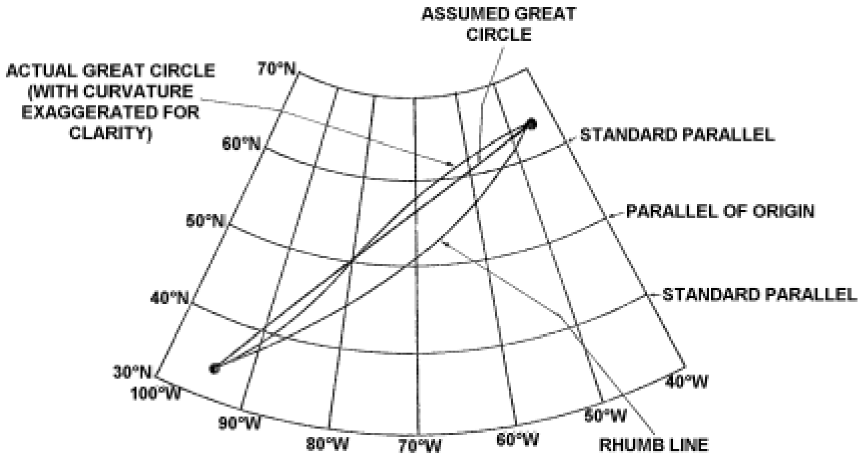


FIGURE 10-7
Rhumb Line and Great Circle Tracks on a Conical Projection (Southern Hemisphere)

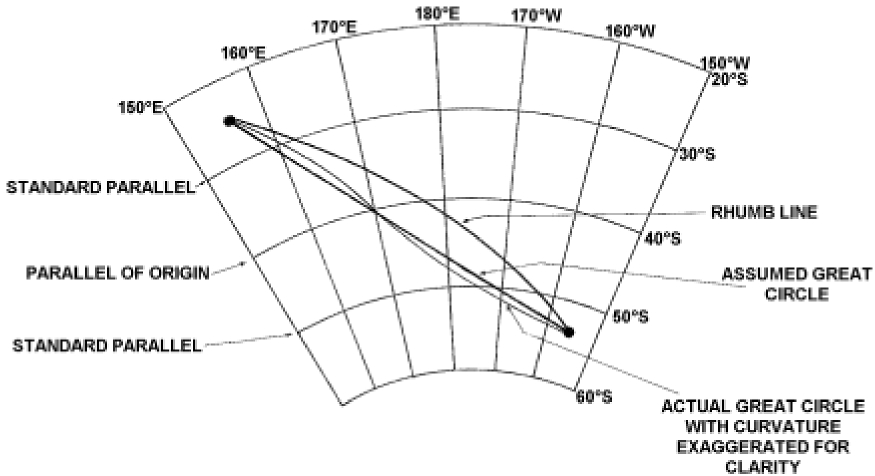


Chart Convergency

12. On a Lambert chart, chart convergency is equal to Earth convergency at the parallel of origin.

This is because:

Earth convergency (EC) = ch long (in degrees) x sine mean latitude and:

13. On a Lambert chart, convergency for a given change of longitude must remain constant, regardless of latitude, since the meridians are shown as **straight lines**. Therefore, for a Lambert chart:

Chart convergency (CC) = ch long (in degrees) x the sine of the parallel of origin

14. From the above it can be seen that Earth convergency is equal to chart convergency **only** at the parallel of origin.

15. At latitudes greater than the parallel of origin Earth convergency will be greater than chart convergency, since the sine of the mean latitude (EC) will be greater than the sine of the parallel of origin (CC).

16. At latitudes lower than the parallel of origin Earth convergency will be less than chart convergency, since the sine of the mean latitude (EC) will be less than the sine of the parallel of origin (CC).

17. The numerical value of the **sine of the parallel of origin** is sometimes referred to as the **Constant of the Cone or Convergence Factor.**

18. Straight lines drawn on a Lambert chart are not precisely great circles and track angles measured on the chart do not correspond exactly with the same track angles on the Earth (other than at the parallel of origin). It might therefore appear that the Lambert chart is of limited use, but in fact the above errors are acceptably small and are normally ignored for navigational purposes.

Calculations Involving the Lambert Projection

19. The examples which follow are typical of the calculations required in the examination.

EXAMPLE 10-1

EXAMPLE

A straight line on a Lambert's chart is drawn from A at 30°N 10°W to B at 50°N 11°E. The direction of this straight line is measured as 035°(T) at A. The parallel of origin of the chart is 38°N. Determine:

(i) The direction of the straight line track B to A as measured on the chart at B.

(ii) The approximate rhumb line bearing of B from A.

SOLUTION

See Figure 10-8.

(1) Straight line track direction A to B at A = 035°(T)

(2) Chart convergency 10°W to 11°E

= ch long x sine parallel of origin

= 21° x sine 38

= 13°

(3) Straight line track B to A at B = 13° + 035° +180°

(i) = 228°

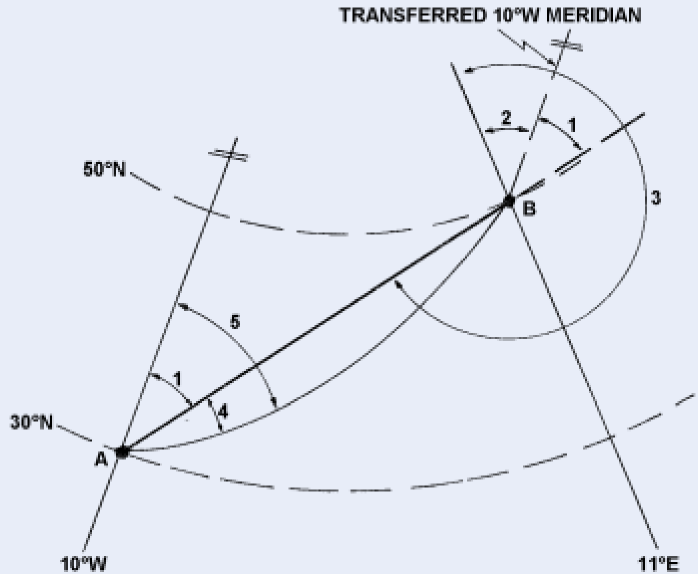
(4) Conversion angle = approximately half chart convergency

= 6.5°

(5) Approximate rhumb line track A-B = 035° + 6.5°

(ii) = 041.5°(T)

FIGURE 10-8



EXAMPLE 10-2

EXAMPLE

A straight line drawn on a Lambert's chart between point C at 65°S 50°W and point D at latitude 55°S. The straight line track C to D, measured at C, is 063°(T). The straight line track D to C, measured at D, is 223°(T). The parallel of origin of the chart is 53°S. Determine:

(i) The longitude at point B

(ii) The approximate rhumb line bearing of D from C

SOLUTION

See Figure 10-9.

(1) Straight line track direction C to D at C = 063°(T)

(2) Straight line track direction D to C at C = 223°(T)

(3) Chart convergency C to D = 063 + 180 - 223

= 20°

Chart convergency = ch long x sine parallel of origin

$$\text{therefore ch long} = \frac{\text{chart convergency}}{\text{sine parallel of origin}} = \frac{20°}{\text{sine } 53°}$$

= 25°

The longitude of D = 50°W - 25°

(i) = 25°W

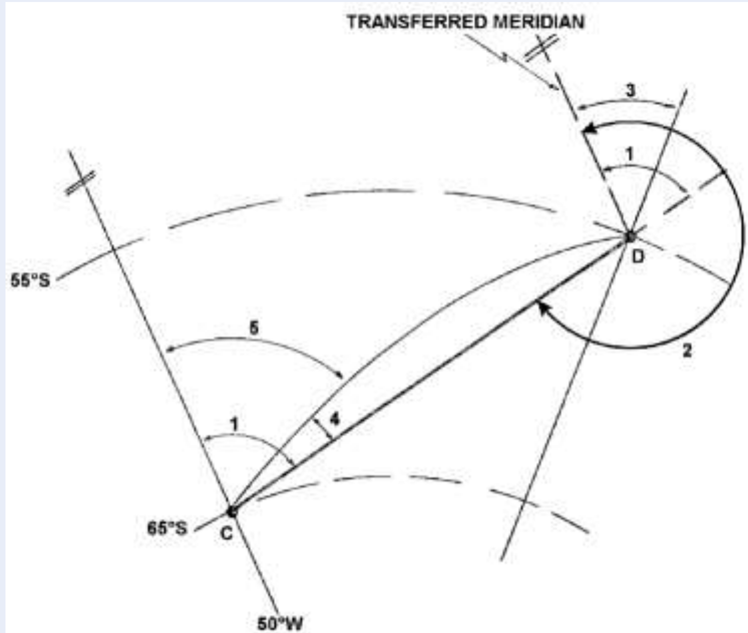
(4) Conversion angle = approximately half chart convergency

= 10°

(5) Approximate rhumb line track C-D = 063° - 10°

(ii) = 053°(T)

FIGURE 10-9



Plotting Radio Bearings on a Lambert Chart

20. Plotting radio bearings on a Lambert chart is very much simpler than on a Mercator chart since radio waves travel over great circle paths which are assumed to be straight lines (or plotting purposes) on the Lambert chart.

21. Again, it is important to appreciate that if a magnetic bearing is obtained but the position line is to be plotted with reference to true north, the variation must be applied at the correct point:

(a) For ADF/NDB bearings, apply the variation **at the aircraft.**

(b) For VOR or VDF bearings, apply the variation **at the station.**

22. VDF and VOR bearings are respectively measured and manufactured at the station. The bearing thus achieved is relative to a north reference at the station.

23. To determine the bearing to plot for VOR or magnetic VDF bearings (QDM or QDR), simply apply the variation **at the station,** add or subtract 180° if required (**QDM** or **VOR TO** indications), and plot the resultant bearing using the meridian passing through the station. In this case there is no requirement to consider convergency since the bearing is measured or manufactured at the station, and it is the meridian passing through the station which is used as the true north reference to plot the bearing.

24. ADF bearings are measured at the aircraft. It is therefore necessary to convert magnetic to true bearings using **variation at the aircraft**. However a problem still exists. The bearing in degrees true at the aircraft is given with reference to the meridian passing through the aircraft. If 180° is applied to this bearing and it is then plotted from the station, with reference to the meridian passing through the station, the resultant position line will be in error by the convergency between aircraft and NDB. Either the bearing to plot must be corrected by convergency or the true north at the aircraft must be transferred to the beacon and this datum used to plot the bearing.

EXAMPLE 10-3

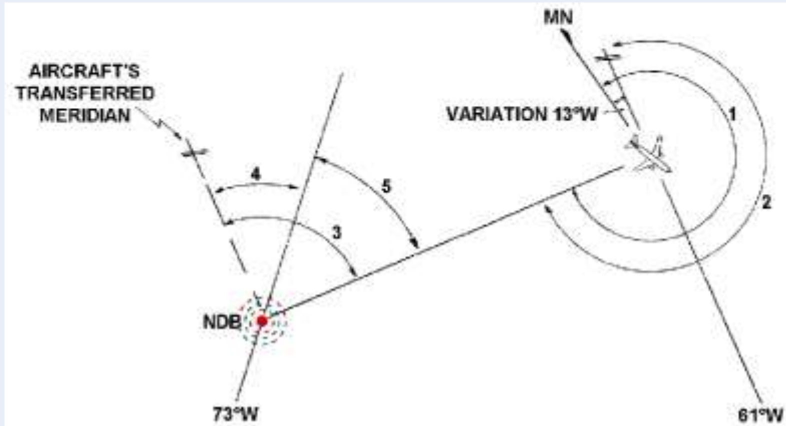
EXAMPLE

An NDB bears 283°(M) from an aircraft. Variation at the aircraft is 13°W. The aircraft's estimated position is 53°N 61°W, and the NDB is at 52°50'N 73°W. Determine the bearing to plot from the meridian passing through the NDB on a Lambert chart, the parallel of origin of which is 58°N.

SOLUTION

(1)	NDB bears 283°(M) from aircraft.	
(2)	NDB bears 270°(T) from aircraft.	
(3)	Aircraft bears 270° - 180°	= 090°from NDB relative to **the aircraft transferred meridian**
(4)	Chart convergency aircraft to NDB	= ch long x sine parallel of origin
		= 12° x sine 58°
		= 10° (approx)
(5)	Bearing to plot (from the **actual** meridian passing through the NDB)	= 090° - 10°
		= 080°

FIGURE 10-10



MN
AIRCRAFT'S
TRANSFERRED
MERIDIAN
VARIATION 13°W
1
2
3
4
5
NDB
73°W
61°W

EXAMPLE 10-4

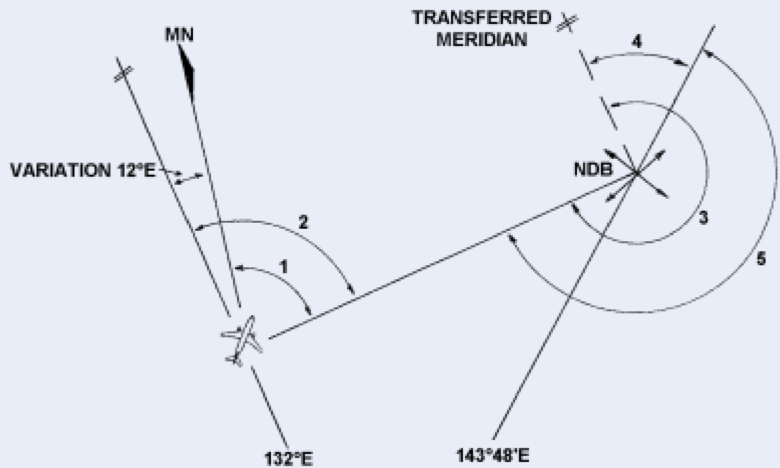
EXAMPLE

An NDB bears 072°(M) from an aircraft. Variation at the aircraft is 12°E. The aircraft's position is estimated as 63°S 132°E, and the NDB is at 61°15'S 143°48'E. Determine the bearing to plot from the meridian passing through the NDB on a Lambert chart, the parallel of origin of which is 50°S.

SOLUTION

(1)	NDB bears 072°(M) from aircraft.		
(2)	NDB bears 084°(T) from aircraft.		
(3)	Aircraft bears 084° + 180°	=	264° from NDB relative to the **aircraft transferred meridian**
(4)	Chart convergency	=	ch x sine parallel of origin
		=	11.8° x sine 50°
		=	9° (approx)
(5)	Bearing to plot (from the **actual** meridian passing through the NDB)	=	264° - 9°
		=	255°(T)

FIGURE 10-11



Uses of the Lambert Chart

25. Lambert charts are mainly used for the following purposes:

(a) Plotting charts where great circle tracks are to be flown.

(b) Topographical charts.

(c) Meteorological synoptic charts.

(d) Radio aid and radio navigation charts.

Self Assessed Exercise No. 6

QUESTIONS:

QUESTION 1.

Refer to FIGURE 89 in the Reference Book, what is the Parallel of Origin of the chart?

QUESTION 2.

At what latitude is the scale of the chart at FIGURE 89 in the Reference Book correct?

QUESTION 3.

At what latitude does chart convergency equal Earth convergency on the chart at FIGURE 89 in the Reference Book?

QUESTION 4.

Referring to FIGURE 89 in the Reference Book, what is the rhumb line track (T) from PWK to THT?

QUESTION 5.

At DR position 60N 015W (variation 19°W) a bearing is obtained from YQX NDB (variation 29°W) of 280°(M) on the RMI; what is the bearing (T) to plot from the NDB, on the chart at FIGURE 89 in the Reference Book?

QUESTION 6.

From the chart at FIGURE 89 in the Reference Book, what is the rhumb line track (T) from YYQ to SF?

QUESTION 7.

From the chart at FIGURE 89 in the Reference Book, what is the great circle track (T) from YYQ to SF measured at SF?

QUESTION 8.

On a route from 60N 120W to 60N 010W, at what latitude and longitude does the great circle reach its most northerly point and what is the track direction (T) at this point?

QUESTION 9.

At DR position 61N 030W (variation 30°W) a bearing is obtained from VG NDB (variation 14°W) of 107°(RMI); what is the bearing (°T) to plot from the beacon, on the chart at FIGURE 89 in the Reference Book?

QUESTION 10.

At DR position 61N 030W (variation 30°W), a bearing was obtained from OZN NDB (variation 36°W) of 299° (RMI); what is the bearing (T) to plot from the beacon on the chart at FIGURE 89 in the Reference Book?

QUESTION 11.

At DR position 55N 045W (variation 32°W) a bearing is obtained from YQX VOR (variation 27°W) of 245°(RMI); what is the bearing (T) to plot from the beacon on the Chart at FIGURE 89 in the Reference Book?

QUESTION 12.

Calculate the difference in values of convergence for 30° of longitude at 40N on the Earth and on the chart at FIGURE 89 in the Reference Book.

Sin 40 = 0.64; cos 40 = 0.77; tan 40 = 0.84

QUESTION 13.

At DR position 45N 105W (variation 20°E) a bearing is obtained from YWG VOR (variation 15°E) of 032°(RMI); what is the bearing (T) to plot from the beacon on the chart at FIGURE 89 in the Reference Book?

QUESTION 14.

On the chart at FIGURE 89 in the Reference Book, what is the rhumb line track (T) from YWR to YQX?

QUESTION 15.

Where on the chart at FIGURE 89 in the Reference Book is a great circle most closely represented by a straight line?

QUESTION 16.

If the Chart distance of 120° of longitude at 45N is measured as 33 cm, what is the scale of the chart at this latitude?

QUESTION 17.

Two aircraft depart overhead LHR en-route YWR. Aircraft A flies the great circle route direct to YWR; aircraft B flies the great circle track to YQX and then the great circle track to YWR. Which aircraft flies the shorter distance and by what percentage of the longer distance does it gain?

ANSWERS:

ANSWER 1.

Convergence factor quoted on the chart as 0.87036; need to find angle for which that is the sine (called inverse sine) i.e. sin x = 0.87036, what is x? – 60°30' 1.8"N

ANSWER 2.

47°N and 73°N (quoted as standard parallels on chart).

ANSWER 3.

At the parallel of origin – 60°30'1.8"N

ANSWER 4.

Great circle track at PWK measured as 335°(T)
CA = ½ x ch. long° x convergence factor (= sine of parallel of origin)
= ½ x 63° x 0.87 = 27.4°
RL TK at PWK = 335° + 27.4 = 307.6°

ANSWER 5.

Chart convergence = ch.long° x convergence factor
= 39° x 0.87 = 33.9° (34° approx)
Bearing = 280°(M) - 19°W
= 261°(T)
Apply convergence = 261° - 34° = 227°(T)
Take reciprocal = 047°(T) - bearing to plot

ANSWER 6.

Great circle track at YYQ measured as 048°(T)
CA = ½ x ch. long° x convergence factor (= sine of parallel of origin)
= ½ x 43° x 0.87 = 18.7°
RL TK at YYQ = 048° + 18.7 = 066.7°(T)

ANSWER 7.

GC Tk measured at SF = 085°(T)

ANSWER 8.

At mid-longitude GC and RL are parallel.

70° 30' N 065° 00' W; 090°(T)

ANSWER 9.

Chart convergence = ch.long° x convergence factor
= 23° x 0.87 = 20°
Bearing = 107°(M) - 30°W
= 077°(T)
Apply convergence = 077° + 20° = 097°(T)
Take reciprocal = 277°(T) - (bearing to plot)

ANSWER 10.

Chart convergence = ch.long° x convergence factor
= 14° x 0.87 = 12° (approx)
Bearing = 299°(M) - 30°W
= 269°(T)
Apply convergence = 269° - 12° = 257°(T)
Take reciprocal = 077°(T) - (bearing to plot)

ANSWER 11.

Bearing = 245°(RMI)
Variation = 27°W (at station)
True Bearing = 245° - 27°
= 218°(T)
Take reciprocal = 038°(T) - bearing to plot

ANSWER 12.

Earth convergency = ch. long° x sin (mean lat.)
= 30° x sin 40
= 19.3°

Chart convergence = ch. long° x convergence factor
= 30° x 0.87
= 26.1°

Difference = 26.1° - 19.3° = 6.8° = 6°48’ (with chart convergence greater)

ANSWER 13.

Bearing = 032°(RMI)
Variation = 15°E (at station)
True bearing = 032° + 15°
= 047°(T)
Take reciprocal = 227°(T) - bearing to plot.

ANSWER 14.

Great circle track at YWR measured as 060°(T)

CA = ½ x ch. long° x convergence factor (= since of parallel of origin)
= ½ x 69° x 0.87 = 30°
RL tk at YWR = 060° + 30° = 090°(T)

ANSWER 15.

Along the parallel of origin (60°30’ 1.8”N)

ANSWER 16.

Departure formula:

Earth distance = ch. long’ x cos (lat)

= 120 x 60’ x cos 45

= 5091 nm

Conversion using CRP-5 = 9437.3 km

= 943,730,000 cm

$$\text{Scale} = \frac{24.5 \text{ cm}}{\text{million cm}}$$

$$= \frac{1}{38.5 \text{ million cm}}$$

Equates to 1 : 38,519,591

ANSWER 17.

Chart distance for A; LHR to YWR = 19.25cm
Chart distance for B; LHR to YQX = 9.6 cm
YQX to YWR = 12.4 cm
Bs total chart distance = 22cm

$\therefore$ A files the shorter distance

$$\text{advantage} = \frac{22 - 19.25}{22}$$
$$= 12.5\%$$

Chapter 11
The Polar Stereographic Projection

The Appearance of Rhumb Lines and Great Circles

Plotting Radio Bearings on a Polar Stereographic Chart

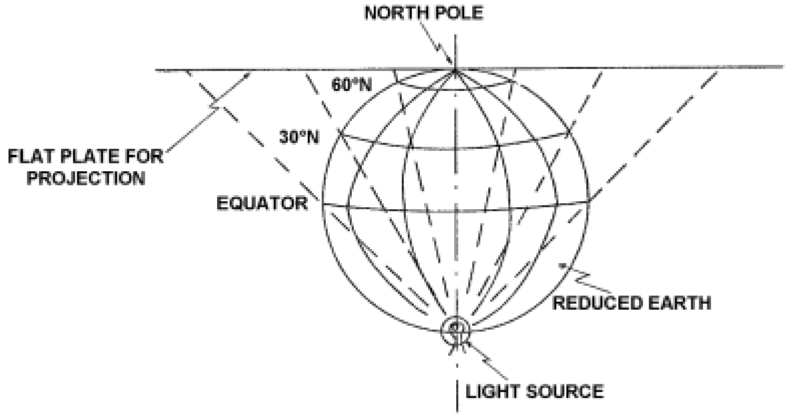
The Polar Stereographic Projection

1. The polar stereographic projection is a logical progression from the Lambert projection. With a Lambert projection the chart convergency is governed by the latitude of the parallel of origin, which is itself governed by the angle at the apex of the cone. As the cone is made flatter (that is to say the apex angle is increased) the latitude of the parallel of origin increases and the value of chart convergency for a given change of longitude increases, since chart convergency is equal to the change of longitude multiplied by the sine of the parallel of origin.

2. If the cone is made as flat as possible it becomes a flat sheet of paper and the parallel (or point) of origin will be the pole (90°N or S). **The chart convergency is now equal to the change of longitude since the sine of 90° is 1.**

3. We have now effectively produced a polar stereographic **flat plate** projection. The sheet is laid on the reduced Earth such that it touches the globe only at one of the poles, being tangential to that pole, see Figure 11-1.

FIGURE 11-1
Basic Polar Stereographic Chart Projection

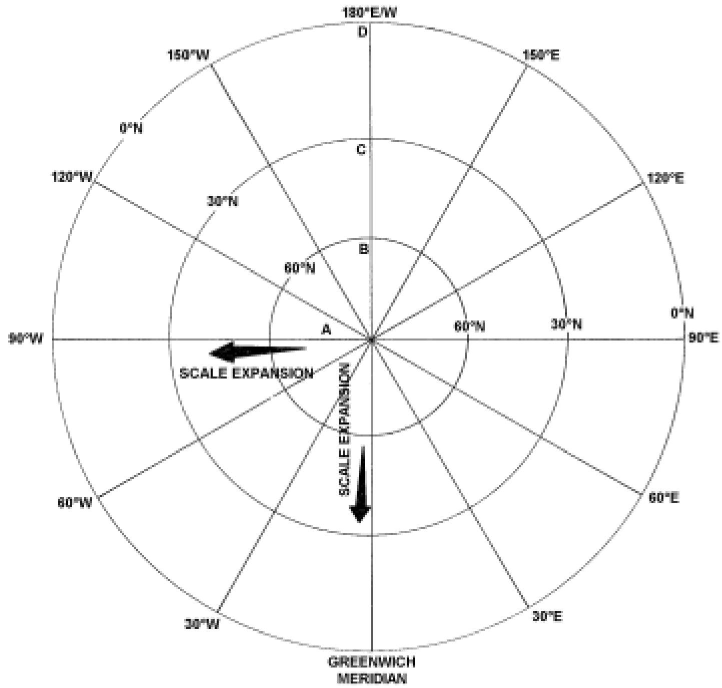


4. In order to reduce the scale expansion at points remote from the pole the light source is placed at the opposite pole and the graticule is fixed on to the flat sheet.

5. The projected graticule will appear as shown at Figure 11-2. The meridians are straight lines radiating from the pole and the parallels of latitude are concentric circles centered on the pole. As with all of the charts considered so far the polar stereographic chart is orthomorphic. It can be seen at Figure 11-2 that all meridians and parallels of latitude cross at right angles. Although the scale is not constant across the chart, the scale in any direction from a given point is constant for short distances.

6. As can be seen at Figure 11-2 the scale expands away from the pole. This is apparent since distances AB, BC and CD are all identical on the Earth (1800 nm) whereas on the chart, distance AB is smaller than distance BC, which is itself smaller than distance CD.

FIGURE 11-2
Polar Stereographic Map Projection



7. The scale expansion on a polar stereographic projection is given by the formula:

$$\text{Scale at any latitude} = \text{Scale at the pole} \times \left(\text{secant of the } \frac{\text{co-latitude}}{2}\right)^2$$

8. The co-latitude in the above formula is 90° minus the latitude in question.

EXAMPLE 11-1

EXAMPLE

Given that the scale of a polar stereographic chart is 1:1,000,000 at 90°N, determine the scale at 70°N.

SOLUTION

$$\text{Scale at } 70° = \frac{1}{1,000,000} \times \left(\sec\left(\frac{90-70}{2}\right)\right)^2$$

$$= \frac{1}{1,000,000} \times (\sec 10°)^2$$

$$= \frac{1}{1,000,000} \times 1.031$$

$$= \frac{1.031}{1,000,000}$$

$$= \frac{1}{969,846}$$

9. It is apparent from this calculation that the scale distortion at 70° latitude (north or south) is limited to only 3%. It is therefore normal to accept this projection as being constant scale **above** this latitude.

The Appearance of Rhumb Lines and Great Circles

10. Rhumb lines appear as curves which are concave to the pole.

11. Any great circle passing through the pole (meridian/anti-meridian) will appear as a straight line. All other great circles (for example the equator at Figure 11-2) will appear as a shallow curve concave to the pole.

12. Great circles are **assumed** to be a straight line on the polar stereo for plotting purposes.

EXAMPLE 11-2

EXAMPLE

A straight line track is drawn on a polar stereographic chart from A (80°N 60°W) to B (80°N 150°E). Determine:

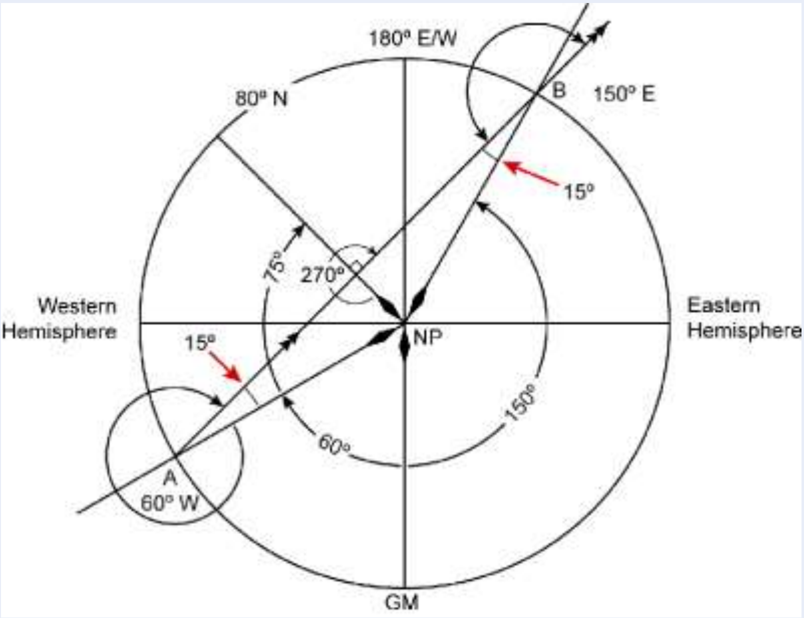
(i) The true track direction at A.

(ii) The true track direction at B.

(iii) The longitude at which the track will measure 270°(T).

SOLUTION

FIGURE 11-3



(i) The triangle PAB is an isosceles triangle since A and B are at the same latitude and hence the same distance from the Pole. The angles at A and B must therefore be the same. The angle at the Pole (angle APB) can be calculated since we know the respective longitudes of A and B.

Angle APB = 360° - (Long A and Long B)
= 360° - (60° + 150°)
= 360° - 210°
= 150°

Any triangle contains a total of 180° therefore, the angles at A and B amount to:

180° - 150° = 30°

But the angles are the same, so each angle will be:

30° ÷ 2 = 15°The track A to B measured at A is the external angle at A ie. measured clockwise from True North (represented by the meridian at A, the line AP)

Therefore: True track at A = 360° - 15°
= 345°(T)

(ii) The true track direction at B is measured from the meridian at B (the line BP) clockwise to the track line

Track at B = Angle PBA + 180°

= 15° + 180°

= 195°T

(It follows, therefore that the track B to A would be 015°T)

(iii) The line bisecting the angle APB crosses the track A to B at right angles. That means the track at that point is 270°T

Te bisecting line represents the meridian at the point where the track is 270°T and by definition must halve the angle APB

Angle APB = 150° (as we have already seen)

Therefore the meridian represented by the bisecting line is 75° round from point A

Therefore the longitude of that point is:

60°W + 75 = 135°W

Note. When considering two points at the same latitude on a Polar Stereo chart, the track at the half-way point will **always** be 090°T or 270°T.

EXAMPLE 11-3

EXAMPLE

A straight line track is drawn on a polar stereographic chart from C (70°S 15°E) to D (70°S 105°W).

Determine:

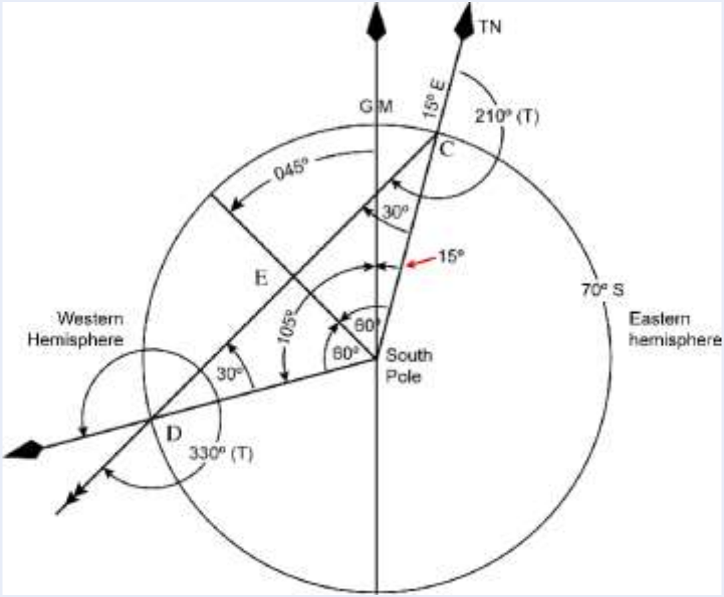
The initial true direction of the track at C.
The final true direction of the track at D.
The longitude at which the track will measure 270°(T).

Also:

State, giving a reason, whether the most southerly point along track is at a higher or lower latitude than 80°S.

SOLUTION

FIGURE 11-4

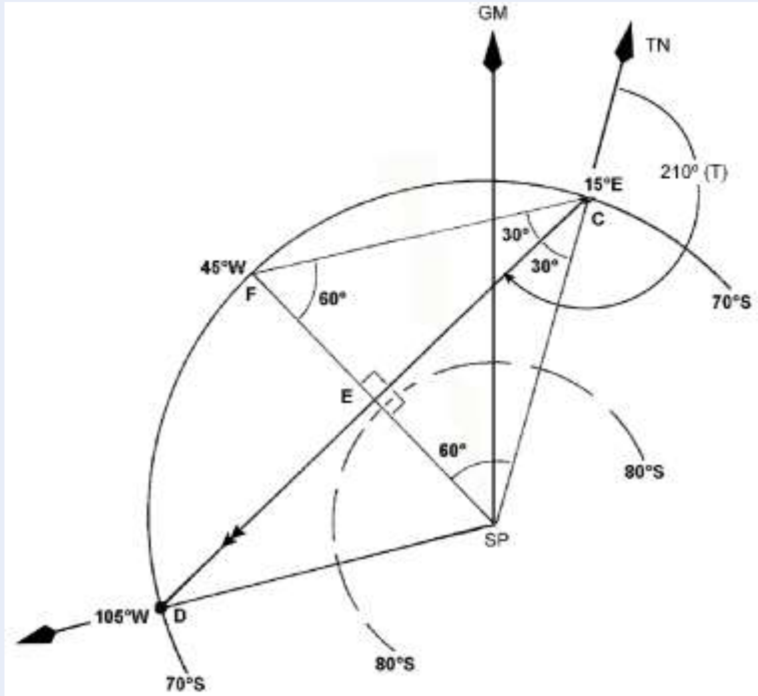


When polar stereographic problems are set in the southern hemisphere it can be confusing. At the south pole all directions are north. The Greenwich meridian (GM) divides the 360° of longitude in two hemispheres, the eastern hemisphere lies to the right and the western hemisphere to the left.

Change of longitude C to D	=	120°
Change of longitude C to mid-point	=	60°

(i)	Track at C	=	210°(T)
(ii)	Track at D	=	330°(T)
(iii)	Longitude for track	=	45°W

FIGURE 11-5



The angle PCE is already known to be 30°.

The straight line track CD is known to measure 210°(T) at C.

By a similar calculation the track CF can be calculated as 240°(T) at C.

Angle ECF is therefore 30°.

Side CE is common to both triangle CEF and triangle CEP.

Triangles CEF and CEP are therefore identical and point E is equidistant between points P and F.

Because of scale expansion the distance **on the chart** from the pole to the 80° parallel of latitude will be slightly less than the distance between the 80°and 70°parallels of latitude.

On the chart the most southerly point on the track **will appear to be** at a latitude of slightly less than 80°S.

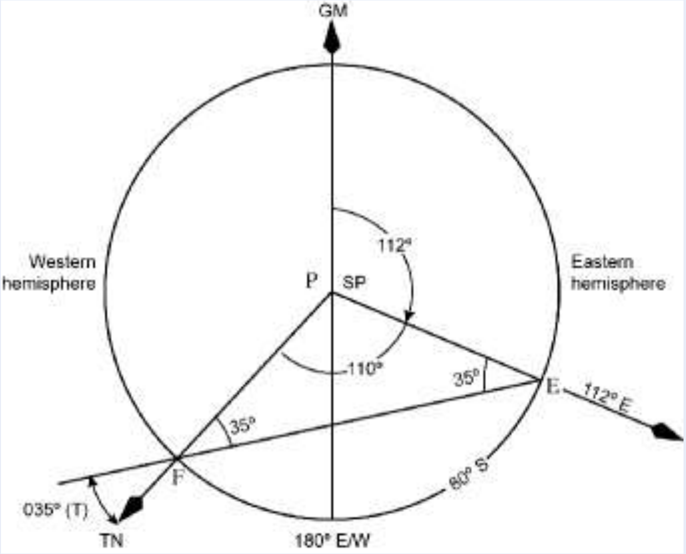
EXAMPLE 11-4

EXAMPLE

On a polar stereographic chart the track from E at 80°S 112°E to F, which is also at latitude 80°S, measures 035°(T) at F. Determine the longitude of F.

SOLUTION

FIGURE 11-6



The track angle at F, 035°(T), is equal and opposite to base angle PFE. Therefore angle FPE = [180° - (2 x 35°)] = 110°. The longitude of F = (360° - 110° - 112°) = 138°W.

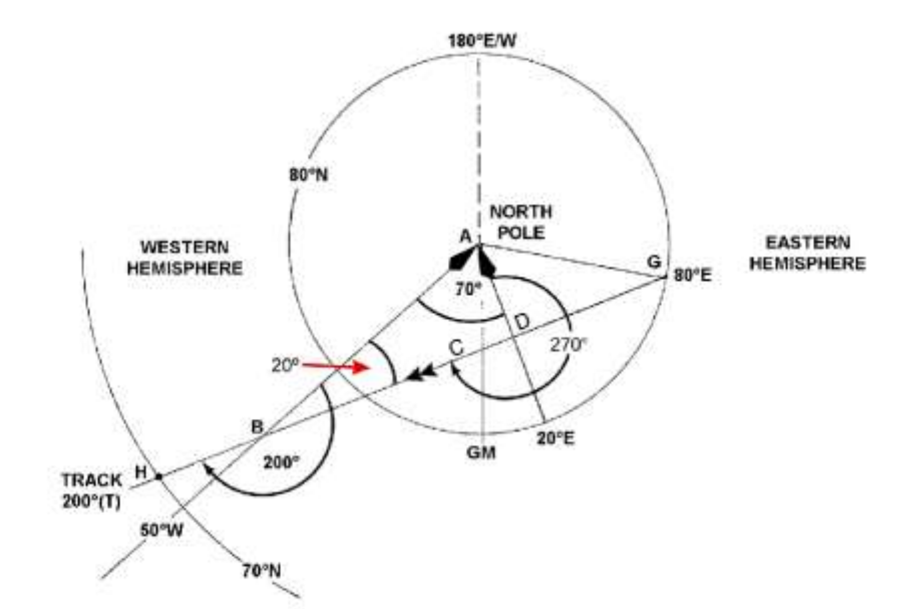
EXAMPLE 11-5

EXAMPLE

An aircraft is to fly from G at 80°N 80°E to H at latitude 70°N. The track, on a polar stereographic chart, passes closest to the north pole at longitude 20°E where it measures 270°(T). Determine the longitude at which the track measures 200°(T).

SOLUTION

FIGURE 11-7



The track will measure 200°(T) at a longitude where the internal angle (angle ABC at Figure 11-7) is 20°.

In triangle ABD, angle ABD is 20° and angle ADB is 90°. Therefore, angle BAD = 70°. This is the change of longitude.

70°W of 20°E = 50W

The longitude of B, at which the track angle will be 200°(T), is therefore determined as 50°W

EXAMPLE 11-6

EXAMPLE

An aircraft flies from J at 80°N 70°W to K at 70N° 160°W. The track on a polar stereographic chart reaches its highest latitude at 94°W where it measures 270°(T).

Determine the track direction at K.

SOLUTION

FIGURE 11-8

In triangle KPL, angle KPL = (160 - 94) = 66°.
Angle LKP = (180 - 66 - 90) = 24°
Track JK measured at K = 180° + 24° = 204°(T).

13. A further type of polar stereographic problem is shown at Example 11-7.

EXAMPLE 11-7

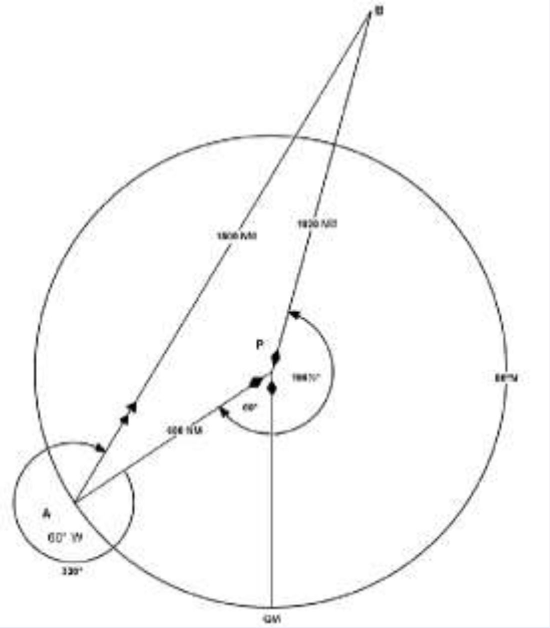
EXAMPLE

An aircraft departs overhead point A (80°N 60°W), and maintains a constant Mach no. of 0.8. The wind component is +25 kt and the temperature -40°C and both of these values remain constant. The aircraft flies along the great circle track to B, taking 3 hours to fly to overhead B. The initial track at A is 330°(T).

Using a polar stereographic construction (and assuming that the great circle track is represented by a straight line on your diagram, and ignoring scale expansion), determine the latitude and longitude of B.

SOLUTION

FIGURE 11-9



M 0.8 on navigation computer @ - 40°C

TAS	475 kt
Wind component	+25 kt
Groundspeed	500 kt
Flight time	3 hours
Distance	1500 nm

The construction is as follows:

Construct a polar stereographic using a scale of 10 mm:100 nm. With this scale the 80°N parallel is drawn as a 60 mm (600 nm) radius circle centred on the pole.

Plot the start position (A) on the chart at 60°W of the Greenwich Meridian on the latitude circle.

Draw in the great circle track on the chart, using the initial track angle of 330°(T) at A.

Measure 1500 nm (150 mm) along the track from A and plot position B using scale 10mm = 100nm.

Construct the meridian passing through B by joining B to the pole.

Measure the angle from the Greenwich meridian to the meridian passing through B to determine the longitude of B (166°30'E). Determine the latitude of B (assuming that there is no scale expansion, as stated in the question). The easiest way to do this is to determine the co-latitude by measuring the distance along the meridian between the pole and B, in this case 102 mm or 1020 nm. Remember that 1 nm is equal to 1 minute of arc of latitude, when the distance is measured along a meridian. Therefore, in this case, the co-latitude is 17° and the latitude of B as 73°N.

Plotting Radio Bearings on a Polar Stereographic Chart

14. The principles applied to Lambert charts in respect of plotting radio bearings equally apply to polar stereographic charts. For VDF/VOR bearings there is no consideration of convergency. For ADF bearings convergency is applied in the normal way, but of course calculation of convergency is much simplified since it is equal in magnitude to the change of longitude between aircraft and NDB.

Uses of the Polar Stereographic Chart

15. The main uses of the Polar Stereograph Chart are:

(a) Polar plotting chart from the pole to 70° latitude

(b) Topographical maps to 70° latitude

(c) Loran plotting charts.

Self Assessed Exercise No. 7

QUESTIONS:

QUESTION 1.

At what latitude on a Polar Stereographic Chart is scale correct?

QUESTION 2.

For the Polar Stereographic Projection, the light source is placed at the opposite pole; what purpose does this serve?

QUESTION 3.

How does scale vary over the chart?

QUESTION 4.

At what latitude does Earth Convergency equal Chart Convergency?

QUESTION 5.

Describe the appearance of rhumb lines and great circles on a Polar Stereographic Chart

QUESTION 6.

Using the chart at FIGURE 94 in the Reference Book, what is the true track of the great circle route from YWR to OZN measured at OZN?

QUESTION 7.

Using the chart at FIGURE 94 in the Reference Book, a bearing of 280°(M) is obtained from the NDB at YQX, where the variation is 29°W. The aircraft's estimated position is 60°N 15°W, where the variation is 19°W. What is the bearing (°T) to plot from the meridian passing through the NDB?

QUESTION 8.

Using the chart at FIGURE 94 in the Reference Book, while en-route from 60°N 120°W to 60°N 160°E, at what longitude does the great circle track reach its most northerly point.

QUESTION 9.

Using the chart at FIGURE 94 in the Reference Book, what is the difference in the values of Earth Convergency and Chart Convergency for a change in longitude of 30° at a latitude of 60°N?

QUESTION 10.

Using the chart at FIGURE 94 in the Reference Book, a great circle track is drawn from 70°N 00°E/W to 70°N 120°E. Is the highest latitude along the track greater or less than 80°N?

QUESTION 11.

If the chart distance representing a change of longitude of 120° along the 60°N parallel of latitude is 25.6 cm, what is the scale of the chart at 60°N?

QUESTION 12.

Using the chart at FIGURE 94 in the Reference Book, an aircraft departs from 50°N 015°W en route to point B somewhere on the 80°W meridian. If the track reaches its highest latitude at the 36°W meridian and measures 270°(T) at that point, what is the latitude of point B?

QUESTION 13.

An aircraft estimates its position to be 55°N 025°W where the variation is 22°W. With the OZN NDB (60°N 043°W) selected, a bearing of 329° is noted on the RMI. What would be the bearing to plot from the meridian passing through OZN NDB on the chart at FIGURE 94 in the Reference Book?

QUESTION 14.

Use an accurate diagram or thumbnail sketch for this question. A straight line track is drawn from point A at 80°N 60°W to point B at 80°N 150°E.

a) What is the track (°T) A to B measured at A?

b) What is the track (°T) B to A measured at B?

c) At what longitude will the track measure 270°(T)

QUESTION 15.

Use an accurate diagram or thumbnail sketch for this question. A straight line track is drawn from point C (80°S 112°E) to point D also on 80°S. If the track D to C measured at D is 135°T, what is the longitude of D?

(Hint: start your construction at point D)

QUESTION 16.

An aircraft sets off from 85°S 160°E on an initial great circle track of 150°(T). The aircraft cruises at M0.75 at FL330 where the temperature is -15°C. The aircraft experiences a 15 kt headwind and the flight takes 3 hours 27 minutes. Using a scale drawing, determine the latitude and longitude of the finish point.

ANSWERS:

ANSWER 1.

At the pole of projection; either 90°N or 90°S. See 061-11-paragraph 6 to paragraph 9.

ANSWER 2.

Placing the light source at the opposite pole significantly reduces scale expansion.

See 061-11-paragraph 4

ANSWER 3.

Scale expands away from the pole of projection at a rate of $\left[\text{secant}\left(\frac{co-lat}{2}\right)\right]^2$.

See 061-11-paragraph 6 to paragraph 9

ANSWER 4.

At and only at the pole of projection.

Earth Convergency = d long x sin mean lat.

Chart Convergency = d long.

For those to be equal, the sine of the mean latitude must equal 1, so the mean latitude is 90°; ie. the pole.

ANSWER 5.

Rhumb lines appear as curves concave to the pole of projection. Great circles are very slightly curved concave to the pole of projection, but we accept that a straight line represents a great circle. Meridians appear as straight lines; parallels of latitude appear as concentric circle. See 061-11-paragraph 10 to paragraph 12

ANSWER 6.

120°(T)

ANSWER 7.

Bearing (°T) = 280°(M) - 19°W = 261°T

Reciprocal = 261° - 180° = 081°

By inspection the bearing to plot is smaller than the bearing at the aircraft, therefore, convergency must be subtracted.

Convergency = d long = 39.5°

Bearing to plot = 081° - 39.5° = 041.5°T

ANSWER 8.

160°W (half way along the track).

ANSWER 9.

Earth Convergency = d lonx x sin mean lat
= 30° x sin 60°
= 26° (approx)
Chart Convergency = d long
= 30°
Difference = 30° - 26°
= 4°

ANSWER 10.

Less than, i.e. south of 80°N, due to chart scale expansion.

ANSWER 11.

$$\frac{1}{F} = \frac{CD}{ED}$$

ED = Departure (rhumb line distance)

= d long’ x cos lat

= 120 x 60 x cos 60°

= 7200’ x 0.5

= 3600 nm

$$\frac{1}{F} = \frac{25.6 \text{ cm}}{3600 \times 1.854 \times 1000 \times 100 \text{ cm}}$$

$$= \frac{25.6}{667{,}440{,}000}$$

$$= \frac{1}{26{,}071{,}875}$$

1 : 26,000,000

ANSWER 12.

On the chart at FIGURE 94 in the Reference Book, construct the 36°W meridian. Place a square protractor so that its edge touches 50°N 015°W and crosses the 36°W meridian at right angles. Mark the track along the edge of the protractor. Using a ruler or straight edge continue the track to the 80°W meridian. The latitude should be 40°N at 80°W.

ANSWER 13.

See FIGURE 90 in the Reference Book

Bearing (T)= 329°M – 22°W
= 307°T
Reciprocal = 307° - 180°
= 127°

By inspection, the bearing to plot at the NDB is smaller than the bearing measured at the aircraft hence convergency is subtracted.

Chart convergency = d long
= 18°
Bearing to plot = 127° - 18°
= 109°

ANSWER 14.

Great circle track at YWR measured as 060°(T)
CA = ½ x ch. long° x convergence factor (= sine of parallel of origin)
= ½ x 69° x 0.87 = 30°
RL Tk at YWR = 060° + 30° = 090°(T)

See FIGURE 91 in the Reference Book

ANSWER 15.

See FIGURE 92 in the Reference Book

1. Start the construction with point D
2. Draw the track D→C (135°T) which establishes the position of point C
3. We know point C is 112°E, so measure 112° west of point C to find the Greenwich Meridian
4. Measure the angle between Greenwich and the meridian at point D (22°)

Longitude of point D is therefore 22°E

ANSWER 16.

See FIGURE 93 in the Reference Book

Mach 0.75 ∴ G/S= 420kt

Temp -5 3hr 27 mins @ 420kt

TAS = 435kt = 1450nm

W/C -15 (equates to 7.25 cm)

SP→B= 5.95cm
= 1190nm

Along a meridian, so can convert to degrees and minutes of lat by ÷ 60 i.e. 1190 ÷ 60 = 19° 50’ ∴
Lat of B = 90° - 19° 50’ = 70° 10’S

Long of B obtained by measuring ∠c = 58° Long of B = 58°W **Position of B = 70°10’S 58°**

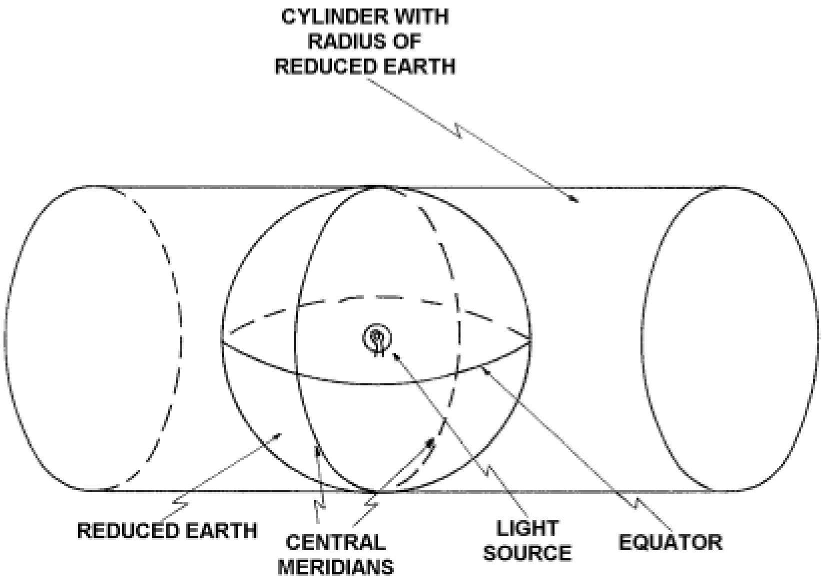
The Transverse Mercator

The Appearance of Rhumb Lines and Great Circles

The Transverse Mercator

1. The Transverse Mercator is a cylindrical projection placed horizontally on the reduced Earth such that the cylinder and the globe touch along a chosen meridian and corresponding anti-meridian, as shown at Figure 12-1. The chosen meridian and anti-meridian are known as the **central meridians**, and the points at which the central meridians cross the equator are known as the points of origin.

FIGURE 12-1
Transverse Mercator Map Projection Cylindrical

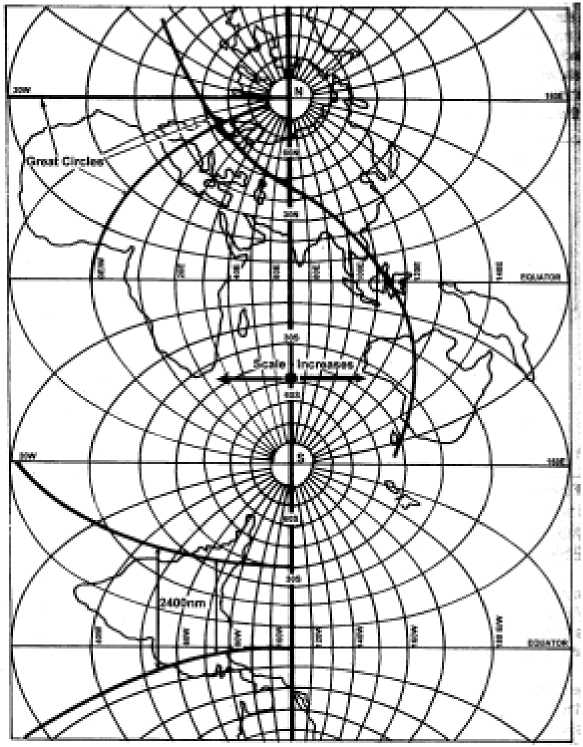


2. The light source is at the centre of the reduced Earth and the graticule of latitude and longitude is fixed on to the cylinder.

3. When the cylinder is unrolled the graticule appears as shown at Figure 12-2. At Figure 12-2 the 70°E and the 110°W meridians have been chosen as the central meridians. Although the chart is orthomorphic, the curved lattice of latitude/longitude limit the value of the chart for plotting purposes.

4. The scale is correct at all points along the central meridians and expands as a function of the secant of the great circle distance in both directions perpendicular to the central meridians. This means that within a distance of 300 nm from the central meridians the scale distortion is acceptably small and the chart can be taken to be constant scale.

FIGURE 12-2
Transverse Mercator Map Projection with Cylinder Unrolled



5. This projection is widely used for producing topographical charts of countries which extend for considerable distances north-south, but not very far in an east-west direction. Such countries are Italy, New Zealand and the United Kingdom where the projection is used for the Ordnance Survey series.

The Appearance of Rhumb Lines and Great Circles

6. Rhumb lines appear on this chart as complex curves which are concave to the nearer pole, for example the parallels of latitude shown at Figure 12-2.

7. Certain great circles, namely the central meridians and any great circle tracks crossing the central meridians at 90°, appear as straight lines. Other great circle tracks appear as curves which are concave to the central meridian.

8. Earth convergency is correctly represented only at the poles and at the equator on this projection.

The Oblique Mercator

The Oblique Mercator

1. The oblique Mercator is a cylindrical projection. In this case the cylinder touches the reduced Earth along a chosen **great circle of tangency** (sometimes referred to as the **False Equator**) see Figure 13-1.

2. The graticule of latitude and longitude associated with an oblique Mercator is complex in appearance, as shown at Figure 13-2.

3. The scale of the chart expands as the secant of the great circle distance from the great circle of tangency and is considered accurate enough for navigational purposes within 300m of the great circle of tangency. As with all Mercators, the oblique projection is orthomorphic. Rhumb lines appear as complex curves and great circles appear as curves which are concave to the great circle of tangency, unless they cross the great circle of tangency at right angles, in which case they appear as straight lines.

4. The oblique Mercator is used to produce 'purpose built' great circle route maps, normally in a strip format, such that the great circle of tangency co-incides with the route to be flown. As with the transverse Mercator, the oblique Mercator is ideal for producing topographical maps of countries which are long and thin, but which this time do not lie predominantly north-south, for example the Malaysian Peninsula.

FIGURE 13-1
Oblique Mercator
Map Projection
Cylindrical

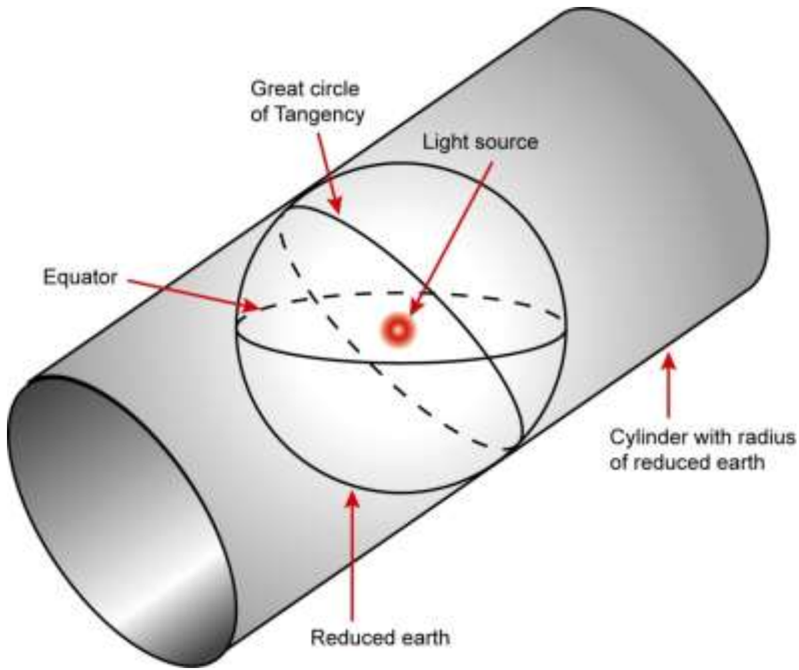
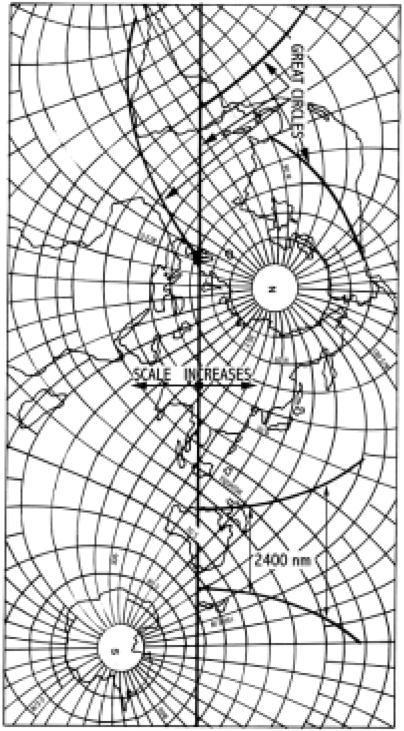


FIGURE 13-2
Oblique Mercator
Map Projection



Grid Navigation

Grid Navigation

1. High latitude flights in Polar regions are subject to specific problems associated with the use of magnetic variation to derive true heading from the output of a gyro magnetic compass. Firstly, in the region immediately surrounding the North magnetic pole, accurate values of magnetic variation are impossible to quantify and therefore navigation charts highlight this area as having magnetic anomalies. Secondly, in high latitudes adjacent to this area, the value of magnetic variation changes so rapidly that the true Great Circle track changes markedly with change of longitude.

2. For the above reasons flights in polar regions may require the use of the gyro magnetic compass in the DG mode plus the adoption of a method of navigation called 'grid navigation'.

3. The following paragraphs give a brief overview of the basics of the grid navigation technique.

Grid Navigation

4. The advantage of using a standard Mercator chart for plotting is that the true track direction remains constant along each leg of the flight. The disadvantage is of course that a rhumb line track is flown and this is not the shortest distance between each turning point.

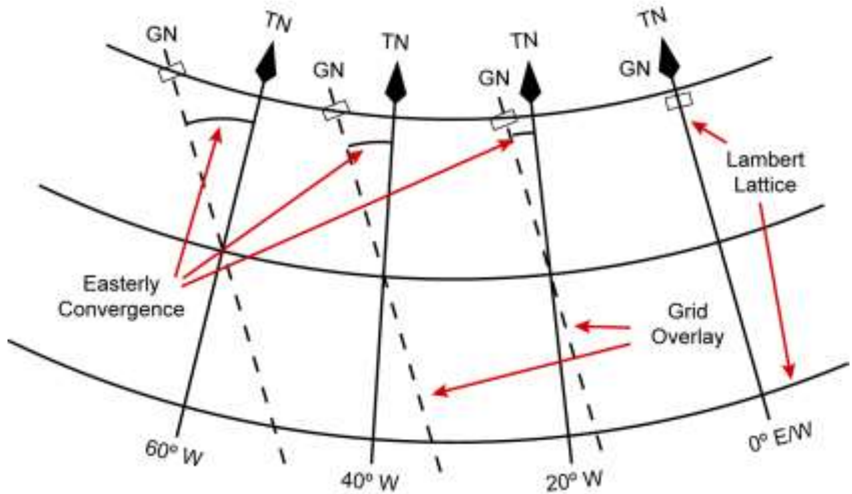
5. When using either a Lambert or a polar stereographic chart any straight line is very close to a great circle, but the true track direction changes with change of longitude particularly at high latitudes.

6. A compromise is achieved by using either a Lambert or polar stereographic chart with a **grid overlay**.

7. The grid may be aligned with true north on any specified meridian. Usually the true north used is that of the Greenwich meridian and occasionally the Greenwich anti-meridian.

8. Figure 14-1 shows a Lambert chart with such a grid overlay. Being a Lambert projection the real meridians are portrayed as straight lines converging towards the pole. True north is defined by the meridians and of course any track direction measured relative to true north will change with change of longitude as a function of chart convergency. The parallel of origin of this chart is 40°N.

FIGURE 14-1



9. The grid overlay shown in Figure 14-1 is aligned with the Greenwich meridian. The grid can in fact be aligned with any convenient **datum meridian**. Notice that the grid lines are parallel to each other and are therefore all parallel to the datum meridian. All of the grid lines define **grid north**. Any track direction expressed in **degrees grid** and measured relative to the grid lines will remain at a constant value for the entire length of the track.The angle between grid north and true north at any point on the chart is known as **convergence**. The value of convergence increases with increasing distance from the datum meridian.

10. Convergence is said to be easterly when true north lies to the east of grid north, as at Figure 14-2. Conversely convergence is said to be westerly when the true north lies to the west of grid north, see Figure 14-3.

FIGURE 14-2

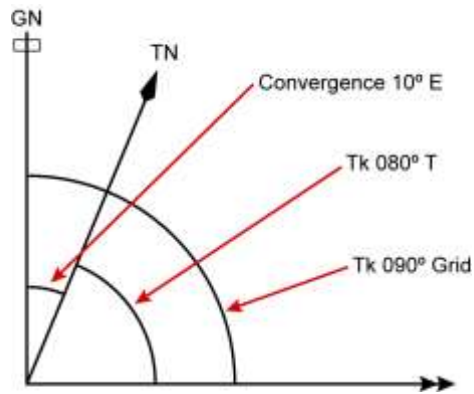
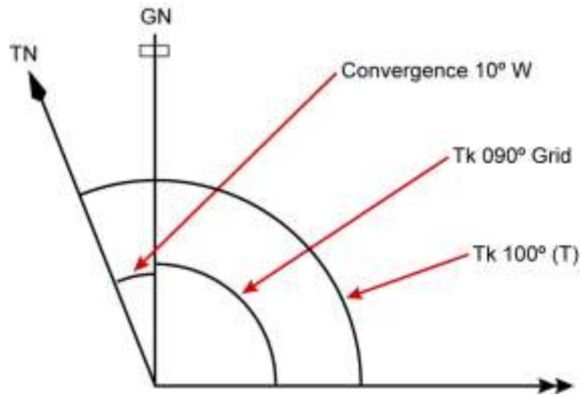


FIGURE 14-3



When convergence is east true track is least.

When convergence is west true track is best

Returning to Figure 14-1, consider the convergence at various longitudes:

At longitude 0ºE/W grid north and true north are aligned and therefore convergence is zero.

At longitude 20ºW the chart convergency (between 0º E/W as the datum meridian and 20º W)is:

Chart convergency = change of longitude x the sine of the parallel of origin

= 20º x sine 40º

= 13º

11. The calculated value of chart convergency between the datum meridian and the meridian at 20º W is also the value for convergence at longitude 20ºW.
Similarly the value of convergence at 40ºW is 26ºE, and at 60ºW is 39ºE.

12. Given that a straight line track drawn on the chart at Figure 14-1 from A at longitude 0ºE/W to B at longitude 60ºW measures 240º(T) at A, it should now be obvious that:

The track will measure (240º - 13º) 227º (T) at 20ºW.

The track will measure (240º - 26º) 214º (T) at 40ºW.

The track will measure (240º - 39º) 201º (T) at 60ºW.

However the grid track will measure 240º at all points along track.

EXAMPLE 14-1

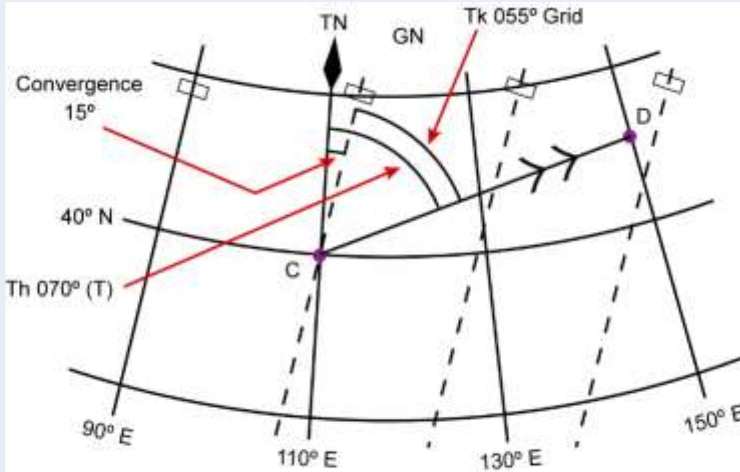
EXAMPLE

A straight line track is drawn on a Lambert chart (constant of the cone = 0.75) from C at 40ºN 110ºE to D at longitude 150ºE. The straight line track measures 070º (T) at C. The chart is overprinted with a grid overlay which is based on a datum meridian at 90ºE. Determine the direction of the track with reference to grid north.

SOLUTION

See Figure 14-4.
Remember that the constant of the cone is simply another way of expressing the value of the sine of the parallel of origin.

FIGURE 14-4



The value of convergence at 110ºE is the same as the value of chart convergency between 110ºE and the datum meridian, which in this case (20º x 0.75) is 15º. By inspection convergence is westerly (true north lies to the west of grid north) and therefore the grid track direction (070º - 15º) is 055º (G).

EXAMPLE 14-2

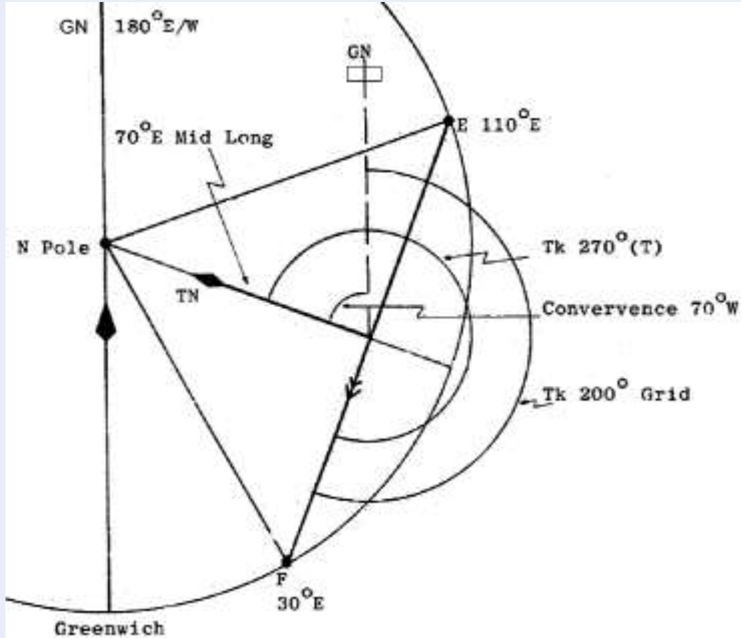
EXAMPLE

A straight line track is drawn on a polar stereographic chart from E at 70ºN 110ºE to F at 70ºN 30ºE. Determine the track direction in degrees grid given that the grid is aligned with the Greenwich meridian.

SOLUTION

See Figure 14-5.
Remember that on a polar stereo chart convergency is equal to change of longitude.

FIGURE 14-5



At the mid longitude between E and F (70ºE) the true track direction will be 270º. At 70ºE the convergence is 70º and by inspection is westerly (true north lies to the west of grid north). The grid track is therefore (270º - 70º), 200º (G).

EXAMPLE 14-3

EXAMPLE

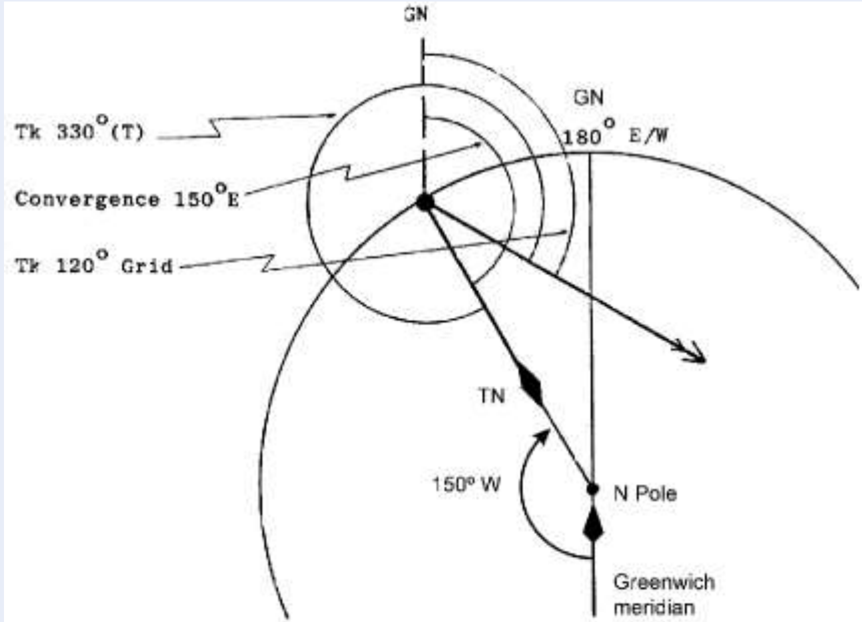
An aircraft is at position 80ºN 150ºW and is making good a track of 330º (T). Express this track in degrees grid assuming that a polar stereographic chart is being used and that the chart is overlaid with a grid which is aligned with the Greenwich meridian.

SOLUTION

See Figure 14-6.

Convergence is equal to the change of longitude between the datum meridian and the merdian in question (150º) and by inspection is easterly. The true track direction is therefore less than the grid track direction by 150º, and the grid track is in this case (330º + 150º) = 120º (G).

FIGURE 14-6



13. Grid navigation techniques really become useful at high latitudes, where magnetic compasses are unreliable due to the proximity of the magnetic pole (weak H field). It is then necessary to steer the aircraft with reference to a rigid unslaved directional gyro.

14. In mid latitudes it is often convenient to use grid technique whilst steering magnetic headings. In order to convert a grid track to the corresponding magnetic track it is necessary to correct for both convergence and variation. To make life easier these two corrections are normally added algebraically together and termed grivation.

15. For example, at a point on the chart where the convergence was 15ºE and the variation 10ºW, the grivation would be given as 5ºE, as illustrated at Figure 14-7. At another point on the chart the convergence might be 12ºW and the variation 25ºW, the grivation would now be 37ºW, as illustrated at Figure 14-8.

FIGURE 14-7

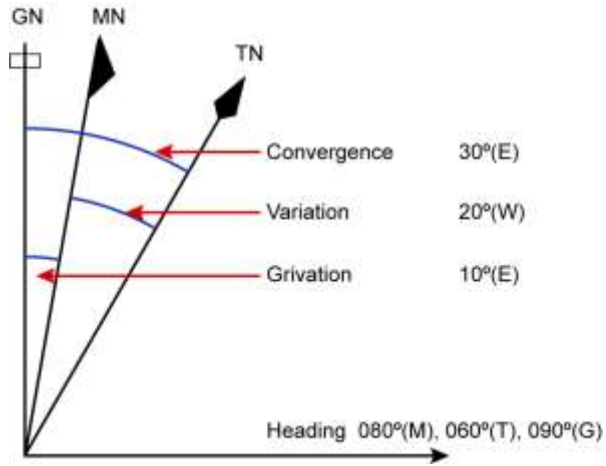
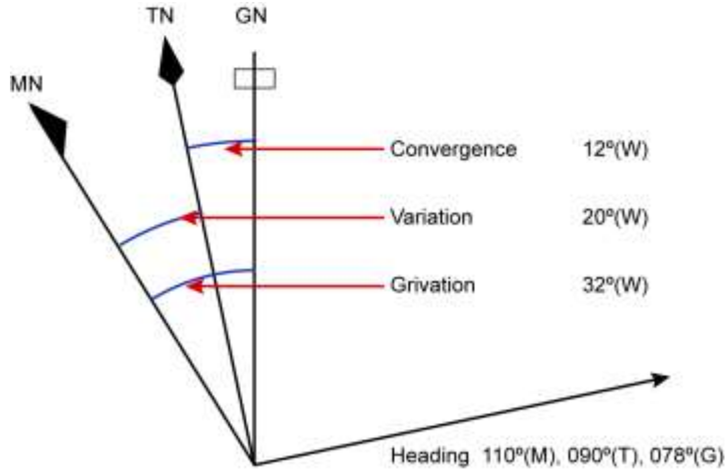


FIGURE 14-8



- **If grivation is east magnetic track is least.**
- **If grivation is west magnetic track is best.**

16. Lines joining places having equal grivation are referred to as isogrives and are depicted on all grid charts.

EXAMPLE 14-4

EXAMPLE

An aircraft - Heading 090(M) is at a position where the variation is 48°W and the convergence is 20°W. Calculate the grid Heading.

SOLUTION

090° - 48° - 20° = 022°(G).

Self Assessed Exercise No. 8

QUESTIONS:

QUESTION 1.

On a Transverse Mercator where is the scale correct and at what rate does it change away from that position?

QUESTION 2.

For what purpose is the Transverse Mercator used and which feature(s) make it suitable for this purpose?

QUESTION 3.

Where on a Transverse Mercator is earth convergency correctly represented?

QUESTION 4.

Describe the appearance of rhumb lines and great circles on an Oblique Mercator.

QUESTION 5.

Why is the Oblique Mercator ideal for the production of strip route charts?

QUESTION 6.

Why is it common practice for trans polar flights to adopt grid navigation techniques?

QUESTION 7.

Why is it necessary to define a datum meridian for a grid overprint?

QUESTION 8.

What formula is used to calculate convergency for a grid superimposed on a Lamberts Conformal chart?

QUESTION 9.

Define grivation and describe how it is depicted on a grid chart.

QUESTION 10.

Complete the following table by filling in the blank spaces.

True Track (°T)	Convergence (°E/W)	Grid Track (°G)
220°	20°E	
240°		170°
	60°W	060°
	34°E	196°
047°	27°E	
310°		355°

QUESTION 11.

Given: grid track 070°(G); convergence 45°W; variation 15°E.

Calculate the magnetic track (°M) and the grivation.

QUESTION 12.

Given: DR position 70°N 110°W; true track 320°(T). Grid datum Greenwich Meridian.

Calculate: (a) the grid track (°G) on a polar stereographic chart and (b) the grid track (°G) on a Lamberts Conformal chart (constant of cone 0.8)

QUESTION 13.

Given: DR position 70°S 110°W; true track 230°(T). Grid north is true north on the Greenwich Anti-Meridian:

Calculate (a) the grid track (°G) on a polar stereographic chart and (b) the grid track (°G) on a Lamberts Conformal chart (constant of cone 0.8)

QUESTION 14.

Given: DR position 80°N 50°E; true track 330°(T). Grid datum Greenwich Anti-Meridian.

Calculate the grid track (°G) on a polar stereographic chart.

QUESTION 15.

Given: DR position 80°S 70°W; grid track 090°(G). Grid north is true north at the Greenwich Meridian.

Calculate the true track (°T) on a polar stereographic chart.

ANSWERS:

ANSWER 1.

Page 12-1 para 4.

ANSWER 2.

Page 12-2 para 5. Because the central meridian would be along the countries north/south axis where the scale is correct and the scale distortion is acceptably small up to 300 nm east and west of the central meridian.

ANSWER 3.

Page 12-3 para 8

ANSWER 4.

Page 13-1 para 3

ANSWER 5.

Page 13-1. The cylinder may be adjusted in such a manner that the great circle of tangency lies along the mean track of the route and permits deviations from the mean track up to 300 nm either side without perceptible error. The scale is therefore accurate enough for navigational purposes.

ANSWER 6.

Page 14-1. There are two major reasons for using grid navigation techniques in polar regions:

1. The horizontal component of the Earth's magnetic field is extremely weak at high latitudes rendering the magnetic compass unreliable. Furthermore when passing close to the magnetic pole the variation changes by large amounts over short distances and would be impossible to accurately compensate.

2. A straight line track in high latitudes changes its true direction rapidly even over short distances. The charts normally used for navigational purposes in these areas are either the Transverse Mercator or the Polar Stereographic, on which a straight line approximates a great circle.

Using a gyro steering technique together with a chart overprinted with a grid overcomes both of these problems.

ANSWER 7.

Page 14-1 paragraph 7. A grid is aligned with true north on the selected meridian, usually the Greenwich Meridian, so that convergence can be determined for any point on the chart. The advantage of using the Greenwich Meridian is that the change of longitude from the datum meridian is equal to the longitude of that position

ANSWER 8.

Page 14-6 paragraph 10. Convergency = change of longitude x sine of parallel of origin.

ANSWER 9.

Grivation is the algebraic sum of convergence and variation. It is shown on a chart by lines joining points of equal grivation, isogrivs, which are normally shown as a broken green line which is labelled with its value.

ANSWER10.

True Track (°T)	Convergence (°E/W)	Grid Track (°G)
220°	20°E	240°
240°	70°W	170°
120°	60°W	060°
162°	34°E	196°
047°	27°E	074°
310°	45°W	355°

ANSWER11.

Grid track = 070°(G)
Convergence = +45°(W) (convergence west, true track best)
True track = 115°(T)
Variation = -15°E (variation east, magnetic least)
Magnetic track = 100°(M)

ANSWER 12.

See FIGURE 95 in the Reference Book

Convergence = 110°E on Polar Stereo
= 110 x 0.8 = 88°E on Lamberts

		Polar	Lamberts
True Track	=	320°(T)	320°(T)
Convergence	=	+110°E	+88°E
Grid track	=	430°(G)	408°(G)
		-360	-360
Grid track	=	070°(G)	048°(G)

(or just measure it)

ANSWER 13.

See FIGURE 96 in the Reference Book

Convergence= 70° on Polar Stereo
= 70 x 0.8 = 56°E on Lamberts

		Polar	Lamberts
True track	=	230°(T)	230°(T)
Convergence	=	+ 70°E	+56°E
Grid track	=	300°(G)	286°(G)

ANSWER 14.

See FIGURE 97 in the Reference Book

Convergence = 130°E

True track = 330°(T)

Convergence = +130°E

Grid track = 460°(G)

-360

Grid track = 100°(G)

ANSWER 15.

See FIGURE 98 in the Reference Book

Grid track = 090°(G)

Convergence = +70°W

Grid track = 160°(T)

The One in Sixty Rule

Uses of the One in Sixty Rule

Altering Heading to Parallel Track

Altering Heading to Regain Track in Equal Time

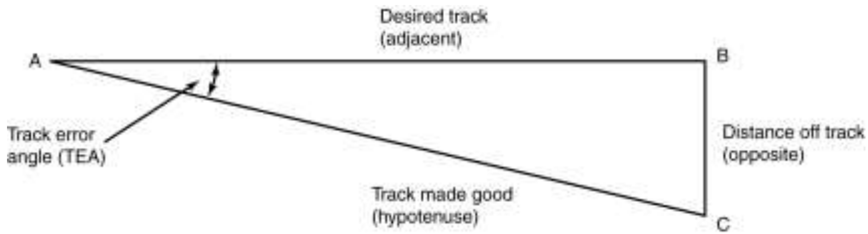
Altering Heading Direct to the Destination

Approach Height and Range Calculations

The One in Sixty Rule

1. The tangent of an angle can be calculated by dividing the length of the side opposite the angle by the length of that adjacent side which is not the hypotenuse. A practical application of this is shown at Figure 15-1, which illustrates a navigation tracking problem.

FIGURE 15-1
Basic Navigation Tracking Problem



2. In Figure 15-1, assuming the distance off track and the distance along track are known, the value of the track error angle (TEA) can be found from the formula:

$$\text{Track error angle} = \frac{\text{distance off} \times 60}{\text{distance along track}}$$

Uses of the One in Sixty Rule

3. There are many types of problem where the 1 in 60 rule will give an acceptably accurate answer. The 1 in 60 rule is a very useful tool for pilot navigation, and various examples follow.

Altering Heading to Parallel Track

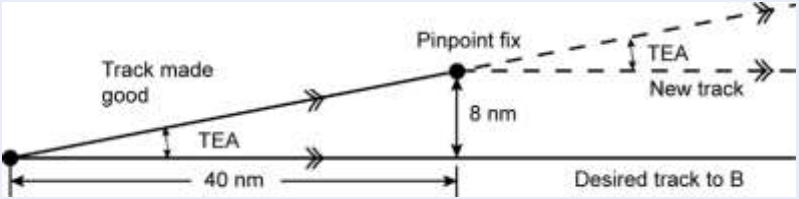
EXAMPLE 15-1

EXAMPLE

An aircraft departs point A, intending to fly the direct track to point B. After travelling 40 nm, the pilot pin-points his position as being 8 nm to the left of track. Determine the alteration of heading required in order to track parallel to the original flight plan track.

SOLUTION

FIGURE 15-2



To find the track error angle, use the formula:

$$\text{TEA} = \frac{\text{distance off} \cdot 60}{\text{distance gone}}$$

$$= \frac{8 \cdot 60}{40}$$

$$= 12°$$

The track error angle is 12°. Accepting that the drift is unlikely to change significantly with an alteration of heading of 12°, a turn to starboard of 12° will cause the aircraft to fly parallel to the original track.

Altering Heading to Regain Track in Equal Time

EXAMPLE 15-2

EXAMPLE

An aircraft departs from point C at time 1230, intending to fly the direct track to point D which is 120 nm away. At 1252 the pilot pin-points his position and finds that the aircraft has travelled 43 nm along track but is 5 nm to the right of track.

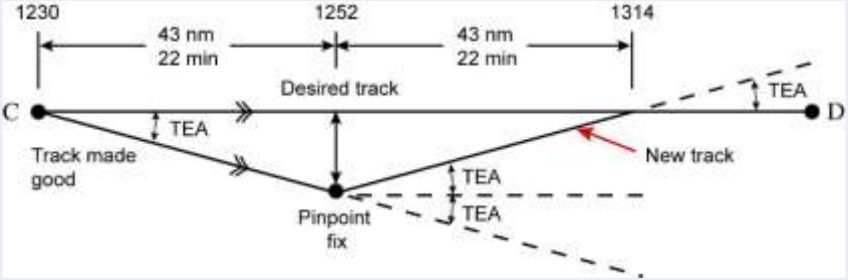
Determine:

The alteration of heading required at 1252 to regain track at 1314.

The alteration of heading required at 1314 to maintain the original track.

SOLUTION

FIGURE 15-3



$$TEA = \frac{\text{distance off}}{\text{distance gone}} \cdot 60$$

$$= \frac{5 \cdot 60}{43}$$

$$= 7°$$

An alteration of heading to port of 7° would cause the aircraft to track parallel to the flight plan track. An alteration of heading to port of 14° (2 x TEA) would cause the aircraft to regain the flight plan track at 1314. Two assumptions are made, firstly, it is assumed that the drift will not change significantly for a change of heading of 14°. Secondly it is assumed that the groundspeed will not change significantly, when the aircraft changes heading by 14°.

At 1314 the aircraft will be back on the original track and an alteration of heading to starboard of 7° should keep the aircraft on the flight plan track.

Altering Heading Direct to the Destination

EXAMPLE 15-3

EXAMPLE

An aircraft departs from point E, intending to fly the direct track to point F, 160 nm away. Having flown for 85 nm, a fix is obtained which puts the aircraft 11 nm to the port of the intended track.

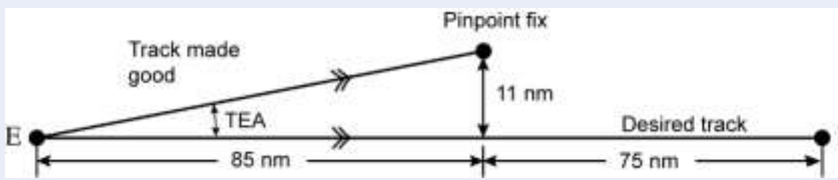
Determine the alteration of heading required, if the aircraft is to track directly from the fix to point F.

SOLUTION

$$TEA = \frac{11 \times 60}{85}$$

An alteration of heading to starboard of 7.8° would cause the aircraft to track parallel to the original track E to F. However the aircraft is required to turn further to track direct to F.

FIGURE 15-4



Required to turn is known as the **closing angle (CA)**, see Figure 15-5. The closing angle is equal to the angle subtended at the destination between the direct track E to F and the new track fix to F.

The CA is effectively a TEA in reverse. The formula therefore becomes:

$$CA = \frac{\text{distance off} \cdot 60}{\text{distance to go}}$$

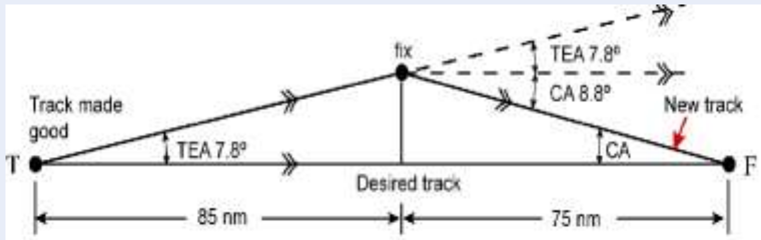
therefore:

$$CA = \frac{11 \cdot 60}{75}$$

$$= 8.8°$$

The additional angle through which the aircraft is:

FIGURE 15-5



The aircraft is therefore required to alter heading at the fix position by:

$$\text{TEA} + \text{CA} = 7.8° + 8.8°$$

$$= 16.6° \text{ to starboard}$$

The ideal time to fix the aircraft's position is around the half-way point. In this case, the track error angle is equal to the closing angle, giving an alteration of heading of twice the track error angle.

Questions are often set whereby the TEA is known and the aircraft's displacement from track is required.

EXAMPLE 15-4

EXAMPLE

An aircraft is tracking outbound from a VOR. The intention is to maintain the centreline of an airway which is defined by the 300° radial from the VOR. A back-bearing from the VOR shows the aircraft to be on the 295° radial, and at the same time the DME gives a range of 85 nm from the VOR. Determine the aircraft's displacement from the airway centreline.

SOLUTION

$$TEA = \frac{\text{distance off}}{\text{distance gone}} \times 60$$

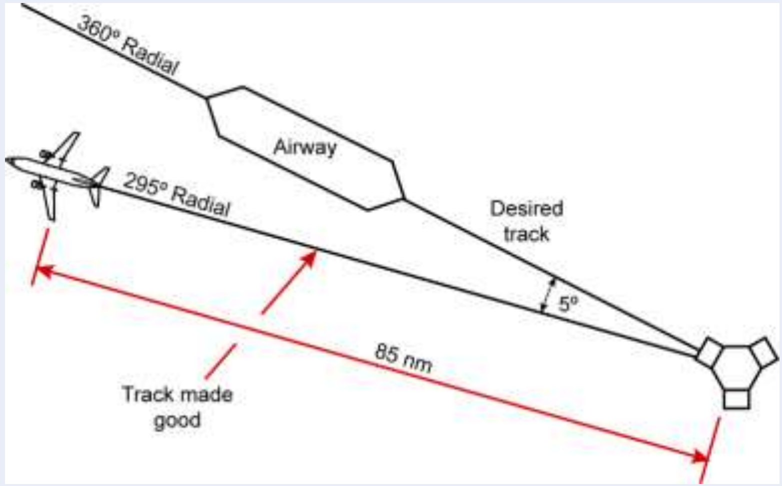
therefore;

$$\text{Distance off track} = \frac{TEA \times \text{distance gone}}{60}$$

$$= \frac{5° \times 85}{60}$$

$$= 7\text{nm}$$

FIGURE 15-6



4. The One in 60 rule forms the basis of calculations in the vertical plane of height against range for an aircraft on the ILS glidepath, and of cloud height determination using the airborne weather radar. These calculations are covered in the relevant sections of the text.

Approach Height and Range Calculations

5. Calculation of required height above touchdown for a given glidepath angle and range from touchdown can be achieved using the 1 in 60 rule formula.

This formula is:

$$\text{Ht above touchdown} = \frac{\text{Range from touchdown (ft) x Glidepath angle}^\circ}{60}$$

Try the following example:

EXAMPLE 15-5

EXAMPLE

Determine the height above touchdown of an aircraft which is on a 3° glidepath at a range of 3 nm from touchdown.

SOLUTION

FIGURE 15-7

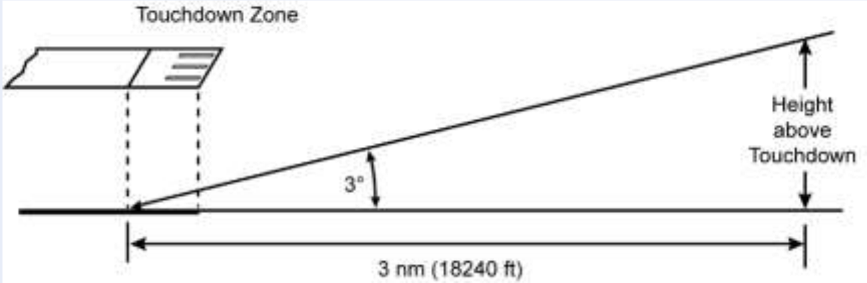


Figure 15-7 illustrates the solution using the 1 in 60 rule:

$$\text{Height above touchdown} = \frac{(3 \times 6080) \times 3}{60} = 912 \text{ ft}$$

EXAMPLE 15-6

EXAMPLE

Determine the height above touchdown of an aircraft which is on a 3.25° glidepath at a range of 3.75 nm from touchdown.

SOLUTION

Using the 1 in 60 rule:

$$\frac{(3.75 \times 6080\ ) \times 3.25}{60} = 1235\text{ft}$$

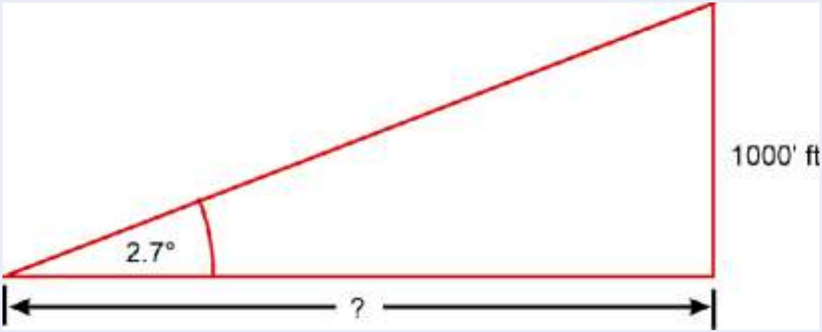
6. It is important to appreciate that in both the previous examples the range has been given from touchdown. If, in the question, the range is given from the runway threshold, the 1000 ft must be added to the range because the touchdown is normally this distance from the threshold and height is required above touchdown.

EXAMPLE 15-7

EXAMPLE

At what range from touchdown must an aeroplane, on a 2.7° glidepath be when at 1000ft above touchdown.

SOLUTION



Using the 1 in 60 rule:

In this case the formula must be re-arranged as follows:

$$\text{Range (ft)} = \frac{\text{Height above Touchdown x 60}}{\text{Glidepath Angle}}$$

$$= \frac{1000 \times 60}{2.7}$$

= 22,222 ft

= 3.65nm

Visual Navigation Techniques

Introduction

Topographical Maps

Map Preparation

Visual Check Points

Selection of Features

Estimating Bearing and Range

Fixing the Aircraft's Position

Track and Time Keeping - Mental DR Techniques

Time Adjustments

Limitations

Visual Navigation Techniques

Introduction

1. Although most modern aircraft are fitted with a comprehensive suite of navigation systems, occasions can arise when map reading is the only appropriate source of positional information. A knowledge of map reading and visual navigation techniques is therefore an essential requirement for any pilot. Map reading is the ability to relate observed ground features to the details portrayed on a topographical map to establish the aircraft's position. Whilst a natural aptitude is an advantage, map reading is a skill that can be learnt through regular practice and careful application.

2. The principle of visual navigation is to compare an estimated or Dead Reckoning (DR) position with visual references in order to determine the actual position of the aircraft and make the necessary adjustments to speed and heading. Therefore the pilot must have the ability to:

(a) Select features

(b) Identify landmarks

(c) Estimate distance

(d) Estimate direction

3. The pilot will need the ability to build a mental picture of the terrain and features along the planned route, estimating ranges and bearings of features relative to each other both in the air and on the map will greatly assist the pilot in this task. It is also necessary to have a sound knowledge of the map being used and the techniques of track keeping and timing. Thorough map preparation will considerably reduce the complexities of map reading.

Topographical Maps

4. Visual navigation will invariably be carried out using a Topographical Map. We tend to use the terms Map and Chart to mean one and the same thing; however, in aeronautical terms, there is an important distinction. On the whole, plotting charts show very little in the way of ground features; the less clutter on the chart when plotting, the better. Charts are therefore generally unsuitable for map-reading. On the other hand, Topographical Maps for aeronautical use show as much detail as the scale will permit and are produced specifically for the purpose of map-reading.

5. Aeronautical series Topographical Maps will show features such as rivers, roads, railways and terrain. They will also contain information of special interest to pilots including the position of airfields, radio beacons and airways. Remember: **the larger the scale, the greater the detail** available on the chart.

6. The choice of map for any particular flight will depend on many factors, including the length of the journey; the cruising height and speed; and other available means of navigation. Bad weather can also considerably affect the choice; the worse the weather, the larger the scale of map that should be used. The choice is then determined by the properties of the maps available for the area to be flown over.

7. The CAA issues a series of 1:500,000 topographical charts; however, apart from some important differences which will be highlighted, these notes will concentrate attention on the Jeppesen 1:500,000 series Aeronautical charts.

Topographical Symbols and use of Colour

8. It is unusual to find two map series which use identical methods of representing topographical information; however, by convention, certain colours are used to denote particular features to aid visual navigation. These are as follows:

Feature	Colour
Water	Blue - whether sea, lake or river
Woods	Green
Roads	Red lines
Railways	Black lines

9. Figure 16-1 gives the symbology used for aeronautical features on CAA Topographical Maps. Figure 16-2 and Figure 16-3 give the symbology currently used on the Jeppesen 1:500,000 series charts.

FIGURE 16-1
CAA Chart Symbology

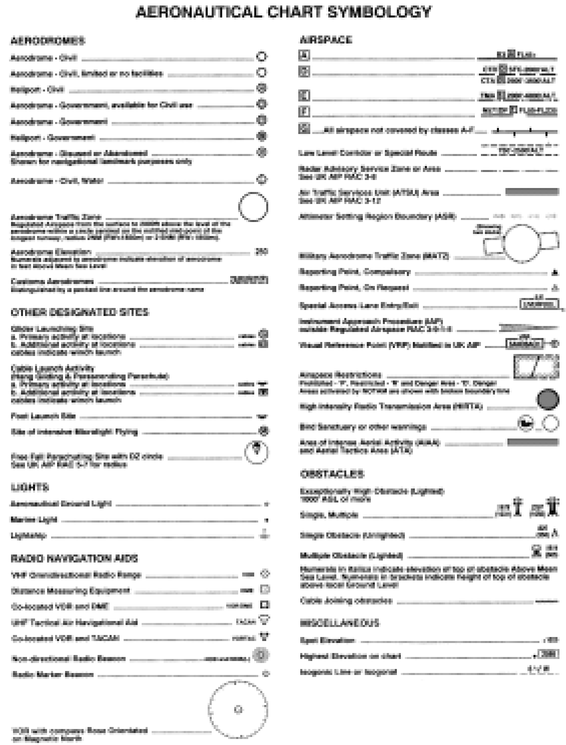


FIGURE 16-2
Jeppesen Chart Symbology

The general purpose of the chart is to provide the optimum amount of relevant information for VFR flights consistent with legibility. However, to achieve this required selective omission of certain highpoints and/or man-made structures in some areas.

PLANNING A SAFE FLIGHT ALTITUDE: When planning and conducting a flight under VFR, we recommend maintaining an altitude at or above the depicted MINIMUM GRID AREA ALTITUDES. When below the Grid Area Altitude (during departure or landing), use charts at more suitable scales, e.g. a Visual Approach and Landing Chart or an Area Chart contained in the Bottlang Airfield Manual or in other Publications.

Latest aeronautical information can be obtained from the following sources: JEPPESEN AIRWAY MANUAL, JEPPESEN BOTTLANG AIRFIELD MANUAL, VFR+GPS CHART NOTAM SERVICE (updated weekly) via **Fax Polling/Retrieval System (+49 6102 5083 00)** or via **Internet http://www.jeppesen.com** or the relevant AIPs, NOTAMs and AICs.

FLYING OVER MOUNTAINOUS TERRAIN: Extreme caution should be used when flying in valleys, due to numerous transmission lines and cableways which may not be depicted on the chart.

GPS LATITUDE AND LONGITUDE DISCREPANCIES: GPS receivers/systems are currently only authorized as supplementary navigation reference systems for VFR and IFR flights. GPS values are based on a geodetic reference system known as "World Geodetic System" (WGS 84). Official government aviation authorities still publish position information in geographical latitude and longitude based on local geodetic reference datums which may not be compatible with WGS 84. This can result in position discrepancies. The geographical coordinates in degrees, minutes and tenth of minutes are based on local geodetic datums as obtained from the most reliable sources available.

MILITARY LOW LEVEL FLIGHTS: Civil VFR Flights should be conducted above 2000ft AGL whenever possible to provide necessary separation from military low level flights.

LOCAL REGULATIONS: In some countries cross country flights under VMC with power-driven aircraft must be conducted at a height of at least 2000ft (600m) above ground or water, unless for safety reasons higher altitudes or heights above ground or water have to be used.

AERONAUTICAL INFORMATION:

AIRPORTS are shown with ICAO four letter code, location name, elevation in feet, longest runway available in meters and communication frequency with VDF if available (v). Heliports and Glider Sites are shown with location name, geographical coordinates, elevation and communication frequency.

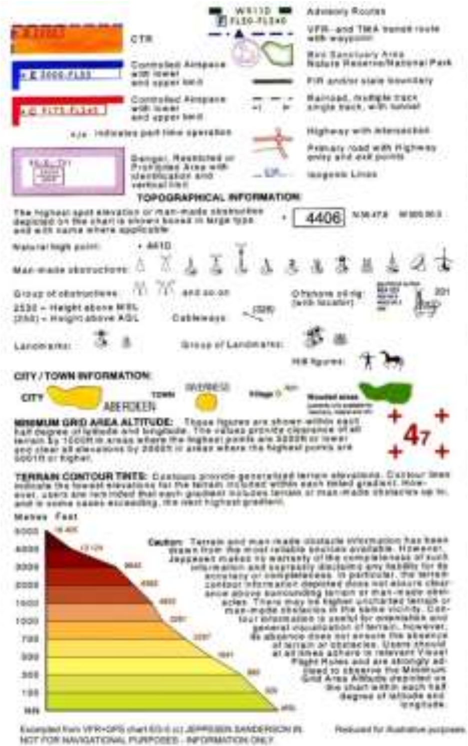
WAYPOINT INFORMATION: Compulsory and Non-Compulsory Reporting Points with assigned names. Geographical coordinates and radial/distance definitions are shown on chart in tabulated form.

NAVAID INFORMATION: Name, frequency and identification. In addition geographical coordinates are shown on chart in tabulated form.

Excerpted from VFR+GPS chart ED-5 (c) JEPPESEN SANDERSON INC. NOT FOR NAVIGATIONAL PURPOSES—INFORMATION ONLY.

Reduced for illustrative purposes.

FIGURE 16-3
Continued.



Representation of Relief

10. Relief information (the location and elevation of high ground) can be shown on topographical charts in a variety of ways; note that this information can be presented either in feet or in metres. It is essential for the pilot to ascertain which unit applies to the chart to be used. The Jeppesen 1:500,000 series charts are marked in feet.

11. The main ways of presenting relief are as follows:

(a) Contours

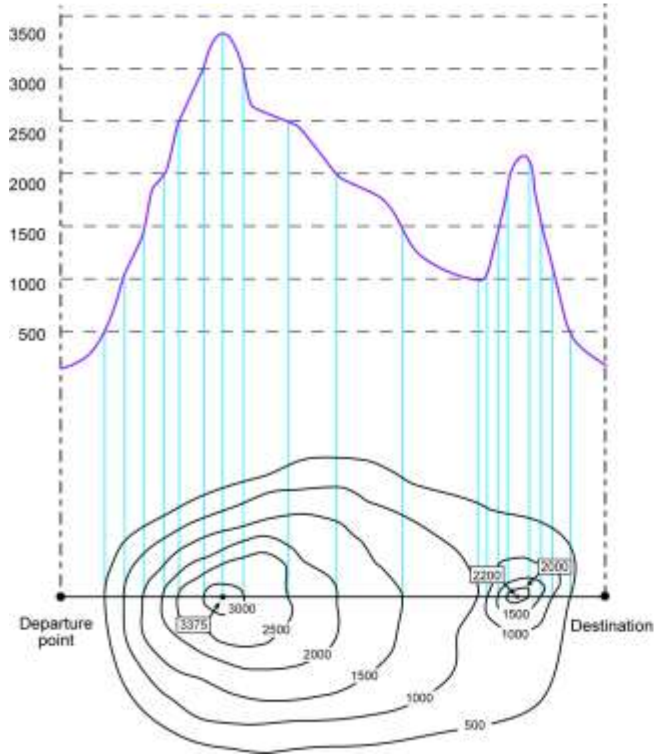
(b) Spot Heights

(c) Layer Tinting

12. **Contours. Contours** are lines on a map joining points of equal elevation. The height interval between adjacent contours is governed to a large extent by the scale of the map. The intervals are standardised for a particular map series although the interval may not necessarily be constant. For instance, small changes in height in a generally flat landscape may result in a significant hill feature; therefore, contours at smaller intervals may be introduced.

13. The height interval between adjacent contours is called the **vertical interval**. The horizontal distance on the earth between each contour is called the **horizontal equivalent**. It should be apparent that dividing the vertical interval by the horizontal equivalent will give the gradient of the slope. The closer together the contours appear, the steeper the slope.

FIGURE 16-4
Interpreting Contours



14. Figure 16-4 depicts how contours might be used to illustrate a hill feature on a map. In order to describe the appearance of a significant feature, it is necessary to produce a cross section through the hill along the line from the departure point to the destination. It is not immediately apparent from the contours alone exactly where the highest point on the hill (3375' AMSL) is located.

15. To show the elevation of prominent peaks and to highlight the position of the highest point on a peak, **spot heights** (given in figures) are used. On Jeppesen charts the highest spot height on the chart is outlined in a white rectangle with a black perimeter and its position is given in the legend.

16. Different colours are used to represent ground elevation between contours in different height bands; this is called **layer tinting**. A key to the layer tinting will always be found in the legend of the chart; by convention, the lighter the colour, the lower the ground. As can be seen from the Jeppesen chart legend, these charts use layer tinting. On the Jeppesen series charts, the colours used vary from green through yellow to darkening shades of brown. This technique provides an immediate impression of the relief in the area shown on the map.

17. **Minimum Grid Area Altitudes** are shown on Jeppesen charts within quadrangles bounded by lines of each half degree of latitude and longitude. They are printed in red and are represented in thousands and hundreds of feet above mean sea level. The values provide clearance of all terrain by 1000ft in areas where the highest points are 5000ft or below and clear all elevations by 2000ft in areas where the highest points are 5001ft or higher. When planning a safe flight altitude, Jeppesen 'recommend' maintaining an altitude at or above the Minimum Grid Area Altitude.

18. On CAA topographical charts, **maximum elevation figures** indicate the highest known feature in a given area. They are shown in quadrangles bounded by lines of each half degree of latitude/longitude and are represented in thousands and hundreds of feet above mean sea level. As the 1:500,000 charts generally do not show obstacles that are less than 300 ft agl, it follows that for any quadrangle, in the absence of a dominant obstacle, the MEF will be 300 ft higher than the highest spot height or highest surface (rounded up to the next highest hundred feet). **MEFs are not a safety height - they indicate the highest known feature in each quadrangle including terrain and obstacles and allowing for unknown features.**

19. The CAA 1:500,000 series charts show the lateral and vertical limits of controlled airspace. Where the controlled airspace has an upper limit which is below FL 245 both the lower and upper limits are printed on the chart. When the upper limit is FL 245, this is indicated by a + (in order to save space and avoid clutter on the chart). For example, an airway with a lower limit of 3000 ft amsl and an upper limit of FL 245 would be annotated as **3000' ALT+** on the chart.

20. The CAA also publish 1:500,000 topographical charts which are specifically designed for low level navigation. On these charts controlled airspace with a base above 5000 ft amsl is not shown.

Map Preparation

21. Careful and thorough preparation of the required maps for a flight can save a lot of valuable time in the air and make the problem of map-reading simpler. Colour and highlighting should be used to reduce the risk of confusion wherever possible. Preparation should include:

(a) Plot, circle and label the turning points.

(b) Draw in the tracks between turning points using fine felt-tipped pen; highlight the tracks; measure the tracks and distances for each leg and write these in a circle near the start of each leg. Be careful not to obscure valuable map information.

(c) Make distance to go marks along each track. (If a constant groundspeed is planned, it will probably be better to make time to go marks.)

(d) Study the route to gain a mental picture of what to expect along the way and highlight features along the track which will be suitable to use as visual check points. The chosen features should be 5-10 minutes flying time apart and, if possible, spaced equally over the leg distance. (Allow plenty of time for route study; initially, you can expect to spend about the same length of time on route study as the flight is planned to take.)

(e) Draw 5° drift lines either side of track extending not only from the start point (which will give an indication of Track Error – TE), but also back from the turning point (which will give an indication of Closing Angle – CA). This will assist with track-keeping and calculating changes of heading. Use a different colour from the track lines to avoid confusion.

(f) Make a note of any positions where the track crosses from one Altimeter Setting Region (ASR) to another.

(g) Mark the forecast wind direction at a suitable place near the track together with the maximum expected drift. (Calculating the maximum drift will be covered later.)

(h) If more than one map is required, arrange them in the order they will be needed. If a large number of maps will be used, number them in the sequence they will be required. Fold the maps so that the required route is visible and easily accessible on each one.

Visual Check Points

22. A visual check point is a significant feature or landmark on or near the aircraft's planned track. It must be large enough to be visible, but small enough to pinpoint the aircraft's position. It should stand out from its surroundings and therefore should be unique. It is not always possible to find such a feature, but combinations of features can provide useful alternatives.

Types of Feature

23. **Coastlines** can usually provide some position information, but an inlet on the coast will be of more use than a long, straight, featureless, sandy beach. It may be possible to use the beach as a line feature for checking track or groundspeed, but it will be difficult to identify exactly one's position on a beach without reference to another feature. An inlet, a promontory, a pier, a small wooded area or some cliffs are the sort of features one would be looking for to pinpoint one's position along a coast. When crossing a coastline, checking the direction in which it lies with reference to the compass, combined with a check feature should dispel any ambiguity in position.

24. **Water features** such as rivers, estuaries, canals and lakes can be invaluable visual checkpoints. But beware: the changing seasons can drastically alter the appearance of such features. A river in flood will be almost unrecognizable from the information presented on a map, whilst drought conditions could cause a lake to disappear completely. In winter, a frozen lake may well be indistinguishable from its surroundings if it is covered in snow.

25. **Mountains and hills** will often provide a valuable source of check points; however, it should be remembered that the higher one flies, the flatter the ground beneath will appear. If the ground is obscured by low cloud or fog, the tops of hills may still be visible and therefore usable as check points. It is also possible for the tops of hills or mountains to be hidden in cloud, not only making identification difficult, but also a potential flight safety hazard to the unwary.

26. When flying at low levels ground contours become more significant because even quite small hills and hillocks become more obvious with the better perspective offered at that height.

27. **Villages**, being quite small, can make good check points, but when flying over an area where many villages are scattered (as in much of the UK) it is important to cross-refer with other features.

28. **Towns**, being that much larger than villages can often be identified simply from their shape, but it is a wise pilot that will cross-check with other features such as the roads and railways associated with a town.

29. On the other hand a **city** can usually be identified from its sheer size, but being a large feature, it also has its drawbacks. It will not be possible to fix one's position accurately from such a large feature and once again, it will only be by comparing smaller nearby features that one's position will be established precisely.

30. **Railways** are prominent features easily visible from the air, but identifying a particular stretch of railway (as with any 'line' feature) may not be possible without reference to other features such as bridges, level crossings etc. Where a railway **is** useful (and other line features for that matter) is when it is parallel or nearly parallel to the desired track, then it will provide useful tracking information. Alternatively, if the line feature crosses the desired track at right angles, then timing and groundspeed information can be derived.

31. **Roads** can also be an invaluable aid to tracking and/or timing as well as being a source of check points; however, a number of points should be borne in mind:

(a) Minor roads are more easily recognisable in sparsely populated areas; in towns there will be too many roads to be able to identify any particular one.

(b) Roads can disappear from view, hidden by terrain or woodlands or urban complexes.

(c) Motorways, like railways are easily distinguishable from the air, but a long straight stretch of motorway presents the same problem as a stretch of railway: it is impossible to pinpoint position without reference to other features. A motorway junction or service area, however, is much more useful.

32. Pinpointing the aircraft's position is achieved by estimating the range and bearing from a check point or by combining bearings from two different check points and referring that position to the map. Adjustments can then be made to heading and speed in order to maintain the desired track and timing.

Selection of Features

33. Selecting the right features along the planned route will make the task of map reading very much simpler. The usefulness of any particular feature depends on how easily it can be identified. It is important to separate significant features from less useful background. Factors to be borne in mind when selecting features are as follows:

(a) The angle of observation.

(b) The size of the feature

(c) The uniqueness of the feature

(d) Contrast and colour

Angle of Observation

34. The altitude at which we fly will affect how features appear. At low levels, masts and aerials can provide very good check points, because of their vertical extent, but the higher one flies the larger the mast needs to be to be clearly visible. Generally speaking, nearer objects will be identifiable from their shape in plan form, whilst features at greater distances will be more easily identified from their elevation. Hence towns, cities and lakes are useful close to track whereas masts, bridges, church steeples and high ground will be easier to identify when some distance away.

Size of Feature

35. Large features are easily distinguished and identified as we have noted with cities, but an accurate position is more likely to be obtained from a small or narrow feature.

Uniqueness of the Feature

36. Avoid choosing common or ambiguous features; choose features with characteristics which make them unique in their vicinity. For example, we have noted that aerials and masts stand out quite well at a distance because of their vertical extent, but beware of aerial 'farms'. Similarly, an isolated clump of trees may provide a useful check point, but a clump of trees in a wooded area will be virtually impossible to distinguish and therefore useless. Be sure to use uniqueness in your criteria for choosing a check point.

Contrast and Colour

37. Variations in contrast, colour, light, the position of the sun, the season of the year and the amount of precipitation etc will all have an effect on the appearance of the features to be found. Recognising and identifying features will be a lot easier if sufficient thought and imagination is applied to these effects.

Estimating Bearing and Range

Bearings

38. There are a number of features on Jeppesen Maps to assist in estimating bearings:

(a) The latitude and longitude grid marked on the map is aligned to true north, so the meridians indicate 000° and 180°. The parallels of latitude cross the meridians at 90° and therefore indicate bearings of 090° and 270°. These lines can assist in estimating bearings. It will also help to orientate the map so that the next turning point is at the top and the track is along the fore and aft axis.

(b) Jeppesen maps mark the locations of VOR beacons with a compass rose (aligned with magnetic north) which can be used to great effect to obtain bearings. Remember,to mentally centre the compass rose on the current position and allow for variation.

39. Using the 'clock code' can help to estimate bearings to or from a visual check point. This method is based on relative bearings and the fact that one hour on a 12 hour clock represents 30°. The aircraft's heading represents 12 o'clock; therefore, an object in the pilot's 2 o'clock position is 60° right of the nose or 60° right of the aircraft's heading. A bearing from the check point can then be calculated and used to help fix the aircraft's position.

40. Another useful technique for estimating bearing is called 'Bisecting the Angle'. This technique relies on the ability to bisect progressively an angle of 90 degrees. In your head, imagine a right angle constructed at the relevant position and then imagine where the 45° line would halve the angle; repeat the exercise to imagine the angle of 22°, then 11° then 5°; with practice this method can achieve quite accurate estimates of bearings and angles.

41. The one-in-sixty rule covered in Chapter 15 is an extremely useful tool for estimating bearings: remember, it is based on the fact that an angle of 1° will be subtended by a distance of 1nm at a range of 60 nm.

Range

42. To assist with estimating range on a map it is useful to have a pencil or something similar marked with 10, 20 and 30 nm ranges for the appropriate scale. The pencil can also be used to measure the required length against the latitude scale.

43. You will be using a 1:500,000 scale map; it will be useful to know what distance on the map your hand span represents; the average span is about 60nm. Measure your hand span against the latitude scale on the chart and remember it. The average thumb from tip to first knuckle equates to 10nm; hence 'Rule of Thumb'.

44. Clues to be obtained from a map include UK MATZ's which are 10nm in diameter; Jeppesen uses a blue circle to denote an airfield which is 5nm in diameter; and, of course, 1 minute of latitude along a meridian equates to 1nm.

45. It is possible, using the one-in-sixty rule, to estimate the range from a feature by taking successive bearings from it. For instance, if the aircraft is doing 120kts groundspeed, then in one minute the aircraft will travel 2nm. Taking two bearings from the feature at an interval of one minute will give a difference in the bearing. If the difference between the bearings is 10°, then we can apply the one-in-sixty rule to find the range from the feature:

$$\frac{\text{Dist Off}}{\text{Dist Gone}} \times 60 = \text{TEA}$$

So, in this case:

$$\frac{\text{Dist Travelled}}{\text{Range to Feature}} \times 60 = \text{Change in Brg}$$

Therefore,

$$\frac{\text{Dist Travelled}}{\text{Change in Brg}} \times 60 = \text{Range to Feature}$$

$$\frac{2}{10} \times 60 = \text{Range to Feature}$$

Therefore, Range to Feature = 12nm

46. Alternatively a technique referred to as ‘Double the Angle on the Bow’ may be used. Observe the relative bearing of an object and note the time. Wait for the relative bearing to double and note the time. Use the groundspeed multiplied by the time interval, this then is the range of the object at the second time.

47. It is also possible to obtain an approximate range from a landmark by using the sighting angle and the flight altitude. For example, let us say that the aircraft is at 3000’ (approx ½ nm) and the sighting or depression angle is 5° ie the landmark is estimated at an angle of 5° below the aircraft’s horizon. Once again, using the one-in-sixty rule:

$$\frac{\text{Dist Off}}{\text{Dist Gone}} \times 60 = \text{TEA}$$

So, in this case:

$$\frac{\text{Aircraft Altitude (nm)}}{\text{Range to Feature}} \times 60 = \text{Sighting Angle}$$

Therefore,

$$\frac{\text{Aircraft Altitude (nm)}}{\text{Sighting Angle}} \times 60 = \text{Range to Feature}$$

$$\frac{1/2}{5^\circ} \times 60 = \text{Range to Feature}$$

Therefore, Range to Feature = 6nm

Fixing the Aircraft's Position

48. Once airborne, it will be necessary to determine the position of the aircraft in order to check progress along the chosen route, make heading alterations and adjust the speed of the aircraft so as to arrive at each turning point on time, and adhere to the flight plan.

49. To fix the position of the aircraft it is necessary to adopt the following procedure:

(a) A few minutes before flying over a pre-planned check point, establish the aircraft's approximate position, ie calculate a DR position.

(b) Determine the position of the check point relative to the DR position and decide where the feature should be relative to the view from the cockpit.

(c) Relate the map information to what can be seen from the cockpit. Look for the chosen feature in the predicted position. If the feature is exactly where you expect it to be, then the aircraft is on track and on time. If, however, the feature is not immediately visible, then the aircraft is either off track, or not on time, or a combination of the two. (Bear in mind that it is possible that the feature may be obscured from view and could become visible later). It is then necessary to widen the scan of the area around the aircraft. Bear in mind that the aircraft's DR position is constantly changing and, therefore, the chosen feature's relative position will be also changing.

(d) When the check feature is sighted, estimate the range and bearing and note the time. Return to the map and using the observations, establish an accurate position of the aircraft.

(e) Having fixed the position of the aircraft, take action to keep or return to track and adjust speed to achieve the required timing.

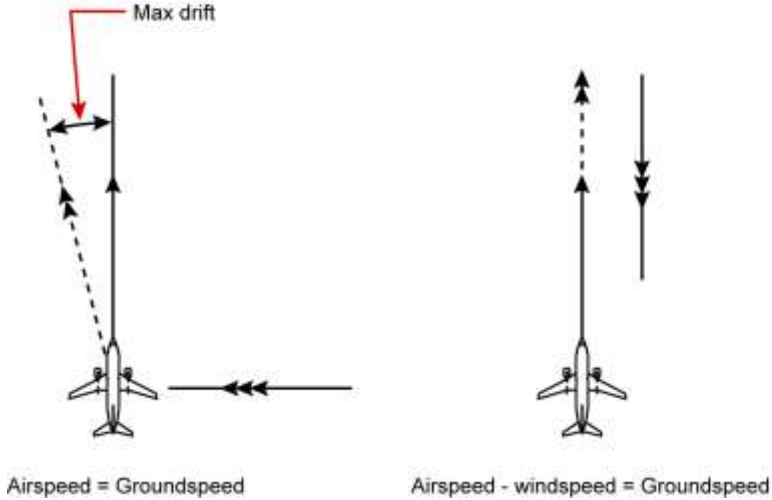
Track and Time Keeping - Mental DR Techniques

50. It is useful to be able to solve mentally the problems normally solved on the CRP-5, even though the answers obtained will be approximate. For visual navigation, only approximate answers are required. The track and time-keeping techniques used in visual navigation are usually referred to as mental dead reckoning. It should be noted that all of these techniques require mental agility and a reasonable level of mental arithmetic.

The Effect of Wind

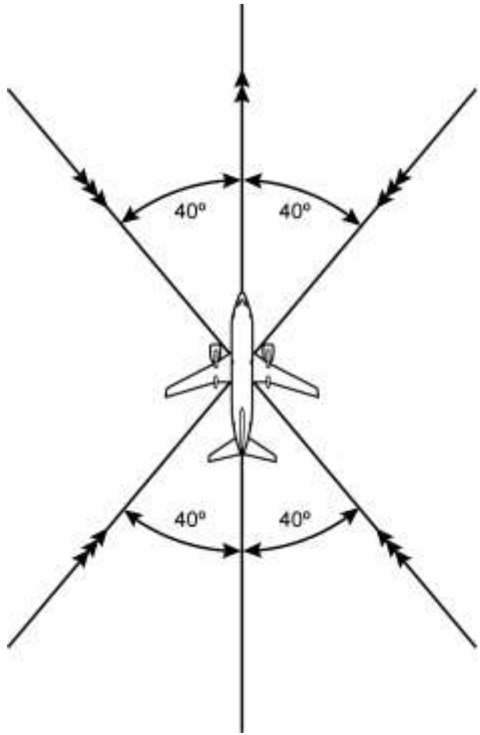
51. Wind affects the aircraft's flight path in two ways, namely drift and groundspeed. The angle between aircraft track and the wind direction is called the wind angle; the actual values of drift and changes to speed will vary depending on the wind angle and wind speed compared to the aircraft's track and TAS.

FIGURE 16-5
The Effect of Wind



52. If the wind angle is 90°, then drift will be at its maximum value, whereas the effect on speed will be nil. If the wind angle is zero, then drift will be zero, but the effect on speed will be at a maximum.

FIGURE 16-6
Wind Angle



53. Remember that there can be 4 wind angles of any given value; the wind could be coming from the left or the right and can be a headwind or a tailwind. Care must be taken, therefore, to apply the wind angle in the correct sense in order to obtain the correct drift (port or starboard) and groundspeed. The diagram at Figure 16-6 shows the 4 possible effects of a wind angle of 40°.

54. **Maximum Drift.** To estimate the drift on a given heading, it is first necessary to calculate the maximum possible value of drift for the forecast wind velocity for the route. It will make the calculations in the air simpler if this is done before flight and a note of the maximum drift made in a suitable place on the map. The method is as follows:

$$\text{Maximum Drift} = \frac{\text{Windspeed}}{\text{TAS in miles per minute}}$$

55. As the wind velocity used is forecast and may not be accurate the use of wind speed will only give an approximate (though reasonable) value for maximum drift. For example, if the forecast wind velocity is 270°/30 knots and the TAS is 180 kts, then:

$$\text{Maximum Drift} = \frac{30}{3} = 10°$$

56. The actual drift expected can be calculated by considering the wind angle and using a clock analogy. The wind angle is considered to be minutes of time and the proportion of an hour that this number of minutes represents is the proportion of maximum drift that would be expected for a given heading.

57. For example, if the planned track is 300°, the wind angle will be 30° which equates to 30 mins or ½ an hour. We would expect, therefore, ½ of the maximum drift to be experienced.

58. In our example, we expected 10° to be the maximum drift and with a wind angle of 30° we would expect ½ of maximum drift. Therefore the drift expected would be 5°.

59. For wind angles of 60° or more it is assumed that the aircraft will experience maximum drift.

60. As mentioned before, care must be taken to apply the drift in the correct sense. Since the wind is blowing from left to right, the drift will be starboard; therefore the heading required to make good a planned track of 300° is 295°.

Groundspeed

61. Groundspeed is calculated in a similar manner to drift. The maximum effect will occur when the wind is directly on the nose (groundspeed = airspeed – wind speed) or the tail (groundspeed = airspeed + wind speed), but will be zero when the wind is at 90° to heading. Once again we can use the clock analogy to determine the amount of head or tailwind to apply, but now we use 90° minus the wind angle.

Following through with our example:

	Wind Velocity	=	270°/30kts
	Planned Track	=	300°
Therefore,	Wind Angle	=	30°
	Applying 90°	-	Wind Angle =
	90° - 30°	=	60°

When applied to the clocking analogy, 60° equates to 60 minutes ie 1 hour

Therefore, we apply **all** the windspeed

In this case we have a headwind component

TAS	=	180kts

Therefore we can expect a groundspeed of 180 - 30 = 150kts

62. When a fix position is established it can be compared with the planned track, the 5° drift lines and timing marks drawn on the map. It will then be necessary to decide what tracking and/or timing action is required to make good the next turning point.

63. When discussing visual check points, it was noted that line features can be used to check tracking when aligned with the track and to check timing when aligned across track. Similarly, a single position line from a feature can be used for the same purposes. Tracking information can be obtained from a feature on or close to track, whilst timing information can be obtained when passing abeam a feature.

Time Adjustments

64. In Chapter 15 it was explained how we can use the one-in-sixty rule as a means of regaining track, but it is often also necessary to maintain the planned timing. There are a number of ways of adjusting a flight in order to arrive at a destination (or some point en route) at a given time, the most commonly used methods are:

(a) Planning the route so that there is a convenient corner before the destination or turning point which can be cut or extended to gain or lose time.

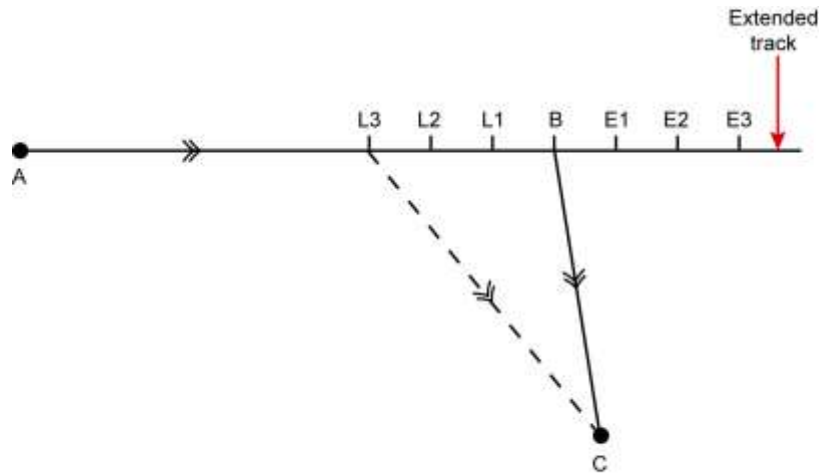
(b) Flying a 'dog leg' or similar alteration of heading to lose time.

(c) Adjusting the airspeed.

Cutting the Corner

65. When there is a large track alteration along the route, say 60° or more, it is possible to adjust timing by extending or cutting the corner at that turning point.

FIGURE 16-7
Extending or Cutting the Corner



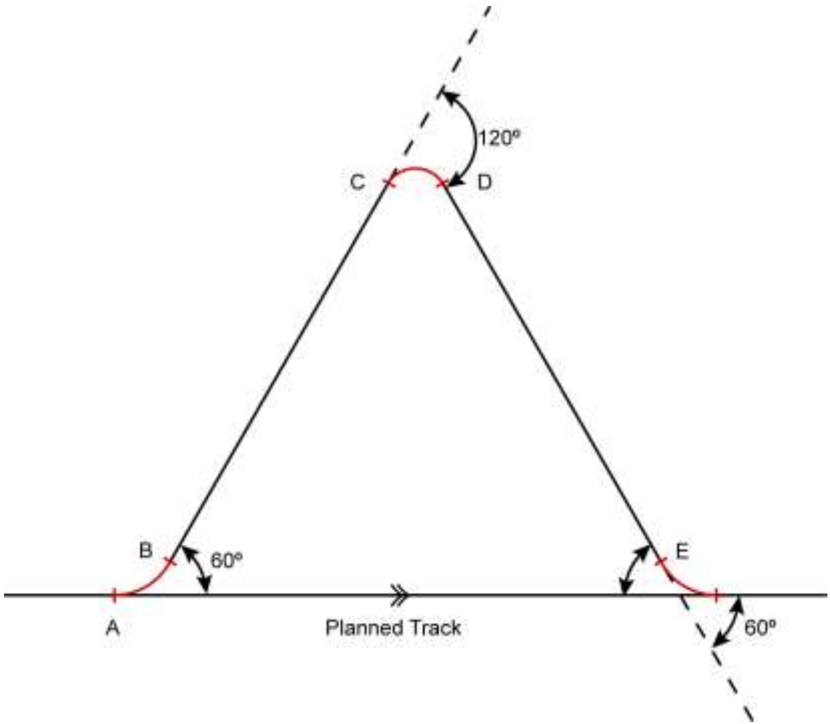
66. Given a route A – B – C as in Figure 16-7 timing is altered by turning at a point other than at point B either early or late. The technique is to extend the track A – B beyond B on the map and then to mark off distances representing 1, 2 and 3 minutes of groundspeed either side of the turning point. If running late, time may be saved by turning at point L_1, L_2 or L_3; if running early, time can be added on by extending beyond point B and turning at point E_1, E_2 or E_3. It should be borne in mind that a heading adjustment will be required, because a different track from that originally planned will be flown from B – C, for instance track L3 – C.

Losing Time by 60° Dog Leg

67. Flying a dog leg can serve to lose a lot of time in a short distance along track; see Figure 16-7. The technique is to alter heading by 60° in either direction for the length of time to be lost, then to turn through 120° in the opposite direction for the same length of time in order to regain track. The aircraft will then have flown two sides of an equilateral triangle, and the time lost will be equal to the time taken to fly one side.

68. The wind effect during the procedure must also be accounted for because this will affect the final ground position at the end of the procedure. This is caused because the heading is altered by 60° and 120° when it should have been the track. Of course in still-air the procedure will be correct.

FIGURE 16-8
60° Dog Leg



69. Heading to the next turning point is then resumed; however, it may be necessary to adjust the heading and/or speed in order to make good the turning point on time.

Adjusting the Airspeed

70. Providing that the speed range of the aircraft permits and any fuel penalty is acceptable, small timing errors can be corrected by altering speed. There are a number of ways of achieving this, but two possible methods are as follows:

(a) Adjust the speed by a number of knots equal to the number of seconds late or early and maintain the new speed for the number of minutes equal to the groundspeed in nm/minute eg 180kts groundspeed, 30 seconds early; reduce speed to 150kts for 3 minutes.

(b) Adjust the speed by an amount equal to 5 times the groundspeed in nm/minute and maintain that speed for a number of minutes equal to one fifth of the number of seconds early or late eg 180kts groundspeed, 30 seconds early; reduce speed to 165kts for 6 minutes.

71. In either case the speed adjustment can be halved and maintained for twice the time (or doubled and maintained for half the time) if necessary.

The DR Turn

72. Although it is not generally desirable, there will be occasions when it is necessary to plan a turning point where there is no obvious visual check point available. In this case it is essential to plan a good fix on track as close to the turning point as possible and to work out in advance the time from the fix to the turning point. It will also be necessary to highlight a number of alternative fix-points after the turn.

73. The procedure in the air would be to fly over the fix point prior to the turn, observe the stopwatch and then fly as accurately as possible the heading and speed required for the turn. At the time worked out in advance, turn onto the next heading and immediately start to look for one of the fix-points planned to be used after the turn. In this way it should be possible to establish very quickly whether the aircraft is on the correct track or not.

Limitations

74. It should be noted that visual navigation is more difficult over some geographical areas than others due to the nature of the terrain, the lack of distinctive features or possibly the lack of detailed and accurate chart data.

75. Map reading in higher latitudes can be considerably more difficult than in lower latitudes because the nature of the terrain is so different, maps are less detailed and less precise, and seasonal changes may drastically alter the appearance of terrain or possibly hide it from view completely.

76. In areas where snow and ice cover the terrain from horizon to horizon and where the sky is covered with a uniform layer of clouds so that no shadows are cast, the horizon disappears, causing earth and sky to blend and making disorientation a real danger.

77. In a ‘white-out’ there is a complete lack of contrast; distance and height above ground are virtually impossible to estimate.

Plotting

Climbing and Descending

Flight on Airways

Fixing Position

Plotting Radio Bearings

VOR Bearings

VDF Bearings

NDB Bearings

DME Range Position Lines

Airborne Weather Radar Fixes

The Non-Simultaneous Fix

Doppler Information

Determining Wind Velocities

Doppler Wind Velocities

Dead Reckoning Following a Fix

Pilot Navigation

Selection of Fixing Aids

Revision of ETA and Fuel Endurance

Plotting

1. Teaching of the various plotting procedures and skills is primarily achieved in this chapter using worked examples. The chart illustrated at various figures in the earlier part of the text is in fact the 'Instructional Plotting Chart - UK' (for training purposes only, NOT for operational use). It is a Lambert conical orthomorphic projection with a published scale of 1: 1,000,000. The latitudes of the standard parallels and of the parallel of origin are not given on the chart.

2. In common with most if not all charts used primarily for airways navigation, this chart gives airway centreline tracks in degrees **magnetic**. Turn to Figure 17-1 and check the track of airway Delta White 11, 080°(M) from the Deans Cross VOR (DCS) to the Newcastle VOR (NEW), and of course 260°(M) in the opposite direction. Additionally, the total distance DCS to NEW is given as 59 nm (22 nm + 37 nm), and this can be checked against the meridian scale using a pair of dividers. On this particular chart the meridian scale is the only means of measuring distance, however on many Lambert airways charts **bar scales** are provided at the top and/or bottom margins.

3. Figure 17-1 shows the symbology used on this particular chart. These symbols are typical of those found on operational charts (Aerad and Jeppesen for example), however there are differences, and the relevant chart legends should always be consulted.

4. To standardise plotting and make it easy to 'read' it is recommended that the standard plotting symbols shown at Figure 17-2 be adopted.

5. So that the plotting shown on the chart should have some meaning it is necessary to keep a record or 'log' of the associated calculations. To keep the log as brief as possible the abbreviations shown at Figure 17-3 should be used. These abbreviations may be amended provided that the meaning of the abbreviated form is obvious.

6. A formal log is not compulsory, although it is of great benefit and assists the logical processes required to maintain the plot. Furthermore, should a mistake be made, it is relatively easy to determine the origin from the log.

FIGURE 17-1
UK Instructional Plotting Chart

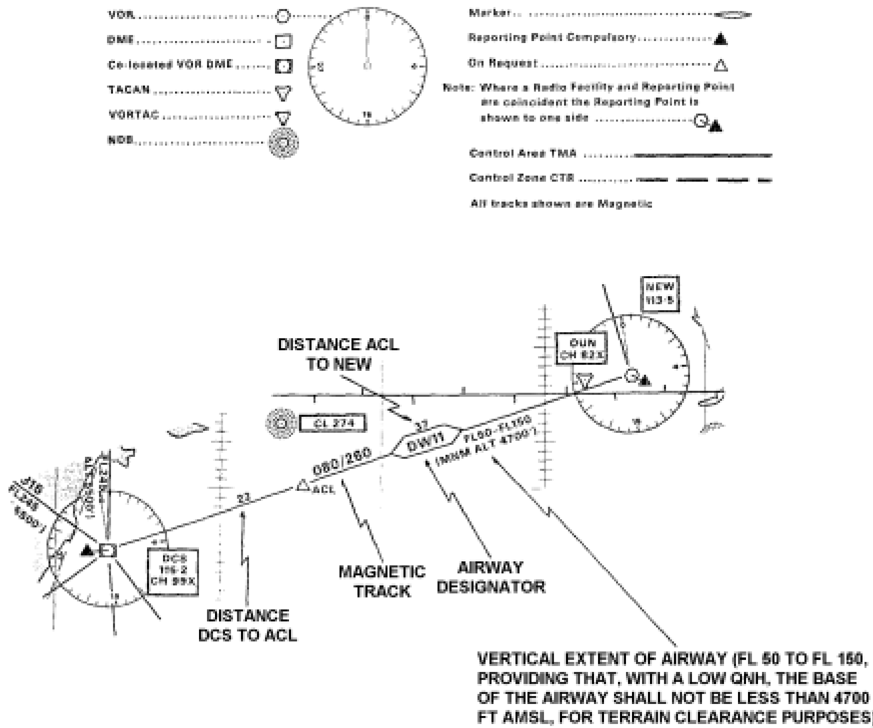


FIGURE 17-2
Standard Plotting Symbols

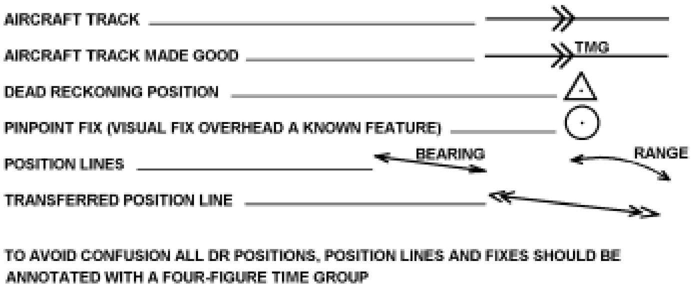


FIGURE 17-3
Standard Abbreviations

Alter heading	A/H
Convergency	Ccy
Conversion Angle	CA
Dead reckoning position	DR
Deviation	Dev
Estimated time of arrival	ETA
Groundspeed	G/S
Overhead (a given position)	O/H
Set heading	S/H
Top of climb	TOC
Top of descent	TOD
Track	Tk
Track made good	TMG
True airspeed	TAS
Variation	Var

7. The track plot method of navigation is that which is required and is covered in detail in the following pages. It is of benefit to practice the techniques involved and the following examples should be completed.

8. A fix is an accurate ground position of the aircraft determined by visual, radio or radar navigation aids. The accuracy of the fix is dependent on the accuracy of the aids used.

9. Dead reckoning (DR) is the practice of estimating the ground position of an aircraft using the most recent information available. The purpose of establishing a DR position is:

(a) If a fix has shown the aircraft to be significantly off the planned track, then it enables reasonably accurate calculations to be made to regain track or recover to the next turning point.

(b) If there are no navigation fixing aids available then a DR position is required for safety reasons, eg. high ground avoidance, maintaining the flight progress chart or calculating the remaining endurance.

10. The information required to construct a DR position is obtained from instruments and equipment contained within the aeroplane and requiring no external source. The DR position is subject to error, the size of which is dependent on the accuracy of the following:

(1) The last fix and time interval since that fix;

(2) The maintenance of heading and speed;

(3) The wind velocity or drift and groundspeed;

(4) The calculation and plotting of the values determined.

11. Because of these inaccuracies, the DR position is only an approximation. There is, therefore, a degree of uncertainty regarding this position. This is referred to as the circle of error. The size of this circle is dependent on the time interval since the last positive fix. The longer the time interval, the greater is the radius of the circle. The radius of the circle of error is approximately equal to 5% of the TAS.

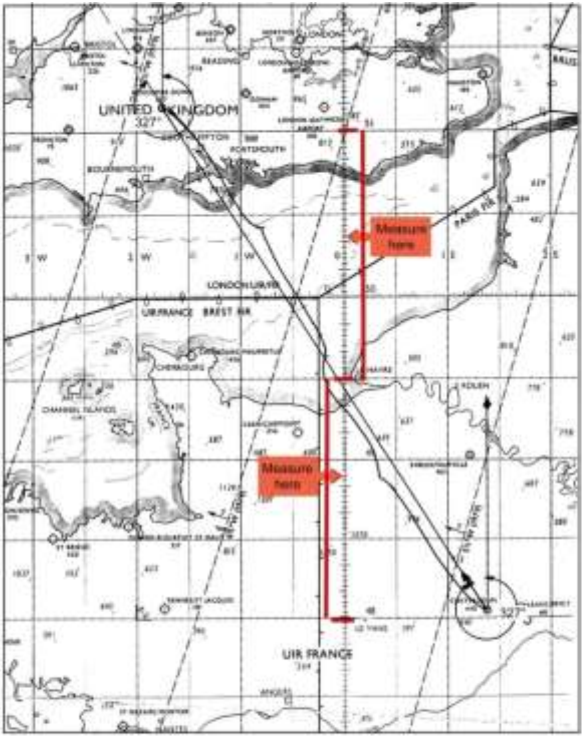
12. To plot a DR position, it is essential to accurately determine the track and distance travelled along that track since the last fix. If the doppler computer is used to determine the DR position, then if it was updated to last positive fix, the error around the DR position is an ellipse. The dimensions of the ellipse are, along track 0.1% of the distance and across track 0.2% since the last fix.

Plotting on a Mercator Chart

13. **Track.** Because a straight-line on a Mercator is a rhumb line, it is only possible to plot rhumb line tracks. If it is necessary to plot the shortest distance between two points a considerable distance apart, then the great circle would have to be approximated by a series of rhumb line tracks; the length of these tracks being dependent on the latitude and the distance between the points. The path of the great circle would have to be transferred as a series of points from a gnomonic projection if the route distance exceeds 1000 nm. Between 12°N and 12°S, the difference between the rhumb line and great circle is small enough to be ignored.

14. **Distance.** The scale expansion of this chart makes the measurement of distance a more difficult task than on other charts. As one minute of latitude equals one nautical mile, the latitude graduations on the meridians must be used for this purpose. The track should be divided into segments of approximately 100 nm to 200 nm. Lengths are measured against the nearest meridian to the segment at the same latitude. An example is shown at Figure 17-4.

FIGURE 17-4

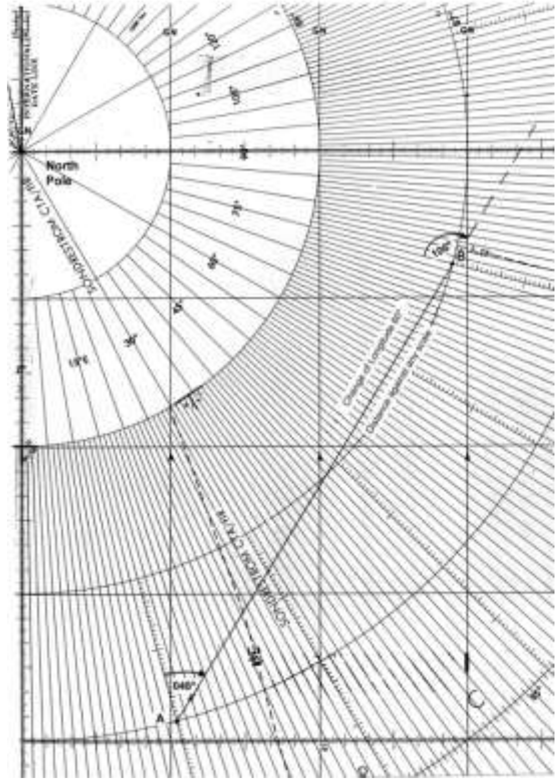

UNITED KINGDOM
327°
PORTSMOUTH
BOURNEMOUTH
READING
LONDON
Measure here
LONDON UIR/FIR
UIR FRANCE BREST FIR
CHERBOURG
CHANNEL ISLANDS
Measure here
UIR FRANCE
PARIS FIR
ROUEN
327°

Plotting on a Polar Stereographic Chart

15. **Track**. Because this chart is not used for navigational purposes at latitudes less than 70°, then great circles are almost straight lines. Those that pass through the pole are exact straight lines. For navigational purposes then, a straight line is considered to be a great circle. Due to the convergence of the meridians, the true direction changes rapidly along track. To accurately plot a track, its direction should be plotted as a mean direction at the mid point of the track. Of course this is not always possible when projecting a DR position ahead of the present position but because this distance is relatively short it is sufficiently accurate to plot the initial track direction at the start time and position.

16. **Distance.** The scale of this chart expands with latitude away from the pole. It is less than 1% in error at 75° latitude. It is therefore considered to be a constant scale chart. Distance may be measured from the latitude graduations on any meridian or from the scale bar in the legend of the chart. See Figure 17-5.

FIGURE 17-5



Plotting on a Lambert Conformal Chart

17. **Track**. The convergence of the meridians on this chart is equal to a fraction of the change of longitude, known as the constant of the cone or convergence factor. This means that the true direction of a track changes progressively along the track. Because the DR ahead is usually only for a short distance, it is accurate enough to use the initial track at the beginning of the track for this purpose.

18. **Distance.** If the separation of the standard parallels (SP's) on any plotting chart is less than 16°, the scale error, either contraction between the SP's or expansion outside of the SP's, is less than 1%. The scale is, therefore, considered to be constant. The latitude graduations on the meridians or the scale bar in the legend may be used for measuring distances.

19. The Lambert Conformal Chart is that which will be used in the Navigation examination. Hence the following examples are given to demonstrate the technique to be adopted.

EXAMPLE 17-1

EXAMPLE

An aircraft is overhead SAB VOR (5555N 02122W) at 1015, heading 150°(T), TAS 180 kt. The W/V is given as 250°/30 kt. At 1032 heading is altered onto 217°(T). At 1038 heading is altered onto 246°(T). Determine the aircraft's DR position at 1045.

SOLUTION

See Figure 17-5 and Figure 17-7. Headings are given in degrees true in this example, and the forecast wind is given in degrees true unless otherwise specified. Always plot track in degrees true. From 1015 to 1032 (17 min) the track is computed as 141°(T) and the groundspeed as 188 kt. The aircraft will therefore cover 53 nm over the ground.

Annotate SAB with the time 1015. Plot 141(T) distance 53 nm. Draw DR symbol annotate with time 1032.

From 1032 to 1038 (6 min) the track is computed as 211°(T) and the groundspeed as 156 kt. The aircraft will therefore cover 15½ nm over the ground.

Plot 211(T) distance 15½ nm. Draw DR symbol annotate with time 1038.

From 1038 to 1045 (7 min) the track is computed as 245°(T) and the groundspeed as 150 kt. The aircraft will therefore cover 17½ nm over the ground.

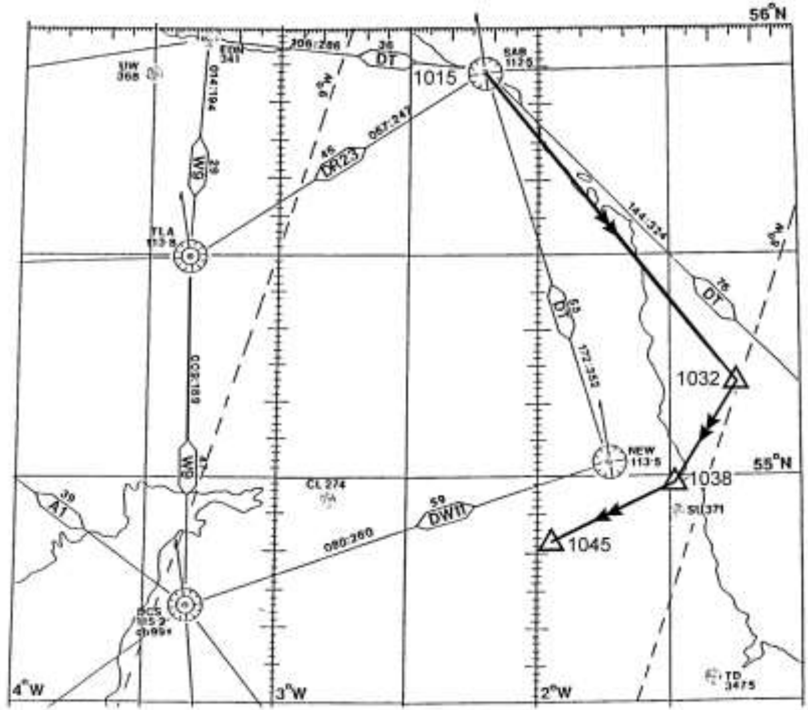
Plot 245(T) distance 17½ nm. Draw DR symbol annotate with time 1045. Measure latitude and longitude.

At 1045 the aircraft DR position is **5452N 0157W.**

FIGURE 17-6

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1015	141	250/30	150					O/H SAB				180	188	53	17	1032
1032	211		217					A/H					156	15½	6	1038
1038	245		246					A/H					150	17½	7	1045
1045								DR 5452N 0157W								

FIGURE 17-7



56°N
55°N
4°W
3°W
2°W
1015
1032
1038
1045
SAB
DT
WG
DR23
DWII
A1
TLA
NEW
CL 274
SU371
TD
144:324
172:352
080:260
057:247
008:188
014:194

EXAMPLE 17-2

EXAMPLE

An aircraft is at DR position 5530N 0330W at time 1417, heading 178°(M), TAS 135 kt, forecast W/V 300°/15 kt. At 1428 heading is altered onto 090°(M).

Determine the aircraft's DR position at 1435.

Determine the magnetic heading required to fly from the 1435 DR position to the NEW VOR (5502N 0144W), and the ETA at the VOR.

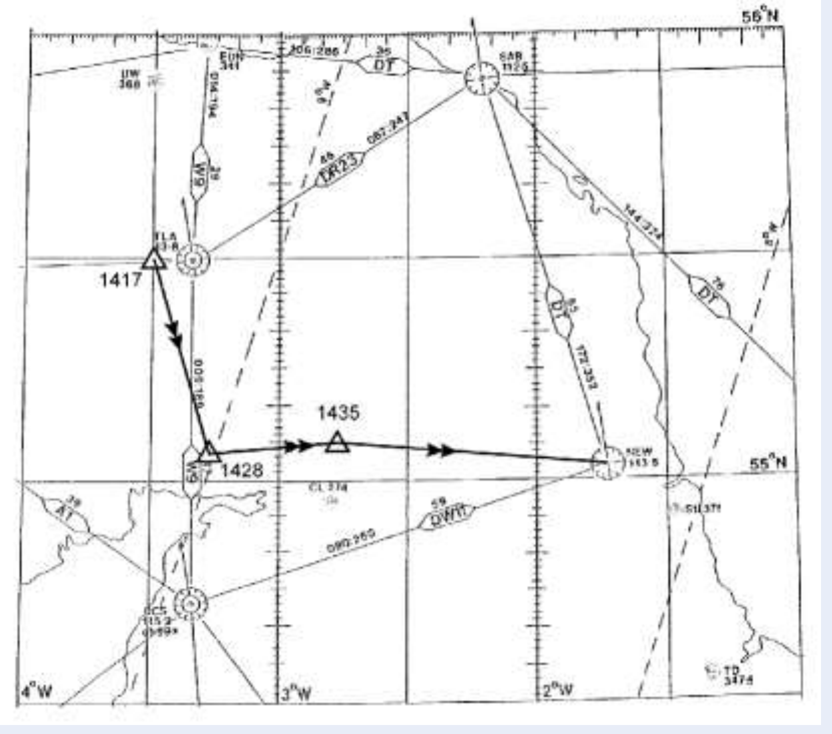
SOLUTION

See Figure 17-7 and Figure 17-8. This time the headings are given in degrees magnetic and the W/V in degrees true. Convert the headings to degrees true (using the dashed line isogonals) and use the true north reference.

FIGURE 17-8

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1417	164	300/15	169	9W	178			DR 5530N 0330W				135	146	27	11	1428
1428	085		081	9W	090			A/H					147	17	7	1435
1435	095		092	9W	101	(ii)		DR 5505N 0247W (i) A/H NEW					150	36	14½	1449½
																(ii)

FIGURE 17-9


56°N
55°N
4°W
3°W
2°W
1417
1428
1435
SAB
NEW
DT
TR23
087:247
144:324
172:352
090:260

From 1417 to 1428 (11 min) heading is 169°(T), track 164°(T), groundspeed 146 kt, and distance covered over the ground 27 nm.

From 1428 to 1435 (7 min) heading is 081°(T), track 085°(T), groundspeed 147 kt, and distance covered over the ground 17 nm.The DR position at 1435 is **5505N 0247W.**

The track from the DR position to the NEW VOR is measured as 095°(T), and the distance as 36 nm. The heading is computed as 092°(T) + variation 9°W giving 101°(M). The groundspeed is computed as 150 kt, the time is therefore 14½ min, and the ETA NEW VOR 1449½.

The heading required at 1435 is **101°(M)**, and the ETA NEW is **1449½.**

Self Assessed Exercise No. 9

QUESTIONS:

QUESTION 1.

The principle of visual navigation is to compare an estimated position with visual references. What skills are required for a pilot to be able to achieve this?

QUESTION 2.

Explain the difference between a map and a chart.

QUESTION 3.

Explain how scale effects the amount of detail on a map.

QUESTION 4.

Explain the colour convention used on topographical maps.

QUESTION 5.

Identify the CAA chart symbols at FIGURE 99 in the Reference Book.

QUESTION 6.

Identify the Jeppesen chart symbols at FIGURE 100 in the Reference Book.

QUESTION 7.

List the 3 main ways of depicting elevation on a Jeppesen map.

QUESTION 8.

Explain what is meant by Minimum Grid Area Altitude on a Jeppesen chart.

QUESTION 9.

List the factors to be borne in mind when selecting features for visual check points.

QUESTION 10.

State the One-in-Sixty Rule formula for Track Error Angle (TEA).

QUESTION 11.

An aircraft is en route from A to B. A and B are 200 nm apart. After travelling 80 nm, the pilot pinpoints the position as being 8 nm right of track.

Determine the change in heading required to parallel the original track.

QUESTION 12.

An aircraft is en route from A to B. A and B are 200 nm apart. After travelling 80 nm, the pilot pinpoints the position as being 8 nm right of track.

Determine the change in heading required to make good the destination, B.

QUESTION 13.

Given that an aircraft must follow a track of 090° and the forecast W/V is 210°/40, calculate the maximum drift, the drift expected and the expected G/S if the TAS is 150kts.

QUESTION 14.

An aircraft is 20 seconds late flying from A to B. If the speed of the aircraft is 150kts, determine the new speed required and for how long it should be flown.

ANSWERS:

ANSWER 1.

The pilot should have the ability to;

(a) Select features from a map

(b) Identify landmarks from the air

(c) Estimate distance on the map and in the air

(d) Estimate direction on the map and in the air

The pilot should be able to build a mental picture of the features along the planned route in order to make it easier to identify those features once in the air.

ANSWER 2.

A chart is used for plotting and shows very little in the way of ground features.

A map on the other hand has as much topographical information as the scale will allow and is designed for map-reading.

ANSWER 3.

The larger the scale the greater the detail available on the map.

ANSWER 4.

Feature	Colour
Water	Blue - sea, lakes or rivers
Woodland Areas	Green
Roads	Red lines
Railways	Black lines

ANSWER 5.

Customs Aerodrome

Military Aerodrome Traffic Zone

Co-located VOR/DME beacons

ANSWER 6.

(a) Glider site

(b) Military Airport with a grass runway

(c) The highest spot elevation or man-made obstruction depicted on the chart

See figures 16-2 and 16-3

ANSWER 7.

(a) Contours

(b) Spot heights

(c) Layer tinting

ANSWER 8.

See 061-16-17 and also the Jeppesen Chart Legend at Figure 16-3.

ANSWER 9.

(a) The angle of observation

(b) The size of feature

(c) The uniqueness of the feature

(d) Contrast and colour

ANSWER 10.

$$\text{TEA} = \frac{\text{Distance Off}}{\text{Distance Gone}} \times 60$$

ANSWER 11.

See FIGURE 101 in the Reference Book

To parallel the original track, the pilot must turn left by the TEA:

$$TEA = \frac{\text{Distance Off}}{\text{Distance Gone}} \times 60$$

$$= \frac{8}{80} \times 60$$

$$= 6°$$

$\therefore$ must turn left 6°

ANSWER 12.

See FIGURE 102 in the Reference Book

To make good the destination, the pilot must turn left through an angle TEA + Closing Angle (CA).

$$TEA = \frac{\text{Distance Off}}{\text{Distance Gone}} \times 60 = \frac{8}{80} \times 60 = 6°$$

$$CA = \frac{\text{Distance Off}}{\text{Distance To Go}} \times 60 = \frac{8}{120} \times 60 = 4°$$

$\therefore$ must turn left 6° + 4° = 10°

ANSWER13.

a. Max Drift $= \dfrac{\text{Windspeed}}{\text{TAS (nm/min)}} = \dfrac{40}{2\frac{1}{2}} = 16°$

b. Expected Drift. Wind Angle = 60° from the right, therefore, expect Max Drift, 16° to the left.

c. Wind Angle = 60° (behind)

90 – 60 = 30° which equates to half an hour therefore apply half the wind speed = 20kts. Since the wind is from behind, this will be a tailwind, therefore expected G/S = 170kts.

ANSWER 14.

Two methods:

Adjust speed by the number of seconds late and maintain for the number of minutes equal to the G/S in nm/minute. i.e. increase speed by 20kts for 2.5 minutes (170kts).

Adjust speed by 5 times the G/S in nm/minute and maintain for the number of minutes equal to one fifth of the number of seconds late. i.e. increase speed by 12.5kts for 4 minutes (162.5kts).

Climbing and Descending

20. Climbs and descents are usually calculated using DR principles, since it is not normal to fix the aircraft's position during changes of altitude.

21. Assuming that an aircraft is required to climb at a constant CAS from one level to another, then it is necessary to calculate the **mean** TAS at a point half way up the climb. Next determine a **mean** wind, either by interpolation or inspection, and again the W/V half way up the climb is used.

EXAMPLE 17-3

EXAMPLE

An aircraft is required to climb from 2,000 ft to 25,000 ft on a track of 350°(T), at a constant CAS of 160 kt. The forecast meteorological information is as follows

Altitude	W/V	Temperature
2,000 ft	270°/30 kt	+5°
25,000 ft	330°/70 kt	-37°

If the rate of climb is 1,500 ft per minute throughout the climb, determine the true heading required and the distance covered during the climb.

SOLUTION

See Figure 17-10. Half way up the climb ((2,000 + 25,000) ÷ 2) is 13,500 ft.

Interpolate the temperature = (+5 - 37) ÷ 2 = -16°C

At 13,500 ft the temperature is -16°C.

An RAS of 160 kt with an OAT of -16°C at a mean altitude of 13,500 ft gives a TAS of 195 kt.

Half way up the climb the wind velocity (by interpolation) is 300°/50 kt. The track is given as 350°(T) and the TAS is 195 kt, therefore the mean groundspeed is computed as 158 kt and the heading required as 338°(T).

Rate of climb of 1,500 ft/min the climb through 23,000 ft takes 23,000 ÷ 1500 = 15½ minutes. At a mean groundspeed of 158 kt the distance covered in the climb is therefore 40½ nm.
The heading for the climb is **338°(T),** and the distance covered in the climb is **40½ nm.**

FIGURE 17-10

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	350	300/50	338					2000 25,000@ 1500'/min				180	188	53	17	1032

EXAMPLE 17-4

EXAMPLE

An aircraft is on the direct track from NEW VOR (5502N 0144W) to SAB VOR (5555N 0212½W) at FL 250. The aircraft is instructed to commence a descent at 5530N to cross the SAB VOR level at FL 90.

The forecast meteorological information is as follows:

Pressure altitude	W/V	Temperature
FL 200	050/60 kt	-20°
FL 100	350°/20kt	-6°

The aircraft will descend at a constant RAS of 140 kt.

Determine the magnetic heading required during the descent.
The minimum rate of descent required.

SOLUTION

See Figure 17-11 and Figure 17-12.

The mean TAS for the descent is calculated at the mid level ([250 + 90] ÷ 2) of FL 170 using a temperature (interpolated for FL 170 from the met information given) of -16°C, and an RAS of 140 kt. The mean TAS is therefore 183 kt. The mean W/V for the descent (interpolated for FL 170 from the met information given) is 032°/48 kt.
The direct track is given on the chart as 352° magnetic which is 344(T). The heading required is 355(T)
The mean groundspeed during the descent is computed as 148 kt, and the heading as 003°(M).

The heading for the descent is **003°(M)**.

SOLUTION

The distance from 5530N to the SAB VOR is measured as 25 nm, and, at a groundspeed of 148 kt, the time taken in the descent is 10 minutes.

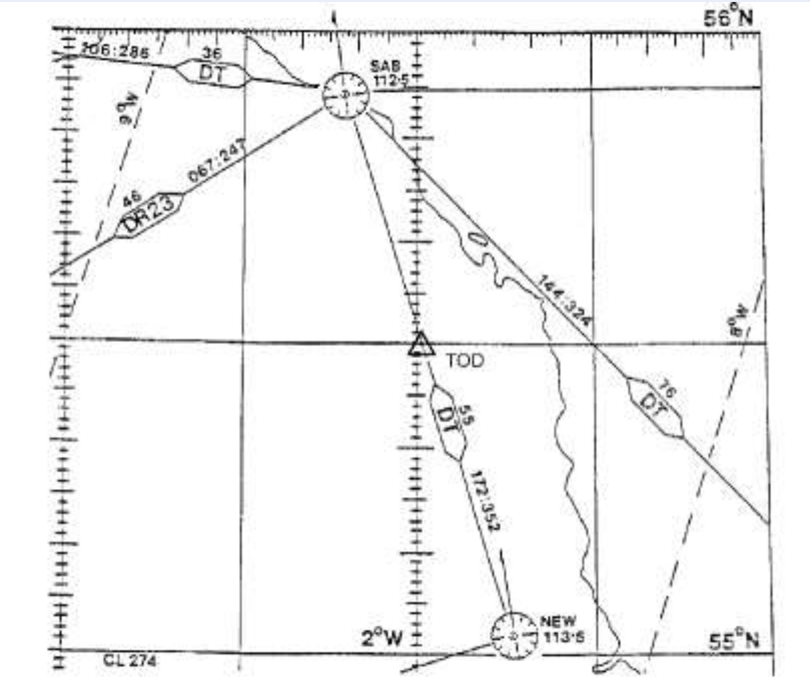
The aircraft must descend (FL 250 to FL 90) through 16,000 ft in 10 minutes, therefore the minimum rate of descent required is 1,600 ft/min.

The minimum rate of descent is **1,600 ft/min**.

FIGURE 17-11

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	(M)	(M)														
	352	240/48			003	(i)		530N DR TOD FL250 FL90	140	17000	-16	183	148	25	10	
								ROD = 1600’/min (ii)								

FIGURE 17-12



56°N
106:286
36
DT
SAB
112·5
9°W
067:247
46
DR23
144:324
TOD
76
DT
55
DT
172:352
8°W
NEW
113·6
2°W
55°N
CL 274

Flight on Airways

22. The centreline tracks of the airways are given in degrees magnetic. Route forecast wind directions are given in degrees true. It is therefore necessary to convert everything to a common north reference, and for flights along airways it is more convenient to convert the wind direction to degrees magnetic.

EXAMPLE 17-5

EXAMPLE

An aircraft is overhead EDN NDB (5559N 0317W) at 1029, FL 120, routing W9 to DCS VOR (54432N 03202W). The W/V is 250°/40 kt and the TAS 210 kt.
Determine: The magnetic heading for each leg. The ETA DCS.

SOLUTION

See Figure 17-13 and Figure 17-14.

The W/V is 259°(M)/40 kt.

From EDN to TLA the track is 194°(M), the heading is therefore computed as 204°(M) and the groundspeed 190 kt.

The distance EDN to TLA is 29 nm and therefore the leg time is 9 min, giving an ETA at TLA of 1038.

From TLA to DCS the track is 189°(M), the heading is therefore computed as 199°(M), and the groundspeed 193 kt.

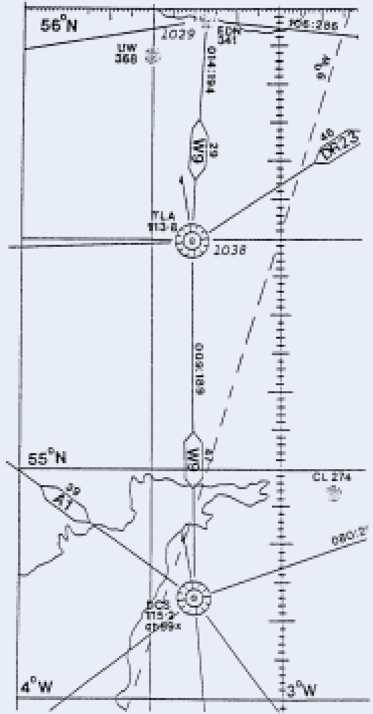
The distance TLA to DCS is 47 nm, and therefore the leg time is 14½ min giving an ETA at DCS of 1052½ (for examination purposes you should calculate ETAs to the nearest half minute, but use only whole minutes on the R/T).

The headings required are **204°(M)** to TLA, and **199°(M)** to DCS. The ETA DCS is **1052.5.**

FIGURE 17-13

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	(M)	(M)														
1029	194	259/40			204	(i)		O/H EDN S/H TLA		F120		210	190	29	9	
1038	189				199	(i)		O/H TLA S/H DCS					193	47	14½	1052½
																(ii)

FIGURE 17-14



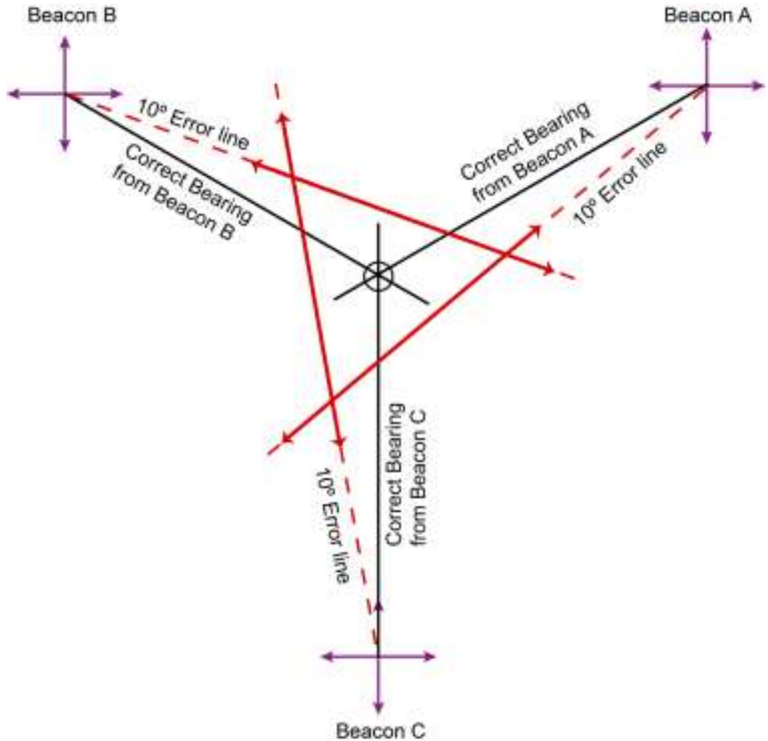
56°N
UW
368
EDN
341
014:194
4.6
W9
29
DR23
TLA
113·8
55°N
W9
A1
CL 274
4°W
3°W

Fixing Position

23. A fix is defined as the exact position of an aircraft at a specified time. It may be derived from visual, radio or radar observations or any combination of these navigational aids. The observations from different sources may be made simultaneously or at different times and transferred to a common time to construct a fix.

24. The most accurate fix is that derived from the aids least likely to be in error and intersecting at a large angle i.e. 60° or 90°. Therefore the most accurate fix is obtained from a twin DME fix because the DME has the least error and the position lines can be chosen to cut at between 60° and 90°. If this is not possible, or the Airborne Search Radar cannot be used to obtain a similar fix, the aids should be selected to produce three position lines intersecting at 120° because this will tend to eliminate any errors inherent in the equipment. See Figure 17-15.

FIGURE 17-15
The Cocked Hat



25. The plotted position lines illustrated do not intersect at one position but form a ‘cocked hat’. However the centre of the cocked hat is at the same position as the correct fix should have been.

Plotting Radio Bearings

26. Refer to the section entitled Radio Aids on the methods of bearing presentation (RBI, RMI, OBI) discussed below.

27. As a convenient abbreviation the Q code may be used when converting from **indicated bearings to bearings to plot**, as shown at Figure 17-16.

FIGURE 17-16

QDM	Magnetic great circle bearing of the station from the aircraft.
QDR	Magnetic great circle bearing of the aircraft from the station (the radial).
QTE	True great circle bearing of the aircraft from the station.
QUJ	True great circle bearing of the station from the aircraft.

VOR Bearings

28. VOR bearings may be presented to the pilot on either an RMI (radio magnetic indicator) or an OBI (omni-bearing indicator).

29. An RMI bearing, for example SAB VOR bears 067° RMI, will always be a QDM, since the sharp end of the RMI needle points **towards** the station. The example given in the preceding sentence would therefore put the aircraft on the centreline of airway DR23 (see Figure 17-14).

30. An OBI bearing may be given as either QDM or QDR (depending on whether the **TO** or the **FROM** flag is showing).

31. Remember that, with a VOR, the variation **at the station** is used (rather than the variation at the aircraft, as with an NDB). Since this is the case, the variation to be applied will remain constant for each VOR regardless of the position of the aircraft. The bearing must be converted to become a true bearing which is plotted against true north at the beacon.

32. To achieve a more accurate plot of the position line, draw the local meridian (true north) passing through the VOR station position, and plot the position line as a QTE (true bearing) from this local meridian.

EXAMPLE 17-6

EXAMPLE

At time 1915 the following VOR bearings are obtained:

DCS VOR (5443½N 0320½W) bears 230° RMI

SAB VOR (5555N 02122W) bears 193° QDR

NEW VOR (5502N 0144W) bears 150° RMI

Determine the position at 1915

SOLUTION

See Figure 17-16 and Figure 17-18.

At time 1915 DCS VOR bears 230° RMI (QDM) - 180° = 050°QDR. Plot 041(T)

At time 1915 SAB VOR bears 193° QDR. Plot 201½(T)

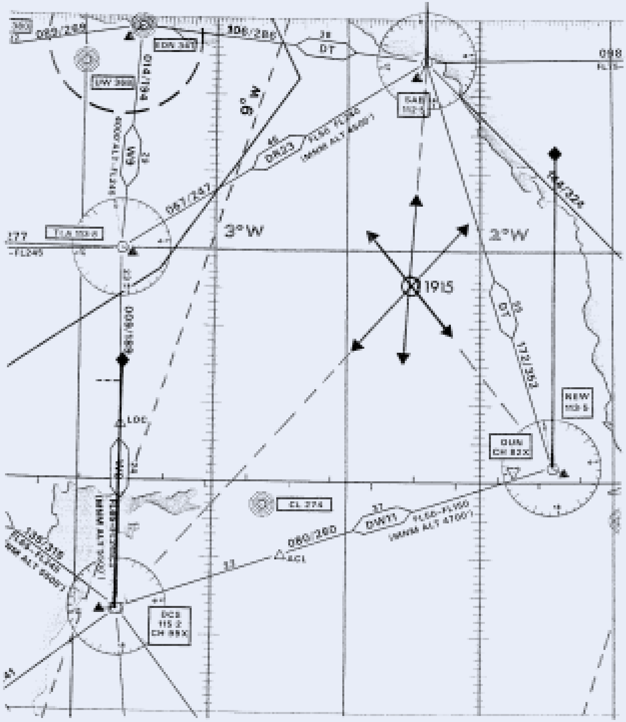
At time 1915 NEW VOR bears 150° RMI (QDM) + 180° = 330° QDR. Plot 322(T).

The aircraft position is fixed as **5526N 0216W** at time 1915.

FIGURE 17-17

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1915								DCS VOR brs 230°QDM, plot 050° QDR or 041°QTE								
								SAB VOR brs 193° QDR or 184½ QTE to plot								
								NEW VOR brs 150°QDM, plot 330° or 322° QTE								
								Fix 5526N 0216W								

FIGURE 17-18



VDF Bearings

33. VHF Direction Finding stations normally provide either QTE or QDM bearings.

34. QTE bearings are plotted as given, using the meridian passing through the DF station as the true north reference.

35. To plot a DF bearing given as a QDM it is necessary to convert to QDR, by adding or subtracting 180° as appropriate, and then to apply the variation **at the station** to obtain the QTE. The QTE is then plotted as before.

EXAMPLE 17-7

At time 0825 the following VDF bearings were obtained:

VDF station A (5515N 0400W) gives QDM 272°

VDF station B (5544N 0305W) gives QTE 185°

Determine the position at 0825.

Plot the positions of the VDF Stations

See Figure 17-19 and Figure 17-20. VDF station A gives QDM 272°, - 180°, QDR 092°, variation at station 9½°W, plot QTE 082½° (using meridian at station).

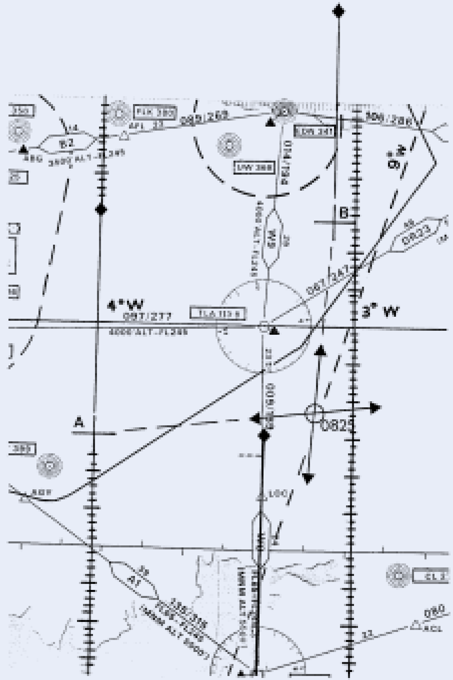
VDF station B gives QTE 185° to plot (using meridian at station).

Aircraft position at 0825 is **5518½N 0308½W**

FIGURE 17-19

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
0825								DF Sta A gives 272°QDM, 092°QDR, var 9½°W, Plot 082½°QTE								
								DF Sta B gives 185°QTE to plot								
								Fix 5518½°N 0308½W								

FIGURE 17-20



NDB Bearings

36. The traditional way of presenting ADF bearing information to the pilot is by means of the Relative Bearing Indicator (RBI). The sharp end of the needle points to the NDB, and the bearing displayed on the RBI is **relative** to the forward end of the aircraft fore and aft axis. In order to obtain a QDM it is therefore necessary to add the aircraft heading (°M) to the relative bearing. In order to obtain a QDR it is then necessary to add or subtract 180° to the QDM. Finally, in order to obtain a QTE (bearing to plot), it is necessary to apply the variation **at the aircraft**.

37. The more convenient method of presenting ADF bearing information is by means of the Radio Magnetic Indicator (RMI). Again the sharp end of the needle points to the NDB and the top of the instrument represents the forward end of the fore and aft axis. The compass rose within the RMI is slaved to a suitable gyro magnetic compass, and therefore the aircraft's magnetic heading is shown against the datum at the top of the instrument. The sharp end of the needle is thus pointing at the QDM on the compass rose. In effect the RMI has done mechanically what the pilot is required to do mentally (i.e. relative bearing + magnetic heading = QDM) when using the RBI.

EXAMPLE 17-8

EXAMPLE

An aircraft is at DR position 5530N 00230W at time 1315. The following ADF bearings are obtained whilst the aircraft is maintaining a heading of 178°(M):

UW NDB (5554N 0330W) bears 133° relative

CL NDB (5457N 0248W) bears 028° relative

Determine the aircraft position at 1315.

SOLUTION

See Figure 17-21 and Figure 17-22.

At 1315 UW NDB bears 133°(R), hdg 178°(M), QDM 311°, - 180°, QDR 131°, variation at aircraft 9°W, plot 122° (using the aircrafts meridian transferred to the NDB).

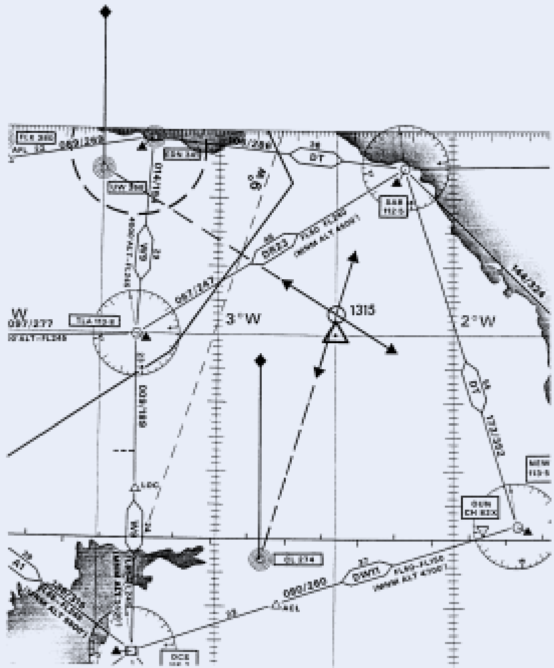
At 1315 CL NDB bears 028°(R), hdg 178°(M), QDM 206°, - 180°, QDR 026°, variation at aircraft 9°W, plot 017° (using the aircrafts meridian transferred to the NDB).

Aircraft position at 1315 is **5533N 0229W**

FIGURE 17-21

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1315			169	9W	178			DR 5530N 0230W								
								UW NDB brs 311°QDM, QDR 131° var 9°W, plot QTE 122°								
								CL NDB brs 206°QDM, QDR 026° var 9°W, plot QTE 017°								
								Fix 5533N 0229W								

FIGURE 17-22



38. To apply a correction for convergency, by far the easiest way is to transfer the meridian passing through the aircraft (where the bearing is measured) to the NDB (where the bearing is plotted). This procedure is shown in the following example. Of course, convergency is never applied to VOR or VDF bearings, since with these position lines the bearing is either manufactured at the station (VOR), or measured at the station (VDF).

EXAMPLE 17-9

EXAMPLE

An aircraft is at DR position 5530N 00230W at time 1315. The following ADF bearings are obtained:

UW NDB (5554N 0330W) bears 311° RMI

CL NDB (5457N 0248W) bears 206° RMI

Determine the aircraft position at 1315.

SOLUTION

See Figure 17-22 and Figure 17-23.

At 1315 UW NDB bears 311° RMI (QDM), - 180°, QDR 131°, variation at aircraft 9°W, plot 122° (using meridian at aircraft transferred to NDB).

At 1315 CL NDB bears 206° RMI (QDM), - 180°, QDR 026°, variation at aircraft 9°W, plot 017° (using meridian at aircraft transferred to NDB).

Aircraft position at 1315 is **5533N 0229W.**

FIGURE 17-23

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1315			169	9W	178			DR 5530N 0230W								
								UW NDB brs 311°QDM, QDR 131° var 9°W, plot QTE 122°								
								CL NDB brs 206°QDM, QDR 026° var 9°W, plot QTE 017°								
								Fix 5533N 0229W								

EXAMPLE 17-10

EXAMPLE

At time 1216 an aircraft at DR position 5255N 00130E, on a heading of 342°(M), obtains the following NDB bearings:

BK NDB (5326N 0013W) bears 303° RMI

WJ NDB (5236N 0028.5W) bears 279° Relative

Determine the aircraft position at 1216.

SOLUTION

See Figure 17-24 and Figure 17-25. To transfer the meridian passing through the aircraft DR position to the NDB, align a protractor with the DR meridian and then slide it along a straight edge to the NDB.

At 1216 BK NDB bears 303° RMI (QDM), - 180°, QDR 123°, variation at aircraft 6°W, plot QTE 117°(T) **using the transferred aircraft meridian at the NDB**.

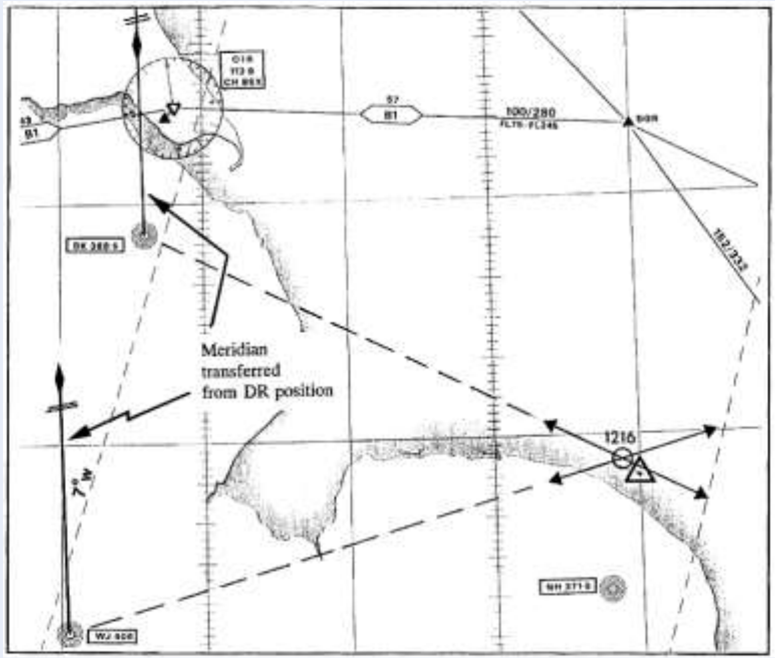
At 1216 WJ NDB bears 279°(R), hdg 342°(M), QDM 261°, - 180°, QDR 081°, variation at aircraft 6°W, plot QTE 075°(T) **using the transferred aircraft meridian at the NDB**.

The aircraft position at 1216 is **5256½N 0127E**.

FIGURE 17-24

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1216			336	6W	342			DR 5255N 0130E								
								BK NDB brs 303°QDM, QDR 123°, var 6°W QTE 117° (ccy on chart								
								WJ NDB brs 279°(R), hdg 342°(M), QDM 261°, QDR 081°, var 6°W, plot QTE 075° (ccy on chart)								
								Fix 5256½N 0127E								

FIGURE 17-25



DME Range Position Lines

39. To plot a DME range position line, arc a curve of the appropriate radius from the DME station, as shown in the following example. It is not normal to account for the difference between indicated (slant) range and true (horizontal) range. Remember that a TACAN is effectively a military DME, and that a VORTAC is effectively a co-located VOR and DME.

EXAMPLE 17-11

EXAMPLE

At time 1847 an aircraft is at DR position 5330N 0120E, heading 089° (C), deviation 2°E. The following fixing information is obtained:

OTR VOR (5342N 0006W) bears 294° RMI

OTR DME (5342N 0006W) gives range 50nm

WJ NDB (5236N 0028W) bears 147° Relative

Determine the aircraft position at 1847.

SOLUTION

See Figure 17-26 and Figure 17-27.

At 1847 OTR VOR bears 294° RMI (QDM), - 180° = QDR 114°. Plot 107(T)

OTR DME 50 nm. Arc a circle radius 50 nm from QTR to cut through the bearing position line.

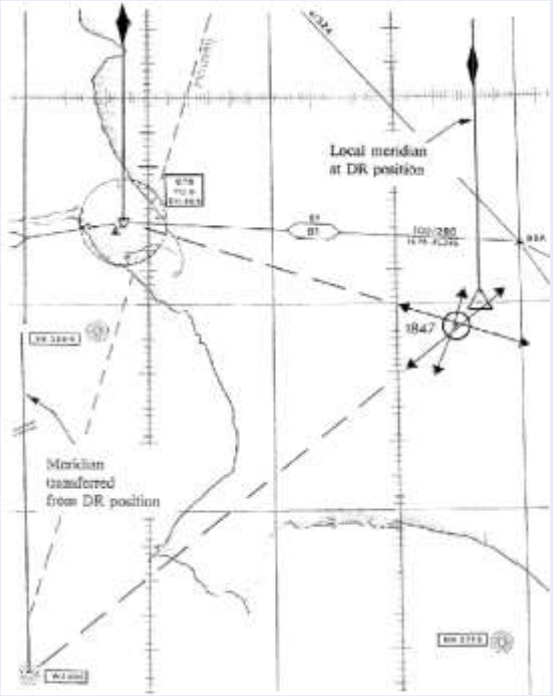
At 1847 WJ NDB bears 147° (R), hdg 091° (M) (dev 2°E), QDM 238°, - 180°, QDR 058°, variation at aircraft 6° W, plot QTE 052° (using the transferred DR meridian at the NDB).

The aircraft position at 1847 is **5326½ N 0115E**.

FIGURE 17-26

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1847			085	6W	091	2E	089	DR 5330N 0130E								
								OTR VOR brs 294°QDM, plot 114° QDR								
								OTR DME range 50nm								
								WJ NDB brs 147°(R), hdg 085°(T) QUJ 232°, plot QTE 052° (ccy on chart)								
								Fix 5326½ 0115E								

FIGURE 17-27



Local meridian
at DR position
1847
Meridian
transferred
from DR position

Airborne Weather Radar Fixes

40. The airborne weather radar may be used as a basic map painting radar to fix the aircraft position. The technique is to set the radar up for ground mapping, to find a unique ground feature which is painting on the radar screen, and to identify this feature on the chart. At a convenient time simply note the relative bearing of the feature using the etched lines on the face of the screen, the range of the feature using the electronically painted range rings, the aircraft heading and the time of the observation. The bearing to plot is calculated as if the bearing were from an NDB, taking the variation at the aircraft, and applying convergency if necessary. This is perhaps unlikely, since the radar is limited to line of sight range. The range arc is plotted as for a DME.

EXAMPLE 17-12

EXAMPLE

At time 1017 an aircraft is at DR position 5305N 00055E, heading 013° (M). The tip of the peninsula at position 5334N 0007E bears 055° left of the nose at a range of 48nm, using the AWR.

Determine the aircraft position at 1017.

SOLUTION

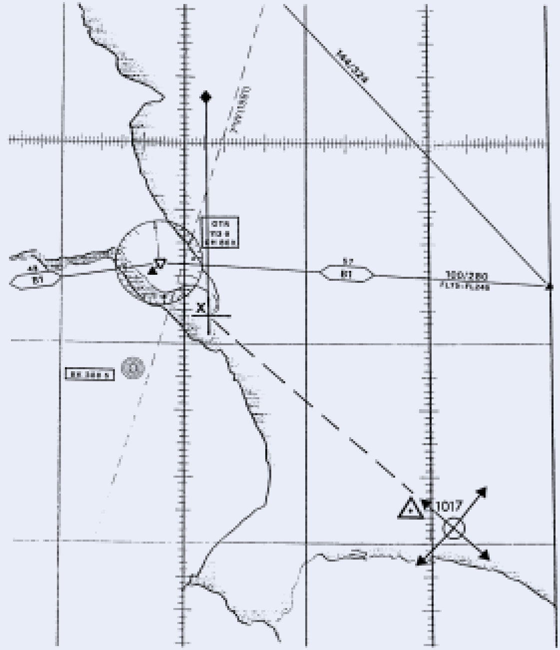
At 1017 point X bears (360° - 55° left) 305° Relative, hdg 013°(M), QDM 318°, - 180°, QDR 138°, variation at aircraft 6°W, plot QTE 132° and range 48 nm.

Aircraft position at 1017 is **5302½N 0106E**

FIGURE 17-28

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1017			007	6W	013			DR 5305N 0055E								
								X (5334N 0007E) brs 055° left, range 48nm. X brs 305°(R), hdg 007°(T), QUJ 312°, plot QTE 132°								
								Fix 5302½N 0106E								

FIGURE 17-29



The Non-Simultaneous Fix

41. In the previous examples the two or three position lines which have together comprised a fix have all been obtained at the same time. More realistically, it is likely that bearings and/or ranges will be obtained one at a time, a few minutes apart, and then brought together to manufacture a fix.

42. The following example illustrates the basic procedure for the **transfer** of position lines using the track-plot technique. The golden rule is to transfer the position line along the **best known track** for the appropriate time, using the **best known groundspeed** to calculate the transfer distance.

EXAMPLE 17-13

EXAMPLE

An aircraft is tracking eastwards along DW 11 with a groundspeed of 240 kt. The following bearings are obtained from the CL NDB (5457N 0248W):

0826	CL NDB bears 040° RMI
0828	CL NDB bears 349° RMI
0830	CL NDB bears 300° RMI

Determine the aircraft position at 0830.

SOLUTION

See Figure 17-30 and Figure 17-31. Since all of the bearings in this example are from one station this non-simultaneous fix is known as a running fix. Invariably the time used for any **manufactured fix** is the time of the **LAST** position line.

For examination purposes the candidate is expected to use **all** of the bearings and/or ranges provided when manufacturing a fix. In reality you might well discard a long range NDB position line of dubious accuracy in favour of a VORTAC range and bearing. In the examination, however, one should assume that if the NDB bearing is given, it is expected that you will use it, together with the other two position lines, to manufacture a three position line fix.

There is no consideration of convergency in this example, since the change of longitude between NDB and aircraft is well below 2°.

At 0826 CL NDB bears 040° RMI (QDM), + 180°, QDR 220°, variation at aircraft 9°W, plot QTE 211°.

At 0828 CL NDB bears 349° RMI (QDM), - 180°, QDR 169°, variation at aircraft 9°W, plot QTE 160°.

At 0830 CL NDB bears 300° RMI (QDM), - 180°, QDR 120°, variation at aircraft 9°W, plot QTE 111°.

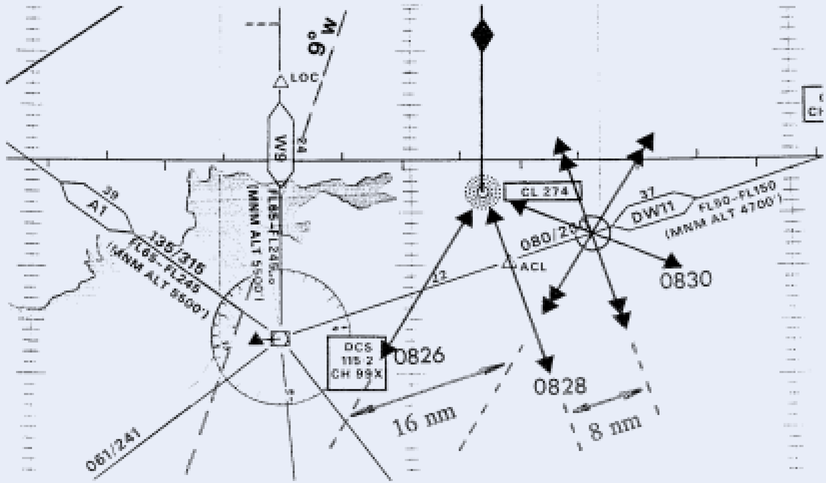
Transfer the 0826 position line by 4 minutes along the best known track (in this case the centre-line of DW 11), using the best known groundspeed of 240 kt, giving a transfer distance of 16 nm.

Similarly, the 0828 position line is transferred along track by 8 nm (2 minutes at 240 kt).
Aircraft position at 0830 is **5453N 0232W.**

FIGURE 17-30

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
0826	071							CL NDB brs 040°QDM, 220°QDR, var 9°W, plot QTE 211°T'fer 4min/240kt = 16nm					240			
0828	✓							CL NDB brs 349°QDM, 169° QDR, var 9°W, plot QTE 160° T'fer 2min/240kt = 8nm					✓			
0830	✓							CL NDB brs 300°QDM,120° QDR, var 9°W, plot QTE 111°					✓			
✓								Fix 5453N 0232W								

FIGURE 17-31



43. There are occasions when a back bearing is used to establish the aircraft's track made good, and this TMG is subsequently used for the transfer of position lines. Similarly, a position line which crosses track at more or less 90° can be used to determine the groundspeed, and this speed used to calculate transfer distances for position lines.

EXAMPLE 17-14

EXAMPLE

At time 1238 an aircraft is overhead the OTR VOR (5342N 0006W), maintaining a constant heading of 090°(M), and a constant TAS of 178 kt. At time 1244 OTR VOR bears 275° RMIAt time 1246 VDF station C (5258N 0035E) gives a QTE of 358°At time 1249 BK NDB (5326N 0013W) bears 252° RMI

Determine the aircraft position at 1249.

SOLUTION

See Figure 17-32, and Figure 17-33

At 1238 the aircraft is overhead OTR VOR.

AT 1244 OTR VOR bears 275° RMI (QDM), - 180°, QDR 095° to plot using the compass rose at the VOR.

The track made good is therefore 095°(M), and it is along this track that position line transfer should be made. Note that the 095° radial is a position line in its own right.

At 1246 DF station C gives QTE 358° to plot.

The distance travelled between 1238 and 1246 (along the TMG of 095°M) is measured as 22.5 nm. The lapsed time is 8 minutes, and therefore the groundspeed is 169 kt. It is this groundspeed which is used for position line transfer.

At 1249 BK NDB bears 252° RMI (QDM), - 180°, QDR 072°, variation at aircraft 7°W, plot QTE 065°(T).

The 1244 position line obviously needs no transfer, since it is itself the best known track.

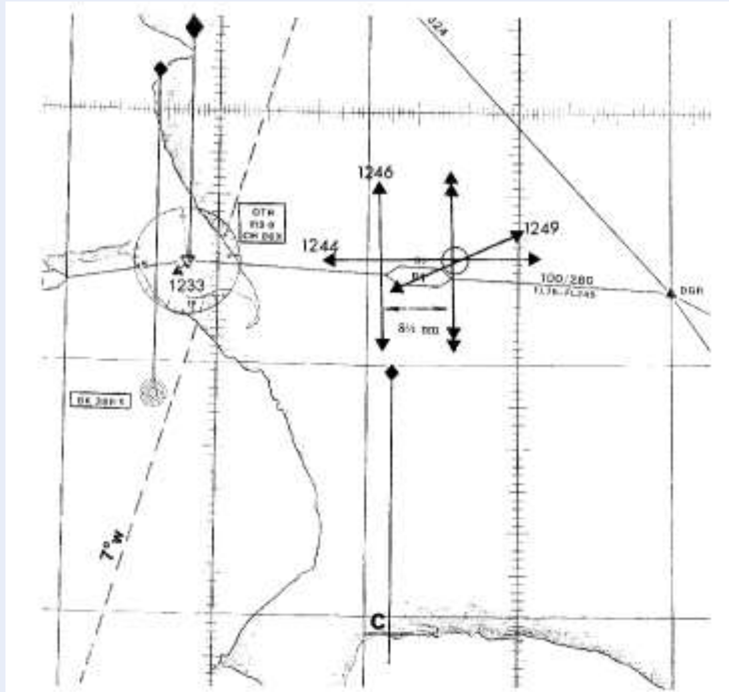
The 1246 position line needs transferring (along the 095° radial) by 8½ nm (3 minutes at 169 kt) to time 1249.

Aircraft position at 1249 is **5342½N 0047E**

FIGURE 17-32

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1238			083	7W	090			O/H OTR				178				
1244	TMG 088			✓				OTR VOR brs 275°QDM, plot 095° QDR								
1246								VDF Stn C gives QTE 358° to plot					169	22½	8	
1249								BK NDB brs 252°QDM, 072°QDR, var 7°W, plot QTE 065° (nil ccy)								
								T'fer 1246 p/l (3min/ 169kt) 8½ nm								
✓								Fix 5342½N 0047E								

FIGURE 17-33



44. DME, Tacan and AWR range lines are curved position lines and therefore require a different approach for transfer. The technique is to transfer the DME (or Tacan, VORTAC, or AWR fix ground feature) in a direction which is parallel to the best known track at the best known groundspeed, for the requisite period of time. The original range arc is now plotted from the transferred origin, thus the resulting position line has been transferred forward for the required time. This technique is illustrated in the following example.

EXAMPLE 17-15

SOLUTION

At 1637 an aircraft is overhead the KNI NDB (5232½N 0314W), routing White 39, LFS (5315N 0530W), maintaining FL 150. The RAS is 140 kt with an outside air temperature of -12°C and a forecast W/V of 270°/40 kt.

Determine the magnetic heading required from KNI to LFS.

Determine the ETA at LFS.

At time 1647 VYL Tacan (5315N 0433W) gives a range of 41 nm. At time 1700 VYL Tacan gives a range of 21 nm

At time 1704 VDF Station D (5231N 0359W) gives a QTE of 314°

Determine the aircraft position at 1704.

SOLUTION

See Figure 17-34, and Figure 17-35

The track required is 306°(M).

The W/V is 279°(M)/40 kt.

RAS is 140 kt, FL 150/-12°C, TAS is 177 kt.

The heading required is computed as **300°(M).**

The groundspeed is 140 kt, the distance KNI to LFS is 93 nm. The lapsed time is computed as 40 min and the ETA LFS is therefore 1717.

The ETA LFS is **1717.** At 1647 VYL Tacan range is 41 nm. The fix is required at 1704, it is therefore necessary to transfer the VYL Tacan by (17 min at 140 kt) 40 nm along a line parallel to the best known track of 306°(M), and then to plot the 41 nm range arc from this transferred origin.

At 1700 VYL Tacan range is 21 nm. Transfer the VYL Tacan by (4 min at 140 kt) 9 nm along the same line which is parallel to track. The 21 nm range arc is then plotted from the transferred origin.

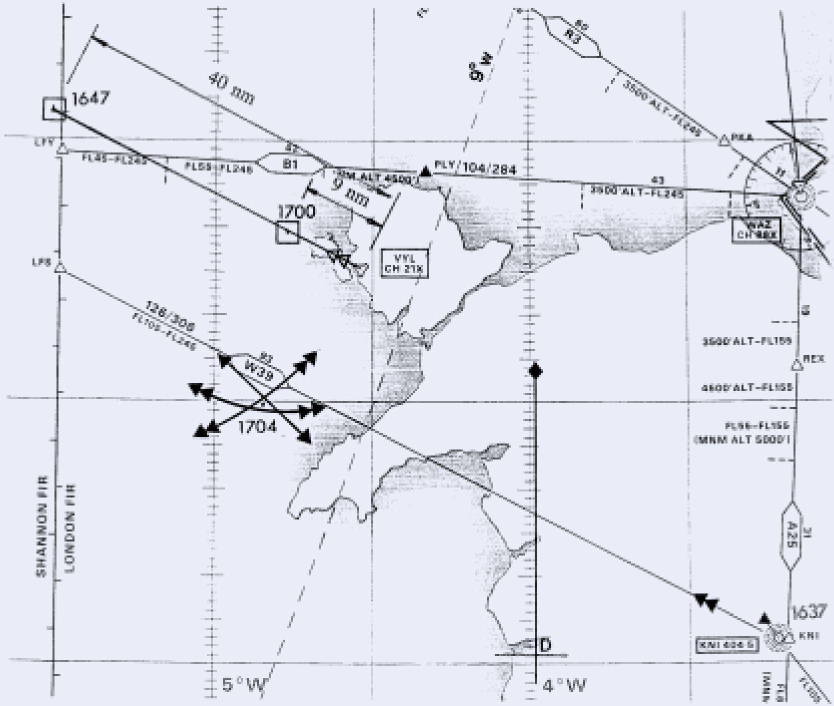
At 1704 DF Station D gives QTE 314°. Plotting the three position lines for time 1704 gives a triangle known as a cocked hat. This is by no means an unusual phenomenon, and is due to slight inaccuracies in the bearings and/or ranges, plus plotting and transfer inaccuracies. For examination purposes take the centre of the cocked hat as the fix position.

The aircraft position at 1704 is **5300N° 0450W**

FIGURE 17-34

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	(M)	(M)														(ii)
1637	306	279/ 40			300	(i)		O/H KNI S/H LFS	140	F150	-12	177	140	93	40	1717
1647	✓							VYL Tacan range 41nm T'fer (17min/140kt) 40nm					✓			
1700	✓							VYL Tacan range 21nm T'fer (4min/140kt) 9nm					✓			
1704								VDF Sta D gives QTE 314° to plot								
✓								Fix 5300N 0450W (iii)								

FIGURE 17-35



40 nm
1647
9 nm
1700
1704
1637
FLY/104/284
VYL
CH 21X
128/308
W39
B1
R3
A25
WAZ
KNI
REX
3500' ALT–FL245
3500' ALT–FL155
4500' ALT–FL155
SHANNON FIR
LONDON FIR
5° W
4° W
D

Doppler Information

45. Doppler gives a continuous readout of drift and groundspeed, and therefore enables the pilot to adjust the heading to maintain the required track, and to periodically update the ETA.

EXAMPLE 17-16

EXAMPLE

At time 2347 an aircraft is overhead the OTR VOR (5342N 0006W), routing B1 eastbound, heading 105°(M). At time 2348 the doppler gives drift 7° left, groundspeed 347 kt.

Determine the required heading to maintain the airway centre-line, and the ETA at DOGGER.

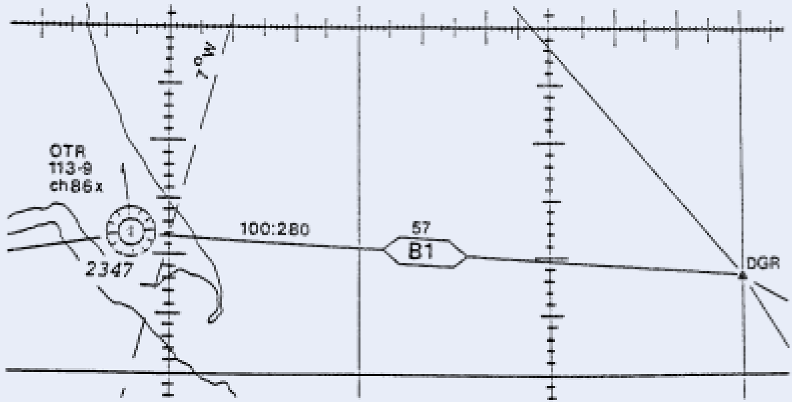
SOLUTION

See Figure 17-36 and Figure 17-37. The centre-line track is 100°(M), doppler drift is 7° left, therefore the required heading is 107°(M). The distance OTR to DGR is 57 nm, the groundspeed 347 kt, the leg time 10 min, and the ETA at DGR 2357.The required heading is **107°(M),** the ETA DGR is **2357.**

FIGURE 17-36

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	(M)															
2347	100				105			O/H OTR S/H DGR					347	57	10	2357
2348	100	Dop 7°P			107			A/H parallel track								

FIGURE 17-37



Determining Wind Velocities

46. To determine the wind velocity between two successive fixes:

    (a) Join the fixes with a straight line and measure the TMG at the mid position against the local meridian.

    (b) Measure the distance between the two fixes.

    (c) Note the mean heading steered between fixes.

(d) Note the TAS.

(e) Use the navigation computer with the above values to find the mean wind velocity.

Note. This procedure assumes no change of heading and/or TAS between the fixes. Usually the minimum time interval between fixes is 20 minutes.

Doppler Wind Velocities

47. The wind velocity determined from the information given by the doppler equipment is only a local wind at the instant the readings are taken. It is not, therefore, a mean wind although it is better than not having a recent wind at all.

Dead Reckoning Following a Fix

48. It takes a certain amount of time to plot a fix. Should the fix indicate that it is necessary to alter heading to make the next turning point it is no use calculating the new heading from the fix position, since the aircraft has by now flown beyond this point. It is therefore necessary to **DR AHEAD,** and to calculate an alteration of heading from a point ahead of the aircraft.

EXAMPLE 17-17

EXAMPLE

At time 1615 an aircraft is overhead NH NDB (5240½N 0123½E), en route to the OTR VOR (5342N 0006W). The aircraft is maintaining a TAS of 130 kt, the forecast wind velocity is 280°/ 20 kt.

Determine the heading (°M) and the ETA for the OTR VOR.

At time 1628 the aircraft is observed to be crossing the coast.

At time 1638 the tip of the peninsula at 5334N 00007E shows at zero degrees relative, range 32nm.

Determine the aircraft position at 1638.

Determine the mean wind velocity affecting the aircraft since 1615.

At time 1644 the aircraft alters heading for OTR.
Give the mean heading required (°M) for OTR, and the revised ETA at OTR.

SOLUTION

At 1615 the aircraft is overhead NH NDB, on course for OTR VOR. Track 319°(T), W/V 280°/20 kt, TAS 130 kt, heading 314°(T), variation 7°W, heading 321°(M), groundspeed 114 kt, distance 82 nm, time 43 min, ETA 1658.

Mean heading **321°(M),** ETA OTR **1658.**

At 1628 the aircraft coasts out (to the best of our knowledge on track), construct the position line (see Figure 17-38).

At 1638 point X bears 0° relative, range 32 nm. Heading 314°(T), 0° rel, QUJ 314°, plot QTE 134°.

Transfer 1628 position line (10 min at G/S 114 kt) 19 nm along track.
The aircraft position at 1638 is **5312N 00045E**.

1615 to 1638:	TMG	324°(T)
	Dist (NH to fix)	39 nm
	G/S (39 nm in 23 min)	102 kt
	Heading	314°(T)
	TAS	130 kt

The mean W/V 1615 to 1638 is **283°/34 kt.**

1638 to 1644 (the DR ahead). The aircraft is maintaining a heading of 314°(T) and a TAS of 130 kt. The wind expected to affect the aircraft is 283°/34 kt. The best available information therefore suggests that the aircraft will continue along a track of 324°(T) at a groundspeed 102 kt. The DR position for 1644 is therefore 10.2 nm beyond the fix on the extended TMG line.

At 1644 the heading is altered for OTR.

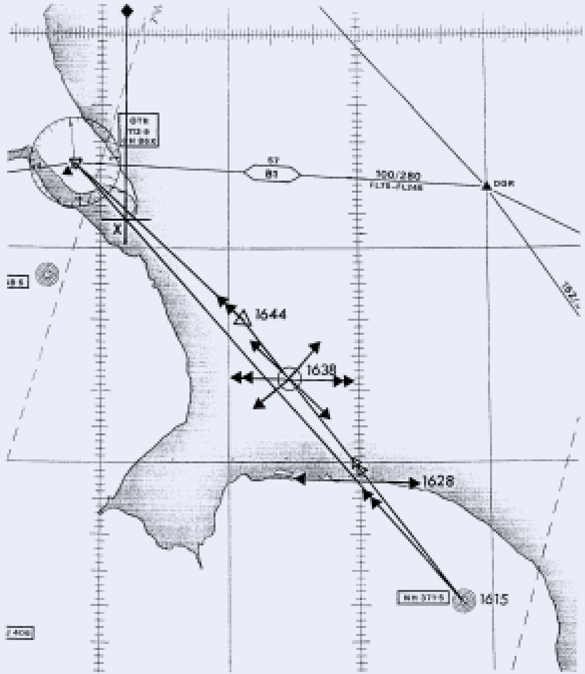
Track 312°(T), TAS 130 kt, W/V 283°/34 kt, heading 305°(T), variation 7°W, heading 312°(M), groundspeed 99 kt, distance 32 nm, time 19½ min, ETA OTR 1703½.

At 1644 mean heading is **312°(M)**, ETA OTR **1703½**

FIGURE 17-38

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1615	319	280/20	314	7W	321	(i)		O/H NH S/H OTR				130	114	82	43	1658
1628								Coasting out T'fer (10min/114kt) 19nm								(i)
1638								X brs 0°(R) range 32nm AWR. QUJ 314° plot QTE 134°								
✓								Fix 5312N 0045E (ii)								
✓	TMG 324	Drift 10°S	314	7W	321			W/V 283°/34kt (iii) (1615 to 1638)				130	102	39	23	
✓	324		314									✓	✓	102	6	1644
1644	312	283/34	305	7W	312	(iv)		DR A/H OTR				130	99	32	19½	1703½ (iv)

FIGURE 17-39



EXAMPLE 17-18

EXAMPLE

At 0342 an aircraft is at DR position 5308N 0305W, heading 271°(M), TAS 125 kt, flight plan W/V 340°/15 kt.

At 0354 VYL Tacan (5315½N 00433W) gives a range of 34 nm.

At 0400 WAL VOR (5323N 00308W) bears 060° RMI.

At 0400 WAZ Tacan (5325N 00308W) gives a range of 44 nm.

Determine the aircraft position at 0400.

At 0406 heading is altered for LFS (5315N 00530W).

Determine the mean heading (°M) for LFS, and the ETA LFS.

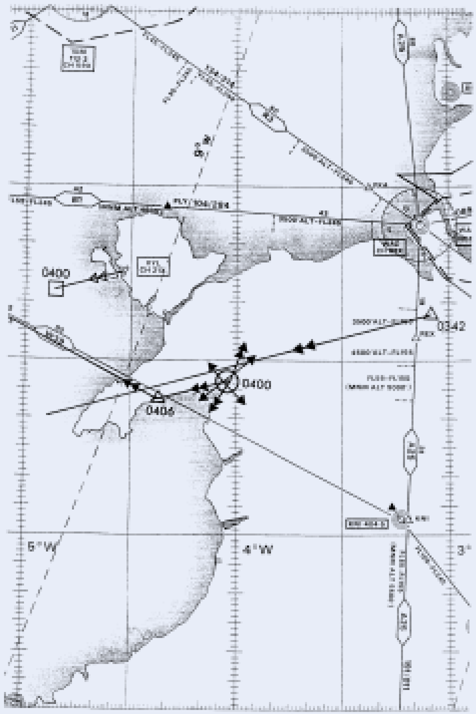
SOLUTION

At 0342 the aircraft is at DR position 5308N 00305W, heading 271°(M), variation 9°W, heading 262°(T), TAS 125 kt, W/V 340°/15 kt, track 256°(T), groundspeed 123 kt. At 0354 VYL Tacan gives range 34 nm. Transfer VYL Tacan (0354 to 0400 = 6 min at 123 kt) 12.3 nm along a line parallel to the track of 256°(T). Plot a 34 nm range arc from the transferred origin. At 0400 WAL VOR bears 060° RMI (QDM), + 180° = QDR 240°. Plot 231½°(T) At 0400 WAZ Tacan gives range 44 nm to plot. The aircraft position at 0400 is **5256½N 00402½W.** 0400 to 0406 (the DR ahead). The aircraft is maintaining a heading of 262°(T) at a TAS of 125 kt. **There is no way of updating the wind velocity since the start position at 0342 was a dead reckoning position and not a fix.** The best available track and groundspeed for the DR ahead leg is therefore, still, a track of 256°(T) and a groundspeed of 123 kt, giving a distance (fix to DR position) of 12.3 nm.

At 0406 the heading is altered for LFS, track 299°(T), TAS 125 kt,W/V 340°/15 kt, heading 303°(T), variation 9°W, heading 312°(M), groundspeed 114 kt, distance 46 nm, time 24 min, ETA 0430. Heading required at 0406 is **312°M,** ETA LFS is **0430.**

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
0342	256	340/15	264	9W	271			DR 5308N 0305W				125	123			
0354								VYL Tacan range 34nm T'fer (6min/123kt) 12.3nm								
0400								WAL VOR brs 060×QDM, plot 240×QDR								
✓								WAZ DME range 44nm								
✓	✓	✓	✓	✓	✓			Fix 5256½n 0402½W (i)				✓	✓	123	6	0406
0406	299	340/15	303	312	(ii)			A/H LFS				125	114	46	24	0430 (ii)

FIGURE 17-41



49. One of the most common mistakes made by candidates in respect of plotting questions involves a failure to understand the subtle but important difference between the situation with the DR ahead considered at Example 17-17 and the DR ahead considered at Example 17-18.

50. At Example 17-17, the aircraft flew from one fix to another, the straight line joining the two fixes accurately represented the track made good (since the heading and TAS had remained constant throughout), and the DR position following the fix lay on an extension of this track made good.

51. At Example 17-18, the start position was a DR position (**an estimated position).** The line joining this start position to the subsequent fix is meaningless (because of the uncertain accuracy of the start position). Therefore the DR position following the fix is unlikely to lie on an extension of this line. In this case the DR position following the fix **must** be determined using the original W/V and the aircraft heading and TAS.

Pilot Navigation

52. The type of plotting discussed so far in this section, namely the construction of fixes, the calculation of wind velocities and the DR ahead, sensibly requires a plotting table and a flight navigator. When flying on airways it is necessary to adopt a more **approximate** approach to the problem, namely **pilot navigation techniques.**

53. Pilot navigation is a sensible combination of such things as the 1 in 60 rule, back bearings, QDMs and cross-cuts, the DME and doppler information. For examination purposes Flight Director Systems, VLF/Omega and INS are not carried aboard the aircraft.

EXAMPLE 17-19

EXAMPLE

At 1307 an aircraft is overhead WAL VOR (5323.5N 00308W) routing B1 to LFY, heading 292°(M). At 1310 WAL VOR bears 107° RMI. At 1310 heading is altered to parallel the centre-line of B1.

Determine the heading required (°M) at 1310.

SOLUTION

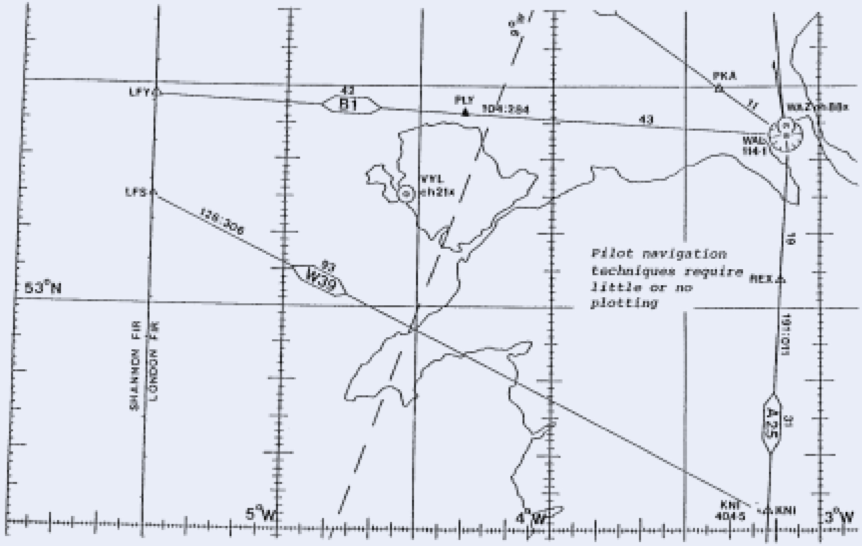
The centre-line of B1 is 284°(M). The back bearing from the WAL VOR gives a track made good of 287°(M). The track error is 3° right, and the actual drift being experienced is therefore 5° left. In order to maintain a track of 284°(M) it is therefore necessary to alter heading on to 289°(M).

The heading required at 1310 is **289°(M).**

FIGURE 17-42

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	(M)															
1307	284				292			O/H WAL VOR S/H LFY								
1310	284	Drift 5°P			289			WAL VOR brs 107°QDM, 287°QDR, A/H 3° left to parallel track								

FIGURE 17-43



LFY
43
B1
FLY
43
PKA
WAL
114·1
LFS
VYL
128·306
97
W39
53°N
SHANNON FIR
LONDON FIR
Pilot navigation
techniques require
little or no
plotting
REX
A25
KNI
404·5
KNI
5°W
4°W
3°W

EXAMPLE 17-20

EXAMPLE

At 0321 an aircraft is overhead KNI NDB (5233N 0314W) routing W39 to LFS (5315N 0530W), TAS 240 kt, forecast W/V 345°/40 kt.

Determine the mean heading (°M) for LFS, and the ETA LFS.

At 0334 KNI NDB bears 169° relative.

Suggest a sensible heading for LFS.

SOLUTION

The required heading is **313°(M)**, ETA LFS is **0347.5.**

The back bearing from KNI NDB gives a track made good of 302°(M), and therefore the track error is 4° left. The back bearing is taken at the halfway time, and therefore a heading correction of twice the track error (8° right) should find LFS.

The heading after 0334 should be **321°(M).**

FIGURE 17-44

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
	(M)															
0321	306	354/40			313	(i)		O/H KNI S/H LFS				240	211	93	26½	0347½
0334					313			KNI NDB brs 169°(R) hdg 313°(M) QDM 122° QDR 302° Track error 4° left half way along track.								
✓					321	(ii)		A/H 8° right								

Selection of Fixing Aids

54. It may be that the candidate is asked to decide which of the available navigation aids should be used to fix the aircraft position, given the aircraft's DR position and altitude. When making this decision, consider the following points:

(a) For bearing information, a VOR is invariably more accurate than an NDB.

(b) Both VOR and DME are limited to line of sight range, therefore aircraft height and range from the station must be checked before electing to use a distant VOR/DME or VORTAC.

(c) For two position line fixing, the best angle of cut between one position line and the other is 90°. The ideal two position line fix is therefore a range and bearing from co-located or adjacent VOR/DME facilities.

(d) If possible, use a VOR/DME or VORTAC situated on the aircraft's track. In this way, the VOR bearing is used for tracking and the DME range to check the groundspeed and ETA.

(e) If you have to use NDB bearings because the aircraft is outside of VOR/DME fixing range, then it is likely that the aircraft will be over the sea. In this case, use NDBs which are close to the coast, to minimise the errors caused by coastal refraction. Alternatively, use an inland NDB such that the bearing line between the NDB and the aircraft crosses the coast at approximately 90°.

55. The previous worked examples have all been based on the UK Instructional Plotting Chart. It is now necessary to consider plotting on a somewhat smaller scale Lambert chart. None of the principles previously discussed change because of the reduced scale, however the need for a sharp pencil and overall accuracy of both plotting and calculation is even more apparent.

56. Plotting on a small scale Lambert simply means that, for a given chart track length, the earth distance covered will be considerably greater. In the event that the track in question is predominantly east-west, the amount of convergency involved (that is to say the difference between the initial and the final track angles) will be larger. The measurement of track angles must therefore be made with care.

57. Similarly, since the convergence of adjacent meridians is quite marked on these small scale charts, plotting a bearing or track line with respect to true north from a station or point which does not lie on a printed meridian on the chart can lead to significant errors. In order to overcome this problem, it is often wise to construct a meridian passing through the station or point in question. This procedure is illustrated in the following examples.

EXAMPLE 17-21

EXAMPLE

Use the Lambert chart at Figure 17-45, and the log forms provided.
1000. An aircraft is overhead STN VOR (5810N 00620W), FL 230, RAS 190, OAT -31°C, W/V 270°/50 kt. Set heading for SXZ (6050N 0052W). The mean heading (°M) for SXZ is:

(a) 055

(b) 052

(c) 046

(d) 049

The ETA SXZ is:

(a) 1046½

(b) 1049½

(c) 1052½

(d) 1043½

1020 A pinpoint fix is obtained, which fixes the aircraft at 5920N 00330W.
The mean W/V affecting the aircraft since 1000 is:

(a) 276°/97 kt

(b) 092°/70 kt

(c) 250°/82 kt

(d) 234°/68 kt

1026 Heading is altered for SXZ. The revised heading (°M) for SXZ is:

(a) 038

(b) 020

(c) 029

(d) 045

The revised ETA for SXZ is:

(a) 1044

(b) 1038

(c) 1048

(d) 1052

1044 The aircraft is overhead SXZ, alter heading for turning point B (6230N 00800W), FL 225, OAT -30°C, W/V 270°/90 kt, RAS 190 kt. The mean heading (°M) for B is:

(a) 300

(b) 308

(c) 305M

(d) 296

The ETA at B is:

(a) 1159

(b) 1202

(c) 1205

(d) 1154

1054 Alter heading onto 320°(M) to avoid weather. 1100 Alter heading onto 300°(M). 1112 Alter heading for B. The aircraft's DR position at 1112 is:

(a) 6148N 00305W

(b) 6139N 00320W

(c) 6202N 00340W

(d) 6125N 00320W

The revised mean heading (°M) for B is:

(a) 297

(b) 285

(c) 301

(d) 292

The revised ETA at B is:

(a) 1200

(b) 1208

(c) 1204.5

(d) 1155.5

1124 AB NDB (6124N 0640W) bears 308° relative, heading 285°(T). 1133 SRE NDB (6204N 0659W) bears 344° relative, heading 285°(T). The aircraft position at 1133 is:

(a) 6225N° 0555W

(b) 6205N° 0615W

(c) 6148N° 0602W

(d) 6140N° 0545W

FIGURE 17-45


65N
60N
10W
AK
FM
SUM
WIK
STW
ADN
Lambert conformal conic projection
Standard Parallels 45N and 68N
Constant of the cone 0.839767

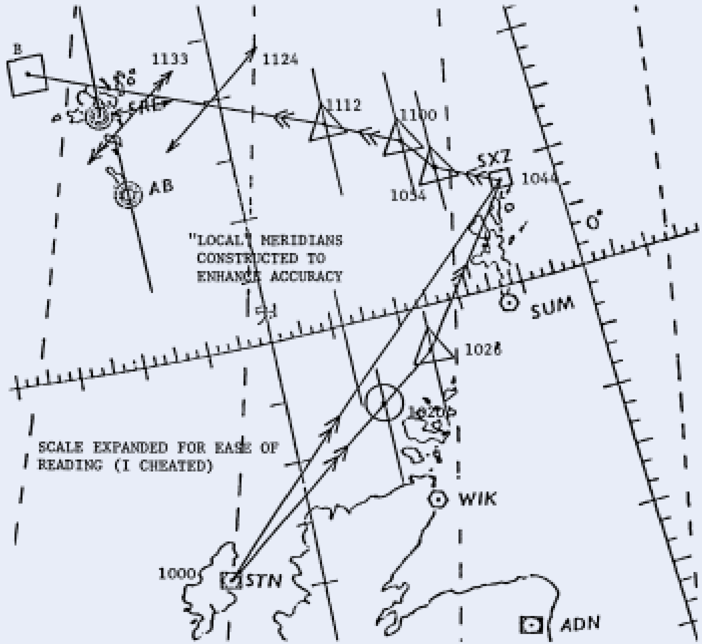
SOLUTION

FIGURE 17-46

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1000	046	270/50	038	11W	042	(i)		O/H STN S/H SXZ	190	230	-31	270	303	234	46½	1046½ (2)
1020	052		038					Fix 5920N 0330W W/V 276/97 (3)				270	330	110	20	
1026	037	276/97	019	10W	029	(4)		DR A/H SXZ				270	305	91	18	1044 (5)
1044	296	270/90	288	12W	300	(6)		O/H SXZ A/H B	190	225	-30	267	183	228	75	1159 (7)
✓												✓	183	30	10	
1054	326	270/90	310	10W	320			A/H 320°(M)				✓	203	20	6	
1110	297	✓	289	11W	300			A/H 300°(M)				✓	184	37	12	
1112	291	✓	284	13W	297	(9)		DR 6139N 0320W (8) A/H B				✓	180	144	48	1200 (10)

1124			285					AB NDB brs 308°(R) + 285° = 593° - 360° = 233° - 180° = 053° to plot (ccy on chart) T’fer (9min/180kt) 27nm								
1133			✓					SRE NDB brs 344°(R) + 285° = 629° - 360° = 269° - 180° = 089° to plot (nil ccy)								
✓								Fix 6205N 0615W (11)								

FIGURE 17-47



B
1133
1124
1112
1100
SXZ
1044
1034
AB
"LOCAL" MERIDIANS
CONSTRUCTED TO
ENHANCE ACCURACY
SUM
1028
1020
SCALE EXPANDED FOR EASE OF
READING (I CHEATED)
WIK
1000
STN
ADN

EXAMPLE 17-22

EXAMPLE

Use the Lambert plotting chart at Figure 17-47, and the log forms provided.
0657. An aircraft is overhead WIK (5828N 00305W) at 4,000 ft and climbing to FL 260, RAS 200 kt, rate of climb 1,500 ft/min. The aircraft is maintaining a heading for 60°N 005°W. The meteorological information is as follows:

F/L 300	090°/80 kt	-45°C

The mean heading (°M) for the top of climb is:

(a) 345

(b) 350

(c) 347

(d) 355

The distance covered over the ground in the climb is:

(a) 59 nm

(b) 69 nm

(c) 71 nm

(d) 63 nm

On reaching the top of climb the aircraft will maintain a RAS of 200 kt.
The mean heading (°M) from the top of climb to 60°N 005°W is:

(a) 350

(b) 345

(c) 355

(d) 343

The ETA for 60°N 5°W is:

(a) 0720

(b) 0723

(c) 0724

(d) 0719

0720 Aircraft is at DR position 60°N 005°W, alter heading for the FM NDB (6350N 01641W). The mean heading (°T) for the FM NDB is:

(a) 318

(b) 322

(c) 312

(d) 330

The ETA at the FM NDB is:

(a) 0829.5

(b) 0835

(c) 0827.5

(d) 0826

0730 STN VOR/DME (5810N 0620W) gives QDR 013°, range 135 nm. The position at 0730 is:

(a) 6045N 00600W

(b) 6125N 00630W

(c) 6025N 00615W

(d) 5945N 00530W

0743 STN DME gives range 195 nm. Assuming that STN DME is at 500 ft amsl, the maximum range at which the DME would unlock is:

(a) 203 nm

(b) 229 nm

(c) 163 nm

(d) 183 nm

0747 SRE NDB (6204N 0659W) bears 096° relative, heading 312°(T). 0752 AB NDB (6124N 0640W) bears 160° relative, heading 312°(T). The position at 0752 is:

(a) 6134N 00905W

(b) 6145N 00920W

(c) 6215N 00910W

(d) 6154N 00900W

The mean W/V affecting the aircraft since 0730 is:

(a) 134°/44 kt

(b) 177°/40 kt

(c) 172°/25 kt

(d) 336°/30 kt

0804 Alter heading for the FM NDB. The mean heading (°M) for the FM NDB is:

(a) 303

(b) 311

(c) 315

(d) 294

The revised ETA for the FM NDB is:

(a) 0835.5

(b) 0845

(c) 0838.5

(d) 0849

It is planned to commence a descent at a range of 80 nm from the FM NDB, to be at an altitude of 4,000 ft on crossing the beacon. The RAS for the descent is 180 kt. The meteorological information is as follows:

FL 200	210°/30 kt	28°C
FL 100	190°/20 kt	-12°C

The minimum rate of descent required is:

(a) 823 ft/min

(b) 1023 ft/min

(c) 1223 ft/min

(d) 1443 ft/min

FIGURE 17-48

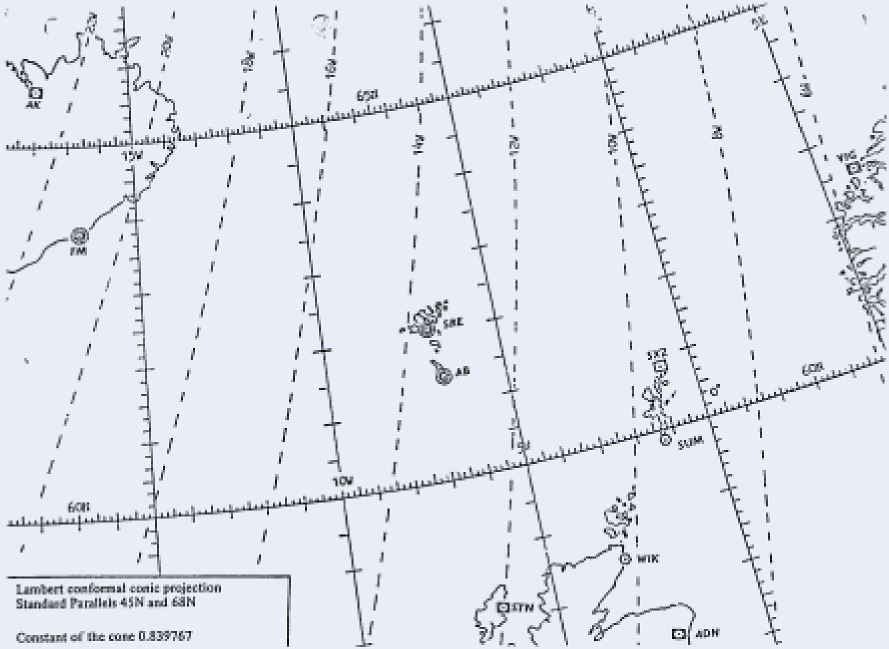

AK
FM
SRE
AB
SXZ
SUM
WIK
STN
ADH
VIG
65N
60N
15W
10W
Lambert conformal conic projection
Standard Parallels 45N and 68N
Constant of the cone 0.839767

FIGURE 17-49

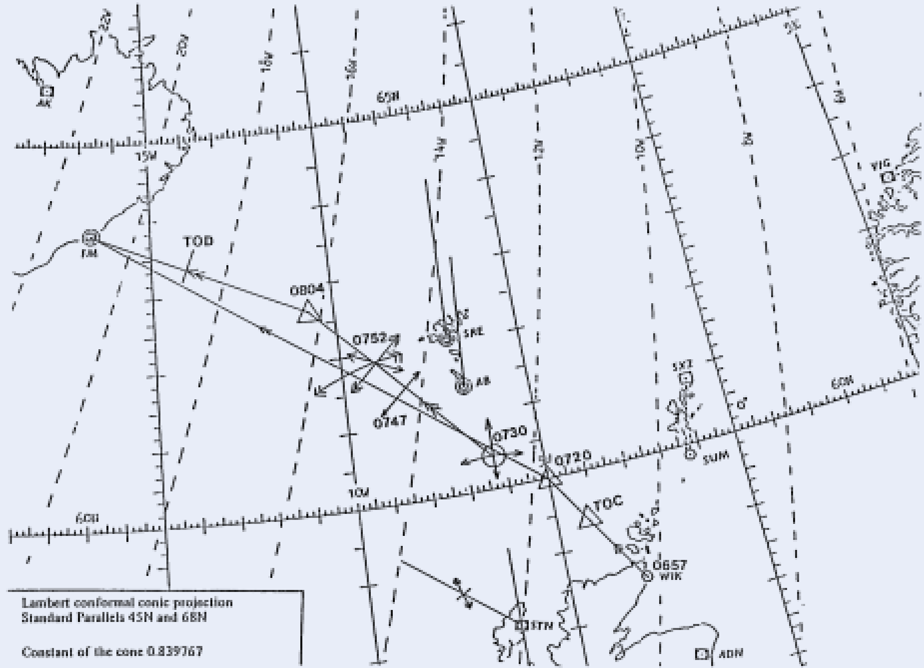
Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
0657	328	075/ 57	340	10W	035 (1)			O/H WIK 4000‰ FL260	200	F150	-15	250	260	63 (2)	14½	0711½
0711½	327	086/ 74	339	11W	350 (3)			DR TOC A/H 60N 5W	200	F260	-37	299	326	47	8½	0720 (4)
0720	304	✓	312 (5)					DR 60N 5W A/H FM	✓	✓	✓	✓	352	408	69½	0829½ (6)
0730								STN VOR/DME brs 013°QDR plot 001°(T) range 135nm								
✓								Fix 6025N 0615W (7)								
0743								STN DME 195nm, t'fer origin (9min/352kt) 53nm								
								Max theoretical range = 1.25 (÷500 + ÷†26000) = 229nm (8)								

0747			312					SRE NDB brs 096°(R) + 312° = 408° - 360° = 048° + 180° = 228°(T) T'fer 5min/352kt) 29nm								
0752			312					AB NDB brs 096°(R) + 312° =408° - 360° = 048° + 180° = 228°(T) to plot (ccy on chart)								
✓	TMG 317	5°R	312					Fix 6154N 0900W (9) W/V 177/40 (10)				299	327	120	22	
0804	292	177/ 40	285	18W	303 (11)			DR A/H FM				299	312	180	34½	0838½ (12)
0823½	292	200/ 25	285					TODÊ 4000ftm 22000ft in 21½ min = 1023 ft/ min(13)	180	F150	-20	222	221	80	21½	

Answers to Example 21-22

1. b	5. c	9. d	13. b
2. d	6. a	10. b	
3. a	7. c	11. a	
4. a	8. b	12. c	

FIGURE 17-50



TOD
0804
0752
0747
0730
0720
TOC
0657
WIK
65N
60N
15W
10W
SUM
STN
ADN
Lambert conformal conic projection
Standard Parallels 45N and 68N
Constant of the cone 0.839767

58. The final example of plotting on a small scale chart uses a Polar Stereographic projection rather than a Lambert.

59. The best way to think of the Polar Stereographic chart is as an easy Lambert chart, in that, on the Polar Stereo, chart convergency is equal to the change of longitude. If you are sensible and apply convergency 'by construction' on the chart, even this subtle distinction ceases to bear any relevance. As with a Lambert, when plotting on a Polar Stereo, convergency is applied **only** when plotting NDB bearings, and **never** when plotting VOR bearings.

EXAMPLE 17-23

EXAMPLE

Use the Polar Stereo chart at Figure 17-51 and the log forms provided. 1825 Aircraft position is fixed as 7920N 01100W, set heading 75°N 010°E,FL 290, JSA -5°C, Mach 0.75, W/V 030°/60 kt. The mean heading (°M) required is:

(a) 143

(b) 146

(c) 149

(d) 152

The ETA 75°N 010°E is:

(a) 1911

(b) 1914

(c) 1917

(d) 1920

1840 VORTAC W (7555N 0830W, not on chart) gives 233° RMI, range 142 nm. The aircraft position at 1840 is:

(a) 7820N 0310W

(b) 7730N 0325W

(c) 7755N 0320W

(d) 7745N 0210W

The mean W/V affecting the aircraft since 1825 is:

(a) 360°/80 kt

(b) 030°/60 kt

(c) 330°/100 kt

(d) 175°/78 kt

846 Alter heading 75°N 010°E. The mean heading (°M) required is:

(a) 139

(b) 132

(c) 156

(d) 147

The revised ETA 75°N 010°E is:

(a) 1913

(b) 1917

(c) 1922

(d) 1919

1914 Aircraft at position 75°N 010°E, set heading VORTAC Y (6940N 01900E), maintaining Mach 0.75, JSA -5°C at FL 290. Use W/V 340°/70 kt. The mean heading (°M) required is:

(a) 165

(b) 171

(c) 186

(d) 179

The ETA VORTAC Y is:

(a) 1954

(b) 1957

(c) 2000

(d) 1951

The range at which you would expect to achieve a range lock-on from VORTAC Y, which is at mean sea level is:

(a) 213

(b) 172

(c) 195

(d) 236

The time at which you would expect to achieve a range lock-on from VORTAC Y, which is at mean sea level is:

(a) 1931½

(b) 1926

(c) 1947

(d) 1934½

The QDM which you would use to track inbound to VORTAC Y is:

(a) 174

(b) 163

(c) 343

(d) 354

1920 VOR X (7430N 01900E) bears 107° RMI. 1929 VOR X bears 072° RMI. 1938 VOR X bears 044° RMI. The aircraft position at 1938 is:

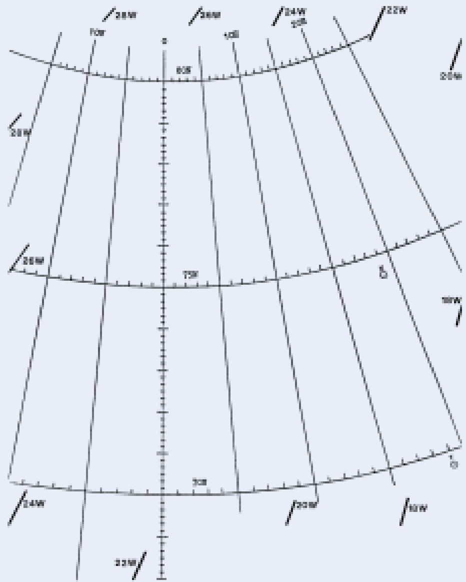
(a) 7133N 01610E

(b) 7237N 01520E

(c) 7142N 01610E

(d) 7207N 01540E

FIGURE 17-51



SOLUTION

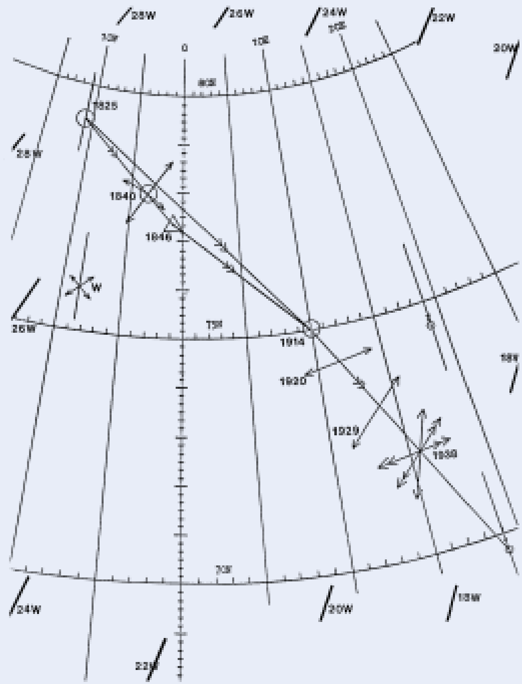
FIGURE 17-52

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1825	133	030/ 60	125	24W	149(1)			O/H 7920N 11W S/ H 75N 10E	MJ.75	F290	-48	438	444	385	52	1917 (2)
1840								VORTAC W brs 233°RMI 053°QDR var at VOR 25°W plot 028°(T) range 142nm								
✓	TMG 133	8°R	125					Fix 7755N 0320W (3) W/V 360/80 (4)	✓	✓	✓	✓	488	122	15	
1846	132	360/ 80	124	23W	147 (5)			A/H 75N 10E	✓	✓	✓	✓	486	219	27	1913 (6)
1914	152	340/ 70	151	20W	171 (7)			O/H 75N 10E S/H VORTAC Y	✓	✓	✓	✓	507	362	43	1957 (8)

								Range = 1.25 ÷29,000 = 213nm (9)					507	149	17½	1931½ (10)
								Inbound track (great circle measured at the VOR)) = 157°(T) var at VOR 17°W inbound QDM 174°(11)								
1920								VOR X brs 107°RMI 287°WDR var at VOR 20°W plot 267° T'fer (18min/507kt) 152nm								

1929								VOR X brs 072°RMI, 252°WDR var at VOR 20°W plot 232° T'fer (9min/507kt) 76nm								
1938								VOR X brs 044°RMI 224° QDR var at VOR 20°W plot 204°								
✓								Fix 7207N 1540E (12)								

FIGURE 17-53



Answers to Example 21-23	
1. c	7. b
2. c	8. b
3. c	9. a
4. a	10. a
5. d	11. a
6. a	12. d

FIGURE 17-54

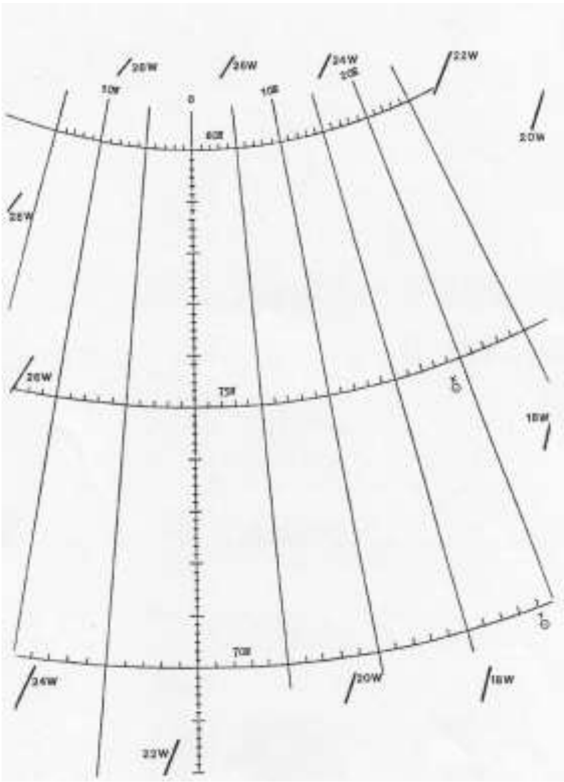


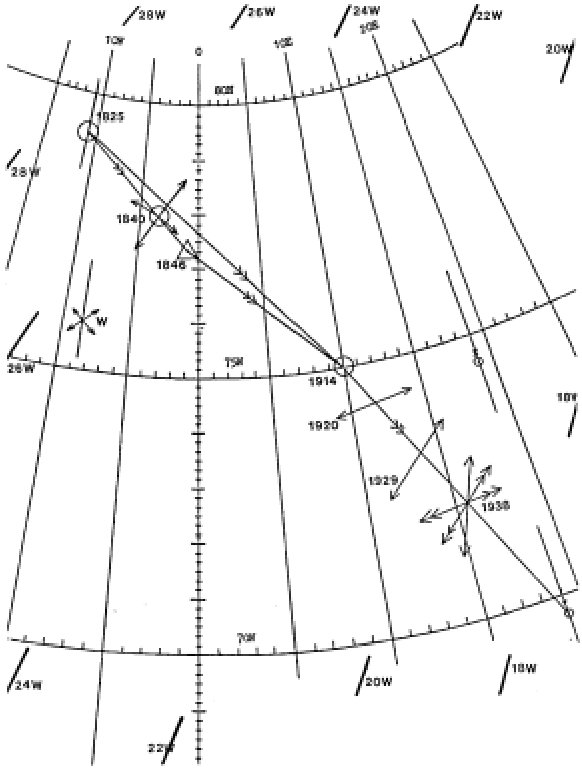
FIGURE 17-55

Time	Tk (T)	W/V	Hdg (T)	Var	Hdg (M)	Dev	Hdg (C)	Observations	RAS	Press Alt	Temp	TAS	Gnd Spd	Dist	Time	ETA
1825	133	030/60	125	24W	<u>149</u> (1)			O/H 7920N 11W S/H 75N 10E	M.75	F290	-48	438	444	385	52	1917 (2)
1840								VORTAC W brs 233'RMI 053c QDR var at VOR 25°W, plot 0.28° (T), range 142nm								
✓	TMG 133	8°R	125					Fix 7755N 0320W (3) W/V 360/80 (4)	✓	✓	✓	✓	488	122	15	
1846	132	360/80	124	32W	<u>147</u> (5)			A/H 75N 10E	✓	✓	✓	✓	486	219	27	1913 (6)
1914	152	340/70	151	20W	<u>171</u> (7)			O/H 75N 10E S/H VORTAC Y	✓	✓	✓	✓	507	362	43	1957 (8)
								Range = 1.25 ÷29,000 = 213am (9)					507	149	17½	1931 ½ (10)

								Inbound track (great circle measured at the VOR) = 157°(T) var at VOR 17°W inbound QDM 174° (11)								
1920								VOR X brs 107° RMI 287°QDR var at VOR 20°W plot 267° T'fer (18 min/ 507kt) 152nm								
1929								VOR X brs 072° RMI 252°QDR var at VOR 20°W plot 232° T'fer (9 min/ 507 kt) 76nm								
1938								VOR X brs 044° RMI 224°QDR var at VOR 20°W plot 204°								
✓								Fix 7202N 1540E (12)								

	1. c 2. c 3. c 4. a 5. d	6. a 7. b 8. b 9. a 10. a	11. a 12. d

FIGURE 17-56



28W
26W
24W
22W
20W
0
1825
28W
1840
1846
W
26W
1914
1920
1929
1938
18W
24W
22W
20W
18W

Revision of ETA and Fuel Endurance

60. We have practised revising ETAs following a fix in many of the preceding examples. The question of fuel endurance, fuel reserves and fuel contingency planning is fully addressed in flight planning (in the chapters dealing with fuel efficiency, the data sheets, the howgozit and computer flight planning). It is important to appreciate that there is a relationship between ETAs and safe fuel margins.

61. The fuel load carried by an aircraft engaged on a public transport flight would typically be made up of taxi fuel; the planned fuel required to fly from departure to destination; a contingency allowance; diversion fuel and holding fuel.

62. The contingency fuel would typically amount to 5% of the departure to destination fuel and 5% of one hour is only 3 minutes. Were the ETAs to slip by 3 minutes in each hour of the flight, due to adverse winds, the contingency allowance would be used up and there would be no margin between the fuel required and the fuel available.

63. Bear in mind that the contingency fuel is carried for reasons other than adverse winds, for example an ATC restriction which puts the aircraft at a flight level other than that planned; an increase in track distance due to weather avoidance, prolonged flight with the anti-icing systems operating; en-route holding. When the aircraft's position is fixed and the ETA revised it is therefore essential that the actual fuel remaining be compared with the planned fuel remaining.

Self Assessed Exercise No. 10

QUESTIONS:

QUESTION 1.

Climb

Plot on FIGURE 108 and also use FIGURE 107 in the Reference Book

1000 Overhead (O/H) 'O' 37°00'N 008°00'W at 4000 ft climbing to FL340. The initial heading for the climb is 210°(M) and the rate of climb is 2000 ft/min at a constant CAS of 200 kts.

The Met Forecast is:

W/V Temp
4000 ft 240/50 -5°C
FL340 300/90 -55°C

Determine:

The mean TAS in the climb

The DR Track (°T)

The time to Top of Climb

The distance covered in the climb

The latitude and longitude of TOC

QUESTION 2.

Descent

Plot on FIGURE 108 and also use FIGURE 107 in the Reference Book

1215 DR position 40°00’N 015°00’W at FL350 and M0.75. Cleared by ATC to descend as required to be O/H ‘O’ 37°00’N 008°00’W at FL 050. Descend at constant CAS 210 kts, with rate of descent 1500 ft/min.

The Met Forecast is:

W/V Temp

FL050 240/40 +5°C

FL350 210/60 -55°C

Determine:

The mean TAS in the descent

The heading (°T) in the descent

The time taken to descend to FL050

The distance covered in the descent

The latitude and longitude of the top of descent (TOD) point

QUESTION 3.

Simultaneous fixing

Plot on FIGURE 108 and also use FIGURE 107 in the Reference Book

1510	O/H Beacon ‘N’ (33° 40’ N 008° 00’ W) en route to 37°00’N 011°00’W. Heading 327°(M), FL310, M0.75, temp –37°C
1525	From AWR, fix point ‘A’ (37° 00’ N 009° 00’ W) bears 045°(R) at a range of 120 nm (ignore convergency).
1545	DR position 37°00’N 011°00’W alters heading (A/H) to 45°00’N 010°00’W, FL330.
1555	‘P’ (39° 00’ N 009° 00’ W) VOR 063°(RMI), DME 90 nm.
1625	From AWR, fix point ‘B’ (42° 56’ N 009° 18’ W) bears 054°(R) at a range of 75 nm (ignore convergency).
1631	DR position 42°56’N 010°45’W.

Determine:

The fix position at 1525

The Track Made Good (TMG) (°T) between 1510 and 1525.

The average G/S between 1510 and 1525.

The mean W/V between 1510 and 1525

The revised ETA for 37°00’N 011°00’W calculated at 1525.

The mean heading (°M) to fly to 45°00’N 010°00’W calculated at 1545 (use a forecast wind of 240/40 and temp of –42°C).

The fix position at 1555.

The fix position at 1625.

The ETA at 45°00’N 010°00’W calculated at 1631 using W/V of 250/50

ANSWERS:

ANSWER 1.

(a) In order to determine the mean TAS in the climb, it is first necessary to find the mid-altitude of the climb and the temperature at that altitude. To work out the mid-altitude we simply calculate the arithmetic average of the base height and the cruising level.

In this case

$$\text{Mid - altitude} = \frac{\text{Base height + Crui sin g level}}{2}$$

$$= \frac{4000 + 34000}{2}$$

$$= 1900 \text{ (or FL190)}$$

To determine the temperature at FL190, we find the arithmetic average of the temperature at the base height (–5°C) and the temperature at the cruising level (–55°C).

Therefore:

$$\text{The temperature at FL190} = \frac{-5 + (-55)}{2}$$

$$= \frac{-60}{2}$$

$$= -30°\text{C}$$

We can now use these figures together with the CAS of 200 kts on the Nav Computer to determine TAS.

TAS = 264 kts

(b) To determine DR track we use the Nav Computer. We need: Hdg (°T), TAS and W/V

Hdg (°M) = 210°(M)

Variation = 7°W

$\therefore$ Hdg (°T) = 203°(T)

We have already calculated TAS (264 kts)

We need to determine mean W/V for the climb, again, an arithmetic average.

$$\text{Wind Direction} = \frac{240 + 300}{2}$$

$$= 270°$$

$$\text{Wind Speed} = \frac{50 + 90}{2}$$

$$= 70 \text{ kts}$$

We can now put this information on the Nav Computer to determine the DR Track.

With these conditions we get 15° Port drift.

Track(°T) = Hdg(°T) – drift(°P)

= 203 – 15 = 188°(T)

(c) The time to Top of Climb (TOC) is obtained from the change in height in this case from 4000 ft to FL340) and the rate of climb (given as 2000 ft/min)

$$\text{Time in Climb} = \frac{34000 - 4000}{2000 \text{ ft/min}}$$

$$= \frac{30000}{2000 \text{ ft/min}}$$

$$= 15 \text{ mins}$$

(d) The distance covered in the climb comes from a speed/distance/time calculation. From (c), the time in the climb is 15 mins. When determining the Track in (b) we should also note the G/S (264 kts).

$$\text{Speed (kts)} = \frac{\text{Dis tance (nm)}}{\text{Time (min)}} \times 60$$

$$\frac{\text{Speed x Time}}{60} = \text{Distance}$$

$$\frac{246 \times 15}{2} = 61\tfrac{1}{2}\text{ nm}$$

(e) To determine the lat and long of TOC must plot a DR position on a bearing of 188°(T) from beacon 'O' at a range of 61½ nm and then extract the position.
Position of TOC = 35°59'N 008°11'W

ANSWER 2.

(a) As for the climb, in order to calculate a mean TAS for the descent, we require the mid-altitude and a mean temperature.

$$\text{Mid - altitude} = \frac{5000 + 35000}{2}$$

$$= 20000 \text{ (FL200)}$$

$$\text{Mean temperature} = \frac{+5 + (-55)}{2}$$

$$= \frac{-50}{2}$$

$$= -25°C$$

Using the CAS given (210 kts) we can now determine the TAS from the Nav Computer.

Mean TAS = 286 kts

(b) In order to determine the heading to fly in the descent, we require Track (°T), mean W/V and mean TAS.

In this case we do not know where the descent will commence, so by convention we use the final track, ie. measure the back bearing of the track at the destination and then take the reciprocal, hence 121°T. The mean W/V is again the arithmetical average of the forecast winds:

$$\text{Mean wind direction} = \frac{240 + 210}{2} = 225°$$

$$\text{Mean wind Speed} = \frac{40 + 60}{2} = 50 \text{ kts}$$

We are now in a position to determine the heading on the Nav Computer:

Hdg(°T) = 131°T

(c) The time taken in the descent comes from the height through which we descend and the rate of descent.

$$\text{Time for the Descent} = \frac{35000 - 5000}{1500 \text{ ft/min}}$$

$$= 20 \text{ mins}$$

(d) In order to reduce the possibility of errors in measurement, the solution in the navigation log shows the total distance between 40°N 15°W and 'O' in a triangle, ie. 376 nm.
From the Nav Computer, the G/S is 293 kts and we know the time in the descent is 20 mins, hence the distance covered in the descent will be 98 nm.
It is then possible to calculate the distance from 40°N 15°W to TOD without even plotting the position of TOD. i.e.
376 – 98 = 278 nm

(e) To find the position of TOD, measure back along track from 'O' the distance covered in the descent, i.e. 98 nm. Mark the position with the DR symbol.

TOD is at 37°50'N 009°46'W

Although not asked for in the worksheet it is now possible to complete the line in the Navigation Log from 45°N 15°W to TOD and hence determine the ETA at TOD.

ETA at TOD = 1253.9

ANSWER 3.

See FIGURE 17 and FIGURE 341 in the reference book.

(a) At 1525 a relative bearing of 045°(R) is obtained on the AWR from point 'A'. The aircraft is heading 327°(M), but we must add a true heading to the relative bearing in order to obtain a true bearing to plot.

Since the bearing is measured in the aircraft we need to apply variation at the aircraft position but we are given no indication of aircraft groundspeed in order to work out a DR position, but we are flying towards the 8°W isogonal, so apply that.

Hdg(°M) = 327°(M)
Variation = 8°W
Hdg (°T) = 319°(T)
Rel Brg = 045°(R)
True Brg = 319°(T) + 045°(R) – 360° = 004°T
Reciprocal = 184°(T)

The bearing to plot from 'A' = 184°(T)

Plot the range line 75 nm from 'A'.

Fix position = 35°00'N 009°10'W

(b) At 1510 we were overhead beacon 'N', at 1525 we know from the fix we are at position 35°00'N 009°10'W. The line joining these 2 positions represents Track Made Good (TMG).

TMG measured at 327°(T)

(c) Measure the distance from 'N' to the 1525 fix position: 109 nm

The aircraft has covered this distance in 15 mins ⟶ G/S = 436 kts.

(d) We now have track (327°), G/S (436 kts) and heading (319°).

We can obtain TAS from Nav Computer using M0.75 and temp –37°C

TAS = 450 kts

∴ mean W/V (1510 – 1525) = 246/64.

(e) Distance from 1525 fix to 37°N 11°W = 148 nm

G/S = 436 kts

∴ time = 20.4 mins

ETA = 1545.4

(f) Mean Tk is measured as 005°T. Using new temp –42°C and M0.75, new TAS is 445 kts.

The mean heading to fly is 001°(T), but the question asked for magnetic heading. Variation at the mid-point is about 9°W

∴ new heading to fly = 001° + 9°W = 010°(M)

(g) VOR brg to 'P' 063°(RMI)

Variation at 'P' is 8°W

True brg to 'P' 055°(T)

Reciprocal 235°(T)

∴ brg to plot = 235°(T)

The range to plot from DME 'P' is given as 90 nm

Fix position = 38°09'N 010°32'W

Relative brg to radar point 'C' = 054°(R)
Aircraft hdg = 001°(T)
True brg to 'C' = 055°(T)
Reciprocal = 235°(T)
∴ Bearing to plot = 235°(T)

The range to plot from 'C' is given as 75 nm

Fix position = 42°10'N 010°44'W

(i) We need to establish a DR position for 1631. To do that we need to know Track and G/S. If we draw a line between the fix positions at 1555 and 1625, that represents the TMG. Measure the distance between the two fixes to establish the distance flown in 30 mins.

Distance between fixes = 241 nm

∴ G/S = 482 kts

To establish the DR position for 1631, extend the TMG beyond the fix position at 1625 for 6 minute's worth of G/S, ie. 48 nm.

From the DR position draw in the new track to 45°N 10°W, measure the new track angle and the distance.

Using the wind given, 250/50 we can find Hdg and G/S from the Nav Computer. Hdg is not required but we need the G/S.

Distance to run to 45°N 10°W = 123 nm
G/S from Nav Computer = 472 kts
∴ time to 45°N 10°W = 15.6 mins
ETA at 45°N 10°W = 1646.6

Point of Equal Time

Multi – Leg Point of Equal Time

Point of Equal Time

1. As its name suggests, the point of equal time (PET), sometimes called the critical point (CP), is that point along track at which it will take equal time to reach either of two nominated points, in the configuration being considered.

2. A PET can be calculated for any configuration for example all engines operating, one engine inoperative, pressurisation failure etc. In other words a PET can be found for any event that could occur in flight which will affect the TAS. It is simply a decision point for the pilot. If the event occurs before the PET it will take less time to reach the departure point than it will to continue to the destination. If the incident happens after the PET it is quicker to continue to the destination than it is to return to the departure point.

3. The PET is particularly important for long oceanic legs or tracks across remote areas such as the polar regions and large desert areas where there are no suitable alternate aerodromes along the route and it is vital to land as soon as possible. Such a case would be a passenger having a heart attack in flight on a route from Los Angeles to Hawaii. It is essential to land as soon as possible. There are no alternate aerodromes along the route. The only course of action is to continue to Hawaii or return to Los Angeles. The position of the incident relative to PET will determine which course of action the pilot must take.

4. The simple case is that of a single track from departure to destination in still air. The PET is exactly at the mid-point between the two because it will take exactly the same time to continue as to return to the departure point.

5. The effect of the along track wind component is to offset the PET. The groundspeed into wind is lower than it is downwind. Therefore the PET will have to be moved from the mid-point **into wind** to maintain the equality of time.

6. The time from the PET to the destination is equal to the distance divided by the groundspeed on to the destination, and the time to the departure point (home) is equal to the distance from the PET to the departure point divided by the groundspeed home.

7. If the total distance from departure to destination is D nm and the distance from departure to the PET is X nm then the distance from the PET to the destination is equal to (D - X) nm.

8. It follows then that if the time on is equal to the time home then the following is true:

$$\frac{X}{H} = \frac{(D - X)}{O}$$

From this formula the distance from departure to PET may be derived;

by cross multiplication $XO = H(D - X)$

open the brackets $XO = DH - XH$

transpose the formula $XO + XH = DH$

isolate X $X(O + H) = DH$

transpose the formula $X = \dfrac{DH}{O + H}$

where:

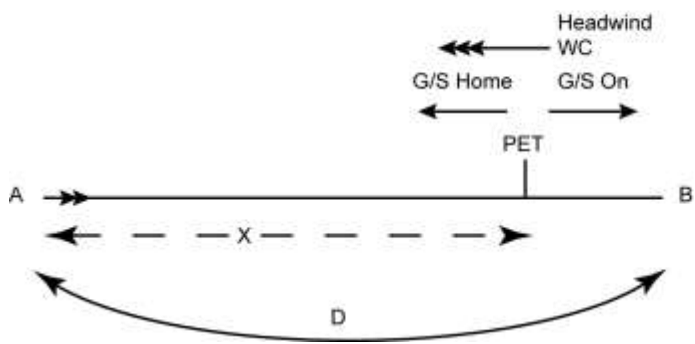
X = distance from departure to PET in nm

D = distance from departure to destination in nm.

O = groundspeed on from PET to destination in Kts

H = groundspeed home from PET to departure on Kts.

FIGURE 18-1
The Single Track PET



EXAMPLE 18-1

EXAMPLE

An aircraft is to fly from A to B. Given:

Distance A to	960 nm
TAS	240 kt
Wind Component Out	+60 kt (Tailwind)

Determine:

(a) The distance from A to the point of equal time between A and B.

(b) The time taken to fly from A to the point of equal time.

SOLUTION

Calculate the G/S(O) and the G/S(H)

G/S(O) = 240 + 60 = 300 kt
G/S(H) = 240 – 60 = 180 kt

Note that the reciprocal value of the wind component is used to calculate the G/S(H) unless a specific value is given. Apply the PET formula to calculate the distance to the PET:

$$\text{Distance to the PET (X)} = \frac{DH}{O + H} = \frac{960 \times 180}{300 + 180} = 360 \text{ nm}$$

Calculate the time from A to the PET. Note that the all engines operating groundspeed out from the departure point, G/S(G), is always used to calculate the time from the point of departure to the PET. In the all engines operating case G/S(G) = G/S(O). This is not so for the one engine inoperative case.

$$\text{Time} = \frac{\text{Distance (x)}}{\text{G/S(G)}} \times 60 = \frac{360}{300} \times 60 = 72 \text{ mins}$$

9. Logically, with a tail wind component from A to B, the point of equal time lies nearer to A than B. A simple check sum can verify this since, by definition, the time taken to fly from the PET to the point of departure should be equal to the time that it would take to fly from the PET on to the destination.

$$\text{Time PET to A} = \frac{\text{Distance PET to A}}{\text{G/S/(O)}} \times 60 = \frac{360}{180} \times 60 = 120 \text{ min}$$

$$\text{Time PET to B} = \frac{\text{Distance PET to B}}{\text{G/S(O)}} \times 60 = \frac{600}{300} \times 60 = 120 \text{ min}$$

10. The point of equal time calculation takes no account of fuel. The fact that the position of the PET has been calculated does not necessarily ensure that the aircraft has sufficient fuel to reach either of the airfields considered.

EXAMPLE 18-2

EXAMPLE

An aircraft is to fly from E to F:

Given:	Distance E to F 990 nm	
	TAS	210 kt
	WC(O)	-33 kt
	WC(H)	+31 kt

Determine the time and distance from E to the PET between E and F.

(a) Calculate G/S(O) and G/S(H).
G/S(O) = 210 – 33 = 177 kt
G/S(H) = 210 + 31 = 241 kt

(b) Calculate distance and time A to PET

$$\text{Distance to PET (x)} = \frac{DH}{O + H} = \frac{990 \cdot 241}{177 + 241} = 571 \text{ nm}$$

$$\text{Time A to PET} = \frac{\text{Distance (x)}}{\text{G/S(G)}} \times 60 = \frac{571}{177} \times 60 = 193.5 \text{ min}$$

Point of Equal Time – 1 Engine Inoperative

11. PETs are calculated so that, in the event of an emergency, the pilot will know in which direction to fly to land in the shortest possible time. The choice is either to continue to the destination or to return to the departure aerodrome.

12. In an emergency which does not affect the aeroplane's performance the normal cruising TAS can be maintained. If such is the case this TAS is used to calculate the G/S(O) and the G/S(H) for use in the critical point formula.

13. On a multi-engine aircraft, an engine failure is an emergency in which the normal cruising TAS **cannot** be maintained. If such is the case, the one-engine inoperative point of equal time is determined using the reduced groundspeed in the formula. The worst case being where the engine failure occurs **at** the one-engine inoperative point of equal time, because flight in either direction is at reduced power for the largest period of time.

14. However, when calculating the time from the point of departure out to the one engine inoperative PET, **the full power G/S Out (G) must be used**. The assumption is that the engine failure does not occur until the one-engine inoperative PET is reached; the journey out to the PET being at the full power TAS.

15. To summarise, when calculating the time and distance to the one-engine inoperative PET:

    (a) The **reduced power G/S(O) and G/S(H)** are used in the formula to calculate the distance to the CP.

    (b) The **full power G/S Out (G)** is used to calculate the time to the PET.

EXAMPLE 18-3

EXAMPLE

An aircraft is to fly from S to T.

Given:	Distance S to T	=	690 nm
	TAS (4Eng)	=	260 kt
	TAS (3Eng)	=	210kt
	WC(O)	=	-50 kt
	WC(H)	=	-50 kt

Determine the time and distance from S to the point of equal time between S and T for the one engine inoperative configuration.

SOLUTION

(a) Calculate the groundspeeds.

	3 ENG		4 ENG
G/S(O)	160 kt	G/S(G)	210 kt
G/S(H)	260 kt		

(b) Calculate the distance to the one-engine inoperative PET. Remember to use the 3 engine groundspeeds in the formula.

$$\text{Distance to PET (x)} = \frac{690 \cdot 260}{260 + 160} = 427 \text{ nm}$$

(c) Calculate the time to the PET using the 4 Engine G/S Out.

$$\text{Time} = \frac{\text{Distance (x)}}{\text{G/S(G)}} \times 60 = \frac{427}{210} \times 60 = 122 \text{ mins}$$

Multi – Leg Point of Equal Time

16. As with the single leg point of equal time, the solution of multi-leg problems is based on the simple premise that the **time** from the PET **on** to the destination must equal the **time** from the PET **home** to the point of departure.

17. In general terms, the method used to resolve the multi-leg problem is to initially eliminate sections of the route to leave a simple single leg problem. This single leg problem is then solved in the manner already detailed in the previous section.

EXAMPLE 18-4

EXAMPLE

An aircraft is to fly from A to C via B. Details are as follows:

Leg	Distance	TAS	Wind Component
A-B	120 nm	120 kt	-30 kt
B-A	120 nm	120 kt	+30 kt
B-C	160 nm	120 kt	-24 kt
C-B	160 nm	120 kt	+20 kt

Determine the time and distance from A to the PET between A and C.

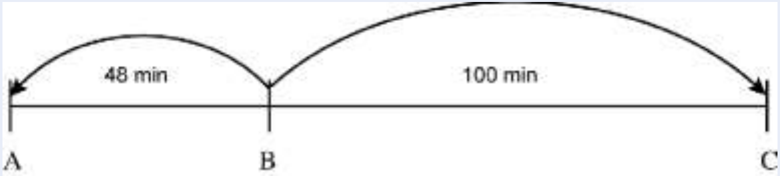
SOLUTION

(a) To accomplish the solution systematically complete the calculation of groundspeeds and times in tabular form as below.

Leg	G/S	Time
A-B	90 kt	80 min
B-C	96 kt	100 min
C-B	140 kt	68.5 min
B-A	150 kt	48 min

(b) Having calculated the time in each direction on each leg follow the convention in the construction of the diagram below.

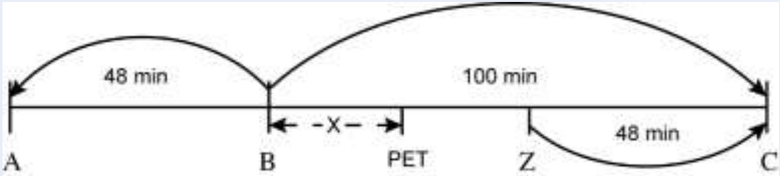
FIGURE 18-2



From Figure 18-2 it can be seen that point B is not the point of equal time between A and C, as the time from B **home** to A is 48 minutes whereas the time from B **on** to C is 100 min.

The next step is to introduce a false point (Z) from which it will take , in this example, 48 min to fly on to C. (The lesser time value of the two used in Figure 18-2 is always used). We now have two equal brackets of time on the diagram; from B **home** to A takes 48 minutes and from Z **on** to C also takes 48 minutes. The times are balanced and need no further consideration

FIGURE 18-3



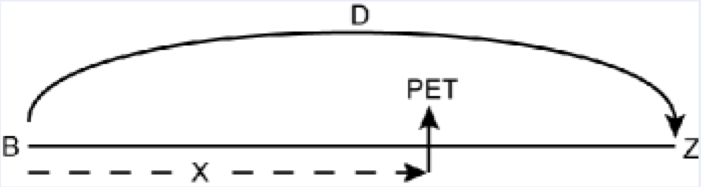
Now consider the leg BZ in Figure 18-3. **If the point of equal time between B and Z is now found it will also be the point of equal time between A and C**.

(c) The leg BZ is now treated as a single leg for which the **point of equal time** formula can be used to solve the problem:

$$\text{Distance to PET (x)} = \frac{DH}{O + H}$$

Figure 18-4 is an extract from the main diagram:

FIGURE 18-4



All factors in the formula are known except the distance (D) which is the distance BZ. The distance BZ can be determined as follows:

(a) Take the difference in the times on the leg BC in the construction: 100 min – 48 min = 52 min = leg time BZ.

(b) Use the groundspeed in the direction of the arrow on the leg BC (i.e. in this case the groundspeed **on** of 96 kt) and the time above to calculate the distance BZ (which remember is D in the formula):

$$\text{Distance BZ} = \frac{96}{60} \times 52 = 83 \text{ nm}$$

All factors in the formula are now known. Remember that the single leg BZ is part of the leg BC and so the G/S(O) and G/S(H) for the leg BC must be used in the formula.

Now calculate the distance B to PET:

$$\text{Distance to PET (x)} = \frac{DH}{O + H} = \frac{83 \cdot 140}{96 + 140} = 49 \text{ nm}$$

Next calculate the time from B to the PET:

$$\text{Time} = \frac{\text{Distance}}{\text{G/S Out}} \times 60 = \frac{49}{96} \times 60 = 30.6 \text{ min}$$

G/S out is the all engines operating groundspeed from the departure point and is that used to calculate the time to the point of equal time irrespective of which type of point of equal time it is. The groundspeeds used to determine the position of the PET are those for the configuration of the PET e.g. one engine inoperative.

The distance and time between B and PET are now known and the original question must now be addressed which was to find the distance and time from A to the PET.

This is found by adding the tabulated values of time and distance for the leg AB to the values calculated over.

Distance A to PET = 120 + 49 = 169 nm

Time A to PET = 80 + 30.6 = 110.6 min

EXAMPLE

An aircraft is to fly from K to N (via L and M). Details are as follows:

Leg	Distance	Track
K to L	175nm	135(T)
L to M	348 nm	105(T)
M to N	197 nm	087(T)

4 Engine TAS is 380 kt   The W/V is 145/45 kt

3 Engine TAS is 315 kt

Determine the time and distance from K to the PET between K and N for the one engine inoperative configuration.

SOLUTION

Leg	Dist	GS(3Eng)	Time(3Eng)	GS(4Eng)	Time(4Eng)
K to L	175	271	38.5	334	31.4
L to K	175	359	29		
L to M	348	280	74.5	342	
M to L	348	347	60		
M to N	197	290	41	352	
N to M	197	335	35.5		

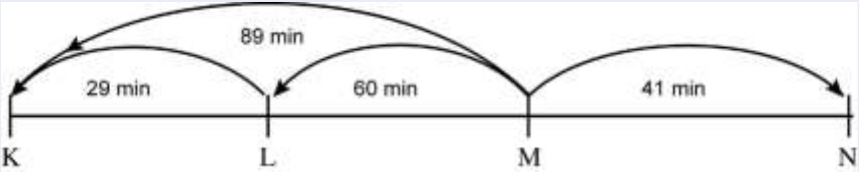
Because the one-engine inoperative PET is required, the 3 engine G/S(O) and G/S(H) is used in the diagram and formula but the 4 engine G/S Out (G) is used to calculate the time to the PET.

The following construction is initially required:

FIGURE 18-5

In this construction there is a greater amount of time ON (i.e. 41 min) than there is HOME (29 min). In an attempt to balance the time ON and HOME the time of leg M to L is now used (i.e. more time going HOME) to attempt to achieve a balance.

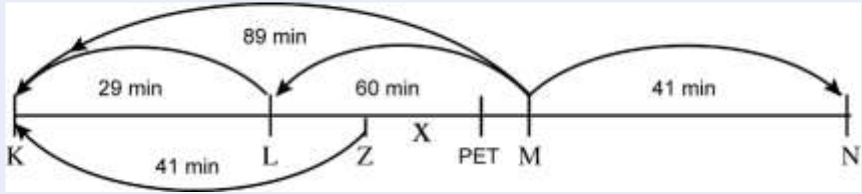
FIGURE 18-6



In Figure 18-6 it can be seen that the total amount of time of 89 minutes **HOME** and 41 minutes going **ON.** Obviously M is not the PET but the procedure is now the same as the previous two leg problem. Take the lesser of the two time values (41 min) and construct point Z as shown in Figure 18-7 below. This identifies the leg Z to M as the single leg problem which can be resolved using the point of equal time formula.

The point of equal time lies somewhere on the leg Z to M and the distance(x) is that between Z and the PET.

FIGURE 18-7



The first step is to calculate the distance Z to M which will become the distance **(D)** in the formula.

To calculate this distance adopt the following procedure:

Take the time difference between 89 min and 41 min; 89min – 41 min = 48 min

Use the groundspeed on this leg (ZM is part of the leg ML) in the direction of the arrow on that leg. In this case this will be the groundspeed ML (347 kt).

$$\text{Distance ZM} = \frac{347}{60} \times 48 = 277.6$$

Next, resolve the single leg problem using the point of equal time formula (remember to use 3 Eng G/S):

$$\text{Distance to PET (x)} = \frac{DH}{O + H} = \frac{277.6 \cdot 347}{280 + 347} = 153.6 \text{ nm}$$

At this stage because the distance (x), Z to PET, lies in the ‘middle’ of the leg L to M there is no point in calculating the time because the time L to PET will have to be calculated later.

Next calculate the TOTAL DISTANCE from K to PET:

Distance K to PET = Distance K to L + Distance L to Z + Distance (x).

Distance K to L is tabulated: 175 nm

Distance L to Z = Distance L to M **minus** Distance Z to M calculated in an earlier step = 348 – 227.6 = 120.4 nm

Distance (x) = 153.6 nm

Therefore, distance K to PET = 175 + 120.4 + 153.6 = 449 nm.

Next, calculate the TOTAL TIME from K to PET **(remember to use 4 Eng G/S OUT on each leg):**

Time K to PET = Time K to L + Time L to PET

Time K to L is tabulated: 31.4 min

$$\text{Time L to PET} = \frac{\text{Distance L to PET}}{\text{4 eng G/S Out}} \times 60 = \frac{274}{342} \times 60 = 48 \text{ min}$$

Therefore Time K to PET = 31.4 + 48 = 79.4 min

Point of Safe Return

R of A Formula Method 1

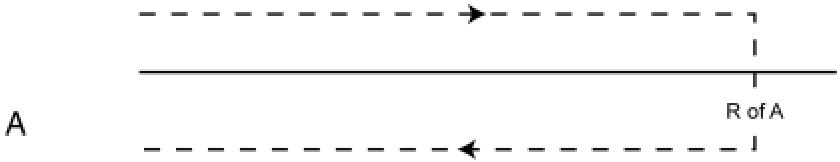
Multi-Leg PSRs

Multi–Leg PSR using the R of A Formulae

Point of Safe Return

1. The point of safe return (PSR) is the most distant point along track from which it is possible to return to a nominated point arriving with a specified reserve of fuel.

FIGURE 19-1
The Radius of Action



2. The radius of action is very similar to the PSR except the return is to the point of departure. For instance an aeroplane en route from Montreal to London overflies Gander which is 300 nm from Montreal. However the whole of Europe is fog bound and the forecasters are unable to predict the time of clearance. The operators instruct the pilot to continue towards Europe until the PSR for Gander is reached and if the fog has not cleared by that time to return to Gander. He/she is subsequently informed that only Montreal has sufficient accommodation available for all the passengers. Therefore the latest time to turn back to Montreal would be at the limit of the radius of action from Montreal allowing for a safe reserve of fuel at Montreal.

3. The determination of a PSR or R of A can be made by calculation, by fuel graph or plotting. In this Chapter only the calculation method is considered. This is by a formula which is the same for both the PSR and R of A. There are two formula methods. The first assumes a constant fuel flow and a constant TAS. The second accounts a change of fuel flow and TAS on the return.

R of A Formula Method I

4. The distance travelled from the departure aerodrome to the limit of the radius of action and back to the departure point are the same. The endurance is calculated in hours. First subtract the reserve fuel from the total fuel available on departure then divide the remainder by the fuel flow. The total of the time outbound and the time inbound must not exceed the calculated endurance.

The formula can be derived from the above as:

Time out + Time home = Safe Endurance

$$\frac{\text{R of A}}{\text{O}} + \frac{\text{R of A}}{\text{H}} = \text{E}$$

Multiply by O and H: $[(\text{R of A}) \times \text{H}] + [(\text{R of A}) \times \text{O}] = \text{EOH}$

Isolate R of A: $\text{R of A} \times [\text{O} + \text{H}] = \text{EOH}$

$$\text{R of A} = \frac{\text{EOH}}{\text{O} + \text{H}}\text{nm}$$

Where E is endurance in hours

O = groundspeed outbound

H = groundspeed inbound (home)

To find the time to the R of A divide the distance by the groundspeed outbound.

$$\text{Time to radius of action} = \frac{EH}{O + H} \text{hours}$$

Because the calculation method can deal only with alternate aerodromes along the route, then the fuel available overhead the alternate must be calculated before this method may be used.

In the following example both formulae are demonstrated:

EXAMPLE 19-1

EXAMPLE

An aircraft is to fly outbound from A on a track of 270(T). Given:

Total Endurance	8 hours
Reserve endurance required in return to A	2 hours
W/V	270/45 kt
TAS	240 kt

Determine the time and distance from A to the R of A.

SOLUTION

R of A Endurance	=	8 hour-2 hours
	=	6 hours
G/S(O)	=	195 kt
G/S(H)	=	285 kt

Time to R of A (in hours)	=	$\frac{6 \cdot 285}{195 + 285}$
	=	3.56 hours
	=	3 hours 34 minutes
	=	3.56 hours @ G/S(O) 195 kt
Distance to R of A	=	695 nm

Logically, as the outbound portion of the flight is into a headwind, the flight out to the R of A will take more than half of the R of A endurance; and this is the case.

A check sum can be made to confirm the R of A principle that the total time taken for the flight out and back should equal the R of A endurance.

The flight out took 3.56 hours. The flight back will take:

Distance R of A to A	=	695 nm
Groundspeed R of A to A	=	285 kt
Time R of A to A	=	2.44 hours
	=	
Total flight time	=	3.56 + 2.44
	=	6 hours
	=	original R of A endurance

Therefore the answers are correct.

The alternate formula gives identical results. Remember to use endurance in hours and decimals in this formula, not minutes.

Example 5-1 is reworked below:

$$\text{Distance to R of A} = \frac{E \times O \times H}{O + H}$$

$$= \frac{6\ (\text{hrs}) \times 195 \times 285}{195 \times 285}$$

$$= 695\ \text{nm}$$

Time to R of A = 695 nm @ G/S(O) 195 k

3.56 hrs

3 hrs 34 mins

5. Both formulae considered above require that the endurance of the aircraft be known. The endurance can only be calculated if the TAS and Fuel Flow throughout the period of flight are constant (or assumed to be). If the TAS or Fuel Flow change then the endurance cannot be calculated and the R of A formulae cannot be used.

6. A number of situations can cause a change in TAS and/or Fuel Flow, the most obvious example being an engine failure which will cause an instantaneous change. As in all of these situations, the engine failure is assumed to take place at the worst possible time i.e. when the aircraft is at the furthest possible point from base. Engine failure is therefore assumed to occur **at** the R of A.

7. As it is not known how long the aircraft has flown on full power and how long it has flown on reduced power then the endurance cannot be calculated and therefore the PNR formulae cannot be used. The problem can now only be resolved by using the **Gross Fuel Flow** concept. This method is the only way to solve the problem if, even without engine failure, the aircraft has to change the cruising level on the return flight for ATC reasons. In such a case the TAS, fuel flow and probably the wind component will all be different.

The Radius of Action Method 2

8. The Gross Fuel Flow (GFF) formula calculates the fuel used (in kg) for each ground nautical mile travelled.

$$\frac{\text{Fuel Flow Kgs/hr}}{\text{Groundspeed kts}} = \frac{F/F}{G/S} = \text{GFF(kg/gnm)}$$

9. If the fuel used for each nautical mile travelled out to the R of A is added to the fuel used for each nautical mile travelled back from the R of A then the result is the fuel used per nautical mile of radius of action along the track considered.

10. The leg out to the R of A and the leg back from the R of A are considered separately in the application of the GFF formula and the subsequent values are added together to give the radius of action GFF. The following basic example illustrates these principles:

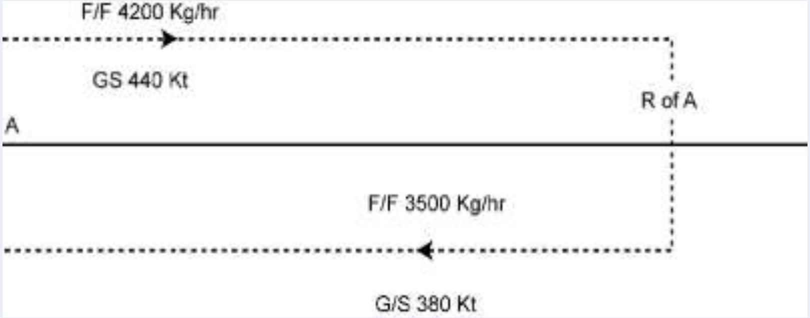
EXAMPLE 19-2

EXAMPLE

Given:

An aircraft is flying from A to B. The F/F and G/S values are indicated in Figure 19-2. R of A fuel available is 36,000 kg.

FIGURE 19-2



Determine the time and distance from A to R of A assuming the change in F/F and G/S occurs at the R of A because of a change of flight level on return.

SOLUTION

A to R of A	GFF Out	$\frac{F/F}{G/S}$	$\frac{4200}{440}$	= 9.5 kg/gnm
R of A to A	GFF Home	$\frac{F/F}{G/S}$	$\frac{3500}{380}$	= 9.2 kg/gnm

Total GFF = 9.5 + 9.2 = 18.7 kg/gnm

If this GFF figure is divided into the R of A fuel available then the distance to the R of A can be calculated

$$\text{Distance to R of A} = \frac{\text{R of A fuel available}}{\text{Total GFF}} = \frac{36000}{18.7} = 1925.1 \text{ nm}$$

If it is required to calculate the time to the R of A:

$$\text{Time to the R of A} = \frac{\text{Dis tance}}{\text{G/S(O)}} = \frac{1925.1}{440} = 4.375 \text{ hrs} = 4 \text{ hrs } 22.5 \text{ mins}$$

Note that the G/S(O) is **always** used to calculate the time **to** the R of A.

EXAMPLE 19-3

EXAMPLE

An aircraft is to fly from Q to R.

Given:	TAS (4 engines)	=	310 kt
	TAS (3 engines)	=	270 kt
	Fuel flow (4 engines)	=	2100 kg/hr
	Fuel flow (3 engines)	=	1700 kg/hr
	Wind component outbound	=	+ 45 kt
	Fuel on board (excluding reserves)		9500 kg

Determine the time and distance to the R of A assuming the loss of one engine at the R of A and consequent change of flight level.

SOLUTION

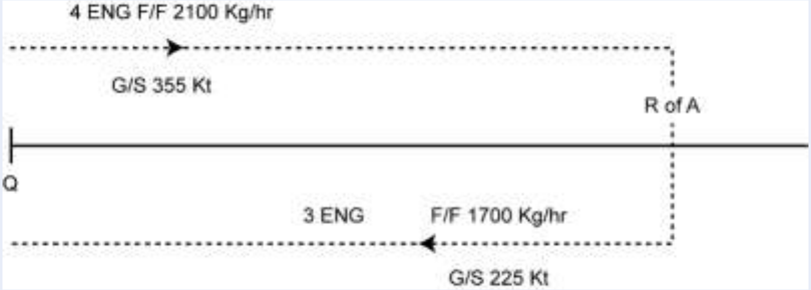
1) Determine the groundspeed out G/S(O) on 4 engines and the groundspeed home G/S(H) on 3 engines.

G/S(O) = TAS 310 kt + 45 kt Wind component = 355 kt.

G/S(H) = TAS 270 kt – 45 kt Wind component = 225 kt

2) The diagram below Figure 19-4 illustrates the flight details:

FIGURE 19-3



Now apply the GFF formula:

Q to R of A	GFF Out	$= \frac{F/F}{G/S}$	$= \frac{2100}{355}$	= 5.92 kg/gnm
R of A to Q	GFF Home	$= \frac{F/F}{G/S}$	$= \frac{1700}{225}$	= 7.56 kg/gnm

Total GFF = 5.92 + 7.56 = 13.48 kg/gnm

3) Calculate the distance to the R of A:

$$\text{Distance to R of A} = \frac{\text{RofA fuel available}}{\text{GFF}} = \frac{9500}{13.48} = 704.7 \text{ nm}$$

4) Calculate the time to the R of A:

$$\text{Time to R of A} = \frac{\text{Distance to R of A}}{\text{G/S(O)}} = \frac{704.7}{355} = 1.99 \text{ hrs} = 1 \text{ hr } 59 \text{ mins}$$

A check sum can be carried out by calculating the fuel used out to the PNR and the fuel used home from the PNR. The sum of the two should approximately equal the PNR fuel.

Time to R of A	=	1.99 hrs
Fuel flow Q to R of A	=	2100 kg/hr
Fuel used Q to R of A	=	4179 kgs
Time R of A to Q (704.7nm @ G/S(H) 225 kt	=	3.13 hrs
Fuel flow R of A to Q	=	1700 kg/hr
Fuel used R of A to Q	=	5321 kgs
Fuel used Q to R of A to Q	=	9500 kgs

Multi-Leg PSRs

11. Multi-leg R of A questions are an additional area of study required at ATPL level; a slightly more complex method of working is required to achieve the solution.

12. The more common examination questions deal with the Gross Fuel Flow formula, covered in the previous single leg R of A section, and this will be dealt with first. The use of the basic R of A formulae will be covered later in this chapter.

13. To solve the multi-leg R of A question requires a process of calculating the fuel required for each leg in turn for the outbound and return flight on that leg. This fuel required is deducted from the total fuel available and, in effect, that leg and its associated fuel required is eliminated from the calculation. The fuel available reduces throughout this process. Each leg is addressed in turn until there is insufficient fuel available to complete the return journey on the next leg addressed. The R of A lies on that leg and the Gross Fuel Formula is used to complete the calculation to determine the time and distance from the point of departure to the R of A.

14. First complete all of the flight details on a standard flight log card for the outbound and the return passage of each leg of the flight. This will also need to include the fuel flow and fuel required for the outbound and return passage on each leg.

15. The following example illustrates the process of compiling this information and how it is used in the elimination process mentioned above.

EXAMPLE 19-4

EXAMPLE

An aircraft is to fly from K to N via two other points at L and M. Flight details are:

Leg	Dist	W/C
K to L	285 nm	+ 28 kt
L to M	380 nm	+15 kt
M to N	340 nm	+23 kt

TAS 4 engines	320 kt
TAS 3 engines	290 kt
Fuel flow 4 engines	4320 kg/hr
Fuel flow 3 engines	3960 kg/hr
Fuel on board at take-off	25,000 kg
Reserve Fuel required on return to K	3,500 kg

Determine the time and distance from K to the R of A from K, assuming the loss of one engine at the R of A.

SOLUTION

1. Calculate the R of A fuel available at K:

R of A fuel available at K for R of A = 25,000 – 3500 = 21,500 kg.

2. Calculate the fuel required on the outbound and return elements of each leg.

Note that the loss of one engine at the R of A means that you fly out to the R of A on 4 engines and return on 3 engines. Therefore, 4 engine values must be used in the calculation of the fuel required on the outbound element of each leg and 3 engine values used for the calculation of the fuel required for the return element of each leg.

Leg	Dist	G/S	Time	F/Flow	Fuel Reqd
K to L	285	348	49	4320	3540
L to K	285	262	65.5	3960	4310
L to M	380	335	68	4320	4900
M to L	380	275	83	3960	5470
M to N	340	343	61	4320	4392
N to M	340	267	76	3960	5016

3. Next, address and eliminate each leg in turn until the leg on which the R of A lies is identified.

Firstly, calculate the fuel required for the leg K to L and L to K.

Leg	Dist	G/S	Time	F/Flow	Fuel Reqd
K to L	285	348	49	4320	3540
L to K	285	262	65.5	3960	4310

FIGURE 19-4

Subtract this 7,850 kg from the R of A fuel available.

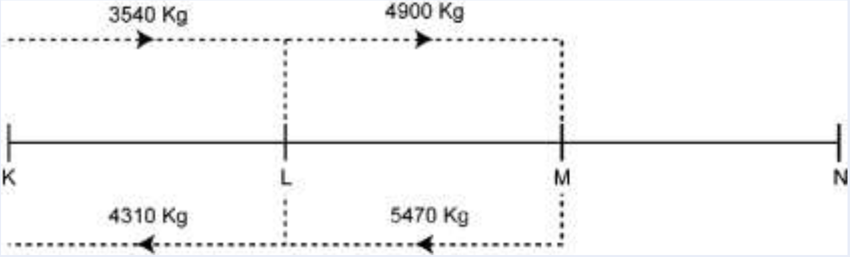
R of A fuel remaining at L = 21.500 – 7850 = 13,650 kg.

The leg K to L, and the fuel required to travel from K to L and L to K, has now been eliminated from the calculation.

Now address the next leg, L to M and calculate the fuel required for the leg L to M and M to L

Leg	Dist	G/S	Time	F/Flow	Fuel Reqd
L to M	380	335	68	4320	4900
M to L	380	275	83	3960	5470

FIGURE 19-5



Fuel required=10,370kg
Subtract this 10,370 kg from the R of A fuel available.

R of A fuel remaining at M = 13,650 – 10,370 = 3,280 kg

The leg L to M, and the fuel required to travel from L to M and M to L, has now been eliminated from the calculation.

The final leg M to N must now be addressed

From the initial log card it can be seen that the fuel required from M to N and back to M is 9,408 kg.

Leg	Dist	G/S	Time	F/Flow	Fuel Reqd
M to N	340	343	61	4320	4392
N to M	340	267	76	3960	5016

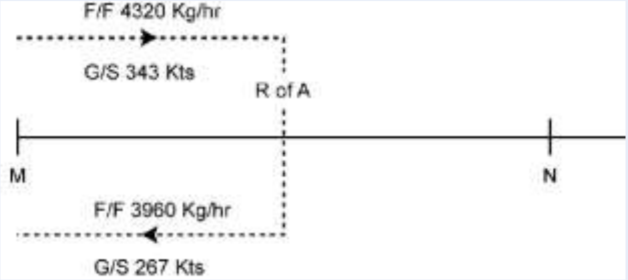
However, as there is only 3,280 kg of fuel available at M then obviously the return journey over the full distance of this leg cannot be completed and therefore the R of A must lie somewhere on the leg M to N.

To determine the exact position of the R of A apply the Gross Fuel Flow formula as before. The Gross Fuel Flow Formula is:

$$GFF = \frac{F/F}{G/S}$$

We can initially treat the leg M to N as a single leg problem to determine the position of the R of A on this leg and then go back and address the multi- leg problem.

FIGURE 19-6



Note. In the elimination of the previous legs we have been dealing with fuel required which we have been subtracting from the R of A fuel available. In the Gross Fuel Flow formula we are now dealing with fuel flow. These figures may be similar in value to one another on your flight log card, as can be seen below, and care must be taken to work with figures from the correct column

Leg	Dist	G/S	Time	F/Flow	Fuel Reqd
M to N	340	343	61	4320	4392
N to M	340	267	76	3960	5016

As in the single leg R of A problem, the values of fuel flow and ground speed on the outbound and on the return leg are considered in the Gross Fuel Flow calculation.

As already determined, the R of A fuel available at M = 3280 kg

$$\text{GFF out} = \frac{F/F}{G/S} = \frac{4230}{343} = 12.59 \text{ kg/gnm}$$

$$\text{GFF home} = \frac{F/F}{G/S} = \frac{3960}{267} = 14.83 \text{ kg/gnm}$$

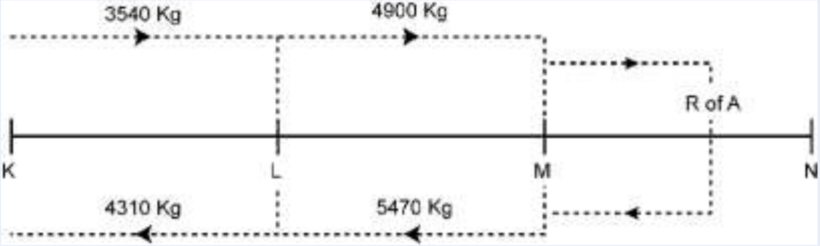
Total GFF = 12.59 + 14.83 = 27.42 kg/gnm

$$\text{Distance M to R of A} = \frac{3280}{27042} = 119.6 \text{ nm}$$

$$\text{Time from M to R of A} = \frac{\text{Dist}}{G/S} \times 60 = \frac{120}{343} \times 60 = 21 \text{ min}$$

This last section of working has dealt entirely with the single leg problem. It is now necessary to address the initial multi–leg problem which was to **determine the time and distance from K to the R of A.**

FIGURE 19-7



From the flight log card the distances and times out for the legs K to L and L to M are tabulated. It is now simply a matter of adding these times and distances to the time and distance calculated from M to the R of A.

The total distance from K to R of A:

Total distance = Dist K to L + Dist L to M + Dist M to R of A
= 285 + 380 + 120
= 785 nm

The total time from K to the R of A (ensure you use the time OUT on each leg):

Total time = Time K to L + Time L to M + Time M to R of A
= 49 + 68 + 21
= 138 min
= 2 hrs 18 mins

Multi–Leg PSR using the R of A Formulae

16. It is possible that an examination question may be orientated towards working with 'endurance available' instead of 'fuel available'.

In this type of question the R of A Formulae can be used in the final stages of the calculation. However, as this is a multi-leg question it will be necessary once again to adopt the same method of eliminating some legs, as described in the previous example, to determine a single leg problem on which the R of A formulae can be used.

EXAMPLE 19-5

EXAMPLE

An aircraft is to fly from A to D, via B and C. The aircraft has sufficient fuel onboard for 6 hours flying but, in the event of returning to A it must have a reserve of 60 minutes fuel onboard on arrival.

Flight details are:

Leg	Dist	W/C
A to B	320 nm	-30 kt
B to C	245 nm	-20 kt
C to D	485 nm	-35 kt

TAS for the flight is 280 kt.

Determine the time and distance from A to the R of A.

SOLUTION

1) R of A endurance = 360 – 60 = 300 min

2) Compile the Flight Log Card as in the previous example without the fuel columns.

Leg	Dist	G/S	Time
A to B	320	250	77
B to A	320	310	62
B to C	245	260	56.5
C to B	245	300	49
C to D	485	245	118.8
D to C	485	315	92.4

3) Address each leg in turn, subtracting the time taken (out and back) from the R of A endurance available.

Leg	Dist	G/S	Time
A to B	320	250	77
B to A	320	310	62
		Total	139 min

Endurance remaining at B = 300 – 139 = 161 min.

Leg	Dist	G/S	Time
B to C	245	260	56.5
C to B	245	300	49
		Total	105.5 min

Endurance remaining at C = 161 – 105.5 = 55.5 min.

Leg	Dist	G/S	Time
C to D	485	245	118.8
D to C	485	315	92.4
		Total	211.2 min

Obviously there is insufficient endurance available at C, only 55 min, to complete the leg C to D and back to C and hence the R of A lies on this leg.

The R of A formula can now be used to determine, on the leg C to D, the distance and time from C to the R of A.

$$\text{Time from C to R of A} = \frac{E \cdot H}{O + H} = \frac{55.5 \cdot 315}{245 + 315} = 31 \text{ min}$$

$$\text{Distance from C to R of A} = \frac{245}{60} \times 31 = 127 \text{ nm}$$

The final step is to then answer the multi-leg question; determine the time and distance from A to the R of A.

From the flight log card details above:

Total distance = 320 + 245 + 127 = 692 nm

For total time remember to use the time OUT on each leg:

Total time = 77 + 56.5 + 31 = 164.5 min.

Self Assessed Exercise No. 11

QUESTION:

QUESTION 1.

See FIGURE 114 and FIGURE 116 in the reference book.

1125	O/H 'W' 52° 00' N 010° 00' W en route to 'S' 43° 25' N 004° 00' W at FL330; MO.70. Forecast Met information: temp -60°C; W/V 230/90.
1135	'W' VOR/DME 350° (RMI) /64nm; TAS 400 kts.
1138	Doppler information: 7° port drift; G/S 390kts. A/H to parallel (//) Track.
1145	'U' DME 139 nm.
1200	'T' VOR 096° (RMI).
1202	A/H // Track.

1215 ‘T’ NDB 049° (RMI) Doppler G/S 380 kts.

1225 ‘S’ VOR 157° (RMI).

1231 DR A/H for ‘S’.

Determine

What is the initial Hdg (°M) calculated at 1125?

What is the fix position obtained at 1135?

What was the Mean W/V 1125-1135 (use mean heading 165°T and TAS 400 kts)?

Given a desired track of 153°(T) what is the heading (°T) to fly calculated at 1138 to parallel track?

What is the fix position obtained at 1200?

What is the fix position obtained at 1225?

What is the mean heading (°M) for ‘S’ calculated at 1231? (use the forecast W/V 210/70; TAS 400 kts).

What is the revised ETA for ‘S’ calculated at 1231?

ANSWER 1.

(a) Initial heading is based on initial Tk (153°T) gives 166°(T) as heading. Variation is 11°W at 'W'; therefore, heading is 177° (M)

(b) Simultaneous fix: 51° 00'N 009° 23'W

(c) TMG from 'W' to fix position is 159°(T); distance covered in 10 minutes is 64 nm giving G/S 384 kts. Heading and TAS are given. Mean W/V (from Nav Computer) is 233/43.

(d) Using the Nav Computer, set 153° (required track) against 7° port drift on the drift scale; read off the heading under the True Heading marker: 160°T

(e) First line is from DME; therefore, must transfer the beacon. Altered heading to parallel track at 1138; therefore, transfer the beacon parallel to planned track for 15 minutes of G/S (390 kts) i.e. 97½ nm. Plot the range (139 nm) from the false origin / DME position. The 2nd position line gives a bearing to plot of 268°T.

The fix position obtained was: 47° 56'N 007° 05'W

(f) The first line is from NDB 'T'; therefore, a DR position must be constructed using latest available Track and G/S from the last fix position. The DR position is required to obtain the variation at the aircraft (8°W) and to transfer the aircraft's meridian to the NDB (006°W) to allow for convergency.

At 1202 the narrative tells us we are paralleling track; the ground speed then was 390kts. The DR position can be constructed starting from the 1200 fix position by drawing the track vector parallel to track for 97½ nm (15 mins at 090 kts)

The bearing to plot from NDB is 221° (remembering to measure from the aircraft's transferred meridian). The NDB position line must be transferred along track for 10 minutes of the G/S noted at 1215 (380 kts – 63 nm).

The bearing to plot from the VOR is 330°T (variation at the beacon = 7°W).

The fix position obtained at 1225 is then: 45° 33'N 005° 44'W.

(g) In order to work out the new heading to fly, must first produce a DR position for 1231 from which the new required track can be drawn to the destination, 'S'.

There is no change in heading between 1225 and 1231 so we can assume we are still paralleling track. The G/S at 1215 was 380kts; 6 minutes at 380kts = 38nm.

A mean heading is required, so measure the mean Tk (148°T); this gives a true heading 157°. Apply variation at the mid-point (7°W), resulting in a heading of 164° (M).

(h) The distance from the DR position to the destination is 111nm; G/S from the Nav Computer is 360kts giving a time of 18.5 minutes. ETA is therefore, 1249½

See FIGURE 66 for completed Flight Navigational Log and FIGURE 180 for Plotting Chart in the Reference Book

Relative Velocity

Collision Risk Principle

Aircraft Velocity Relative to a Fixed Point

Relative Velocities by Calculation

Changing Speed Problems

Relative Velocity

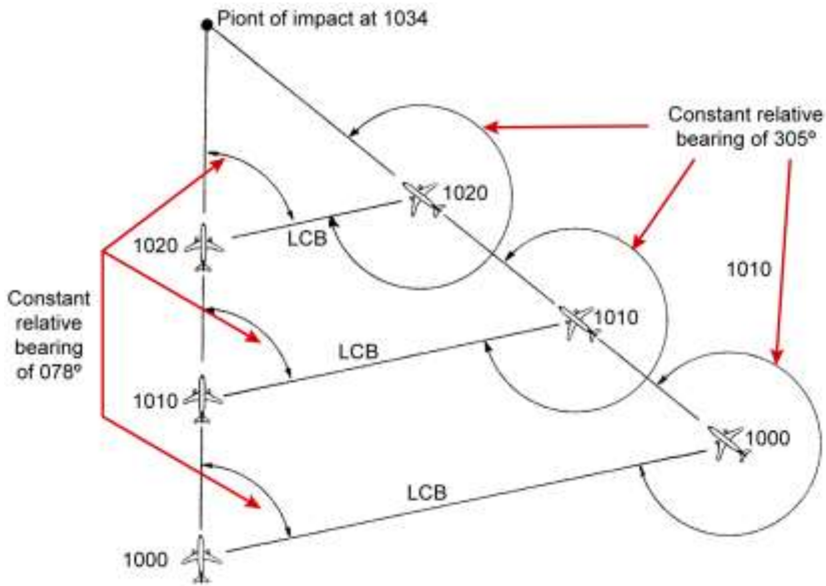
1. Relative velocity problems, solved either by construction or by calculation, are dealt with in this chapter.

Collision Risk Principle

2. The first type of problem is the determination of the point and/or time of collision of two aircraft having constant velocities.

3. The solution is by scale diagram. There is a risk of collision if **the relative bearing of another aircraft at the same height remains constant.** The line joining the two aircraft is known **as the line of constant bearing (LCB)**, Figure 20-1 illustrates. Any wind which is present is assumed to equally affect both aircraft, and is therefore ignored.

FIGURE 20-1



Aircraft Velocity Relative to a Fixed Point

4. This type of problem, which is also solved by construction, concerns an aircraft's changing position relative to a fixed point on the ground. These problems may be set in the context of fixes obtained using the airborne weather radar in the mapping mode.

EXAMPLE 20-1

EXAMPLE

The following observations were obtained of an unidentified ground feature using an airborne weather radar in the mapping mode:

Time	Relative Bearing	Range
1234	015°	95 mm
1255	060°	40 mm

Assuming that the aircraft TAS is 350 kt and that the heading is 105°(T), determine

(a) The average groundspeed

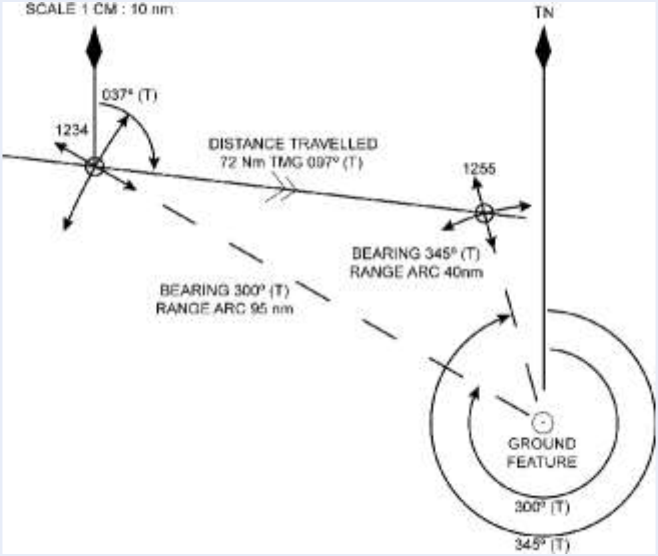
The mean wind velocity

SOLUTION

See Figure 20-2.

The method of solution is as follows:

FIGURE 20-2



(i)	Determine the bearings to plot from the ground feature to the aircraft at 1243 & 1255.			
	1243	Hdg 105°(T), Rel brg 015°	=	120° +180°
			=	300°(T) to plot
	1255	Hdg 105°(T), Rel brg 060°	=	165° +180°
			=	345°(T) to plot

(ii) Position the unidentified ground feature at a convenient point on the paper, in this case the bottom right hand corner since the bearings to plot are to the north-west.

(iii) Choose a suitable scale and plot the aircraft fix positions for times 1243 and 1255.

(iv) Join the two fix positions and measure the track made good, in this case 097°(T). Measure the distance between the fixes and calculate the groundspeed.

(a)	72 nm in 12 mins, groundspeed		=	360 kt.
(v)	Knowing that:	Hdg = 105°		TAS = 350 kt
		Tk = 097°(T)		G/S = 360 kt

(b) Compute the W/V as 200°(T)/50 kt.

EXAMPLE 20-2:

EXAMPLE

An aircraft is heading 016°(T), drift 10° starboard, groundspeed 350 kt. Two bearings of a ground feature are obtained using the airborne weather radar. The first bearing is 330° (relative) and the second bearing, which is taken 6 minutes later, is 270° (relative). Determine the true bearing and the range of the aircraft from the ground feature at the time of the second bearing.

See Figure 20-3. The method of solution is as follows:

SOLUTION

(i) Determine the bearings to plot, this time from the aircraft's estimated ground positions at the times of both observations.

Time 0	Hdg 016°(T), Rel brg 330°	= 346°(T) (to the feature)
Time 0+6	Hdg 016°(T), Rel brg 270°	= 286°(T) (to the feature)

(ii) Plot the aircraft's ground position for time 0 at bottom centre of the paper.

(iii) Draw a line representing the bearing of 346°(T), originating at the aircraft's position.

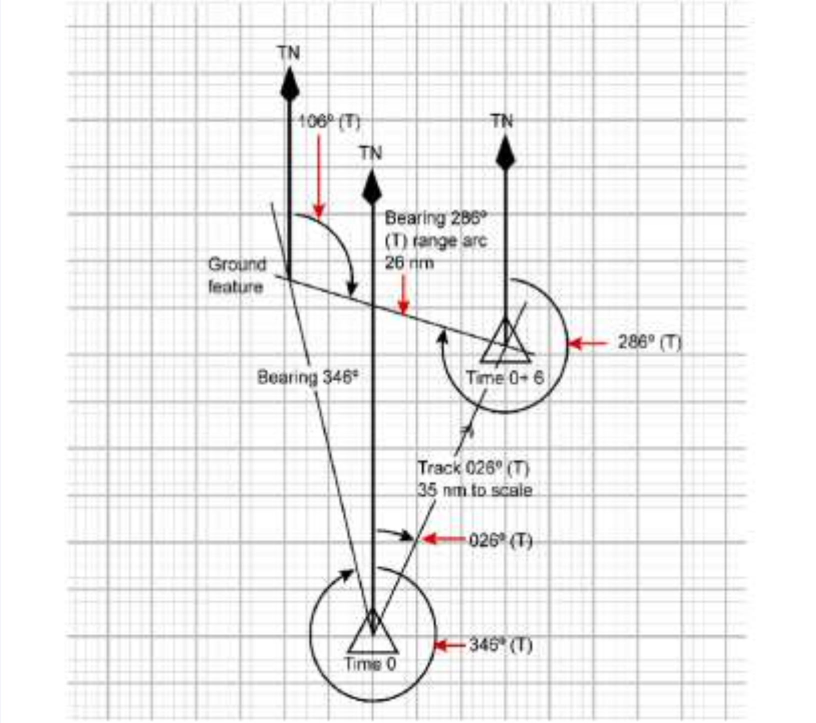
(iv) Draw a line representing the aircraft's track (hdg 016° + 10° stbd) of 026°(T), again originating at the aircraft's position at time 0.

(v) Using a sensible scale, measure a distance of 35 nm (6 minutes at a groundspeed of 350 kt) from the aircraft's original position, along the track to establish the aircraft's position at the time of the second observation.

(vi) From the second aircraft position draw a line representing the bearing of 286°(T). The point where the two bearing lines cross represents the ground feature.

(vii) Using the original scale, measure the distance from the aircraft's position at the time of the second observation to the ground feature. In this case the range is measured as 26 nm. The true bearing of the aircraft from the ground feature is of course (286 - 180) 106°(T).

FIGURE 20-3



TN
106° (T)
TN
TN
Bearing 286° (T) range arc 26 nm
Ground feature
286° (T)
Bearing 346°
Time 0+ 6
Track 026° (T) 35 nm to scale
026° (T)
345° (T)
Time 0

Relative Velocities by Calculation

5. Aircraft travelling along the same track in the same or opposite directions have a relative velocity. For this type of problem the solution may be found by calculator. Plotting is unnecessary although it is beneficial to draw a diagram.

6. If the aircraft are travelling in the same direction along a given track, the relative velocity is the **difference between their groundspeeds** but in opposite directions it is the **sum of their groundspeeds**.

EXAMPLE 20-3

EXAMPLE

At time 1130 aircraft A is overhead point X, tracking 090°(T) and maintaining a groundspeed of 320 kt.

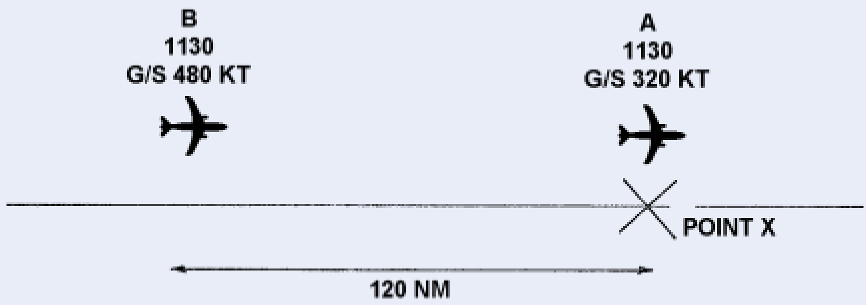
At time 1130 aircraft B is 120 nm west of point X, tracking 090°(T) and maintaining a groundspeed of 480 kt.

Calculate the time and distance from X at which aircraft B will pass aircraft A.

SOLUTION

See Figure 20-4

FIGURE 20-4



The relative velocity of B to A is (480 - 320) = 160kt. The distance to close = 120 nm. The time to close = $\frac{120}{160} \times 60$ = 45 mins. Ground distance travelled in 45 mins is:

A = 45 mins @ 320kt = 240 nm
B = 45 mins @ 480kt = 360 nm

The aircraft will pass at 1130 + 45 = 1215 at a position 240 nm of point X.

EXAMPLE 20-4

EXAMPLE

Aircraft C at FL 360, TAS 460 kt, wind component -75 kt, estimates point Q at 1825.

Aircraft D at FL 320, TAS 500 kt, wind component -60 kt, estimates point Q at 1830.

Both aircraft are on the same track.

Determine the time at which aircraft D will pass aircraft C.

SOLUTION

See Figure 20-5

Aircraft C, groundspeed (460 - 75) = 385 kt. At 1825 the aircraft should be overhead point Q.

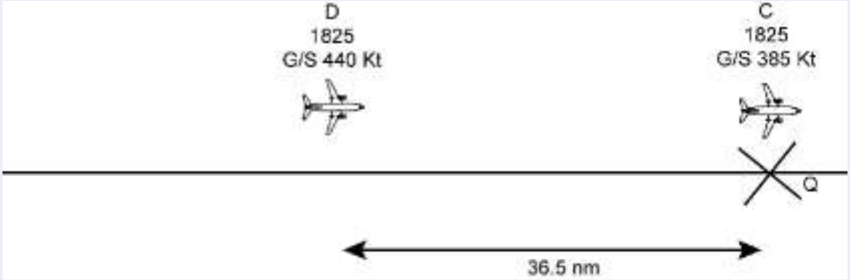
Aircraft D, groundspeed (500 - 60) = 440 kt. At 1825 the aircraft should be (5 minutes at 440 kt) 36.5 nm short of point Q.

The speed of D relative to C is (440 - 385) = 55 kt.

At this relative speed (55 kt) it will take aircraft D (36.5 nm at 55 kt) 40 minutes to catch aircraft C.

The aircraft will pass at 1905

FIGURE 20-5



If two aircraft are maintaining reciprocal tracks, the **relative** speed of one aircraft with respect to the other is the **sum of the groundspeeds.**

EXAMPLE 20-5

EXAMPLE

Points R and S are 2875 nm apart.

Aircraft E sets heading from R to S at 1430 hrs and maintains a groundspeed of 450 kt.

Aircraft F sets heading from S to R at 1442 hrs and maintains a groundspeed of 360 kt.

Determine the time and distance from R at which the aircraft will pass.

SOLUTION

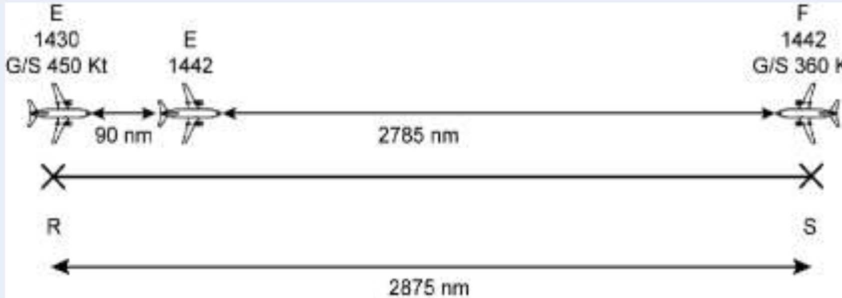
At time 1442 aircraft E will be (12 minutes at 450 kt) 90 nm from point R. The distance between the aircraft at 1442 is (2875 - 90) 2785 nm.

The relative (closing) speed of the aircraft (450 + 360) is 810 kt. The aircraft will therefore pass in (2785 nm at 810 kt) 206 minutes.
The aircraft will pass at time (1442 hrs + 206 min) 1808.

At time 1808 aircraft E will have travelled (218 minutes at G/S 450 kt) 1635 nm from point R.

The aircraft will pass at a point 1635 nm from R.

FIGURE 20-6



Changing Speed Problems

7. The final type of problem involves a change of speed either, of one aircraft relative to another, or of an aircraft relative to a fixed point.

EXAMPLE 20-6

EXAMPLE

Aircraft G has a groundspeed of 300 kt and is overhead point X at 1100. Aircraft H is following aircraft G at a groundspeed of 360 kt. Aircraft H estimates point X at 1125. Both aircraft are following the same track to point Y which is 220 nm distant from point X.

(a) Determine the time at which the separation between the aircraft will be 120 nm.

(b) Determine the minimum reduction in groundspeed which the faster aircraft would have to make at X in order to ensure a separation of 120 nm when the slower aircraft is overhead point Y.

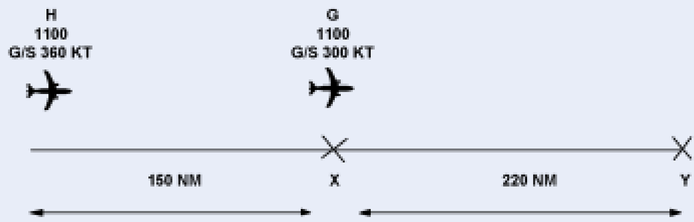
SOLUTION

a) See Figure 20-10

At time 1100 aircraft H will be (25 minutes at G/S 360 kt) 150 nm short of point X.

For a separation of 120 nm the distance to close (150 - 120) is 30 nm at a relative speed (360 - 300) of 60 kt. The time to the 120 nm separation situation is therefore (30 nm at 60 kt) 30 minutes, and this will occur at 1130

FIGURE 20-7



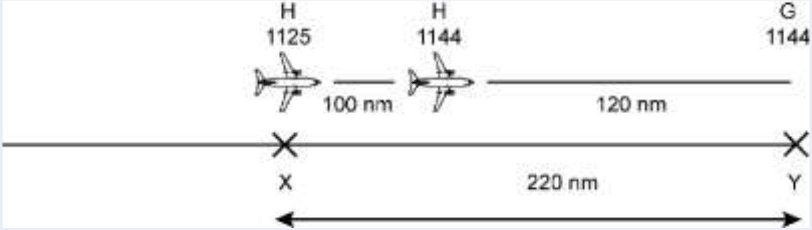
b) See Figure 20-8

At 1125 aircraft H will be overhead at point X. Aircraft G will be overhead point Y at 1100 + $\left(\frac{220}{300} \times 60\right)$ = 1144. Aircraft H must be 120 nm from Y at 1144. Therefore aircraft H must travel (220 - 120) = 100 nm in (1144 - 1125) = 19 mins.

Groundspeed required = $\frac{100}{19} \times 60$= 315.8kts

Reduction in groundspeed = (360 - 315.8) = 44.2 kts.

FIGURE 20-8



EXAMPLE 20-7

EXAMPLE

On a flight from A to B, distance 800 nm, an aircraft whose groundspeed is 480 kt is instructed to delay arrival over B by 8 minutes. It is decided that this will be accomplished by reducing speed to give a groundspeed of 360 kt. Determine the maximum distance from A at which this reduction must be carried out.

SOLUTION

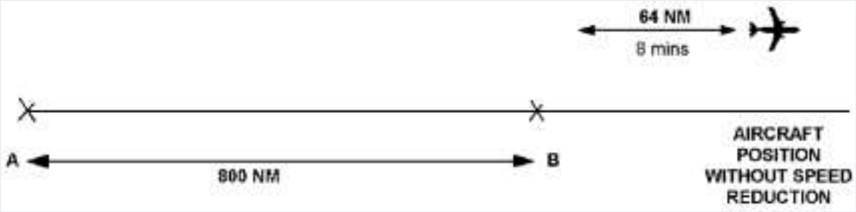
See Figure 20-9.

Were the aircraft to continue at a groundspeed of 480 kt, at the required time for overhead B it would be (8 minutes at 480 kt) 64 nm beyond B.

The aircraft is to slow down to a groundspeed of 360 kt, the **relative** difference in groundspeed is therefore 120 kt. At 120 kt it would take 32 minutes to lose the distance of 64 nm.

The aircraft must therefore reduce speed (32 minutes at reduced speed 360 kt) 192 nm before point B. Speed reduction must occur not after a point (800 - 192) 608 nm from A.

FIGURE 20-9



EXAMPLE 20-8

EXAMPLE

At 0823, when 500 nm from point C, you are requested by ATC not to cross point C before 0947. Your present cruise conditions are FL 310, temperature -46°C, Mach 0.83 and wind component -85 kt. Assuming no change in ambient conditions, determine:

The latest time at which speed may be reduced to Mach .69 in order to comply with ATC.

The distance from point C at the instant of speed reduction.

SOLUTION

Before speed reduction:

M .83/-46°C, TAS 487 kt

W/C -85 kt, G/S 402 kt

After speed reduction:

M .69/-46°C, TAS 405 kt

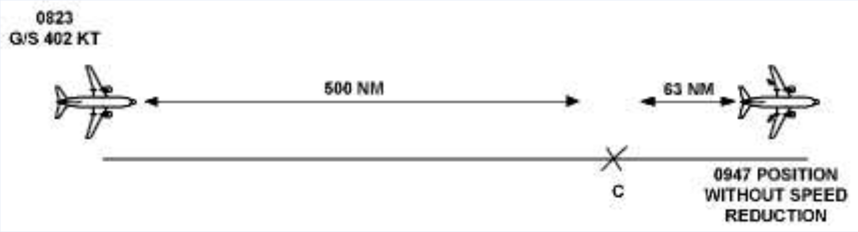
W/C -85 kt, G/S 320 kt

Without a speed reduction, by 0947 the aircraft would have covered 563 nm, and would therefore be 63 nm beyond C.

The aircraft is to slow down to a groundspeed of 320 kt, the **relative** difference in groundspeed is therefore 82 kt. At 82 kt it would take 46 minutes to lose 63 nm.

The aircraft must reduce speed (46 minutes at reduced speed of 320 kt) 245 nm before point C. The time to the speed reduction point is 46 mins before 0947 = 0901

FIGURE 20-10


0823
G/S 402 KT
500 NM
63 NM
C
0947 POSITION
WITHOUT SPEED
REDUCTION

EXAMPLE 20-9

EXAMPLE

An aircraft at FL 120, OAT -5°C, IAS 200 kt, wind component 30 kt tail is required to arrive overhead the next reporting point 100 nm away 5 minutes earlier than planned. Determine the immediate increase in IAS that is required.

SOLUTION

Original TAS	=	241 kt
Original G/S	=	271 kt
Original time to reporting point	=	100 nm @ 271 kt
	=	22.1 min
Revised time to reporting point	=	17.1 min
Revised G/S to reporting point	=	100 nm in 17.1 min
	=	351 kt
Revised TAS	=	351 - 30 kt
	=	321 kt
Revised IAS	=	265 kt
Increase in IAS	=	265 - 200
(ignoring compressibility)	=	65 kt

The Solar System

The Solar System

The Earth's Place in Space

1. The Universe is made up of a countless number of galaxies, each galaxy consisting of a vast number of stars. One such galaxy is our own which we know as the Milky Way.

2. Our Sun is a relatively small star within the Milky Way and, in common with many other stars, is accompanied by a number of planets of which the Earth is one. The planets are often accompanied by moons. Our system of the Sun, 9 major Planets and their associated Moons is known as the Solar System.

3. The huge distances involved are such that, to an observer on Earth, the stars (except for the Sun) all appear to be positioned at the same distance from the Earth. In addition, although the stars are moving through space at different speeds and in different directions, they apparently maintain constant positions relative to each other thus forming the recognisable patterns we call constellations. This gives rise to the notion that the stars are positioned on the inside surface of a sphere whose centre is the same as that of the Earth; this imaginary sphere is termed the **celestial sphere**.

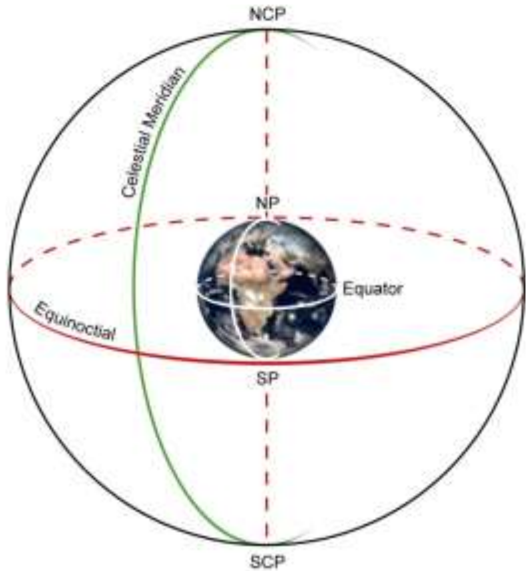
The Celestial Sphere

4. The relative positions of stars on the celestial sphere change only very slowly and therefore we consider them to be fixed. The positions of the Sun, planets and Moon move relative to the stars; the Planets revolve around the Sun in their respective orbits and the Moon orbits the Earth. The positions of these bodies can be predicted and hence plotted on the celestial sphere at any time. To do this we must first produce a position reference system for the celestial sphere.

5. In order to define position on the celestial sphere, we need to establish a datum against which we can determine position. Due to the Earth's daily rotation from west to east, the celestial sphere **appears to rotate from east to west about the Earth**. Extending the Earth's spin axis to the celestial sphere creates the North and South Celestial Poles. By projecting the earth's equator and meridians onto the celestial sphere we create the '**celestial equator**' ('**equinoctial**') and '**celestial meridians**'. See Figure 21-1.

6. Each celestial body is positioned on an '**hour circle**' which is a semi-great circle between the celestial poles and which passes through the body in question. Although hour circles are similar to celestial meridians there is one major difference: **hour circles move with the celestial sphere** as it rotates (or appears to rotate) about the Earth; whereas the **celestial meridians remain fixed**, thus the stars and other celestial bodies continually move with time relative to the celestial meridians.

FIGURE 21-1
Polar Axis, Equinoctial and Celestial Meridian

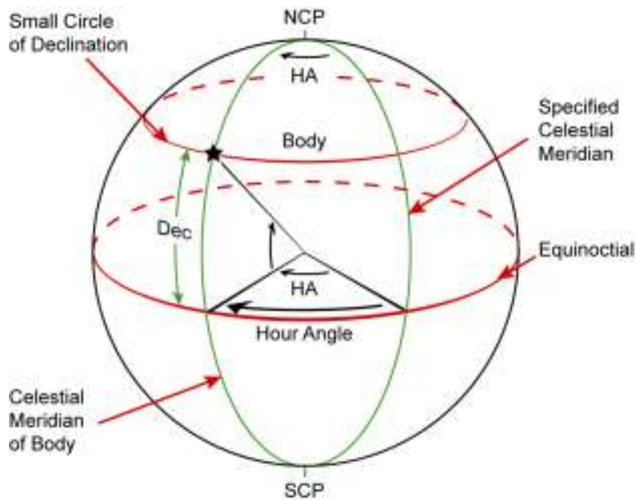


Declination and Hour Angle

7. It is now possible, within the framework of the equinoctial and the celestial meridians, to establish position on the celestial sphere. The first system we look at is similar to the latitude and longitude system we use on the Earth.

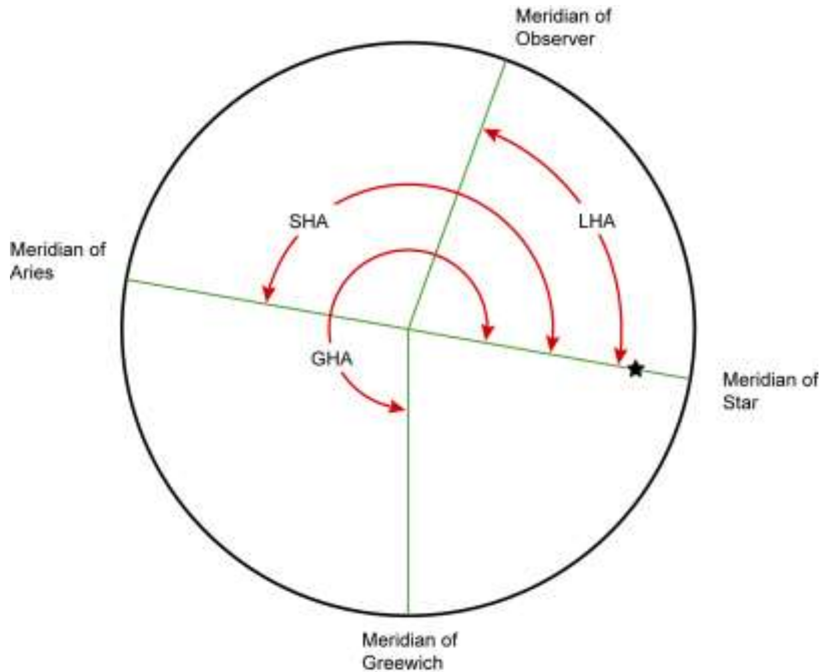
8. **Declination**. Latitude is replaced by declination. Declination is defined as the angular distance to the celestial body measured in degrees north or south of the equinoctial along the hour circle of the body; it is measured from 0° at the equinoctial to 90° at the celestial poles as shown in Figure 21-2.

FIGURE 21-2
Declination and Hour Angle



9. **Greenwich Hour Angle.** Longitude is replaced by 'hour angle'. The hour angle is the angular distance between a datum hour circle and the hour circle of the celestial body; it is measured west of the datum hour circle from 0° to 360° along the equinoctial. A number of datum hour circles are available; one such datum is the Greenwich Meridian projected on to the celestial sphere which is termed the celestial meridian of Greenwich. The hour angle measured from the celestial meridian of Greenwich to the celestial body is referred to as the Greenwich Hour Angle (GHA). See Figure 21-3.

FIGURE 21-3


Meridian of Observer
SHA
LHA
Meridian of Aries
GHA
Meridian of Star
Meridian of Greewich

10. **Sidereal Hour Angle**. An alternative datum which is commonly used is the **First Point of Aries**. The First Point of Aries is defined as the point of intersection of the ecliptic (the path the Sun follows around the Celestial Sphere) and the equinoctial where the sun is moving from south to north declination. In simple terms, it is the position of the sun on the Celestial Sphere on 21st March. The hour angle measured westwards from the celestial meridian of the First Point of Aries to the celestial meridian of the body is referred to as the Sidereal Hour Angle (SHA). See Figure 21-3.

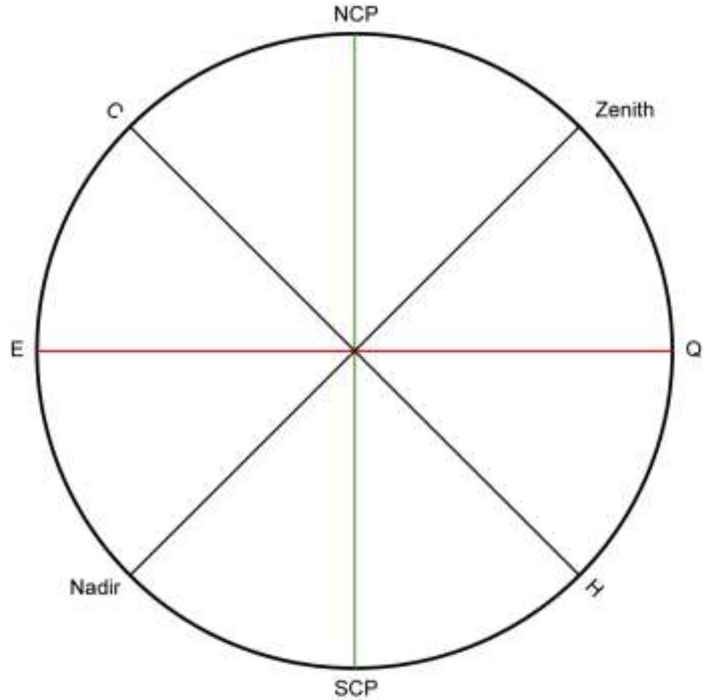
11. Local Hour Angle. Another common term in astronomy is Local Hour Angle (LHA) which is defined as the arc of the equinoctial measured westwards from an observer's celestial meridian to the Earth Co-ordinates.

Azimuth and Altitude

12. An alternative method of defining position on the Celestial Sphere can be derived from an observer's position on the Earth.

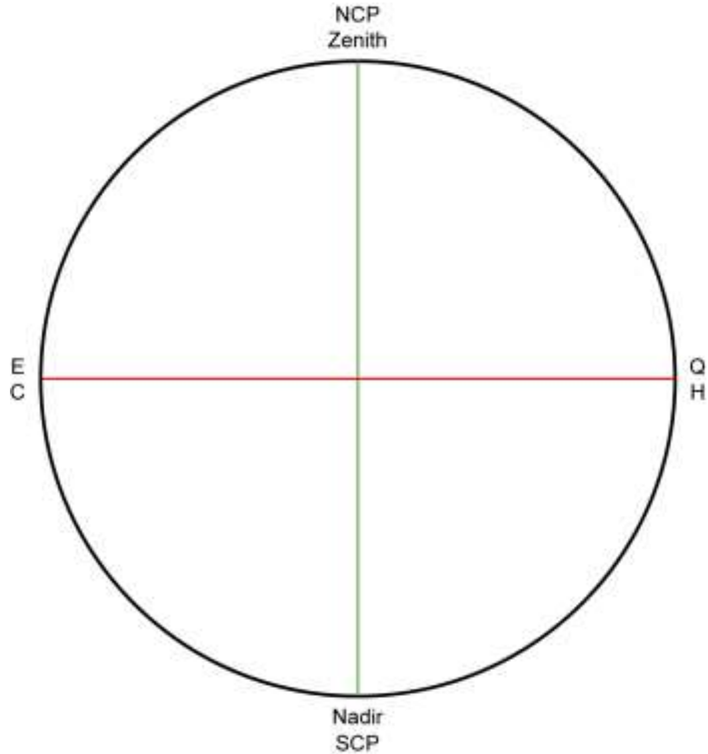
13. **Zenith and Nadir.** The zenith is the point on the celestial sphere immediately overhead the observer. The nadir is the point diametrically opposite the zenith. This establishes an axis between the zenith and nadir; as shown in Figure 21-4, the axis passes through the observer's position and the centre of the Celestial Sphere.

FIGURE 21-4



For an observer situated at the North Pole, the Zenith/Nadir diagram would be shown in Figure 21-5.

FIGURE 21-5

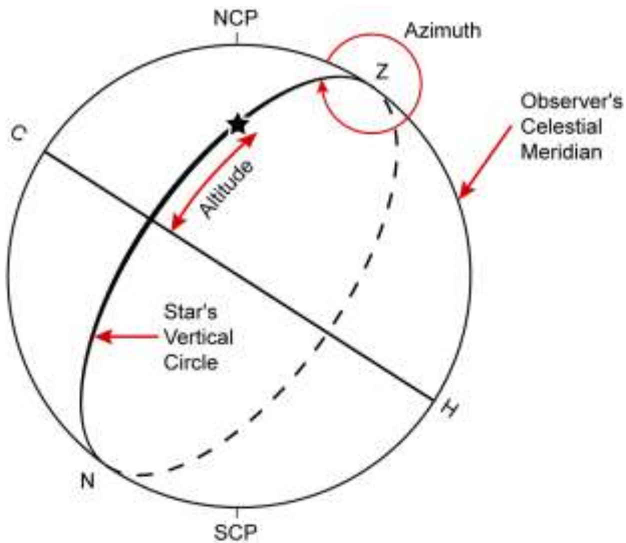


14. The observer's **celestial horizon** is then defined as a great circle whose plane is perpendicular to the zenith/nadir axis and also passes through the centre of the Celestial Sphere. The zenith/nadir axis combined with the observer's celestial horizon now forms the framework of a system that uses **azimuth** and **altitude** to define position. Just as the north pole is joined to the south pole by meridians, so the zenith is joined to the nadir by semi-great circles which are called **vertical circles**.

15. The observer's vertical circle passes through the north and south celestial poles and hence is the same line as the observer's celestial meridian. A celestial body's altitude can then be defined as the angle between the observer's celestial horizon and the body; the angle is measured at the centre of the celestial sphere along the body's vertical circle.

16. The azimuth of the body is the angle at the zenith measured clockwise from the observer's celestial meridian to the body's vertical circle, as shown in Figure 21-6.

FIGURE 21-6



NCP
Azimuth
Z
Observer's
Celestial
Meridian
C
Altitude
Star's
Vertical
Circle
H
N
SCP

The Solar System

17. The Earth is one of the planets in the Solar system. Each of the planets describes an elliptical orbit around the Sun. The Sun, therefore, can be considered to be the stationary centre of the solar system. Each planet takes a year to complete one orbit of the Sun, but since the planets are positioned at different distances from the sun and they are travelling at different speeds through space, each planet's year is different. The Earth's year is equivalent to 365 days 5 hours 48 minutes and 45 seconds.

Kepler's Laws of Planetary Motion

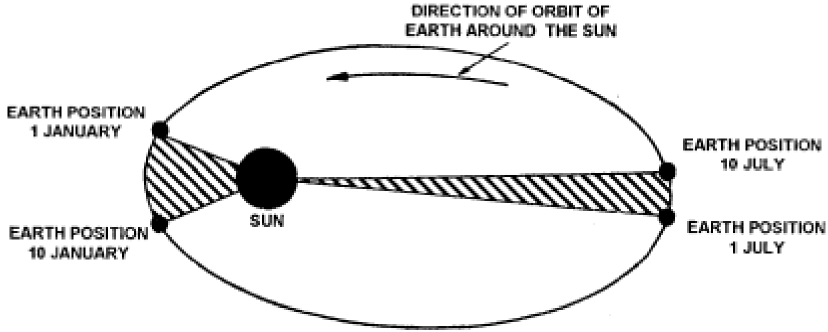
18. The astronomer, Kepler, developed 2 laws to describe planetary motion:

    (a) Kepler's First Law states that the orbit of each planet describes an ellipse around the sun, with the Sun positioned at one of the foci of the ellipse;

    (b) Kepler's Second Law states that the radius vector of any planet sweeps out an equal area in equal time.

19. These Laws are illustrated at Figure 21-7.

FIGURE 21-7
Kepler's Laws



20. The average distance of the Earth from the sun is approximately 93 million miles, but Figure 21-9 shows that the Earth is closest to the sun during January (ie during the northern hemisphere's winter). Figure 21-9 also suggests that for Kepler's Second Law to hold true, the Earth must be moving through space at a greater rate in January than it is in July. (This can be explained by the fact that the sun's gravity will accelerate the Earth as it moves through space towards the Sun and decelerate the Earth as it moves away. In simple terms, this means that the Earth will be travelling fastest when it is closest to the Sun ie the 'slingshot effect'; this fact has implications as discussed in Chapter 22.)

The Seasons

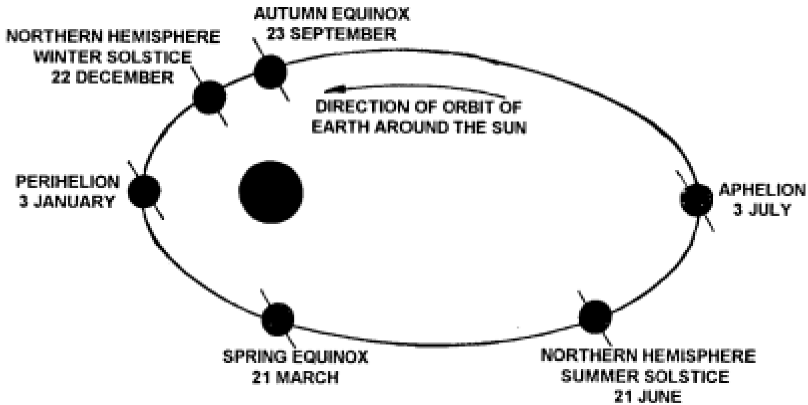
21. During the course of the year, the position of the sun relative to the equator varies; this is because the Earth's spin axis is tilted at an angle of 66.5° to the plane of the Earth's orbit. If the spin axis was perpendicular to the plane of the orbit, the Sun would remain directly above the Equator the whole year round and there would be no seasons. The orbit is inclined at an angle of 23.5° to the celestial equator and it is this inclination which causes the seasons during the course of a year. As already mentioned the apparent path of the sun across the celestial sphere is called the **ecliptic**. The angle between the earth's celestial equator and the ecliptic (23.5°) is known as the '**Obliquity of the Ecliptic**'; Figure 21-8 refers.

FIGURE 21-8
The Obliquity of the Ecliptic

Figure 21-9 shows the position of the Earth on its orbit at some significant dates during the year together with an indication of the effect of the Earth's inclination to its orbital plane.

FIGURE 21-9
Seasonal Dates

22.



23. The **northern hemisphere summer solstice** (June 21) occurs when the northern end of the Earth's spin axis (ie the north pole) is inclined to its maximum value **towards** the Sun. It is on this day of the year that the Sun appears to be directly above an observer at latitude 23° 30'N (the Tropic of Cancer); northern hemisphere days are longest and the nights shortest on this date.

24. The **northern hemisphere winter solstice** (December 21) occurs when the northern end of the Earth's spin axis is inclined to its maximum value **away from** the Sun. It is on this day of the year that the Sun appears to be directly above an observer at latitude 23° 30'S (the Tropic of Capricorn); northern hemisphere days are shortest and the nights longest on this date.

25. The point on the Earth's elliptical orbit where it is closest to the sun is called the **perihelion** and occurs on January 3rd; the point where the earth is furthest from the sun is called the **aphelion** and occurs on July 3rd.

26. Between the summer and winter solstices the sun crosses the celestial equator from north to south on or about September 21; this is called the **autumn equinox**. Similarly, the sun will cross the celestial equator from south to north on or about March 21; this is called the **vernal or spring equinox**. On these dates the Sun appears to be vertically above an observer on the Equator and day and night are of equal duration in both hemispheres. The vernal equinox is also known as the **First Point of Aries**. See Figure 21-10.

FIGURE 21-10
The First Point of Aries (Spring Equinox) and Autumn Equinox

The Year

27. A period of one year is defined as the time taken for a planet to describe one orbit around the sun. For the earth, this orbit takes approximately 365.25 days. A leap year every 4 years corrects for the fractional quarter of a day of orbit time.

28. This familiar leap year correction is, however, not quite exact. There is a further correction needed which will lose 3 days every 400 years. This is achieved by suppressing 3 leap years every 4 centuries. Every century year is classified as an ordinary year of 365 days, except when the century number is divisible by 4, in which case it is classified as a leap year. So 1700, 1800, 1900 were not leap years, while 2000 was a leap year.

29. **Sidereal day.** A 'sidereal' day refers to one rotation of the earth about its axis where the time interval is measured from a star which is not the sun. The star chosen for this definition is the First Point of Aries. The definition of a 'sidereal day' is the time interval between two consecutive transits of the First Point of Aries over a given meridian. Since the First Point of Aries is assumed to be fixed in space, it follows that a 'sidereal day' is of constant duration.

Time and Time Conversions

Background

Local Mean Time

Co-ordinated Universal Time

The International Dateline

Standard Time

Sunrise, Sunset and Twilight

Extacts from the Air Alamanac

Time and Time Conversions

Background

Origin of the Solar Day

1. Before the invention and widespread use of the clock, time was usually kept by noting the position of the sun. Midday (12 noon) was the time the sun reached its highest elevation in the sky and coincided with the time at which its azimuth was along the observer's meridian. Midnight was halfway between noons.

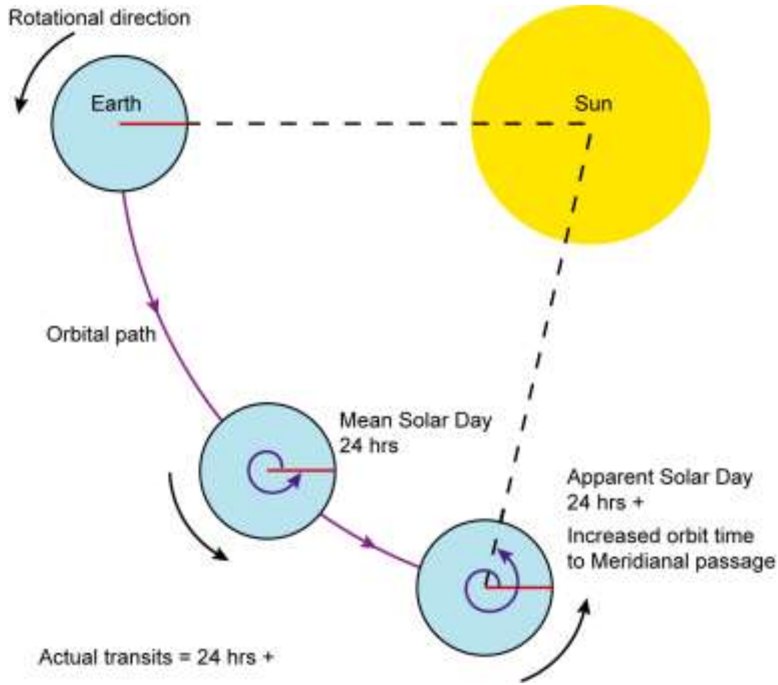
2. Time, therefore, varied with location even in parts of the same country at different longitudes. The length of the day measured in this way i.e. between successive transits of the real sun across the observer's meridian (noon to noon) is known as the '**apparent solar day**'.

Variations in the Length of the Apparent Solar Day

3. The period between successive transits of the sun over a given meridian varies because the Earth is orbiting the sun following an elliptical path, and also because the speed of movement of the Earth along the ellipse is continuously changing (Kepler's second law). See Figure 22-1.

4. Although the average length of an apparent solar day is 24 hours, the actual length will vary about this mean value.

FIGURE 22-1
The Mean and Apparent Solar Day in January



The Mean Solar Day

5. To overcome the variations in the length of the day, the **mean solar day** is used, which is always exactly 24 hours long. The mean solar day may be considered as being the time between successive transits of a non-existent average or **mean sun** over a given meridian.

The Mean Sun

6. The 'mean sun' travels at a constant angular velocity in the plane of the Equator. This means that the position of the mean sun is always overhead the Equator. It follows that an observer in the northern hemisphere will 'see' the apparent (visual) sun higher in the sky than the mean sun between the spring and autumn equinoxes, and lower in the sky between the autumn and spring equinoxes.

7. The time (period) of the orbit of the mean sun is the same as that of the orbit of the apparent sun. However, as described by Kepler's Second Law, the angular velocity of the apparent sun is not constant. At perihelion (3 January) the angular velocity is at a maximum, while at aphelion (3 July) it is at a minimum.

8. At perihelion, the mean sun and the apparent sun will transit a given meridian simultaneously. After 3 January, the apparent sun begins to draw ahead of the mean sun. By aphelion (3 July), the mean sun has slowed down and both suns will again transit a given meridian simultaneously. After aphelion, the apparent sun falls behind the mean sun until by the following 3 January, after the apparent sun has speeded up again, the two suns are again in a line on a meridian.

The Equation of Time

9. The 'Equation of Time' is the name given to the time difference between the noon transit times of the two suns across a given meridian and therefore represents the difference in the length of the apparent solar and mean solar days. The Equation of Time is not an equation in the conventional sense, but merely represents a time difference. As a result, the maximum time differences occur in mid-February when the real sun is approximately 14 minutes behind the mean sun and in early November when the real sun is 16 minutes ahead of the mean sun. Since a change of around 30 minutes occurs over these three months, it follows that the maximum rate of change in the length of the apparent solar day is about the third week in December (which is around the winter solstice in the northern hemisphere).

10. The position of the real sun is of importance to astronomers, however all calculations concerning time are based on the position of the mean sun and therefore relate to mean time.

Local Mean Time

11. The beginning of a day at any point on the Earth is midnight, local mean time (0000 LMT). At midnight LMT, for an observer at 050°W, the mean sun is overhead the meridian at 130°E (the observer's anti-meridian). Twelve hours later, the mean sun will be overhead the observer's meridian (050°W). By now it will be noon LMT (1200 LMT) for the observer and the time at the observer's anti-meridian (130°E) will be midnight LMT. The local mean time at any point is therefore governed by the passage of the mean sun.

Arc to Time

12. The mean sun is assumed to travel around the Earth once every 24 hours. Since the Earth is a spheroid, that apparent movement represents 360°, therefore, in one hour the sun travels through 15 degrees of arc, and in four minutes through one degree of arc. (In one minute of time the sun travels through 15 minutes of arc, and in four seconds of time through one minute of arc).

13. Arc to time therefore, permits the calculation of the LMT at one longitude on the earth from a knowledge of the LMT at another, based on the difference of longitude between the two points.

Co-ordinated Universal Time

14. To provide one time reference for the whole world it is necessary to choose one meridian to which all LMTs are referenced. The chosen meridian is the Greenwich or Prime meridian (0° east/ west). Local mean time at Greenwich meridian is termed Greenwich Mean Time (GMT).

15. GMT is based on the motion of the Earth, which fluctuates very slightly each day. The advent of highly accurate atomic clocks (based on the fundamental properties of the caesium atom) has, since 1 January 1972, provided the basis for international time-keeping, on that date, Co-ordinated Universal Time (UTC) came into effect.

16. UTC runs at the rate of International Atomic Time. The International UTC scale is coordinated in Paris by the International Bureau of Weights and Measures. When the difference between UTC and GMT approaches one second, an adjustment (a 'leap second') is made in UTC. For this reason UTC and GMT can be regarded as one and the same thing for all normal navigation purposes.

17. 0000 UTC occurs when the mean sun is in transit with the anti-meridian of Greenwich (180° east/west). Since the sun appears to travel from east to west, it will transit easterly meridians before crossing the Greenwich meridian. **LMT at positions east of the Greenwich meridian is therefore ahead of UTC, and LMT at positions west of the Greenwich meridian is behind UTC**.

Putting this simple but important relationship another way:

Observer east (of Greenwich) UTC least (slow on LMT)

Observer west (of Greenwich) UTC best (fast on LMT)

The numerical difference between LMT and UTC at any point is related to the rate of rotation of the Earth and is determined using the **arc to time** conversion factors previously described.

18. Here are some examples of conversions from LMT to UTC and vice versa:

EXAMPLE 22-1

EXAMPLE

An observer is at position 29°N 94°30'W at time 1215 LMT on the 1st January. Express the time in UTC.

SOLUTION

The observer is west of Greenwich and therefore UTC is best (fast on LMT). 94°30' of arc of longitude expressed in time gives:

94° of arc at 4 minutes per degree	= 376 minutes
30’ of arc at 1 minute per 15’ of arc	= 2 minutes
Therefore, 94°30’ of arc	= 378 mins total arc to time
	= 6 hours and 18 minutes
Therefore,	
	1215 LMT on 1 January
Arc to time (west)	= +0618
	= **1833 UTC on 1 January**

Note. When carrying out these calculations, it is advantageous always to show the date next to the time, changing the date appropriately as midnight is passed.

EXAMPLE 22-2

EXAMPLE

An observer is at position 47°32'S 126°17'E at time 0237 LMT on 4th April. Express the time in UTC.

SOLUTION

The observer is east of Greenwich and therefore UTC is least (slow on LMT)
126°17’ of arc of longitude expressed in time gives;

126° of arc at 4 minutes per degree	= 504 minutes
17’ of arc at 1 minute per 15’ of arc	= 1 minute (to nearest min)
Therefore, 126°17’ of arc	= 505 mins total arc to time
	= 8 hours and 25 minutes
Therefore,	0237 LMT on 4 April
Arc to time (east)	= -0825
	= **1812 UTC on 3 April**

In this example arc to time has taken us through midnight into the previous day, hence the change of date.

EXAMPLE 22-3

EXAMPLE

An observer is at position 73°21'N 74°57'W at time 2117 on the 17th July. Express the time in UTC.

SOLUTION

The observer is west of Greenwich and so UTC is best (fast on LMT). Arc to time for 74°57'W gives:

74°	= 296 minutes
57'	= 4 minutes (to nearest minute)
Therefore, 74°57' of arc	= 300 minutes total arc to time
	= 5 hours and 00 minutes
Therefore,	
	2117 LMT on 17 July
Arc to time (west)	= +0500
	= **0217 UTC on 18 July**

In this example arc to time has taken us through midnight into the following day, hence the change of date.

19. The following examples go one step further, and involve calculating LMT at one point given LMT at another point. The important thing to remember is **always to work through UTC**.

EXAMPLE 22-4

EXAMPLE

The time is 1437 LMT on 3rd May at position A (27°31'N 127°43'E). Determine the LMT at position B (43°26'S 40°15'W).

SOLUTION

At position A	1437 LMT 3 May
Arc to time (127°43’E)	= -0831
	= 0606 UTC 3 May
Arc to time (40°15’W)	= -0241
At position B	**0325 LMT 3 May**

EXAMPLE 22-5

EXAMPLE

The time is 2219 LMT on 16th June at position C (00°15'N 163°21'W). Determine the LMT at position D (05°17'N 83°13'W).

SOLUTION

At position C		2219 LMT 16 June
Arc to time (163°21'W)	= +1053	
	= 0912 UTC 17 June	
Arc to time (83°13'W)	= -0533	
At position D		**0339 LMT 17 June**

Some problems include flight time. If such is the case, calculate the T/O time in UTC. Then add the flight time to obtain the landing time UTC and if required, convert to LMT. ALWAYS WORK IN UTC.

EXAMPLE 22-6

EXAMPLE

An aircraft departs point E (23°16'N 47°20'E) at 1921 LMT on 21st August. The flight time to F (46°29'N 83°16'E) is 8 hours 30 minutes. Determine the LMT of arrival at F.

SOLUTION

Departure E		1921 LMT 21 August
Arc to time (47°20’E)	= -0309	
Departure E		1612 UTC 21 August
Flight Time	= +0830	
Arrival F		0042 UTC 22 August
Arc to time (83°16’E)	= +0533	
Arrival F		**0615 LMT 22 August**

EXAMPLE 22-7

EXAMPLE

An aircraft departs airfield G (43°16'N 169°35'E) and flies to airfield H (36°59'N 147°26'W). The aircraft departs G at 2015 LMT on 15th January. The flight time is 10 hours 25 minutes. Determine the LMT of arrival at H.

SOLUTION

Departure G		2015 LMT 15 January
Arc to time (169°35'E)	= -1118	
Departure G		0857 UTC 15 January
Flight Time	= +1025	
Arrival H		1922 UTC 15 January
Arc to time (147°26'W)	= +0950	
Arrival H		**0932 LMT 15 January**

20. At first glance it may seem odd that the aircraft arrives before it departs! In fact, the aircraft has crossed the **International Dateline.** It is **by working through UTC**, the change of date on crossing the dateline takes care of itself.

EXAMPLE 22-8

EXAMPLE

An aircraft departs airfield J (63°16'S 153°45'W) and flies to airfield K (58°01'N 150°19'E). The aircraft departs J at 1935 LMT on September 13. The flight time is 8 hours 30 minutes. Determine the LMT of arrival at K.

SOLUTION

Departure J		1935 LMT 13 September
Arc to time (153°45'W)	= +1015	
Departure J		0550 UTC 14 September
Flight Time	= +0830	
Arrival K		1420 UTC 14 September
Arc to time (150°19'E)	= +1001	
Arrival K		**0021 LMT 15 September**

21. Now the flight appears to have spanned two whole days! In fact the aircraft has crossed the dateline, this time in the opposite direction. Again, **by working through UTC**, the change of date has taken care of itself.

22. Having asked you to solve the conversion of arc of longitude to time using simple mathematics, we can now reveal that you will find at the end of this chapter, extracts from (see Figure 22-11) the Air Almanac a table which will do the conversion for you. The use of the table is quite straightforward, the columns of the table come in pairs, and all but the last pair of columns convert whole degrees of longitude into hours and minutes of time. The last column converts minutes of arc into minutes and seconds of time. There are even instructions at the foot of the table which tell you how and which way to apply arc to time conversions. Try reworking some of the previous examples in order to become familiar with the use of this table.

The International Dateline

23. When travelling eastwards from Greenwich, an observer would eventually arrive at longitude 179°59'E, where the LMT is about to become 12 hours more than UTC. Similarly, an observer travelling westwards from Greenwich would eventually arrive at 179°59'W, where the LMT is about to become 12 hours less than UTC. There is a full day of 24 hours difference between the two observers, although they are both about to cross the same meridian.

24. When the anti-meridian of Greenwich is crossed, one day is gained or lost, depending upon the direction of travel. When travelling west, a day is gained (added); when travelling east, a day is lost (subtracted).

As already emphasised in Example 23-7 and Example 23-8, the problem of the date difference, on either side of the International Dateline (principally the 180° east/west meridian) resolves itself, providing that you solve all time problems by reverting to UTC before moving across the surface of the Earth.

Note. The dateline does not follow the Greenwich anti-meridian entirely. For administrative reasons, certain groups of islands in the Pacific Ocean are included in what should be the opposite date zone according to their longitudes eg Chatham Island is 12 hours 45 mins.

Standard Time

25. LMT changes with change of longitude. If LMT were to be used as a means of timekeeping nationally, it would be necessary to adjust one's watch when travelling east-west across a country. Obviously, this is not satisfactory and so countries use **Standard Time**, sometimes referred to as 'clock time', which usually apply to the whole country. For countries having a large East/West extent such as the USA, however, several different standard times apply; these are broken down broadly speaking into 15° bands of longitude or by states.

26. The Standard Time difference for any given country, or state within a country, is usually determined by a combination of geographical position (ie longitude) and political will. For instance, much of France is in the same geographical span of longitude as the United Kingdom and so might be expected to be on the same time 'zone'. However, the French Government has decided that it would be more convenient for France to be on the same time as the majority of her trading partners within the European Union. Therefore France is one hour ahead of the United Kingdom.

27. At the end of this chapter you will find extracts from an Air Almanac. Included in these extracts are three tables or 1 'Lists' for Standard Time difference with respect to UTC (which is still referred to as GMT in the almanac):

(a) List 1 deals with countries whose Standard Time is fast on UTC/GMT (mainly those at easterly longitudes). See Figure 22-7.

(b) List 2 deals with countries normally keeping UTC/GMT. See Figure 22-9.

(c) List 3 deals with countries whose Standard Time is normally slow on UTC/GMT (mainly those countries at westerly longitudes). See Figure 22-9.

Notice that, at the top of lists 1 and 3, there is guidance as to how to apply the relevant correction. At the bottom of each list you will find various footnotes. When you are looking up the standard time difference for a given location you must pay particular attention to the symbol or footnote number which may follow the location name (for example Crete*, Marshall Islands[1]) and comply with the footnote, if possible.

28. Some instructions accompanying the footnotes are non-specific eg 'Summer time may be kept in these countries', but equally, summer time may **not** be kept in those countries; we do not know for sure and therefore we cannot apply a correction. On the other hand, the information for some countries is quite specific such as for USA and UK. The footnote applying to the United Kingdom is specific in terms of how and when to apply British Summer Time and therefore we must comply with the footnote in order to arrive at the correct Standard Time. The Standard Time being kept in a country should be checked in the appropriate Flight-information Handbook or AIP.

EXAMPLE 22-9

EXAMPLE

Given that an aircraft departs the Azores at 2215 standard time on Jan. 15th to fly to New York, and that the flight time is 7 hrs 20 mins, determine the Standard Time of arrival in New York.

SOLUTION

As with previous examples, the key to the correct solution to example 9 is to work through UTC.

Departure Azores		2215 ST 15 January
Standard Time difference	=	+0100
Departure Azores		2315 UTC 15 January
Flight Time	=	+0720
Arrival New York		0635 UTC 16 January
Standard Time difference	=	-0500
Arrival New York		**0135 ST 16 January**

EXAMPLE 22-10

EXAMPLE

An aircraft departs Djibouti at 0215 standard time on March 21st. The flight to Morocco takes 7 hours 15 minutes. Determine the standard time of arrival in Morocco.

SOLUTION

Departure Djibouti		0215 ST 21 March
Standard Time difference	= -0300	
Departure Djibouti		2315 UTC 20 March
Flight Time	= +0715	
Arrival Morocco		0630 UTC 21 March
Standard Time difference	= 0000	
Arrival Morocco		**0630 ST 21 March**

Sunrise, Sunset and Twilight

Sunrise and Sunset

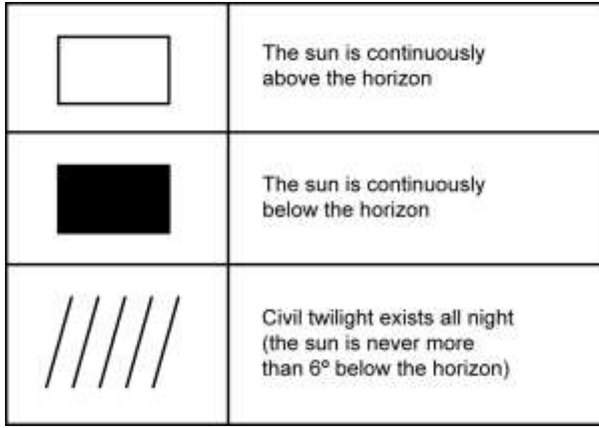
29. Turn to the extract of the Air Almanac dealing with sunrise, sunset and twilight; a quick perusal of the tables will indicate that the time of any of these events varies with both the date considered and the latitude of the observer. **The times tabulated are LMT**.

30. Check the tables and confirm that the sun rises later and sets earlier with increase of latitude in the winter hemisphere, but rises earlier and sets later with increase of latitude in the summer hemisphere. The extreme examples of this are the midnight sun (24 hours of daylight) in the polar regions of the summer hemisphere, and the continuous periods when the sun never rises in the polar regions of the winter hemisphere.

31. Note that the latitude band covered by these tables extends from 72°N to 60°S. At higher latitudes it is necessary to use graphs to determine the time of sunrise, sunset and twilight, and these are outside the scope of this syllabus.

32. Three symbols are used in these tables to replace the four-figure time group. The meaning of these symbols is given at Figure 22-2.

FIGURE 22-2
Symbols found in Sunset, Sunrise and Twilight Tables



33. Refer now to the Air Almanac extracts and check the use of these symbols. Note, for example, that the white rectangle symbol never appears at a lower latitude than 66° (in June in the northern hemisphere, and assumed to be the same latitude in December in the southern hemisphere). Whereas the black rectangle symbol never appears at a lower latitude than 68° (in December in the northern hemisphere).

34. The sunrise, sunset and twilight tables list the **Local Mean Time** for the phenomenon. When the upper limb (edge) of the sun appears to be co-incident with the horizon of an observer it is sunrise or sunset. Twilight is assumed to begin or end when the centre of the sun is 6° below the sensible horizon.

35. Atmospheric refraction causes an observer to observe objects which are actually below his/her sensible horizon. At sunrise and sunset, the sun's upper edge is 34' of arc below the observer's sensible horizon at mean sea level.

36. Sunrise at the Equator is always at approximately (within 20 minutes of) 0600 LMT, and sunset is always at approximately 1800 LMT.

37. When interpolating between dates and latitudes in the tables, work to the nearest whole minute of time. The tables are divided in columns listed every 3 days over a 45 day period on each page. The latitude lines are listed at 2° intervals from 72N to 50N, then at 5° intervals to 30N and at 10° intervals to the equator. The same latitude intervals are used in the Southern Hemisphere. The latitude spacing indicates the degree of accuracy required with the interpretation of latitude. For instance, a high latitude, such as 6243N requires a greater accuracy of interpretation than 1043N because a few minutes of latitude can make a considerable difference to the answer. At low latitudes, rounding to the nearest ½ of latitude is accurate enough.

38. The following examples are based around sunrise and sunset times:

EXAMPLE 22-11

EXAMPLE

An aircraft departs Gibraltar (36°07'N 05°21'W) one hour after sunrise on 14 July and flies to Hamilton, Ontario (43°10'N 79°56'W). The flight time is 7 hours 45 minutes. Determine the LMT of arrival at Hamilton.

SOLUTION

Sunrise Gibraltar		0453 LMT 14 July
Take off 1 hour after sunrise		+0100
Depart Gibraltar		0553 LMT 14 July
Arc to time (05°21'W)	=	+0021
Depart Gibraltar	=	0614 UTC 14 July
Flight Time	=	0745
Arrive Hamilton	=	1359 UTC 14 July
Arc to time (79°56'W)	=	0520
Arrive Hamilton		**0839 LMT 14 July**

EXAMPLE 22-12

EXAMPLE

An aircraft leaves Gander, Newfoundland (48°57'N 54°34'W) one hour before sunset on July 5th. The flight time to Los Angeles, California, USA is 8 hours 45 minutes. Determine the standard (summer) time of arrival at Los Angeles.

SOLUTION

Sunset Gander		2007 LMT 5 July
Take off 1 hour before sunrise		-0100
Depart Gander		1907 LMT 5 July
Arc to time (54°34'W)	=	+0338
Depart Gander		2245 UTC 5 July
Flight Time	=	0845
Arrive Los Angeles		0730 UTC 6 July
Standard Time difference	=	-0800
		= 2330 ST 5 July
Summer Time adjustment (See footnote 3 at list III (continued))	=	+0100
Arrive Los Angeles		**0030 ST 6 July**

39. An examination of the complete Air Almanac would show that at mid-latitudes the time interval between sunrise and sunset changes most rapidly around the spring and autumn equinoxes.

Twilight

40. When the sun is just below the horizon, an observer on the Earth will still receive light which has been refracted and scattered by the atmosphere; this light is called twilight. Figure 22-3 shows the four degrees of twilight:

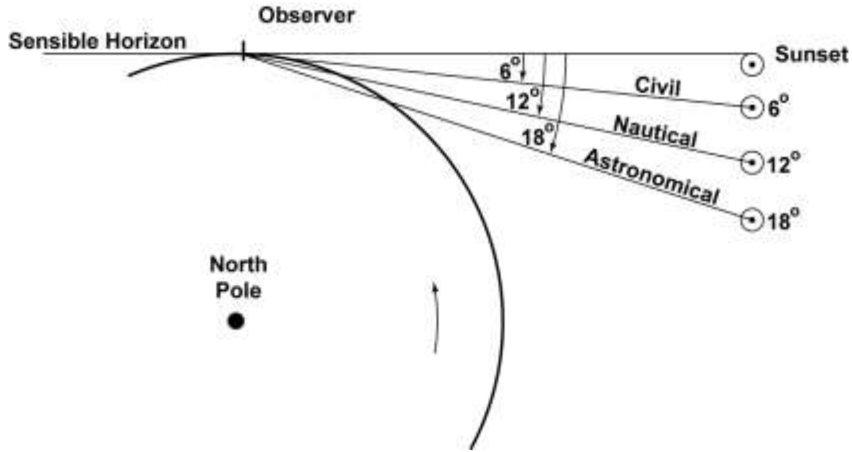
(a) When the sun is between the horizon and a point six degrees below the horizon, **civil twilight** is said to exist. During Civil Twilight, the use of artificial light is not necessary. The time of the beginning of morning civil twilight and the end of evening civil twilight are tabulated in the almanac. The times listed are for when the centre of the sun's disc is 6° below the horizon.

(b) When the sun is between 6 and 12 degrees below the horizon, **nautical twilight** is said to exist.

(c) When the sun is between 12 and 18 degrees below the horizon, **astronomical twilight** is said to exist.

(d) **Legal twilight** is defined by individual states (for the United Kingdom it exists from 30 minutes before sunrise, until sunrise, and from sunset to 30 minutes after sunset).

FIGURE 22-3



41. As with the sunrise and sunset tables, the twilight tables in the almanac give the **LMT** of the event for an observer at mean sea level. When interpolating between dates and latitudes in the tables, work to the nearest whole minute of time.

The Duration of Twilight

42. The duration of civil twilight is the time period from the beginning of morning civil twilight to sunrise or from sunset to the end of evening civil twilight. It will be the same at both ends of the day. On occasions the combination of sun's declination and observers latitude may cause one minute of difference between the two durations.

43. The duration of twilight is affected by the following factors:

(a) The declination of the sun, and therefore by implication the time of year. When the sun is in observer's hemisphere the duration of twilight increases as the sun's declination increases.

(b) The observer's latitude. As latitude increases so the duration of twilight increases. At high latitudes, it is possible for civil twilight to exist all night.

(c) The observer's altitude. Although only MSL tables are provided for the course and the examination, there are tables available at 1000ft intervals up to 50,000ft above MSL. The high altitude tables show that twilight commences earlier and ends later than at MSL but has a shorter duration. This is because of the rarer atmosphere and decreased scattering effect with increased altitude.

EXAMPLE 22-13

EXAMPLE

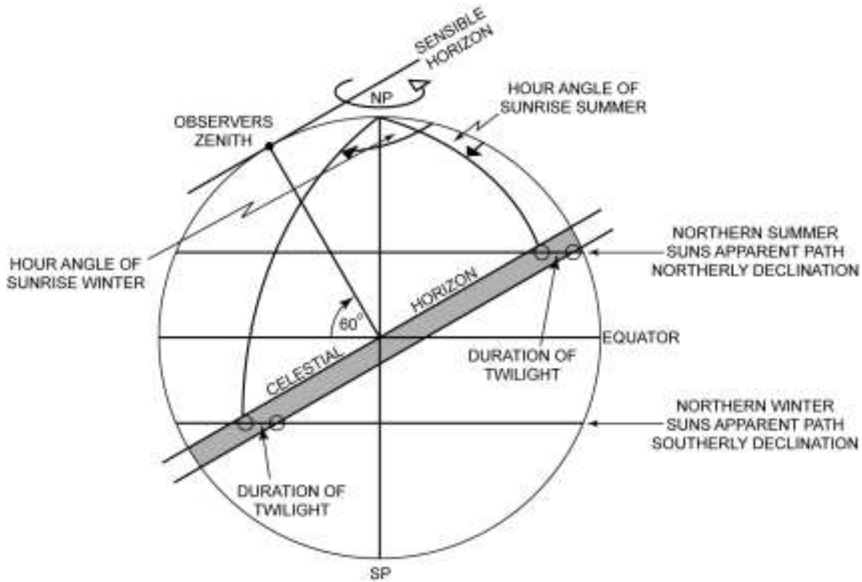
Determine the duration of morning civil twilight at Nicosia, Cyprus (35°09'N 33°16'E) on 6 August.

SOLUTION

Sunrise (35°09'N)		0512 LMT 6 August
Start of MCT (35°09'N)		-0445 LMT 6 August
Therefore, duration of MCT	=	27 minutes

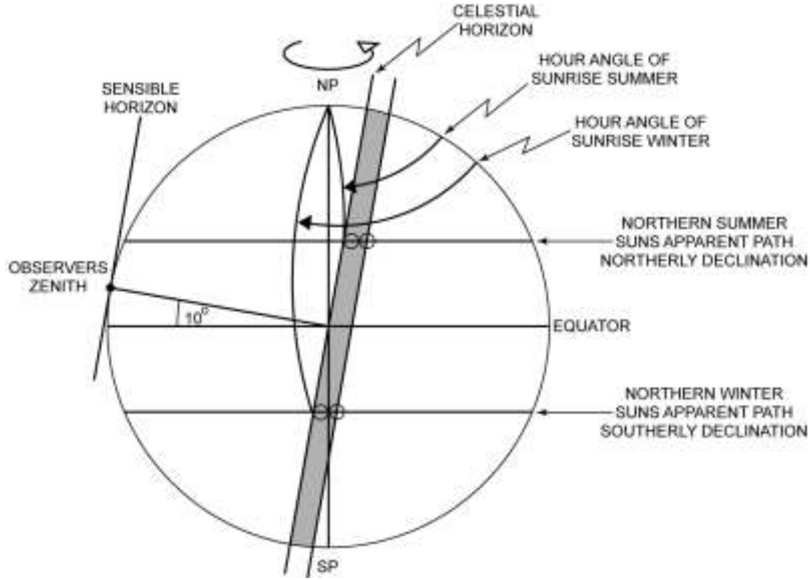
44. Figure 22-4 depicts the effect of declination for an observer at 60°N on June 21st and December 21st. Notice for June 21st, sunrise is at 0230 LMT and the beginning of morning civil twilight is at 0045 LMT. Hence the duration of twilight is 1 hour 45 mins. Whereas on 21st December sunrise is at 0910 LMT and the beginning of MCT 0813 LMT so the duration of twilight is 57 minutes.

FIGURE 22-4



45. Figure 22-5 depicts the effect of declination for an observer at 10°N on 21st June and 21st December. For 21st June sunrise is at 0540 LMT and the beginning of morning civil twilight is at 0517 LMT. The duration of twilight is 23mins. On 21st December sunrise is at 0615 LMT and the beginning of MCT occurs at 0552 LMT. Therefore the duration of twilight is only 23mins.

FIGURE 22-5



46. Comparison of the two sets of times reveals that at high latitudes with the sun in the same hemisphere sunrise occurs earlier and the duration of twilight is longer than at low latitudes. The higher the latitude of the observer and the greater the declination of the sun in the same hemisphere the greater is this effect. When the sun is in the opposite hemisphere i.e. in winter sunrise occurs later and the duration of twilight is shorter. The effect being less at the lower latitude.

FIGURE 22-6

	21st June		**21st December**	
	Sunrise	Duration of twilight	Sunrise	Duration of twilight
60 N	0230 LMT	1 hr 45 mins	0910 LMT	57 mins
10 N	0540 LMT	23 mins	0615 LMT	23 mins

47. If the latitude is above 72° then in summer an observer can experience 24 hours of daylight and in winter 24 hours of darkness.

Self Assessed Exercise No. 12

QUESTIONS

QUESTION 1.

What is the definition of the 'declination' of a star?

QUESTION 2.

What is the definition of the 'hour angle' of a star?

QUESTION 3.

State Kepler's First Law.

QUESTION 4.

State Kepler's Second Law.

QUESTION 5.

Define an apparent solar day.

QUESTION 6.

Why is an apparent solar day sometimes longer and sometimes shorter than a mean solar day?

QUESTION 7.

What is the duration of morning civil twilight at Rashim in Korea (4230N, 13020E) on 16th July?

QUESTION 8.

What is the LMT of sunrise on Lord Howe Island (3131S, 15904E) on 17th July.

QUESTION 9.

It is 1200 LMT on 4th May at 1000N, 09000E. What is the corresponding UTC time?

QUESTION 10.

It is 1104 ST in Mexico City (1930N, 09900W) on 14th February. Express this time in UTC.

QUESTION 11.

What is the UTC time corresponding to 2115 LMT at 6115S, 17235W on 2nd February?

QUESTION 12.

What is the UTC time equivalent to 0011 ST in Hong Kong (2219N, 11412E) on 21st December?

QUESTION 13.

What is the standard time of sunrise at Wellington, New Zealand (4110S, 17445E) on the 19th July?

QUESTION 14.

At 0900 LMT on 22nd July at 1000N 05000W, what is the LMT at 1000S 09000W?

QUESTION 15.

When it is 1210 ST in Kuwait on 15th July, what is the standard time in Algeria?

QUESTION 16.

An aircraft departs at 1910 LMT on 16th March from 'A' (47°N, 63°W) and flies to 'B' (37°N, 49°W). If the flight time is 3hours 20minutes, what is the LMT and date of arrival at B?

QUESTION 17.

An aircraft departs at 1230 ST on 14th August from Los Angeles, California, USA and flies to Tokyo, Japan. If the flight time is 11 hours 10 minutes, what is the standard time and date of arrival in Tokyo.

QUESTION 18.

An aircraft leaves C (longitude 175°E) at 0900 LMT on 7th July to fly to Samoa. If the flight time is 6hours 20 minutes, what is the standard time and date of arrival in Samoa?

QUESTION 19.

An aircraft departs Hong Kong (2219N), 11412E) one hour before sunset on 24th July to fly to Guam (1330N, 14445E). If the flight time is 6 hours 30 minutes, what is the standard time and date of arrival in Guam?

QUESTION 20.

An aircraft arrives at Karachi, Pakistan at 1615 ST. If the time of departure from Ankara, Turkey (3958N, 3225E) for this direct flight was 0825 LMT, what was the flight time?

ANSWERS:

ANSWER 1.

The 'declination' of a star is the angular distance (along the hour circle) of the star measured in degrees north or south of the equinoctial. (It is similar to latitude.)

ANSWER 2.

The 'hour angle' of a star in the angular distance measured in degrees west along the equinoctial between a datum hour circle and the hour circle of the star. (It is similar to longitude except that it is measured west from 0° to 360°.)

ANSWER 3.

The orbit of each planet describes an ellipse around the sun, with the sun positioned at one of the foci of the ellipse.

ANSWER 4.

The radius vector of any planet sweeps out an equal area in equal time.

ANSWER 5.

An apparent solar day is the time between successive transits of the real (visual) sun across the observer's meridian.

ANSWER 6.

A mean solar day in always exactly 24 hours long. The length of an apparent solar day varies because the earth's orbit around the sun is an ellipse (Kepler's first law) and because the earth's speed along its orbit is continuously changing (Kepler's second law).

ANSWER 7.

Start of MCT	0402 LMT
Sunrise	0436 LMT
Duration of MCT	34 minutes

ANSWER 8.

	16th July	**(17th July)**	**19th July**
30° S	0654	(0654)	0653
31°S 31'S		0657	
35° S	0705	(0705)	0704

Answer: 0657 LMT

ANSWER 9.

	1200	LMT	4th May
Arc to time (90°E)	0600	-	
	0600	UTC	4th May

ANSWER 10.

	1104	ST	14th February
ST difference lists	0600	+	
	1704	UTC	14th February

ANSWER 11.

	2115	LMT 2nd February	
Arc to time (172° 35'W)	1130	+	
	(3245)		
	0845	UTC	3rd February

ANSWER 12.

	0011	ST	21st December
ST difference list	0800-		
	1611	UTC	20th December

ANSWER 13.

Sunrise (tables)	0719½	LMT	19th July
Arc to time (17445E)	1139	-	
Sunrise	1940½	UTC	18th July
ST difference lists	1200	+	
	(3140½)		
Sunrise	0740½	ST	19th July

ANSWER 14.

	0900	LMT	22nd July
Arc to time (50°W)	0320	+	
	1220	UTC	22nd July
Arc to time (90°W)	0600	-	
	0620	LMT	22nd July

ANSWER 15.

Kuwait	1210	ST	15th July
ST difference lists	0300	-	
	0910	UTC	15th July
ST difference lists	0100	+	
Algeria	1010	ST	15th July

ANSWER 16.

Depart 'A'	1910	LMT	16th March
Arc to time (63°W)	0412	+	
Depart 'A'	2322	UTC	16th March
Flight time	0320	+	
	(2642)		
Arrive 'B'	0242	UTC	17th March
Arc to time (49°W)	0316	-	
Arrive 'B'	2326	LMT	16th March

ANSWER 17.

Depart Los Angeles	1230	ST	14th August
ST difference lists	0700	+	
Depart Los Angeles	1930	UTC 14th August	
Flight time	1110	+	
	(3040)		
Arrive Tokyo	0640	UTC 15th August	
ST difference lists	0900	+	
Arrive Tokyo	1540	ST	15th August

The apparently long duration of this flight is explained by its crossing the prime anti-meridian from east to west.

ANSWER 18.

Depart 'C'	0900	LMT 7th July	
Arc to time (175°E)	1140	-	
Depart 'C'	2120	UTC 6th July	
Flight time	0620	+	
	(2740)		
Arrive Samoa	0340	UTC 7th July	
ST difference lists	1100	-	
Arrive Samoa	1640	ST	6th July

The fact that this flight apparently lands before it has taken off is explained by its crossing the prime anti-meridian from west to east.

ANSWER 19.

Sunset in Hong Kong 1844 LMT (from tables)

Depart Hong Kong	1744	LMT 24th July	
Arc to time (114°12E)	0737	-	
Depart Hong Kong	1007	UTC 24th July	
Flight time	0630	+	
Arrive Guam	1637	UTC 24th July	
ST difference lists	1000	+	
	(2637)		
Arrive Guam	0237	ST	25th July

ANSWER 20.

Depart Ankara	0825	LMT
Arc to time (32° 25'E)	0210	-
Depart Ankara	0615	UTC
Arrive Karachi	1615	ST
ST difference lists	0500	-
Arrive Karachi	1115	UTC

The flight time is the difference between departure times in UTC. In this example, the flight time is 5 hours.

Extracts from the Air Alamanac

Extracts from

THE AIR ALMANAC

(July - December 1977)

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FIGURE 22-7

STANDARD TIMES (Corrected to December 1977)

LIST I—PLACES FAST ON G.M.T. (mainly those EAST OF GREENWICH)

The times given below should be { added to G.M.T. to give Standard Time. / subtracted from Standard Time to give G.M.T.

	h	m
Admiralty Islands	10	
Afghanistan	04	30
Albania*	01	
Algeria*	01	
Amirante Islands	04	
Andaman Islands	05	30
Angola	01	
Annobon Island‡	01	
Arabian Emirates, Federation of	04	
Australia		
Australian Capital Territory*	10	
New South Wales[1]*	10	
Northern Territory	09	30
Queensland	10	
South Australia*	09	30
Tasmania*	10	
Victoria*	10	
Western Australia	08	
Austria*	01	
Balearic Islands*	01	
Bangladesh	06	
Belgium*	01	
Benin (Dahomey)	01	
Botswana, Republic of	02	
Brunei	08	
Bulgaria	02	
Burma	06	30
Burundi	02	
Cambodia (Democratic Kampuchea)	07	
Cameroon Republic	01	
Caroline Islands, east of long. E. 160°	12	
west of long. E. 160°	10	
Truk, Ponape	11	
Central African Empire	01	
Chad	01	
Chagos Archipelago	05	
Chatham Islands‡	12	45
China*	08	
Christmas Island, Indian Ocean	07	
Cocos Keeling Islands	06	30
Comoro Islands	03	
Congo Republic	01	
Corsica‡*	01	
Crete*	02	
Cyprus, Ercan*	02	
Larnaca*	02	
Czechoslovakia	01	
Denmark	01	
Djibouti	03	
Egypt (United Arab Republic)	02	
Equatorial Guinea, Republic of	01	
Estonia	03	
Ethiopia	03	
Fernando Póo‡	01	
Fiji	12	
Finland	02	
France*	01	
Friendly Islands	13	
Gabon	01	
Germany, East	01	
West[3]	01	
Gibraltar‡	01	
Gilbert Islands	12	
Greece*	02	
Guam*	10	
Holland (The Netherlands)*	01	
Hong Kong	08	
Hungary	01	
India	05	30
Indonesia, Republic of		
Bali, Bangka, Billiton, Java, Lombok, Madura, Sumatra	07	
Borneo, Celebes, Flores, Sumba, Sumbawa, Timor	08	
Aru, Kei, Moluccas, Tanimbar, Irian Jaya	09	
Iran‡	04	
Iraq	03	
Israel*	02	
Italy*	01	
Japan	09	
Jordan	02	
Kamchatka Peninsula	12	
Kenya	03	
Korea, North	09	
Republic of (South)	09	
Kuril Islands	11	
Kuwait	03	
Laccadive Islands	05	30
Ladrone Islands	10	
Laos	07	
Latvia	03	

* Summer time may be kept in these countries.
‡ The legal time may differ from that given here.
[1] Except Broken Hill Area which keeps 09h 30m.
[2] All the coast, but some areas may keep summer time.
[3] Including West Berlin.

FIGURE 22-8

STANDARD TIMES (Corrected to December 1977)

LIST I—(continued)

	h	m
Lebanon	02	
Lesotho	02	
Libya‡	02	
Liechtenstein	01	
Lord Howe Island	10	
Luxembourg*	01	
Macao	08	
Malagasy Republic	03	
Malawi	02	
Malaysia		
Malaya	07	30
Sabah, Sarawak	08	
Maldive Republic	05	
Malta*	01	
Manchuria	09	
Mariana Islands	10	
Marshall Islands†	12	
Mauritius	04	
Monaco*	01	
Mozambique	02	
Namibia (South West Africa)	02	
Nauru	11	30
Netherlands, The*	01	
New Caledonia	11	
New Hebrides	11	
New Zealand*	12	
Nicobar Islands	05	30
Niger	01	
Nigeria, Republic of	01	
Norfolk Island	11	30
Norway	01	
Novaya Zemlya	05	
Ocean Island	11	30
Okinawa	09	
Oman	04	
Pakistan	05	
Papua New Guinea	10	
Pescadores Islands	08	
Philippine Republic*	08	
Poland*	01	
Réunion	04	
Rhodesia	02	
Romania	02	
Rwanda	02	
Ryukyu Islands	09	
Sakhalin	11	
Santa Cruz Islands	11	
Sardinia*	01	
Saudi Arabia	03	
Schouten Islands	09	
Seychelles	04	
Sicily*	01	
Singapore	07	30
Socotra	03	
Solomon Islands	11	
Somalia Republic	03	
South Africa, Republic of	02	
Southern Yemen	03	
South West Africa (Namibia)	02	
Spain*	01	
Spanish Morocco*	01	
Spitsbergen (Svalbard)	01	
Sri Lanka	05	30
Sudan, Republic of	02	
Swaziland	02	
Sweden	01	
Switzerland	01	
Syria* (Syrian Arab Republic)	02	
Taiwan	08	
Tanzania	03	
Thailand	07	
Tonga Islands	13	
Truk	11	
Tunisia*	01	
Turkey*	02	
Tuvalu Islands	12	
Uganda	03	
Union of Soviet Socialist Republics§		
west of long. E. 40°	03	
long. E. 40° to E. 52° 30′	04	
long. E. 52° 30′ to E. 67° 30′	05	
long. E. 67° 30′ to E. 82° 30′	06	
long. E. 82° 30′ to E. 97° 30′	07	
long. E. 97° 30′ to E. 112° 30′	08	
long. E. 112° 30′ to E. 127° 30′	09	
long. E. 127° 30′ to E. 142° 30′	10	
long. E. 142° 30′ to E. 157° 30′	11	
long. E. 157° 30′ to E. 172° 30′	12	
east of long. E. 172° 30′	13	
Vietnam, Northern	07	
Southern‡	07	
Wrangell Island	13	
Yugoslavia	01	
Zaire		
Kinshasa, Mbandaka	01	
Haut-Zaïre, Kivu, Shaba, Kasai	02	
Zambia, Republic of	02	

* Summer time may be kept in these countries.
‡ The legal time may differ from that given here.
† Except the islands of Kwajalein and Eniwetok which keep a time 24h slow on that of the rest of the islands.
§ The boundaries between the zones are irregular; the longitudes given are approximate only.

FIGURE 22-9

STANDARD TIMES (Corrected to December 1977)

LIST II—PLACES NORMALLY KEEPING G.M.T.

Ascension Island	Great Britain[1]	Irish Republic*	Morocco	Senegal
Canary Islands*	Guinea Bissau	Ivory Coast	Portugal*	Sierra Leone
Channel Islands[1]	Guinea Republic	Liberia	Principe	Tangier
Faeroes, The	Iceland	Madeira	Rio de Oro‡	Togo Republic
Gambia	Ifni	Mali	St. Helena	Tristan da Cunha
Ghana	Ireland, Northern[1]	Mauritania (Dakhla)	São Tomé	Upper Volta

* Summer time may be kept in these countries.
‡ The legal time may differ from that given here.
[1] Summer time, one hour in advance of G.M.T., is kept from March 19^{d} 02^{h} to October 29^{d} 02^{h} G.M.T.

LIST III—PLACES SLOW ON G.M.T. (WEST OF GREENWICH)

The times given below should be | subtracted from G.M.T. to give Standard Time. added to Standard Time to give G.M.T.

	h	m
Argentina‡	03	
Austral Islands[1]	10	
Azores	01	
Bahamas*	05	
Barbados	04	
Belize*	06	
Bermuda*	04	
Bolivia	04	
Brazil, eastern[2]	03	
Territory of Acre	05	
western	04	
British Antarctic Territory[3]	03	
Canada		
Alberta*	07	
British Columbia*	08	
Labrador*	04	
Manitoba*	06	
New Brunswick*	04	
Newfoundland*	03	30
Northwest Territories*		
east of long. W. 68°	04	
long. W. 68° to W. 85°	05	
long. W. 85° to W. 102°	06	
west of long. W. 102°	07	
Nova Scotia*	04	
Ontario*, east of long. W. 90°	05	
west of long. W. 90°	06	
Prince Edward Island*	04	
Quebec*, east of long. W. 63°	04	
west of long. W. 63°	05	
Saskatchewan*		
east of long. W. 106°	06	
west of long. W. 106°	07	
Yukon	08	

	h	m
Cape Verde Islands‡	01	
Cayman Islands	05	
Chile‡	04	
Christmas Island, Pacific Ocean	10	
Colombia	05	
Cook Islands, except Niue	10	30
Costa Rica	06	
Cuba*	05	
Curaçao Island	04	
Dominican Republic‡	04	
Easter Island (I. de Pascua)*	07	
Ecuador	05	
Falkland Islands[4]	04	
Fanning Island	10	
Fernando de Noronha Island	02	
French Guiana‡	03	
Galápagos Islands	06	
Greenland, Scoresby Sound[5]	02	
Angmagssalik and west coast	03	
Thule area	04	
Grenada	04	
Guadeloupe	04	
Guatemala	06	
Guiana, French‡	03	
Guyana, Republic of‡	03	
Haiti	05	
Honduras	06	
Jamaica*	05	
Jan Mayen Island	01	
Johnston Island	10	

* Summer time may be kept in these countries.
‡ The legal time may differ from that given here.
[1] This is the legal standard time, but local mean time is generally used.
[2] Including all the coast and Brasilia.
[3] Except South Georgia which keeps 02^{h}.
[4] Except Port Stanley which keeps 03^{h} ‡.
[5] Scoresby Sound may keep 03^{h} in summer.

FIGURE 22-10

STANDARD TIMES (Corrected to December 1977)

LIST III—(continued)

	h	m
Juan Fernandez Islands	04	
Leeward Islands	04	
Low Archipelago	10	
Marquesas Islands[1]	09	30
Martinique	04	
Mexico‡ * [2]	06	
Midway Islands	11	
Miquelon	03	
Nicaragua‡	06	
Niue Island	11	
Panama Canal Zone	05	
Panama, Republic of	05	
Paraguay*	04	
Peru	05	
Puerto Rico	04	
Rarotonga	10	30
St. Pierre and Miquelon	03	
Salvador, El	06	
Samoa	11	
Society Islands[1]	10	
South Georgia	02	
Surinam	03	30
Tobago	04	
Trindade Island, South Atlantic	02	
Trinidad	04	
Tuamotu Archipelago[1]	10	
Tubuai Islands[1]	10	
Turks and Caicos Islands*	05	
United States of America		
Alabama[3]	06	
Alaska[3], east of long. W. 137°	08	
long. W. 137° to W. 141°	09	
long. W. 141° to W. 161°	10	
long. W. 161° to W. 172° 30′	11	
Aleutian Islands	11	
Arizona	07	
Arkansas[3]	06	
California[3]	08	
Colorado[3]	07	
Connecticut[3]	05	
Delaware[3]	05	
District of Columbia[3]	05	
Florida[3, 4]	05	
Georgia[3]	05	
United States of America (continued)		
Hawaii	10	
Idaho[3, 4]	07	
Illinois[3]	06	
Indiana[3]	05	
Iowa[3]	06	
Kansas[3, 4]	06	
Kentucky[3, 4]	05	
Louisiana[3]	06	
Maine[3]	05	
Maryland[3]	05	
Massachusetts[3]	05	
Michigan[3, 4]	05	
Minnesota[3]	06	
Mississippi[3]	06	
Missouri[3]	06	
Montana[3]	07	
Nebraska[3, 4]	06	
Nevada[3]	08	
New Hampshire[3]	05	
New Jersey[3]	05	
New Mexico[3]	07	
New York[3]	05	
North Carolina[3]	05	
North Dakota[3, 4]	06	
Ohio[3]	05	
Oklahoma[3]	06	
Oregon[3, 4]	08	
Pennsylvania[3]	05	
Rhode Island[3]	05	
South Carolina[3]	05	
South Dakota[3], eastern part	06	
western part	07	
Tennessee[3, 4]	06	
Texas[3, 4]	06	
Utah[3]	07	
Vermont[3]	05	
Virginia[3]	05	
Washington, D.C.[3]	05	
Washington[3]	08	
West Virginia[3]	05	
Wisconsin[3]	06	
Wyoming[3]	07	
Uruguay	03	
Venezuela‡	04	
Virgin Islands	04	
Windward Islands	04	

* Summer time may be kept in these countries.
‡ The legal time may differ from that given here.
[1] This is the legal standard time, but local mean time is generally used.
[2] Except the states of Sonora, Sinaloa, Nayarit, and the Southern District of Lower California which keep 07h, and the Northern District of Lower California which keeps 08h.
[3] Summer (daylight-saving) time, one hour fast on the time given, is kept in these states from the last Sunday in April to the last Sunday in October, changing at 02h 00m local clock time.
[4] This applies to the greater portion of the state.

FIGURE 22-11

SUNSET

Lat.	June 28	July 1	4	7	10	13	16	19	22	25	28	31	Aug. 3	6	9	12	15
°	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	□	□	□	□	□	□	□	□	□	□	23 12	22 35	22 08	21 45
70	□	□	□	□	□	□	□	□	□	□	23 36	22 53	22 29	22 07	21 48	21 30	21 13
68	□	□	□	□	□	□	□	23 25	22 58	22 37	22 19	22 03	21 47	21 32	21 18	21 04	20 50
66	□	23 48	23 26	23 11	22 57	22 44	22 32	22 20	22 07	21 55	21 43	21 31	21 19	21 08	20 56	20 44	32
64	22 30	22 26	22 21	22 16	22 09	22 02	21 54	21 45	21 36	27	21 18	21 08	20 58	20 48	38	28	18
62	21 53	21 51	21 47	21 43	21 38	21 33	27	20	21 13	21 05	20 58	20 50	41	33	24	15	20 06
N 60	21 27	21 25	21 23	21 20	21 16	21 12	21 07	21 01	20 55	20 49	20 42	20 35	20 27	20 20	20 12	20 04	19 55
58	21 07	21 06	21 04	21 01	20 58	20 54	20 50	20 45	40	34	28	21	15	20 08	20 01	19 54	46
56	20 50	20 49	20 48	20 46	43	40	36	32	27	22	17	11	20 05	19 59	19 52	45	39
54	36	35	34	32	30	27	24	20	16	11	20 06	20 00	19 56	50	44	38	32
52	24	23	22	21	18	16	13	10	20 06	20 02	19 58	19 53	48	43	37	31	25
N 50	20 13	20 13	20 12	20 10	20 08	20 06	20 04	20 01	19 57	19 54	19 50	19 45	19 41	19 36	19 31	19 25	19 20
45	19 51	19 50	19 50	19 49	19 47	19 46	19 44	19 41	39	36	31	28	25	21	17	12	19 06
40	33	33	32	32	31	29	28	26	24	21	19	16	12	19 09	19 06	19 02	18 58
35	18	18	18	17	16	15	14	13	11	19 09	19 07	19 04	19 00	18 56	18 56	18 53	49
30	19 05	19 05	19 05	19 05	19 04	19 03	19 02	19 01	19 00	18 58	18 57	18 55	18 52	50	47	45	42
N 20	18 43	18 43	18 43	18 44	18 43	18 43	18 43	18 42	18 41	18 40	18 39	18 38	18 36	18 35	18 33	18 31	18 29
N 10	24	25	25	25	26	26	26	25	25	25	24	23	22	22	21	19	18
0	18 07	18 07	18 08	18 08	18 09	18 09	18 10	18 10	18 10	18 10	18 10	18 10	18 10	18 09	18 09	18 08	18 08
S 10	17 49	17 50	17 51	17 52	17 52	17 53	17 54	17 54	17 55	17 56	17 56	17 56	17 57	17 57	17 58	17 58	17 58
20	31	31	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
S 30	17 10	17 11	17 12	17 13	17 15	17 16	17 18	17 20	17 21	17 23	17 25	17 27	17 28	17 30	17 32	17 34	17 35
35	16 58	16 59	17 00	17 01	17 03	17 05	17 07	17 09	17 11	13	15	17	19	22	24	26	28
40	43	45	16 46	16 48	16 50	16 52	16 54	16 56	16 59	17 01	17 04	17 07	17 09	12	15	18	20
45	27	28	30	32	34	37	39	42	45	16 48	16 51	16 54	16 58	17 01	17 05	17 08	11
50	16 06	16 08	10	12	15	18	21	24	28	31	35	39	43	16 48	16 52	16 56	17 00
S 52	15 56	15 58	16 00	16 03	16 06	16 09	16 12	16 16	16 20	16 24	16 28	16 32	16 37	16 41	16 46	16 50	16 55
54	45	47	15 50	15 52	15 55	15 58	16 02	16 06	11	15	20	24	29	34	39	45	50
56	33	35	37	40	44	47	15 51	15 56	16 01	16 05	11	16	21	27	32	38	43
58	18	20	23	27	30	34	39	44	15 49	15 55	16 00	16 06	11	18	24	30	37
S 60	15 01	15 03	15 06	15 10	15 14	15 19	15 24	15 30	15 36	15 42	15 48	15 55	16 01	16 08	16 15	16 22	16 29

SUNRISE

Lat.	June 28	July 1	4	7	10	13	16	19	22	25	28	31	Aug. 3	6	9	12	15
°	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	□	□	□	□	□	□	□	□	□	□	00 43	01 27	01 56	02 19
70	□	□	□	□	□	□	□	□	□	□	□	01 09	01 37	01 59	02 18	02 36	02 52
68	□	□	□	□	□	□	□	00 35	01 08	01 30	01 49	02 06	02 22	02 36	02 50	03 03	03 16
66	□	00 03	00 37	00 53	01 09	01 23	01 36	01 49	02 02	02 14	02 27	02 38	02 50	03 02	03 13	24	35
64	01 33	01 40	01 45	01 52	01 59	02 07	02 16	02 25	34	02 43	02 53	03 02	03 12	21	31	40	03 49
62	02 12	02 16	02 20	02 25	02 30	36	02 43	02 50	02 57	03 05	03 13	21	29	37	45	03 54	04 02
N 60	02 39	02 41	02 45	02 49	02 53	02 58	03 04	03 10	03 16	03 23	03 29	03 36	03 43	03 50	03 58	04 05	04 12
58	02 59	03 01	03 04	03 08	03 11	03 16	21	26	31	37	43	03 49	03 56	04 02	04 08	15	21
56	03 16	18	20	23	27	31	35	40	44	03 50	03 55	04 00	04 06	12	17	23	29
54	30	32	34	37	40	43	47	03 52	03 56	04 00	04 05	10	15	20	26	31	36
52	42	44	46	48	03 51	03 55	03 58	04 02	04 06	10	14	19	23	28	33	38	43
N 50	03 53	03 54	03 56	03 58	04 01	04 04	04 08	04 11	04 15	04 19	04 23	04 27	04 31	04 35	04 39	04 44	04 48
45	04 15	04 17	04 18	04 20	22	25	28	30	33	37	40	43	46	04 50	04 53	04 57	05 00
40	33	34	36	38	39	41	44	46	04 49	04 51	04 54	04 56	04 59	05 02	05 05	05 08	11
35	04 48	04 49	04 50	04 52	04 54	04 55	04 57	04 59	05 01	05 03	05 06	05 08	05 10	12	15	17	19
30	05 01	05 02	05 03	05 05	05 06	05 07	05 09	05 11	12	14	16	18	20	21	23	25	27
N 20	05 23	05 24	05 25	05 26	05 27	05 28	05 29	05 30	05 31	05 32	05 34	05 35	05 36	05 37	05 38	05 39	05 40
N 10	42	05 43	05 43	05 44	05 45	05 46	05 46	05 47	05 48	05 48	05 49	05 49	05 50	05 50	05 50	05 51	05 51
0	05 59	06 00	06 01	06 01	06 02	06 02	06 02	06 02	06 03	06 03	06 03	06 03	06 03	06 02	06 02	06 02	06 01
S 10	06 17	17	18	18	18	18	18	18	18	17	17	16	15	15	14	12	11
20	35	35	36	36	36	35	35	34	34	33	32	31	29	28	26	24	22
S 30	06 56	06 56	06 56	06 56	06 56	06 55	06 54	06 53	06 52	06 50	06 48	06 47	06 44	06 42	06 40	06 37	06 34
35	07 06	07 06	07 06	07 06	07 07	07 06	07 05	07 04	07 02	07 00	06 58	06 56	06 53	06 51	48	45	41
40	23	23	22	22	21	19	18	16	14	12	07 09	07 06	07 03	07 00	06 57	06 53	49
45	07 39	07 38	38	38	36	35	33	31	28	25	22	19	15	11	07 07	07 03	06 58
50	08 00	08 00	07 59	07 58	07 56	07 54	07 51	49	45	42	38	34	30	25	20	15	07 09
S 52	08 10	08 09	08 08	08 07	08 05	08 03	08 00	07 57	07 54	07 50	07 46	07 41	07 36	07 31	07 26	07 20	07 15
54	21	20	19	18	15	13	10	08 06	08 02	07 58	07 54	49	44	38	32	26	20
56	33	33	31	29	27	24	21	17	13	08 08	08 03	07 58	07 52	46	40	33	26
58	08 48	08 47	08 45	08 43	40	37	33	29	24	19	13	08 07	08 01	07 55	48	41	33
S 60	09 03	09 04	09 02	09 00	08 56	08 52	08 48	08 43	08 38	08 32	08 25	08 19	08 12	08 05	07 57	07 49	07 41

FIGURE 22-12

EVENING CIVIL TWILIGHT

Lat.	June 28	July 1	4	7	10	13	16	19	22	25	28	31	Aug. 3	6	9	12	15
	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
N 72	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	////	////	////	////
70	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	////	////	////	////	////	////	////
68	▭	▭	▭	▭	▭	▭	▭	////	////	////	////	////	////	////	[illegible]	[illegible]	[illegible]
66	▭	////	////	////	////	////	////	////	////	////	////	////	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
64	////	////	////	////	////	////	////	////	////	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
62	////	////	////	////	////	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

MORNING CIVIL TWILIGHT

Lat.	June 28	July 1	4	7	10	13	16	19	22	25	28	31	Aug. 3	6	9	12	15
	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
N 72	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	////	////	////	////
70	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	▭	////	////	////	////	////	////
68	▭	▭	▭	▭	▭	▭	▭	////	////	////	////	////	////	////	////	[illegible]	[illegible]
66	▭	////	////	////	////	////	////	////	////	////	////	////	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
64	////	////	////	////	////	////	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
62	////	////	////	////	////	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
N 10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
S 60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

FIGURE 22-13

CONVERSION OF ARC TO TIME

°	h m	°	h m	°	h m	°	h m	°	h m	°	h m	′	m s
0	0 00	60	4 00	120	8 00	180	12 00	240	16 00	300	20 00	0	0 00
1	0 04	61	4 04	121	8 04	181	12 04	241	16 04	301	20 04	1	0 04
2	0 08	62	4 08	122	8 08	182	12 08	242	16 08	302	20 08	2	0 08
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12	3	0 12
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16	4	0 16
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20	5	0 20
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24	6	0 24
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28	7	0 28
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32	8	0 32
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36	9	0 36
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40	10	0 40
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44	11	0 44
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48	12	0 48
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52	13	0 52
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56	14	0 56
15	1 00	75	5 00	135	9 00	195	13 00	255	17 00	315	21 00	15	1 00
16	1 04	76	5 04	136	9 04	196	13 04	256	17 04	316	21 04	16	1 04
17	1 08	77	5 08	137	9 08	197	13 08	257	17 08	317	21 08	17	1 08
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12	18	1 12
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16	19	1 16
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20	20	1 20
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24	21	1 24
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28	22	1 28
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32	23	1 32
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36	24	1 36
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40	25	1 40
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44	26	1 44
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48	27	1 48
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52	28	1 52
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56	29	1 56
30	2 00	90	6 00	150	10 00	210	14 00	270	18 00	330	22 00	30	2 00
31	2 04	91	6 04	151	10 04	211	14 04	271	18 04	331	22 04	31	2 04
32	2 08	92	6 08	152	10 08	212	14 08	272	18 08	332	22 08	32	2 08
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12	33	2 12
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16	34	2 16
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20	35	2 20
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24	36	2 24
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28	37	2 28
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32	38	2 32
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36	39	2 36
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40	40	2 40
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44	41	2 44
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48	42	2 48
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52	43	2 52
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56	44	2 56
45	3 00	105	7 00	165	11 00	225	15 00	285	19 00	345	23 00	45	3 00
46	3 04	106	7 04	166	11 04	226	15 04	286	19 04	346	23 04	46	3 04
47	3 08	107	7 08	167	11 08	227	15 08	287	19 08	347	23 08	47	3 08
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12	48	3 12
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16	49	3 16
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20	50	3 20
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24	51	3 24
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28	52	3 28
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32	53	3 32
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36	54	3 36
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40	55	3 40
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44	56	3 44
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48	57	3 48
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52	58	3 52
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56	59	3 56

The above table is for converting expressions in arc to their equivalent in time; its main use in this Almanac is for the conversion of longitude for application to L.M.T. (added if west, subtracted if east) to give G.M.T., or vice versa, particularly in the case of sunrise, sunset, etc.

Inertial Navigation Systems

Stable Platform Systems

Accelerometers

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Inertial Data - Processing and Presentation

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System Accuracy

The Development of Inertial Systems

Inertial Navigation Systems

1. Inertial Navigation Systems (INS) are self-contained navigation systems which give continuous and accurate information regarding the position of the aircraft to which they are fitted.

2. No inertial system can operate without accelerometers. These acceleration measuring devices sense any change in the aircraft velocity (acceleration/deceleration) very accurately. This information is then **integrated** once to give velocity (speed in a given direction) and a second time to give change of position (distance travelled in a given direction).

3. Necessarily the aircraft to which the INS is fitted will itself pitch, roll and yaw, and accelerations due to these manoeuvres must not be processed by the integrators. Furthermore, the aircraft is moving across the surface of a sphere which is itself moving through space. It is therefore a complex procedure to isolate those accelerations which are relevant to the aircraft's changing horizontal position relative to the Earth.

4. Two approaches for isolating the required horizontal accelerations are available, the first of which utilises a stable platform.

Stable Platform Systems

5. Stable platforms themselves come in two forms, the **north aligned system** and the **wander angle** system.

North Aligned Platforms

6. The north aligned system involves a stable platform upon which two accelerometers are mounted at right angles to each other. One accelerometer is maintained in alignment with true north. The second accelerometer, which is set at 90° to the first, will therefore be aligned east-west. There may be a third (vertical) accelerometer, but for the time being we will ignore this.

7. As well as being stable in direction, that is to say that the north-aligned accelerometer will continue to point to true north regardless of the aircraft heading, the platform is also maintained in a state of **horizontal stability**. This means that the platform, and therefore the accelerometers, will be maintained **Earth horizontal** regardless of any pitching or rolling movements of the aircraft. The accelerometers will not therefore measure the effect of gravity and erroneously interpret this as an acceleration. Since the Earth is itself rotating, and the aircraft moving across the Earth's surface, the INS computer is required to calculate the rate at which to **topple** the platform with respect to space in order to maintain the platform in an Earth horizontal attitude.

Wander Angle Platforms

8. A development of the north aligned system described above is the wander angle system. The hardware is exactly the same as for a north aligned platform and again requires that two accelerometers are mounted at right angles to each other on a platform which is maintained Earth horizontal. With some systems the operator has the choice of operating in either the north aligned or the wander angle mode.

9. In the wander angle system no effort is made to maintain north alignment. Instead, the angle by which the platform would have to be skewed (in the horizontal plane) in order to re-align the platform with north is calculated, and used to correct the outputs of the accelerometers so that the outputs **appear** to be coming from a north aligned system.

10. The notes which follow deal principally with the north aligned system.

Accelerometers

11. Newton's laws of motion give a clue as to the principle of operation of accelerometers:

(a) **Newton's First Law**. A body continues in a state of rest or uniform motion unless compelled to change that state by a force acting upon it.

(b) **Newton's Second Law**. The rate of change of motion of a body is proportional to the applied force causing the change, and takes place in the direction of the applied force.

(c) **Newton's Third Law**. To every applied force there is an equal and opposite reaction.

12. The most commonly used accelerometer is based on a pendulum and a closed loop feedback system and is called a force balance or force re-balance accelerometer. A schematic diagram of such an accelerometer is shown at Figure 23-1.

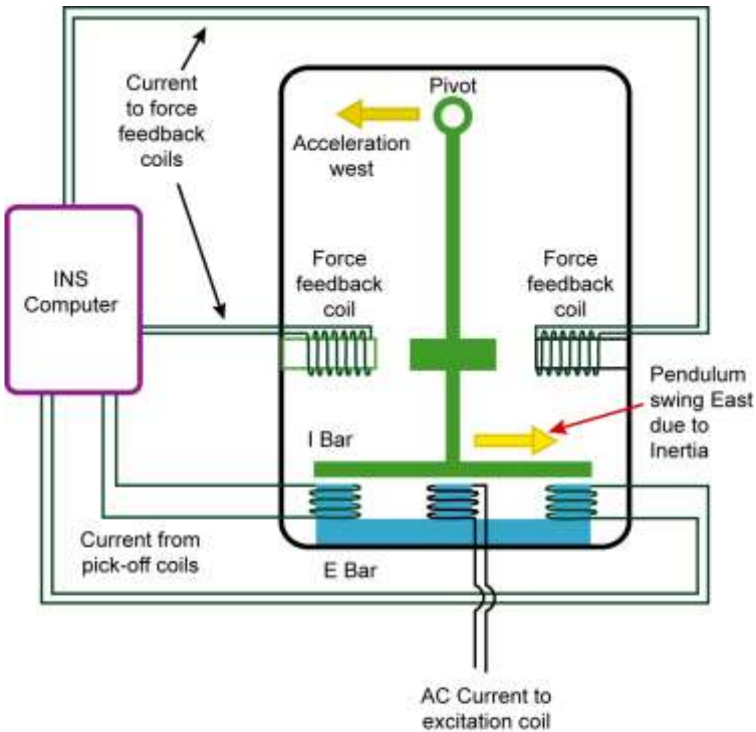
13. The pivot of the pendulum allows it to swing along only one axis, known as the **alignment axis.** In the example shown at Figure 23-1 the pendulum is aligned with an east-west axis and this accelerometer is therefore insensitive to any acceleration/deceleration which is entirely in a north-south direction.

14. The greater the rate of change of speed, the greater is the displacement of the pendulum as it lags behind the pivot. As the pendulum swings, the state of equilibrium at the **E and I bar detector** is disturbed and a greater current is induced to flow in one or other of the **pick-off coils**.

15. At Figure 23-1 the acceleration is to the west, the pendulum swings to the east (right) and a greater current flows to the left-hand pick-off coil, since the air gap between the E and the I bars is now smaller at the left hand end.

16. The difference in pick-off coil currents is sensed within the INS computer, which produces a DC current in the appropriate **force feed-back coil**. This current energises the associated electro-magnet which attracts the pendulum back to the vertical. The strength of the current required to maintain a vertical pendulum is proportional to the acceleration attempting to displace the pendulum, and it is the magnitude of this current which is used by the computer to measure the acceleration along the sensitive axis of the accelerometer.

FIGURE 23-1
Force Balance Accelerometer

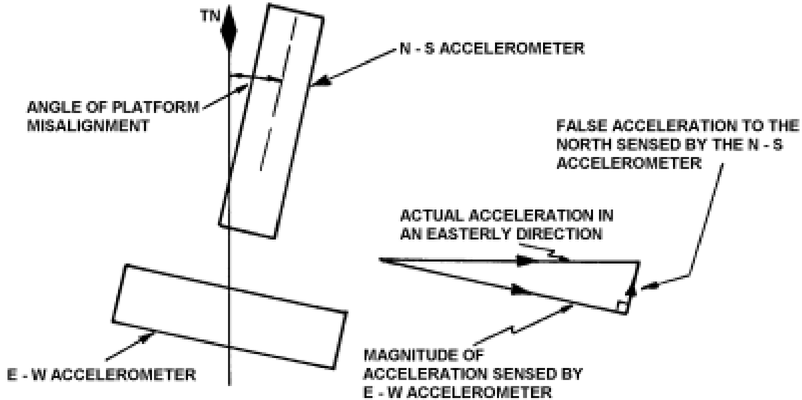


17. The accelerometer itself is extremely accurate and will measure precisely even the smallest change in aircraft velocity, subject to two very important constraints:

(a) If the accelerometer is not **Earth horizontal** a continuous current will be required at one or other of the force-feedback coils to maintain the equal air gaps between the I bar and the outer arms of the E bar. This **bias** current is caused to flow because of gravity, but unfortunately the computer will integrate this bias current as an erroneous groundspeed. It is therefore essential that the stable platform is maintained Earth horizontal.

(b) If the accelerometer is directionally misaligned, that is to say that the sensitive axes of the north-south and east-west accelerometers are not lying precisely in north-south and east-west planes, then the direction of acceleration will be incorrectly computed, as illustrated at Figure 23-2. It is therefore essential that the platform maintains a high degree of **directional alignment**.

FIGURE 23-2 Effect of Mis-Alignment of Accelerometers



Rate Integrating Gyros

18. The sensing gyros mounted on the platform are **rate type** gyros. Unlike the rate gyro found in the turn and slip indicator, the inertial rate gyros have no springs to inhibit the gyro wheel as it precesses. As the aircraft manoeuvres the relevant gyro (pitch, roll or azimuth) is allowed to precess, and this precession is sensed by a **pick-off coil** on the inner can pivot. The output of the pick-off coil is fed via the INS computer to the associated motor to cancel the platform movement as it attempts to follow the changing attitude of the aircraft, which would cause it to depart from an Earth horizontal and/or north aligned condition.

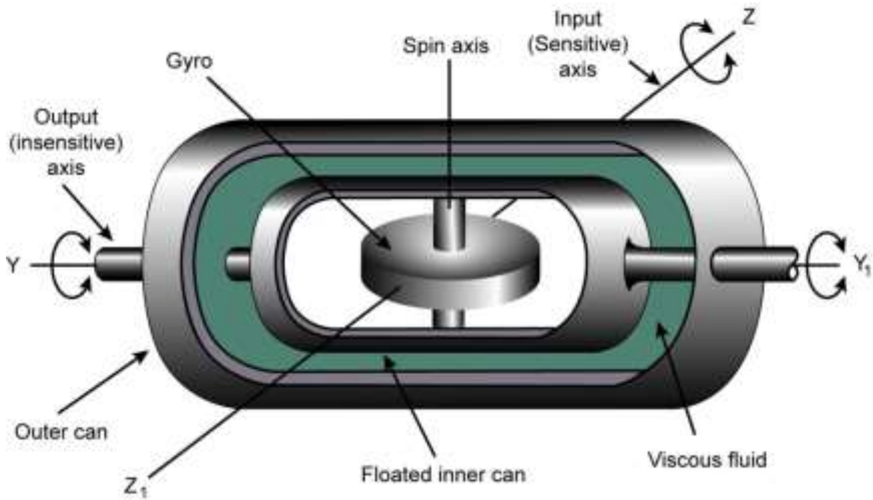
19. It is necessary that the gyros used in inertial systems are very accurate indeed. A real wander rate of 0.01°/hr is about all that is acceptable. In order to achieve this level of accuracy the gimbal and rotor assemblies of the gyro are floated in a fluid. The viscosity of the fluid controls the amount of precession which results from a given input. As viscosity varies with temperature, it is necessary that the gyros reach their operating temperature before the platform is switched into the navigation mode.

20. Like the rate gyro in the turn and slip indicator, the INS rate gyros have freedom of movement about only one axis (apart from the spin axis). Figure 23-3 shows a schematic breakdown of a typical rate integrating gyro. Note that the spin axis, the sensitive axis and the insensitive axis are all at right angles to each other.

21. The sensitive axis of the gyro at Figure 23-3 is $Z - Z^1$. Any movement about this axis results in precession of the gyro about the axis $Y - Y^1$. This rotation is picked off electrically at the pivot (the output axis $Y - Y^1$).

22. Also located at the pivot is a torque motor. This motor is fed from the INS computer with a signal which causes the motor to apply a torque to the gyro to correct for Earth rate and transport wander, in order to keep the platform Earth horizontal and north aligned.

FIGURE 23-3
INS Rate Integrating Gyro



The Stable Platform

23. In order that the platform can remain Earth horizontal and directionally aligned as the aircraft manoeuvres, it is necessary to mount the platform within a two-gimbal system, as illustrated at Figure 23-4.

24. The platform is kept Earth horizontal and in directional alignment by means of sensing gyros, error signals from which are used to drive motors attached to the gimbal spindles, as shown at Figure 23-4. The sensing gyros and motors correct for any and all aircraft movements in the pitching, rolling and yawing planes.

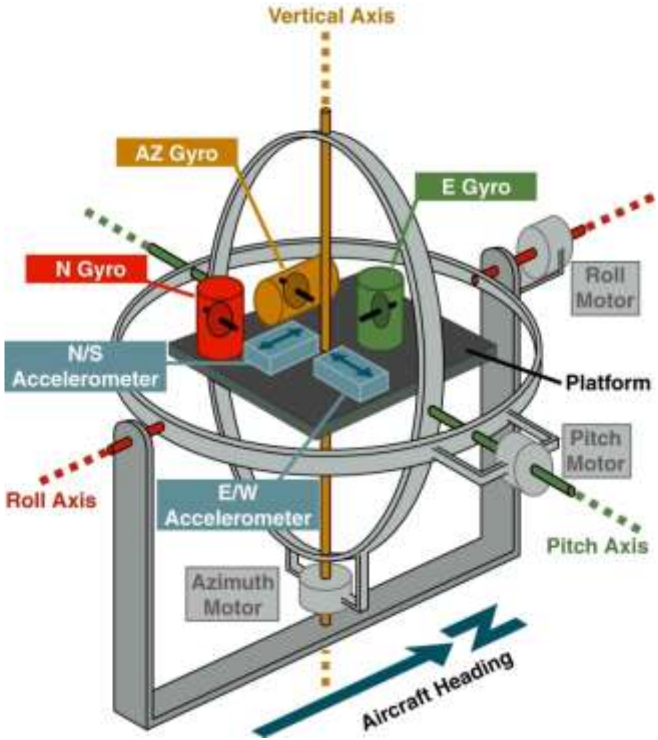
25. In the diagram at Figure 23-4 the aircraft is heading north. Any movement of the aircraft about its rolling axis will be sensed entirely by the **north gyro**, the output of which will be fed via the INS computer to the **roll motor**. On this heading any movement of the aircraft about its pitching axis will equally be entirely sensed by the **east gyro**, the output of which is used to drive the **pitch motor**.

26. Were the aircraft to be heading due east, the north gyro would sense only pitching movement and the east gyro would sense only rolling movement.

27. On all non-cardinal headings the pitch and roll motors are driven by composite outputs from both the north and east gyros, the computer takes care of the mathematics.

28. The azimuth gyro is sensitive to any yawing motion, and its output is always fed via the computer to the azimuth motor. The azimuth motor is also used to isolate the platform from aircraft heading changes.

FIGURE 23-4
The Stable Platform - Aircraft Heading True North



Corrections for Earth Rate and Transport Wander

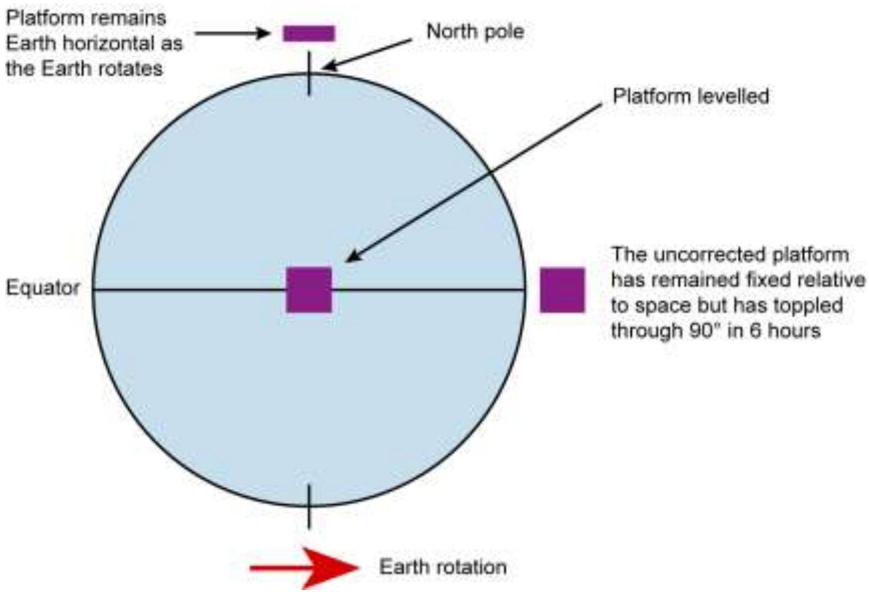
29. In order to maintain the platform Earth horizontal and north aligned, corrections are required to compensate for the rotation of the Earth, otherwise the platform will be controlled by the gyros to remain rigid in space rather than rigid with reference to the Earth. The gyro torque motors must therefore be continuously fed with Earth rate corrections. These corrections are considered in terms of topple and drift.

30. A stable platform which was levelled at either pole and remained there would not topple as a result of the rotation of the Earth. Another platform which was levelled at the equator and neither moved or corrected for Earth rotation would topple through 90° in 6 hours as a result of the Earth's rotation, as shown at Figure 23-5.

31. At the equator, the correction required to maintain the platform Earth horizontal is 15°/hour. At any other latitude the correction required is **15 x the cosine of the latitude °/hr**.

32. Conversely, a platform which was aligned with true north at the equator and neither moved or corrected for Earth rate will continue to point true north as the Earth rotates, since the meridians are parallel at the equator. At latitudes other than the equator it will be necessary to correct the alignment of the platform by applying a current to the torque motor on the azimuth gyro so that the azimuth motor can maintain north alignment. The magnitude of this correction is **15 x the sine of the latitude °/hr.**

FIGURE 23-5
Platform Topple at the Equator and Poles

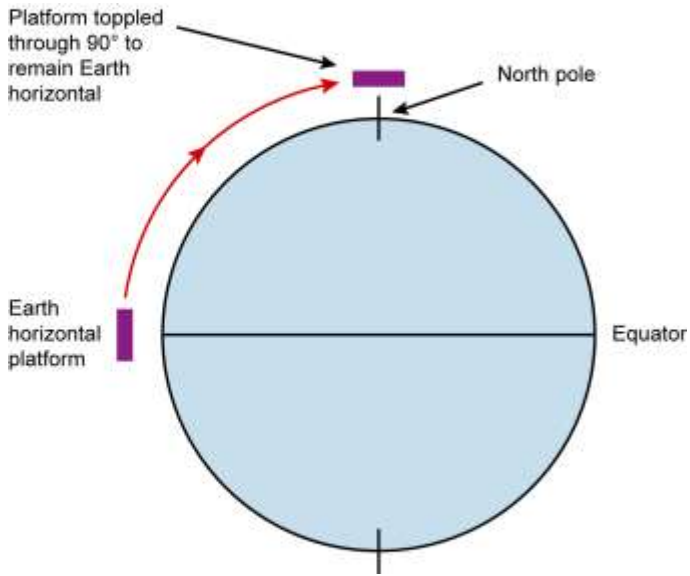


33. The problem of keeping the platform level and aligned is of course complicated by the fact that the platform is being transported across the surface of the Earth. The mathematics for transport wander as it affects the alignment of the INS platform are the same as the transport wander which we previously considered when looking at unslaved directional gyros. The computation of the correction required to maintain north alignment is based on the formula:

$$\text{The alignment rate} = \frac{\text{East/West component of groundspeed} \times \text{Tan latitude}}{60} \,°/\text{hr}$$

34. Transporting the platform also produces a requirement to topple the platform in order to keep it in the Earth horizontal. For example, in Figure 23-6 a platform is flown from the equator due north to the pole. In order that it remains horizontal throughout, the platform must be toppled through a total of 90°. In this case the rate of required topple is a function of the groundspeed north/ south. Suppose that the aircraft's groundspeed was 600 kt. The journey time from the equator to the pole (5400 nm @ 600 kt) would be 9 hours and the required topple rate would be 10°/hr. The correction required can therefore be expressed as **groundspeed north/south ÷ 60 °/hr**. Similarly, any component of the groundspeed in an east/west direction would require a correction of **groundspeed east/west ÷ 60 °/hr**. On anything other than a true cardinal track, the required correction would be a composite of both of the above and the correction would be achieved by means of error signals fed to the torque motors attached to both the north and east gyros which would result in activity at both the pitch and roll motors.

FIGURE 23-6
Platform Topple - Equator to Pole Transit



Coriolis Correction

35. Accelerometers mounted on a stable platform are subject to errors caused by coriolis.

36. Coriolis effect, put simply, is the force which acts on any body which is moving across the surface of the Earth, and which tends to deflect that body to the right in the northern hemisphere and to the left in the southern hemisphere. Coriolis force is due to the Earth's rotation about its own spin axis, and is fully described in the Meteorology Theory syllabus, where its effect on moving bodies of air is considered. Any force which acts upon the accelerometers will be assumed to be a change of velocity (acceleration or deceleration), and so it is necessary for the system to 'compute out' the coriolis effect on the accelerometers. In order to do this the INS computer requires values of aircraft latitude, track and groundspeed, but of course this presents no problem, since all of these values are themselves computed by the system.

Levelling and Gyro-compassing

37. As already emphasised, the output of the INS will only be of a high standard of accuracy if the platform is maintained Earth horizontal and correctly aligned with true north. The first step is obviously to set up the platform on the ground. **This procedure can only be satisfactorily completed whilst the aircraft is stationary**.

38. Initially the platform is levelled to aircraft horizontal and if necessary coarsely aligned by manually slewing the platform so that it is aligned with the aircraft's compass heading, corrected for variation (in modern systems the coarse manual alignment is not normally required). With the platform selected to the **align mode** fine levelling is now commenced. During this process the accelerometer outputs are assumed to be due to gravity, hence the requirement that the aircraft must not be moved at this stage. The outputs of the accelerometers are used in a servo loop principle to level the platform. This process is also known as the accelerometer null technique and is completed in 1 to 1½ minutes.

39. Once the fine levelling procedure is complete, the platform automatically commences the gyro-compassing process **for which it is essential that the present position is correctly inserted into the INS**. In fact it is an incorrect latitude which will prevent successful gyro-compassing and consequent accurate north alignment, we will see why shortly.

40. During the gyro-compassing process the outputs of gyros and accelerometers are combined to achieve a precise alignment. Refer again to Figure 23-4 and let us examine the gyro-compassing process for the platform shown, which is in an aircraft which is facing north. Were the platform to be precisely aligned with true north, the east gyro (which has a north/south spin axis) would be insensitive to movement of the platform about the north/south (Earth) axis as the platform is toppled by the torque motor on the north gyro, which (in this case) is driving the roll motor to keep the platform Earth horizontal.

41. Suppose now that the platform is mis-aligned, and that therefore the spin axis of the east gyro is no longer exactly north/south oriented. The east gyro will now detect a component of the Earth rate topple, and will precess. This precession is picked off and used by the INS computer to tilt the platform. The platform is now no longer Earth horizontal and the north/south accelerometer will sense gravity. The output from the north/south accelerometer is used by the computer to feed an error signal to the azimuth motor to re-align the platform and an error signal to the pitch motor (in this case since the aircraft is facing north) to re-level the platform. This process continues until the output from the north/south accelerometer is zero. Now the east gyro is insensitive to Earth rotation, since the platform is precisely aligned. The two basic inputs which are used to achieve gyro-compassing are therefore **Earth rate** and **gravity**.

42. Since the Earth rate corrections which are fed from the computer are at a rate of 15° x the cosine of the latitude °/hr, it follows that gyro-compassing will not be achieved at high latitudes (the cosine of 90° is zero). With modern systems 70° N/S is about the limit. Furthermore, the time taken to achieve alignment will increase as the latitude at which gyro-compassing occurs increases. If the computer is using an incorrect latitude during the gyro-compassing process, the platform will not align. In the best case, if the latitude which has been entered as the start (ramp) position is wildly in error, the computer will realise that something is wrong and will invite you to check the ramp position. In the worst case, if the entered latitude is only slightly in error, a false north alignment will occur, which will give rise to an ever increasing (unbounded) error in the INS position as the flight progresses.

43. Once the levelling and alignment processes are complete the **platform ready** light will illuminate and the **navigation mode** can be selected. The aircraft can now be moved. The story of what happens next (the way in which the outputs from the platform are processed and presented to the pilots) is continued in the Inertial Data - Processing and Presentation section.

Strapped Down Systems

44. The strapped down type of inertial navigation system became feasible with the advent of high speed large capacity digital microprocessors and the introduction of the ring laser gyro.

45. There is **no stabilised platform** and **three accelerometers are mounted rigidly** inside the inertial navigation unit, which is simply bolted to the aircraft structure. The accelerometers are therefore effectively fixed to the airframe, and are aligned with the aircraft's pitch, roll and yaw axes. **Three ring laser gyros** are also mounted in the same unit, and again their sensitive axes are aligned with the aircraft's pitch, roll and yaw axes.

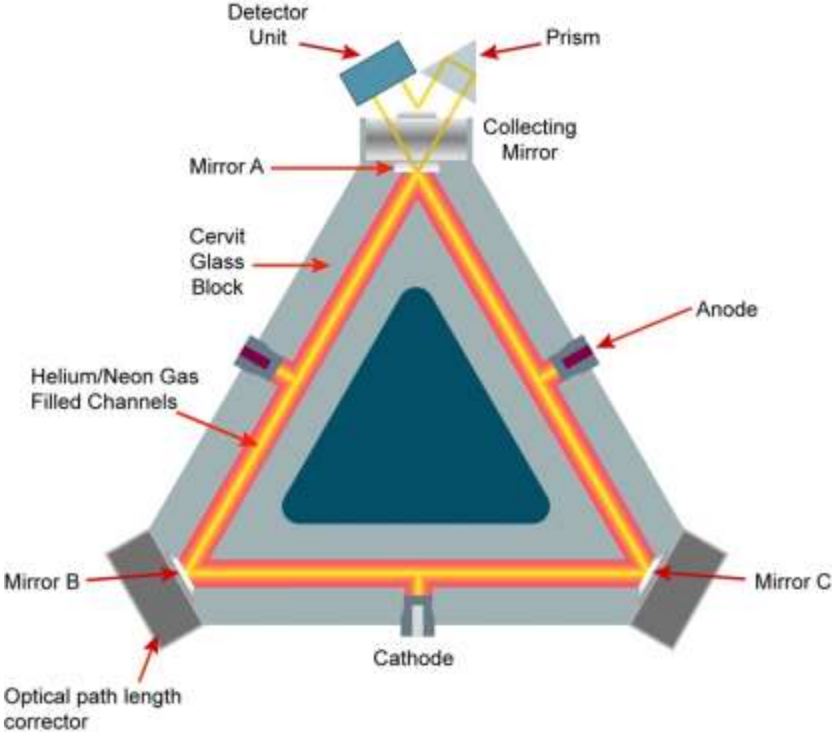
Solid State or Ring Laser Gyros

46. The ring laser gyro (RLG) is just about as different from a conventional gyro as it is possible to get. The RLG operates on the principle of the relative movement of two beams of laser light, whereas a conventional gyro operates on the principle of stored mechanical energy (inertia). RLGs are a solid state alternative to the conventional rate integrating gyro.

Construction

47. The construction of a triangular ring laser gyro is shown at Figure 23-7. Figure 23-7 shows a cross section through the sensitive plane of the RLG, the sensitive axis passes perpendicularly through the page. Square RLGs are also manufactured, the principle of operation is much the same.

FIGURE 23-7
Ring Laser Gyro



48. The RLG is formed from a solid block of cer-vit (glass/ceramic) material which is used because it has a very low temperature co-efficient. It does not distort with age, which would degrade the accuracy and perhaps destroy the gas tight seal necessary to contain the helium and neon gases. Channels are drilled very accurately in the block to form two triangular laser paths which are filled with a low pressure helium/neon mixture.

49. Two anodes and a common cathode are used to create electrical discharges within the gases which cause the channels to act as 'gain tubes', producing laser beams. These beams travel in opposite directions around the triangular block.

50. At each of the three corners of the block are flat mirrors which reflect the laser beams into the next channel. Two of these mirrors (B and C at Figure 23-7) are movable and are controlled by piezo-electric actuators operating in a closed loop system to adjust the optical path length to within precise limits, and so to correct for the small expansions/contractions of the block with changing temperatures within the permitted operating temperature band. It is necessary that the optical path is maintained at a length which is an integral multiple of the lasing wavelength, that is to say an exact number of wavelengths of the light waves at the frequency achieved when the lasers are operating at peak power. The third mirror (A at Figure 23-7) permits a small portion of the laser light beams to pass through the mirror and to be presented at the photoelectric cells of the detector. One of these beams is necessarily routed via a prism and a further mirror (as shown at Figure 23-7) so that the beams approach the detector from the same direction.

Principle of Operation

51. As in many applications of laser oscillators, the RLG makes use of the high sensitivity of the laser's oscillating frequency to variations of the dimensions of the resonant structure. The resonant structure, in this case the triangular block, is designed such that each beam will only sustain oscillation (that is to say resonate) at one particular spot frequency. If the RLG is stationary, the resonant frequency for both clockwise and anti-clockwise laser beams will be identical since the path lengths travelled by the beams is identical. When the beams combine at the detector they interfere with each other to form a fringe pattern of light bars at the photoelectric cells of the detector.

52. Assume now that the RLG shown at Figure 23-7 is rotating in a clockwise direction. The light beam which is travelling in the same direction as the rotation must travel a slightly longer path to complete one revolution, whilst the opposite beam will travel over a correspondingly shorter path. The resonant frequencies of the two beams will therefore be different and there will be a resultant change in the interference pattern, causing the light bars produced at the photoelectric cells of the detector to move. The direction of movement of the light bars will depend upon the direction of rotation of the RLG and the distance by which the light bars move will depend on the rate of rotation.

53. At first glance it might appear that the RLG shown at Figure 23-7 cannot work in the manner described above. The laser beam which is travelling clockwise is travelling further than the laser beam which is travelling anti-clockwise, since it is necessarily routed via the prism and one further mirror. In fact the clockwise beam, having passed through mirror A, travels a precise extra distance which ensures that it arrives at the detector at exactly the same phase as it would have done with a direct path. The effect of the additional path length is therefore cancelled.

54. The output of the photoelectric cells which comprise the detector unit are converted to pulse signals which are representative of the rate and direction of rotation of the RLG in the sensitive plane.

55. One problem remains to be solved. When the RLG is not rotating in its sensitive plane, scatter from the mirrored surfaces causes the opposing beams to lock together in a phenomenon known as **frequency lock**. The beams then tend to remain locked together until the rotation of the RLG reaches a certain rate, preventing small rates of rotation from being detected. To overcome this problem a **dither** is introduced by an external vibration device which vibrates the gyro at a resonant frequency. This breaks the frequency lock and allows much smaller rotations to be measured.

56. If we consider that the sensitive axis of the RLG shown at Figure 23-7 lies parallel to the lateral axis of the aircraft, then the gyro would measure the rate and direction of roll of the aircraft, since the aircraft would be rolling in the sensitive plane of the gyro.

57. The laser gyro is employed to perform the same functions as a conventional rate integrating gyro, however because of the high cost of RLGs it is likely that their use will continue to be restricted to systems requiring high levels of accuracy combined with low weight. In modern inertial navigation/reference systems (strapped down systems), three RLGs mounted orthogonally are used to provide the same outputs as the three mechanical rate integrating gyros which were used in older (stable platform) inertial navigation systems.

Advantages of the RLG

58. The principle advantages of the RLG over its mechanical counterpart are summarised below;

(a) high reliability (up to 60,000 hours before failure has been demonstrated)

(b) sensitive to a wide range of rotation rates (0.004 to 400 degrees/second)

(c) the RLG does not suffer from mechanical precession, and cannot topple

(d) much lower power inputs are required

(e) there is no spin up time required, and the stable operating temperature is quickly achieved

Strapped Down System Operation

59. In the strapped down system the accelerometers will clearly measure total acceleration, which will now be due to gravity (the accelerometers are not stabilised Earth horizontal), to aircraft manoeuvres and to the aircraft movement over the surface of the Earth. Of these only the third is required, and the other two output components must be isolated.

60. In order to achieve this the INS computer needs to determine the difference between the horizontal plane of the strapped down unit (aircraft horizontal) and Earth horizontal, and also the angle between the aircraft's roll (fore and aft) axis and true north during what is still referred to as the alignment (initial setting up) process. It does this by analysing the outputs of the accelerometers due to gravity. As with platform systems the aircraft must not be moved with the equipment in the align mode.

61. Once in the navigation mode the fast/high capacity processor within the INS computer uses the outputs of the RLGs in order to isolate the outputs of the accelerometers which are due solely to the movement of the aircraft relative to the Earth's surface and to relate these outputs to the Earth's north/south, east/west and vertical axes. Corrections for Earth rate and transport wander are achieved by adjusting the perceived angles between aircraft horizontal and Earth horizontal and between true north and the aircraft's roll axis.

62. The main advantage of the strapped down system over the stable platform system is that there are virtually no moving parts, making the system far more reliable, and much lighter. If a strapped down system does go unserviceable, replacement of the navigation unit is a much simpler procedure than with a stabilised platform. Initial alignment times are faster with a strapped down system since there is no platform to be aligned and the ring laser gyros achieve their stable operating temperatures much faster than their mechanical counterparts.

63. All strapped-down systems work on the same principles described above. Whereas many use the triangular ring-laser gyroscope previously described, some use a square or four-sided RLG. A further type patented by Litton Aero Products uses a four-mode laser gyroscope, with four laser paths in two planes, which together with circular light polarisation effectively combines two LRGs into one and eliminates the requirement for dither.

Inertial Data - Processing and Presentation

64. Having considered the nuts and bolts of inertial platforms and briefly discussed strapped down systems, it is now time to consider how the raw output from the system is converted into useful data, and how this is presented to the pilots.

Integration

65. Regardless of which type of inertial system is considered, they all have one thing in common. They all detect components of acceleration which are truly sensed in the Earth horizontal plane along the north/south and east/west axes (the north aligned platform). Alternatively the accelerometer outputs are corrected so that they give an output which represents that which would occur were the accelerometers so aligned (wander angle platforms and strapped down systems).

66. To produce the navigation parameters of track, groundspeed, distance and position the raw inputs of acceleration north/south and east/west must be summed (integrated) by time. Acceleration integrated with respect to time is speed, and speed integrated with respect to time is distance. These are the two necessary stages of integration which are required and which are shown at Figure 23-8.

67. Note that the distance travelled north/south in nautical miles equates to change of latitude, however in order to obtain change of longitude from the east/west distance travelled requires a further stage of computation, which is achieved within the **secant unit** of the computer.

68. Remember that east/west distance is known as departure and:

$$\text{Departure (nm)} = \text{Change of longitude (minutes of arc)} \times \text{Cosine Latitude}$$

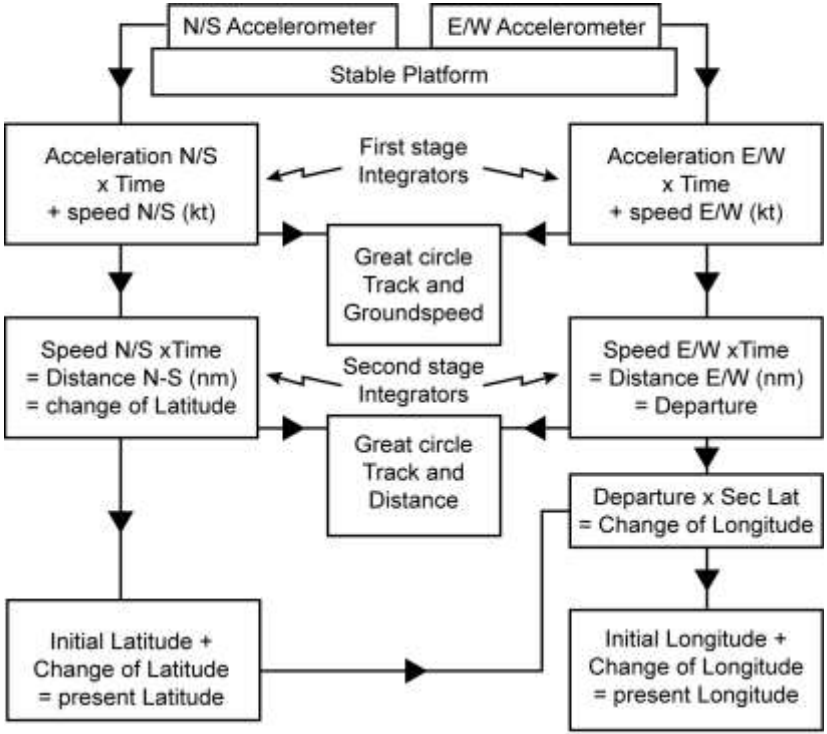
This formula must be re-arranged since in this case it is the change of longitude which is the unknown quantity, therefore:

$$\text{Change of longitude (minutes of arc)} = \frac{\text{Departure (nm)}}{\text{Cosine latitude}}$$

or:

$$\text{Change of longitude} = \text{Departure (nm)} \times \text{Secant latitude}$$

FIGURE 23-8
Stages of Integration



The Schuler Cycle

69. The key factor involved in the Schuler principle is that a stable platform which is maintained Earth horizontal behaves like a pendulum with a length equal to the radius of the Earth. The time taken for a pendulum to swing through one cycle is directly proportional to its length. Where this length is the same as the Earth's radius, the time taken for one cycle of the pendulum is 84.4 minutes.

70. Stable platforms possessing this property are said to be **Schuler tuned**.

71. In theory, once levelled the platform should remain stable and undisturbed, however vibration and turbulence shocks in flight are likely to create small disturbances. Consequently the platform is likely to be swinging continuously, hopefully by only a small amount.

72. The effect of this swing is that the outputs from the accelerometers will be in error by a maximum amount when the platform is at the extremity of its swing. The period of the swing is 84.4 minutes regardless of the magnitude of the disturbance which caused it, and so two important facts emerge:

(a) The Schuler tuned platform produces its maximum error at 21.1 and 63.3 minutes through each 84.4 minute cycle.

(b) The magnitude of the maximum error depends on the size of the disturbance which caused it, however the **mean error remains at zero** (assuming no inherent accelerometer error).

73. From the above it should be apparent that any error in the outputs of the accelerometers which is caused by Schuler tuning is **bounded**, that is to say that the error does not increase with time beyond its original maximum value.

74. Furthermore, since the output error of any accelerometer will not increase with time, the output of the first stage integrator associated with it (velocity) will also be bounded. For example, for an aircraft flying at a groundspeed of 600 kt the INS output of groundspeed might be 600 kt at minute zero, 602 kt at minute 21.1, 600 kt at minute 42.2, 598 kt at minute 63.3 and again 600 kt at minute 84.4. The mean output of groundspeed is correct at 600 kt.

75. We have discussed Schuler tuning as it applies to stable platforms which are maintained Earth horizontal. Appreciate, however, that strapped down systems are designed to deliver accelerometer outputs which are corrected (processed) to be representative of those which would be achieved were the accelerometers to be maintained Earth horizontal rather than aircraft horizontal. Strapped down systems are therefore also considered to be Schuler tuned, and to suffer similar bounded errors as a consequence.

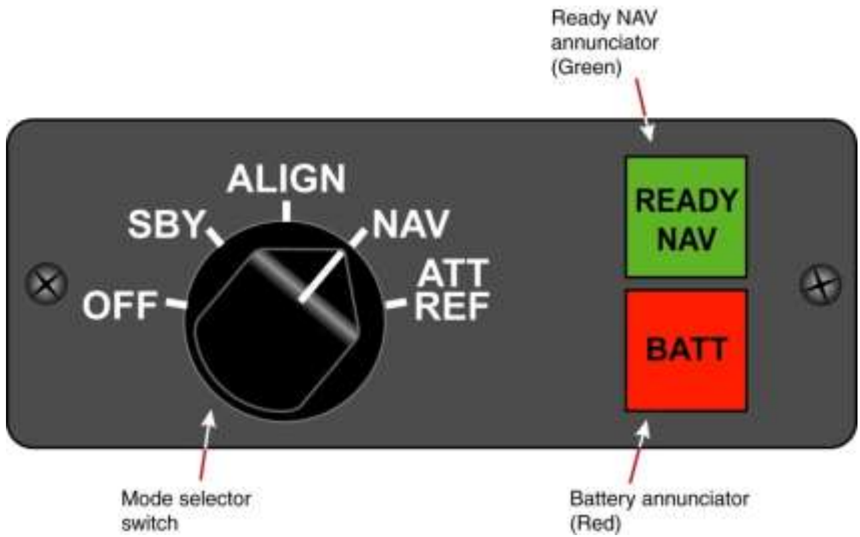
INS Control and Display Panels

76. There are many types of inertial navigation systems currently in use and the final paragraphs of this section summarise the evolution of this invaluable aid to navigation. Suffice for the moment to say that the system which is described in the following pages is an older type of INS, the operation of which is perhaps easier to understand than that of its more sophisticated successors.

The Mode Selector Panel

77. The traditional INS system comprises two control panels for control and display plus an **Inertial Reference Unit (IRU)** which contains the gyros, accelerometers, integration circuits etc. The simpler of the two control panels, **the mode selector unit (MSU)**, is shown at Figure 23-9.

FIGURE 23-9
Mode Selector
Unit (MSU)



78. The positions of the mode selector switch and the purpose of the two lights (Ready Nav and Batt) are discussed in the following paragraphs.

79. **Standby.** In this mode power is supplied to all parts of the system. It is normal to insert the start position (the aircraft's ramp position in latitude/longitude to the nearest tenth of a minute of arc) at this stage. Remember that an accurate latitude is essential for successful platform alignment.

80. **Align.** Having inserted the start position, with the aircraft stationary and with a stable power source, the align mode can now be selected to enable levelling and gyro-compassing to commence. If necessary, the system's progress through this phase can be checked by selecting status (STS) on the control display unit. Numbers will appear in the left numerical display window, starting at 90 as the levelling procedure commences and reaching zero as gyro-compassing is completed. The power source should remain constant during this period, switching from external to internal aircraft power at this stage can persuade the INS to restart the levelling/alignment process.

81. **Ready Nav**. When the levelling/alignment process is complete the green Ready Nav light will be illuminated, to indicate that the system is now ready for use in the navigation mode.

82. **Nav.** Selection of the navigation mode means that the INS is open for business, and the aircraft can now be moved. The nav position on the control panel is detented or spring biased to prevent an inattentive first officer from de-selecting the navigation mode in flight and ruining his command prospects for a year or two.

83. **Att Ref.** This position is normally for use following a computer/processing failure. All navigation computations are removed, however the platform may still be used to supply pitch, roll and heading information.

84. **Batt**. In the event that the power supply to the INS is interrupted, the system will continue to operate on its own internal (battery) DC supply, for approximately 15 minutes. The red **Batt** light on the mode selector panel will illuminate when the battery supply is exhausted.

The Control and Display Unit (CDU)

85. The operation of the INS computer is controlled by **the control and display unit (CDU)**: on a modern aircraft this unit may also be referred to as the **Inertial System Display Unit (ISDU)**.

86. The INS computer is able to determine aircraft heading. In a north aligned platform system the true heading is simply the angle between the north/south axis of the platform and the aircraft fore and aft axis. The INS computer's memory may contain a ‘look up table’ of magnetic variation worldwide. Using this memory and the INS position enables the INS to display heading in degrees magnetic. For commercial transport operations it is normal for the INS to express both headings and tracks in degrees magnetic, however appreciate that the INS is referenced to true north.

87. The other primary outputs of the system are track made good, drift, groundspeed and aircraft present position expressed as latitude and longitude.

88. **With an input of TAS from the air data computer,** the INS computer can also resolve the triangle of velocities and give an output of wind velocity.

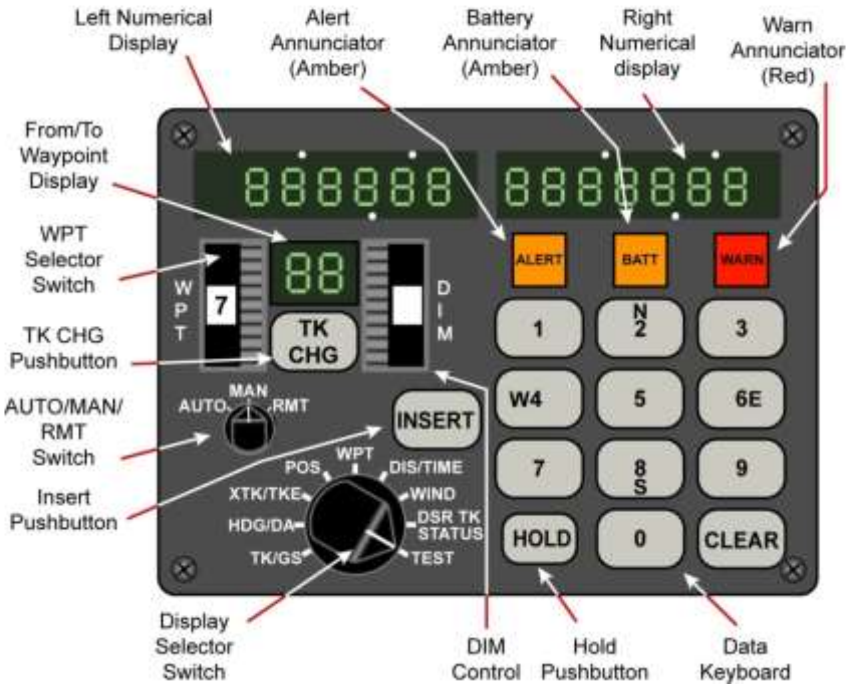
89. Providing that the INS computer is fed with details of the route, in other words it is fed with **waypoints** (the latitude and longitude of all points on the route at which the aircraft is required to change track), the computer uses spherical trigonometry logic to determine the **great circle** tracks and distances between one waypoint and the next. Should the aircraft be cleared at any time to a waypoint which is not the next waypoint, the computer will re-compute the great circle track and distance from the present position to the specified waypoint. Waypoints are identified by number (departure aerodrome as waypoint 1, the first turning point as waypoint 2 and so on) and, by convention, waypoint 0 is a floating waypoint which is the aircraft's present position. When cleared to a distant waypoint (for example waypoint number 5) the operator input would therefore request a track change 0 to 5.

90. Rather than express aircraft position in terms of latitude and longitude, the computer will normally be required to express the position of the aircraft relative to the desired track between the waypoints which define the route.

91. The CDU also provides a means of monitoring the operation of the INS by means of status displays and annunciator lights.

92. A typical CDU is shown at Figure 23-10.

FIGURE 23-10
Typical Control
and Display Unit

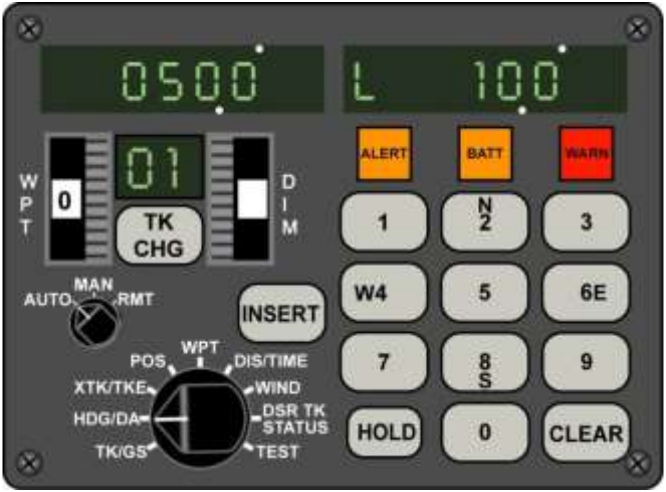


93. The functions of the display selector switch and of other controls and displays on the CDU will now be discussed in general terms

FIGURE 23-11
Track and Groundspeed Selected

94. **TK/GS (Track and groundspeed).** The INS derived aircraft track (against a north reference specified by the operator on the bench prior to installation, normally magnetic north) is shown to the nearest tenth of a degree in the LH window. The INS derived groundspeed is shown to the nearest knot in the RH window. The track is 040.0° and the groundspeed 502 kt at Figure 23-11.

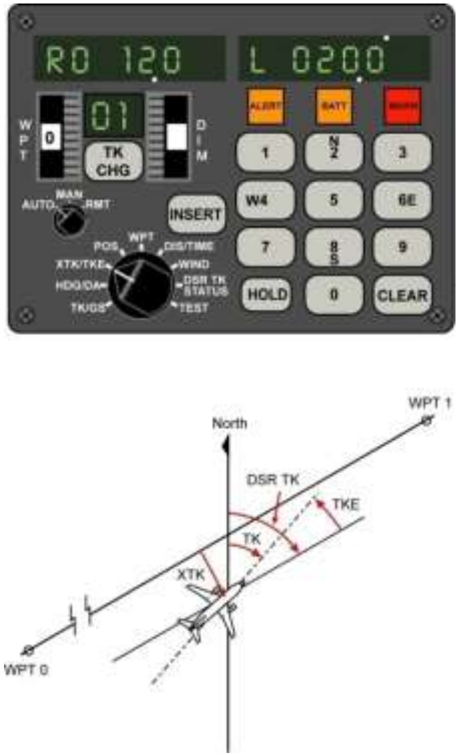
FIGURE 23-12
Heading and Drift Angle Selected



95. **HDG/DA (Heading and drift angle).** The INS derived heading is shown to the nearest tenth of a degree in the LH window. The INS derived drift angle is shown to the nearest tenth of a degree is shown in the RH window, and is preceded by an **L** (left/port drift) or an **R** (right/starboard drift). The heading is 050.0° and the drift 10° left at Figure 23-12.

96. **XTK/TKE (Cross track distance and track error angle)**. The cross track distance (the displacement of the aircraft perpendicularly from the direct great circle track between the two waypoints selected) is shown to the nearest tenth of a nautical mile in the LH window. This figure is preceded by an **L** or an **R** to indicate that the aircraft is left or right of the direct track. The track angle error (the angle between the track which the aircraft would require to make good were it flying along the great circle route between the specified waypoints and the track which it is actually making good) is shown to the nearest tenth of a degree in the RH window. The **L** or **R** which precedes this value indicates that the actual track is to the left or right of the required track. The cross track error is 12.0 nm to the right and the track angle error is 20.0° to the left at Figure 23-13. The geometry of the situation is also shown at Figure 23-13.

FIGURE 23-13
Cross Track Distance and Track Error Angle Selected



97. **POS (Present position).** The aircraft's present latitude is shown to the nearest tenth of a minute of arc, followed by an **N** (north) or **S** (south) as appropriate, in the LH window. The aircraft's present longitude is shown to the nearest tenth of a minute of arc, again followed by an **E** (east) or **W** (west) as appropriate, in the RH window. The aircraft's position is shown as 34° 31.5'N 117° 11.3'W at Figure 23-14.

FIGURE 23-14
Present Position Selected

98. **WPT (Waypoint positions).** The waypoint positions are shown in latitude (LH window) and longitude (RH window) to the nearest tenth of a minute of arc. In the system which we are considering there are 10 possible waypoint selections (0 through 9). Waypoints 1 through 9 are simply selected turning points, and are normally punched into the system by the operator before the flight. Waypoint 0 represents the aircraft's position at the last time a track change from present position to a specified waypoint was selected by the operator. The position of waypoint one is shown as 36° 01.4'N 115° 00.0'W at Figure 23-15.

FIGURE 23-15
Waypoint Position Selected

99. **DIS/TIME (Distance and time to the next waypoint)**. The distance to go from the aircraft's present position to the next selected waypoint is shown to the nearest nautical mile in the LH window. The lapsed time from the aircraft's present position to the next waypoint is shown to the nearest tenth of a minute in the RH window. The distance to go is shown as 140 nm and the time to go as 16.7 minutes at Figure 23-16.

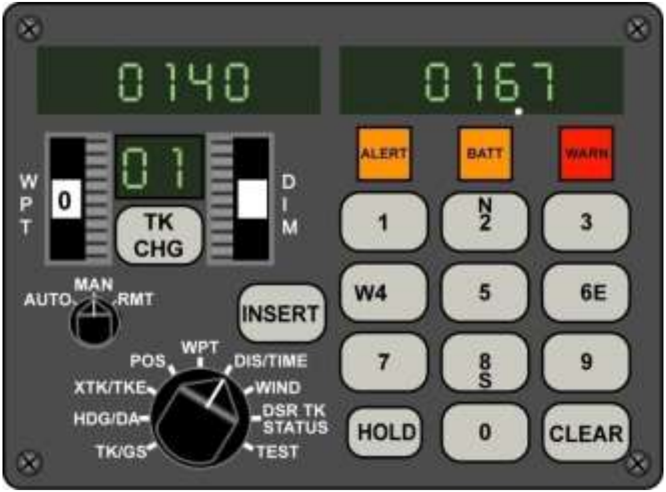
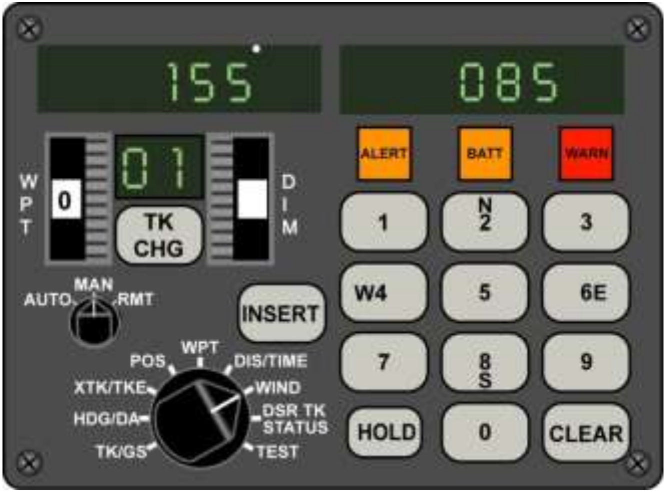


FIGURE 23-16
Distance/Time Selected

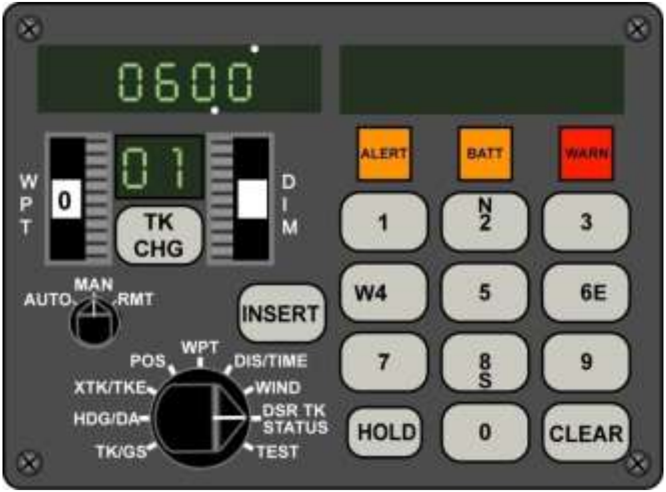
100. **WIND (Wind velocity).** INS derived wind direction is shown to the nearest degree in the LH window. INS derived wind speed is shown to the nearest knot in the RH window. The W/V is shown as 155°/85 kt at Figure 23-17.

FIGURE 23-17
Wind Direction and Speed Selected



101. **DSR TK/STS (Desired track and status).** Assume for the moment that the aircraft is on the great circle track between two specified waypoints. The desired track is the track required to fly from one waypoint to the next and is shown to the nearest tenth of a degree in the LH window. The value of the desired track will change as the aircraft travels from one waypoint to the next. Remember that the INS uses great circle tracks and appreciate that therefore the desired track angle will change to account for Earth convergency. Assuming that a magnetic north reference has been specified by the operator on installation, the desired track will also change as the variation change with aircraft position is pulled from the INS computer's memory. Appreciate that the desired track always assumes that the aircraft will remain on the great circle track between specified waypoints. Should the aircraft depart from this track the desired track readout will show the track angle which is parallel to the original track as defined by the great circle between the chosen waypoints and **not** (as you might expect) the track from the present position to the next waypoint. The desired track is 060.0° at Figure 23-18. You may by now have reached the conclusion that the programme upon which the INS computer operates assumes that the INS will normally operate coupled to the flight director/autopilot, so that across track errors do not occur.

FIGURE 23-18
Desired Track and
Status Selected

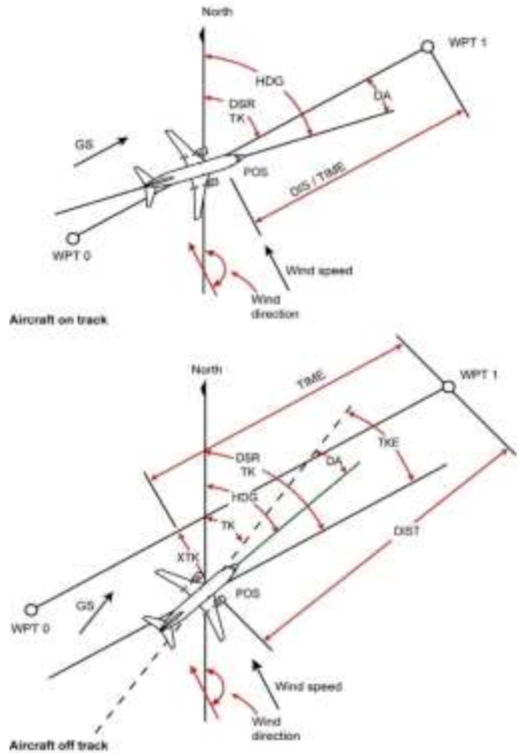


102. The use of the status function has already been discussed in terms of initial alignment. When airborne the status (RH) window should be blank, however, should the INS develop a fault, the status window will give a value which will identify the nature of the problem after reference to the system operating handbook.

103. **Test (Light emitting diode test).** The CDU illustrated at Figure 23-10 shows the function switch in the test position, and consequently all of the digits on the various displays are illuminated and show a number 8. This enables the operator to check that all of the LEDs are operating satisfactorily.

104. This concludes the discussion of the functions of the display selector switch. Before we continue to consider the purpose of the other buttons, selectors and annunciators on the CDU, take a look at the diagrams at Figure 23-19 and ensure that you agree with the INS navigation geometry shown there, both for the on track and the off track situations.

FIGURE 23-19
INS Navigation Geometry



105. With reference to Figure 23-10, the functions of the remaining controls are described in the following paragraphs.

106. The **waypoint selector switch** is thumbed to the appropriate waypoint number (shown in the window to the left of the thumbwheel) when loading the waypoint latitude/longitudes before flight, reloading new waypoints in flight, or checking that waypoints are correctly loaded.

107. The **from/to waypoint display** shows the two waypoints between which the INS assumes that it is flying. All digital readouts, flight director displays and autopilot commands will be based on this information, and so you can imagine the consequences of either giving the equipment the wrong to/from waypoint numbers or feeding the system with the wrong waypoint latitude/longitude to begin with.

108. The **track change push button** enables the operator to tell the system between which two waypoints the aircraft is required to fly (in the event that the system is not set up to fly sequentially through the loaded waypoints). This would normally be used to tell the INS to fly from the present position (waypoint 0) to a waypoint which is not the next waypoint, following a direct routing from ATC.

109. The **dim control** governs the brightness of the LED displays and the panel lighting.

110. The **alert annunciator** flashes to warn the operator that the aircraft is approaching the next waypoint. Typically, the alert light will come on with 2 minutes to run to the waypoint and will flash when the waypoint is crossed. If for some reason the INS is not in the automatic mode (see below), the alert annunciator will continue to flash until cleared (cancelled) by the pilot.

111. As already discussed, the red battery light on the mode selector panel illuminates when the back up power supply is exhausted. The purpose of the **battery annunciator** on the CDU is to warn the operator that the INS is operating on battery power. When the INS is initially powered up prior to levelling/alignment the internal back up power source and associated circuitry are self tested and during this test the battery annunciator is illuminated. In the event that the test is unsatisfactory the battery annunciator will remain on.

112. The **warning annunciator** illuminates when a system malfunction occurs, in this event refer to the status display, which will give a number that will identify the nature of the malfunction.

113. The **auto/manual/remote switch** determines the level of pilot intervention necessary to fly the aircraft. In the **automatic** mode the INS will automatically switch from one track to the next as each waypoint is overflown. In the **manual** mode the pilot is required to switch to the next track as a waypoint is overflown. The **remote** position enables the pilot to cross load route information (waypoints) from this INS to a second INS (if fitted) in order to avoid having to load the second INS manually. If this facility is to be used, it is even more important that the accuracy of the waypoints is checked. If a waypoint is incorrectly loaded into one INS and the data is then cross loaded the error will be duplicated. Apart from redundancy (the failure of one of the two systems is allowed for), the main advantage of having two INS is that one monitors the other. If both are loaded with erroneous route data, no discrepancy will exist between the systems (and no warning given to the pilots) despite the fact that the aircraft will at some point be heading towards an incorrect geographical location.

114. The **insert pushbutton** is used in conjunction with the **data input keyboard** to enter information into the system.

115. Finally, the **hold pushbutton** is primarily used for manually updating the INS position with a reliable fix, for example a radio fix or GPS position. The hold button is depressed as the fix position is noted, the function switch is placed in the POS (position) mode, the exact latitude/longitude of the radio fix is punched into the machine, and the hold button is then released. Appreciate that if this is done, the radial error rate assessment (discussed shortly) will be invalid, unless the position update vector is accounted for.

Checking Manually Entered Positions

116. At the initial setting up stage the start position must be fed into the INS computer with a high degree of accuracy.

117. If the initial latitude is slightly in error the platform will not remain Earth horizontal once the equipment is switched into the navigation mode, since the torque motors will be tilting the platform at an inappropriate rate, due to computer calculations based on an incorrect latitude. Likewise, and for the same reasons, the platform will not remain directionally aligned with respect to north. The same problems will apply to wander angle and strapped down systems, but for different reasons.

118. If the initial latitude setting is grossly in error the system will detect the error and warn the operator (this is one of the principal functions of the warning annunciator on the CDU whilst the equipment is in the align mode). The equipment is able to sense a gross latitude input error since the apparent drift and topple rates sensed by the rate gyros will not correspond to the corrections being applied by the torque motors.

119. An incorrect operator input of longitude will not affect the stability of the platform, but obviously the track and distance from the departure point to the first waypoint will be incorrectly computed. Furthermore, all subsequent indications of longitude will be in error by the amount of the initial input error.

120. For the reasons discussed above it is obviously essential that the ramp position is carefully entered and checked (ideally entered by one pilot and checked by the other).

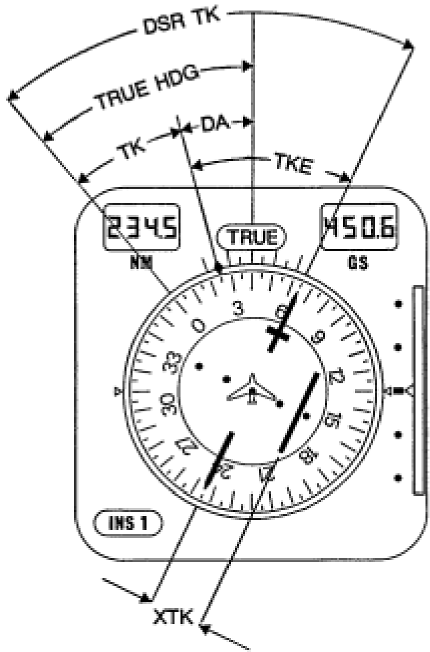
121. An incorrect input of the latitude/longitude of any of the waypoints will have serious consequences. The INS will navigate very accurately between waypoints, but it is incapable of detecting operator malfunctions. A favourite error occurs when punching in a waypoint where either the latitude or the longitude is a whole number of minutes of arc, say 50° 30'N. If you punch in 5030 (instead of 50300) your waypoint is in error by 2,727 nm! It would be unwise to assume that, because a waypoint is stored in the computer memory (data base), it is necessarily correct. A waypoint position may have been incorrectly inserted in the data loader, it may have been corrupted during the data transfer, or it may have been incorrectly entered/amended by one of your colleagues.

122. In order to check that the waypoints have been correctly inserted they should be recalled from store onto the LED display, and rechecked before flight.

123. A second check is to call up the initial great circle track and distances between consecutive waypoints, and to compare these values against those shown on the computer/manual flight plan.

Visual Presentation of INS Data

FIGURE 23-20
INS Data
Displayed on HSI



124. We have previously discussed how INS data is shown on the numerical displays of the CDU. Much of this data may be displayed on either a conventional horizontal situation indicator (HSI) or on an electronic horizontal situation indicator (EHSI) in the map mode (in a glass cockpit aeroplane). HSIs and EHSIs are described subsequently in the chapter entitled Flight Directors and Electronic Flight Information Systems, however for now we will look at a conventional HSI which is coupled to an INS output. Appreciate that the instrument (which is illustrated at Figure 23-20) is essentially a compass rose together with a track deviation indicator. The track deviation indicator can give deviations from tracks defined by VOR, ILS or (as in this case) INS. The vertical scale on the right hand side of the instrument gives glidepath deviations (fly up or fly down) and is active only when an ILS is tuned and selected to the HSI.

125. **HDG**. Aircraft heading, in this case 040° true, is displayed against the central lubber index at the top of the display.

126. **TK**. The solid diamond, in this case against 025° true, is the present track.

127. **DA.** This is the drift angle (the angle between the track diamond and the true heading lubber index line), in this case 15° left drift.

128. **DSR TK**. The desired track is indicated by the course bar or arrow, presently showing 065°T.

129. **XTK.** The horizontal deviation bar shows the distance that the aircraft is displaced from desired track (the cross track distance) with reference to the five dots. Each dot represents 3.75 nm, and therefore the display shown indicates that the aircraft is to the left of desired track by approximately 7 nm.

130. **TKE**. The track error is the angle between the track diamond and the desired track course arrow or bar. In this case the track error is 40° to the left (tracking 025°T, DSR TK 065°T).

131. Distance to go to the next waypoint is indicated in the top left window, and presently-computed groundspeed in the top right window. The window in the bottom left of the display indicates that the HSI is presently coupled to No 1 inertial navigation system, since the INS equipment is often duplicated or even triplicated.

INS Errors

132. The following may be considered, in addition to the Schuler tuning bounded errors already discussed, as likely to produce small but significant errors in the system.

Initial Levelling

133. As already discussed, if the platform is not truly Earth horizontal at the outset (when switched from the align to the nav mode) the accelerometers will sense a component of gravity which will cause a bounded error in groundspeed at the first integration stage and an unbounded error in distance travelled at the second integration stage. With a strapped down system, a similar error will exist in the event that any difference which exists between Earth horizontal and aircraft horizontal is not correctly assessed during the initial levelling/alignment procedure.

Initial Alignment

134. Again, as already discussed, if the platform is not correctly aligned with north at the outset the direction of acceleration sensed will be in error by the amount of misalignment. This will give rise to a bounded error in all track made good computations, leading to an unbounded error in indications of aircraft position. With a wander angle or strapped down system a similar error will exist in the event that the angular difference between the platform/aircraft fore and aft axis and the local meridian (true north) is incorrectly computed during the initial alignment procedure.

Real Wander of the Rate Integrating Gyros

135. The rate integrating gyros used on inertial platforms are of a very high order of accuracy, and are normally required to give real wander rates (those due to engineering imperfections) of less than 0.01°/hr. Despite this high degree of accuracy it is these real wander rates within the gyros which give rise to the most significant unbounded errors in the system. The ring laser gyros which are used with strapped down systems do not suffer from any form of mechanical precession.

Accelerometer and Integrator Errors

136. Slight imperfections in these components can give rise to errors at all stages of computation. These errors are normally much smaller than those caused by real wander of the rate gyros.

Latitude and Height Errors

137. The INS computer is programmed to correct for the varying length of the nautical mile (one minute of arc of latitude at the surface) due to the shape of the Earth. Similarly, aircraft altitude will also introduce small distance errors and, on platforms which are designed to correct for this error, it is necessary to employ a third accelerometer with its sensitive axis lying along the platform/Earth vertical axis, so that the INS computer can integrate vertical accelerations to determine altitude. All strapped down systems necessarily employ an aircraft vertical accelerometer, the output of which is integrated to give aircraft altitude.

System Accuracy

138. The minimum standards of accuracy which are specified for inertial navigation systems require a maximum circular position error rate of 2 nm/hr (on 95% of occasions) on flights of up to 10 hours duration. For flights of over 10 hours duration a cross track error of ±20 nm and an along track error of ±25 nm (on 95% of occasions) is permitted.

139. Modern systems achieve accuracies which are well within these limits and terminal errors of less than 1 nm after 10 hour flights are common, using integrated navigation systems which are discussed shortly. Large errors can occur in individual inertial systems following component failure, however INS equipped aircraft normally employ two (737) or three (747, 757 and 767) independent inertial systems, the outputs of which are continuously compared in order to identify system malfunctions.

Radial Error Rates

140. At the end of each flight the actual ramp position should be checked against the indicated ramp position shown on the INS. A **radial error rate** may then be calculated using the formula:

$$\text{Radial error rate (nm/hr)} = \frac{\text{Distance ramp position to INS position (nm)}}{\text{Time in the navigational mode (hours)}}$$

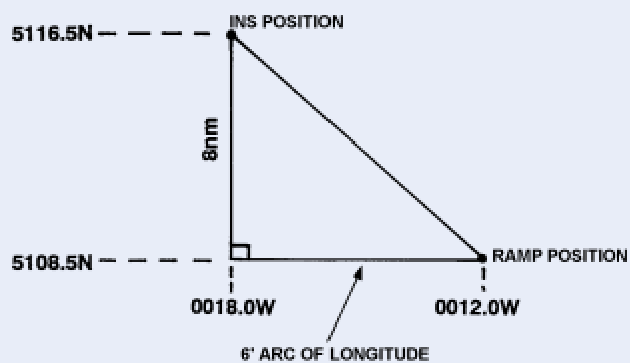
EXAMPLE 23-1

EXAMPLE

Following a flight from New York to London the INS showed a position of 51° 16.5'N 00°18.0'W when the aircraft was stationary on the ramp at London Gatwick. The ramp position was given as 51° 08.5'N 00° 12.0'W. The time in the navigation mode was 5 hours and 24 minutes. Using this information determine the INS radial error rate.

SOLUTION

Whilst the INS computer bases all calculations on spherical trigonometry, the human solution of radial rate error can be achieved to a satisfactory degree of accuracy using two-dimensional trigonometry



The distance in longitude between the INS position and the ramp position is calculated using the departure formula:

$$\text{distance (nm)} = \text{d long (')} \times \cos\text{ (mid) lat}$$

$$= 6 \times \cos 51°12.5'$$

$$\cos 51°12.5' = 0.626$$

$$\text{distance (nm)} = 3.76\text{ nm}$$

Using Pythagoras:

$$\text{distance ramp to INS position}^2 = 8^2 + 3.76^2$$

$$= 78$$

$$\text{distance ramp to INS position} = \sqrt{78}$$

$$= 8.8\text{ nm}$$

$$\text{the radial error rate} = \frac{8.8\text{ nm}}{5.4\text{ hrs}}$$

$$= 1.63\text{ nm/hr}$$

If, in the examination, you are not given the value of the cosine of the precise mid latitude, use the cosine of the nearest whole degree from the table provided.

141. The examiner is also fond of asking questions of the type illustrated in the following example. Providing that you appreciate that inertial systems always navigate along great circle tracks, you should have no difficulty in answering this type of question.

EXAMPLE 23-2

EXAMPLE

An aircraft is flying from waypoint 4 at 35° 00.0'S 20° 00.0'W to waypoint 5 at 35° 00.0'S 30° 00.0'W. The INS is programmed to display tracks and headings with reference to true north. The INS is coupled to the flight control system/autopilot. Using this information answer the following questions:

1) With DSR TK/ST selected on the CDU the desired track readout which is displayed as the aircraft overflies waypoint 4 is:

(a) 270.0°

(b) 090.0°

(c) 272.9°

(d) 267.1°

2) With DIS/TIME selected on the CDU the distance to go which is displayed as the aircraft overflies waypoint 4 is:

(a) 491.5

(b) 600

(c) less than 491.5

(d) more than 491.5

3) With POS selected on the CDU and the longitude readout showing 25° 00.0'W the latitude readout will show a latitude which is:

(a) 35° 00.0'S

(b) south of 35° 00.0'S

(c) north of 35° 00.0’S

4) With XTK/TKE selected on the CDU, when the aircraft is mid way between the waypoints the cross track readout distance will show:

(a) 00.0

(b) R05.7

(c) L05.7

5) The total change in the track angle between waypoints 4 and 5 is:

(a) less than 10°, decreasing

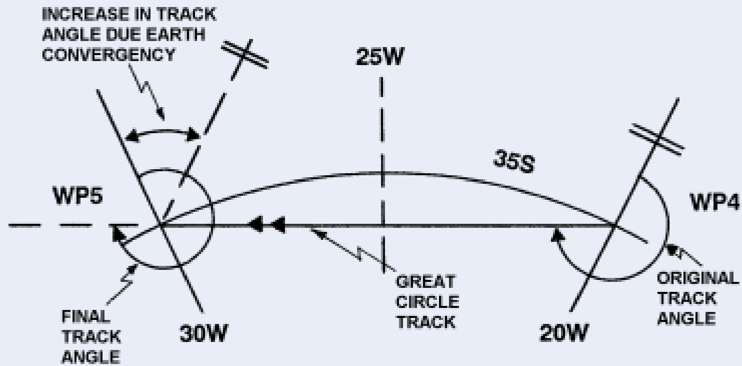
(b) more than 10°, decreasing

(c) less than 10°, increasing

(d) more than 10°, increasing

SOLUTION

1) The answer is (d). The parallel of latitude defines 270°(T), whereas the INS flies the great circle track which lies to the south of the parallel. There is no need to calculate the value of the conversion angle since only one of the options given can be correct.



2) The answer is (c). The distance of 491.5 nm, which is achieved by means of the departure formula, represents the distance between the waypoints along the parallel of latitude (the rhumb line distance). The great circle distance must therefore be less than 491.5 nm.

3) The answer is (b), as shown above.

4) The answer is (a). With the INS coupled to the autopilot the aircraft will remain on the great circle track and there will be no cross track displacement.

5) The answer is (c). Earth convergency increases the track direction as the aircraft flies west. The total change of track direction is obviously less than 10° since the change of longitude is only 10° and Earth convergency = change of longitude x sine latitude.

The Development of Inertial Systems

142. The preceding notes have dealt with inertial systems as stand alone aids to aircraft navigation. Furthermore, for the sake of simplicity, the mode selector panel, control display unit and associated computer hardware/software which were considered were of a fairly early vintage. Over the last 20 years aircraft navigation systems have evolved very rapidly in parallel with other aircraft systems, which is why the 747-400 was designed to be operated with only two people on the flight deck.

143. What follows is a potted history of the way in which inertial navigation, as it applies to civil transport aeroplanes, has evolved.

144. The first INS systems used north aligned platforms and relatively slow processors with limited memory banks. Reliability was a problem and the limited storage capacity of the computer memory meant that the system was limited to 10 waypoints which had to be replaced in flight as they were overflown on routes which required more than 10 waypoints. It was not possible at this time to store the look up table of variation and consequently the system operated only in °(T).

145. Faster processors and larger memory banks meant that the next generation of inertial systems employed wander angle platforms and had the ability to display track and heading information in degrees magnetic. This is about where the system which we discussed fits into the story.

146. Still larger memory capacities enabled the next generation of INS to 'store' typically a thousand waypoints which could be used to produce up to something in the order of 100 routes, which could also be stored in the memory. By now the systems were becoming more reliable and duplicated INS platforms were replacing the navigator on even long oceanic sectors.

147. With the advent of strapped down systems, very high speed processors and very large memories the INS has evolved into the IRS, the inertial reference system. Now the inertial system is no longer a stand alone equipment, but rather the inertial input into an integrated system (the flight management system or FMS) which not only looks after all of the navigation requirements but also has many other functions.

148. With two or three highly reliable IRS in the system it is now possible to dispense with the gyro slaved compasses and the remote gyro compasses, since each IRS is capable of giving accurate outputs of aircraft heading and attitude, the outputs of each IRS being constantly monitored against the other(s).

149. Updating of the position determined by the inertial reference systems is now continuously achieved by the navigation module within the active flight management computer (obviously redundancy is built in here as well), providing that the aircraft is within range of suitable VOR/DME stations**.** The updating process is achieved using either VOR/DME or preferably DME/DME fixes which are more accurate, especially when slant range corrections are applied. Suitable VOR/DME stations are selected by the navigation module using a map of suitable stations which is automatically loaded into the active memory when the pilot types in the route number prior to departure.

150. The number of waypoints which comprise an individual route is now virtually unlimited. The basic route is entered, followed by the relevant standard instrument departure once the clearance is received. Similarly, the standard instrument approach is entered once this is known.

151. VOR/DMEs are automatically selected and the aircraft is navigated using composite data comprising the IRS outputs as updated by DME/DME or VOR/DME inputs. With a glass (EFIS) cockpit the normal mode for the screen which replaces the horizontal situation indicator (HSI) is the map mode, which gives a pictorial representation of the route as defined by the IRS waypoints; the VOR/DME stations which are stored in the navigation module (the stations currently in use are highlighted); the aircraft's present position; the output of the airborne weather radar and various other codes and symbols to indicate the operational status of the system. Navigation errors do occur, and when these are due to operator errors (principally the incorrect insertion of data) they may not be identified by the system, which cannot therefore warn the pilots. The role of the pilots as the final arbiters of the integrity of the system is absolutely paramount.

152. In addition to providing accurate navigation data, strapped down inertial systems have now replaced the remote gyro compasses and the vertical gyros and supply heading, pitch and roll information to the pilots (human and automatic). In the event an individual IRS suffers a component failure in flight which means that it is no longer capable of supplying position information (for example an integrator), or is deprived of its AC power input whilst airborne for a period which exceeds the duration of the internal battery pack, it might appear that the system will no longer be capable of supplying heading or attitude data. By selecting the attitude reference position the IRS reverts to a basic mode which isolates the navigation functions and supplies only heading and attitude data, albeit of a degraded accuracy. The important thing to appreciate is that, in the basic mode, the system is capable of achieving an acceptable level of accuracy following an airborne levelling/alignment process, but that the information supplied by the system is now limited to aircraft heading and attitude.

153. What comes next? With the advent of satellite navigation systems which are capable of determining aircraft position which is accurate to within metres anywhere in the world and for which the aircraft equipment is light, simple (reliable) and inexpensive, the future of inertial reference systems which provide the primary navigation data would appear to be limited. It is likely that, in the not too distant future, strapped down systems will be used to supply attitude and heading data (AHRS, or attitude and heading reference systems), whilst satellite systems will be used to supply primary navigation data.

Purposes of a Flight Management Systems (FMS)

- The Flight Management and Guidance Computer
- The FMC Data Base
- Modes of Operation for Dual FMC Installations
- Lateral Navigation Guidance
- Vertical Navigation Guidance
- The Use of the MCDU

Purposes of a Flight Management Systems (FMS)

1. The **Flight Management System (FMS)** is an integration of the aircraft subsystems, the purpose of which is to assist the flight crew in controlling and managing the flight path of the aircraft. The flight path is divided into lateral and vertical profiles, commonly known as LNAV and VNAV. The system allows the pilots to select the degree of automation required at all stages of flight and consequently the need for many routine tasks and computations is eliminated.

2. Primarily the FMS provides automatic three-dimensional navigation, fuel management and fuel monitoring together with the optimising of aircraft performance. It also provides information to the appropriate displays, including the electronic map, which is fully described in the Flight Director and Electronic Flight Information Systems (EFIS) part of the syllabus. FMS also provides airspeed and engine thrust cues.

3. The main components of an FMS are:

(a) Flight Management and Guidance Computer (FMC)

- uses both manual and automatic inputs of data to compute 3 dimensional position, performance data etc in order to fly the aircraft accurately and efficiently along a pre-defined route.

(b) Multipurpose Control and Display Unit (MCDU)

- the interface between the pilots and FMC.

(c) Flight Control Unit

- supplies the commands to control the lateral and vertical flight path of the aircraft.

(d) Flight Management Source Selector

- selects the sources of input to be used by the FMC.

(e) Display System

- any means of displaying the required data/ information to the pilots.

The Flight Management and Guidance Computer

4. A schematic diagram of the component parts of a typical flight management system is shown at Figure 24-1. The heart of the system is the **Flight Management Computer (FMC)** and its associated **Multipurpose Control and Display Unit (MCDU).** A CDU of the type found in the Boeing 737 is illustrated at Figure 24-2.

FIGURE 24-1
A Typical FMS

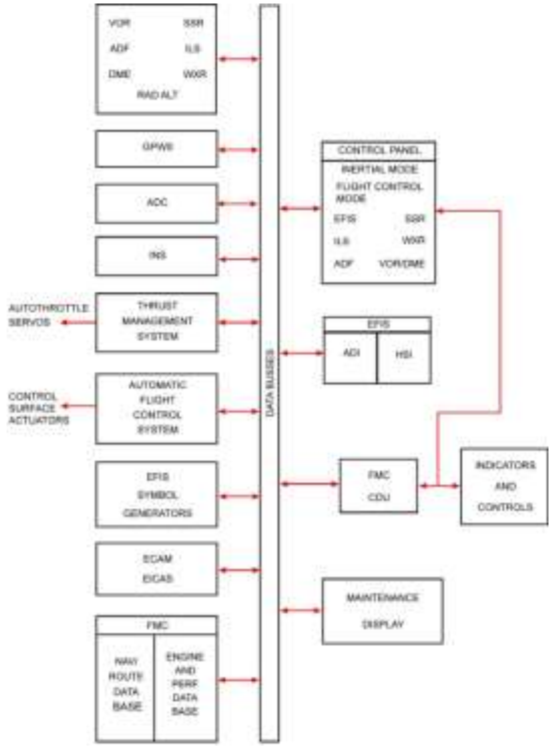
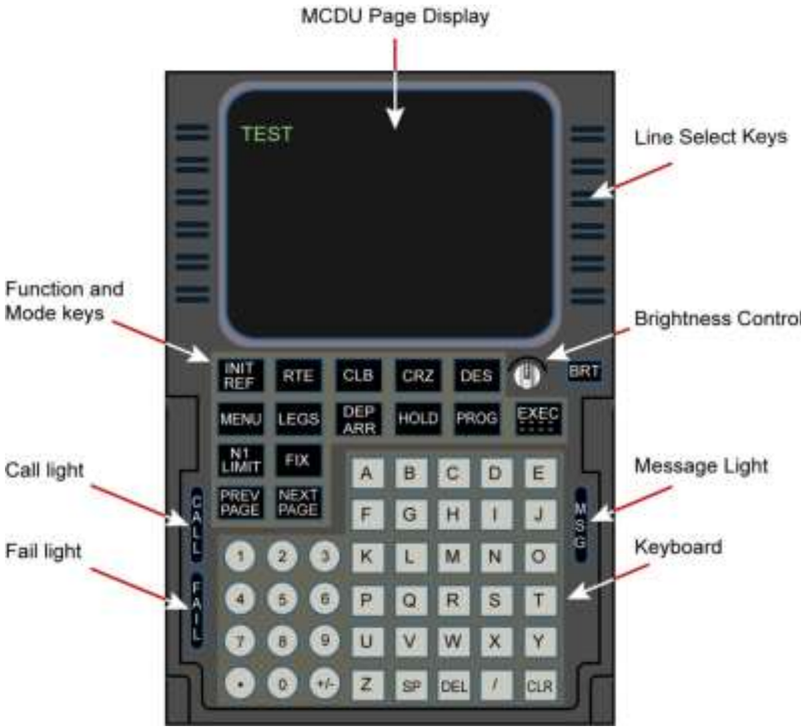


FIGURE 24-2
A Typical FMS Multipurpose Control and Display Unit (MCDU)



5. The MCDU combines flight plan information entered by the pilots with information supplied from supporting systems and information contained in memory. This enables the FMC to determine the aircraft position and to provide pitch, roll and thrust information in order to fly the profile required. Commands are sent by the FMC to the autopilot, the flight director and the autothrottle (autothrust) system. FMC navigational and performance computations are displayed on the MCDUs for reference or monitoring. Related FMC commands for lateral and vertical navigation are coupled to the AFDS and Autothrottle through the Mode Control Panel (LNAV and VNAV). The IRSs and other aircraft sensors provide additional required data. MCDUs also permit interface with the Aircraft Communications Addressing and Reporting System (ACARS). Additionally, map information is sent to the Electronic Horizontal Situation Indicator (EHSI) and displayed in the manner described in the PPSC notes dealing with EFIS.

The FMC Data Base

6. The information which is stored in the FMC data base is divided into two main sections, namely **navigation information** and **aircraft performance information.**

7. The navigation data includes the location of radio navigation aids, SIDs, STARs, company routes, airports, runways, approach aids and airways structures. The data base is tailored to the needs of the individual carrier. This navigation data base is produced by a specialist agency (such as Jeppesen) and is normally updated on a 28 day cycle. Data transfer hardware (using a magnetic tape cassette) is provided to enable the operator to load a new data base into the aircraft FMCs. In order that flight operations do not come to a grinding halt at midnight on the last day of validity of the expiring data base, the current data base together with the next effective data base are both stored in the FMCs. For the pilot then, step one when setting up the FMCs is to ensure that the correct data base for the date of the flight is the operational one.

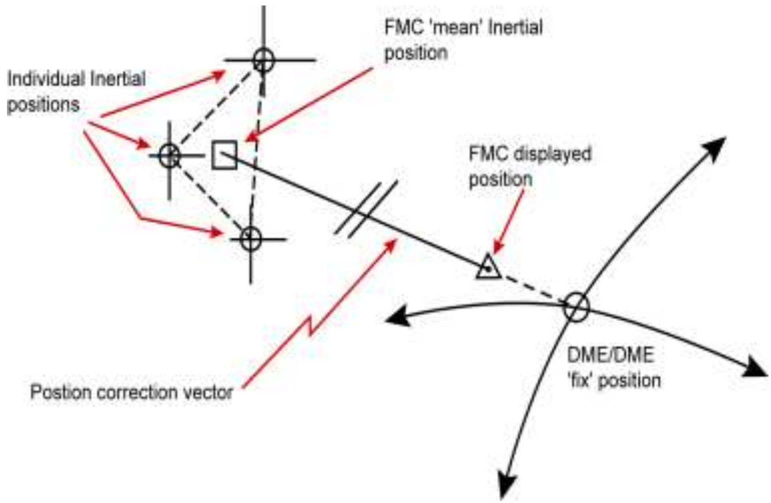
8. Within a given 28 day period it is likely that certain information contained in the navigation data base will become invalid, for example NOTAMs may inform us that a given VOR is out of service for a period of time. The pilot can access the data base and delete that VOR, but only for the duration of the flight. It is therefore impossible for the pilot to corrupt the data base itself. It is important to remember that the data base is produced by another human being and may therefore contain errors. Because of the high degree of automation involved when, basically, the FMC is driving the aeroplane, it is essential that the pilots monitor the aircraft's progress using conventional navigation techniques (raw data), and also that any errors in the data base are fed back through reporting channels so that they can be remedied.

9. During flight the FMC will search the navigation data base and automatically select the best two DME stations with which to determine the aircraft's present position. In the absence of suitable DME/DME crosscuts the system will use co-located VORs and DMEs. When DME/DME or DME/VOR fixing is not possible, for example on an oceanic leg, the aircraft's position is determined by the inertial reference systems plus a correction vector that has been developed by a **Kalman filter** over a period of time. In those systems that use GPS position as an input into the FMC, it is usually possible for the pilot to delete any satellite that has automatically been selected by the GPS receiver, in order to obtain the best fix geometry.

10. The Kalman filter uses **hybrid navigation techniques.** It takes, for example, position information from a number of sources and then statistically analyses that data (taking into account the possible errors) to produce a final solution which, in the case of position, would be the FMC position. The filter also produces the correction vector discussed in paragraph 9.

11. Take the situation where an aircraft, equipped with say 3 inertial systems, is flying from Europe to the USA. As the aircraft crosses the UK, on its way to join the NAT track system, the FMS will be using DME/DME radio ranges to assist in determining position. Figure 24-3 gives a pictorial presentation of the computations involved.

FIGURE 24-3
Position Determination by an FMC



12. In simple terms the FMC first averages out the 3 IRS positions to determine a ‘mean’ inertial position. Secondly, it compares the mean inertial position with the radio aid position (in this case DME/DME ranging is used) and, taking account of the likely error in each position, it computes a final FMC position which is used to steer the aircraft along the planned track.

13. The **position correction vector** in the above example stretches between the mean inertial position and the final FMC computed position. (In an aircraft equipped with a single inertial system the vector would obviously start from that single position).

14. It will be obvious from the above explanation that, in order to develop the position correction vector over a given time, there must be a continuous supply of radio information. However, once the aircraft leaves the area of ground based radio aids the FMC can still use the 'history' of the vector to develop it further, and hence continues to provide the best possible estimate of position. As the aircraft coasts in again over the USA radio aid fixing will once again be used to 'tie down' the FMC position.

15. The accuracy of a Kalman filtering system such as the one described is dependent upon two main factors :

(a) The quality and complexity of the Kalman filter design.

(b) The error characteristics of the various 'navigation' sensors used by the system must be complementary. (i.e. any single system input which is subject to a lot of 'noise/ variation', or 'drifts' in value, may cause a significant error in FMC computed position).

16. The FMCs will automatically select the VOR/DME stations which are displayed on the EHSI needles, the standby RMI needles and the DME range readouts. The system will decode the morse identifier and display letters on the screen. If a satisfactory identifier decode is not achieved, the frequency will be displayed rather than the identifier. In this event it is up to the pilot to identify ground station in the conventional manner. Similarly, providing that the FMC has been informed that the intention is to fly an ILS approach to a given runway at the destination/alternate aerodrome, the

relevant ILS will be autotuned and identified, again with the morse identifier displayed to the pilot, but this time on the Electronic Attitude Direction Indicator (EADI). Where the departure is from an ILS runway, the FMC will again autotune the ILS in order to provide centre line guidance immediately after take-off. When NDBs form part of a SID, STAR approach procedure or (unusually these days) an airways structure, these are also autotuned and identified by the FMC. The option always exists for the pilot to override the automatics by 'hard tuning' stations of his or her choice.

17. The performance data base contains all of the information normally contained within the performance manual, such as engine characteristics, the aircraft limiting speeds for the various configurations, optimum/maximum cruise altitudes and an aerodynamic model of the aeroplane. The data base may be individually tailored for an individual aeroplane within a fleet. Variables such as fuel quantity, zero fuel weight and a company cost index are entered by the flight crew. This data is peculiar to the next sector only and is automatically dumped by the FMC following the next landing and engine shutdown. The simplest explanation of the cost index is that it is a numerical value which tells the FMC whether the operator considers that fuel economy (with longer sector times) or minimum sector times (with a resultant higher fuel burn) is the preferred option. Please note that fixed costs, as a general rule, remain the same no matter what speed is flown. The cost index can therefore be altered on a sector by sector basis to account for the circumstances of that flight.

Modes of Operation for Dual FMC Installations

18. FMC systems are normally duplicated and each FMC has its own CDU. There are 4 modes of operation:

Dual Mode. With the system operating normally the two CDU/FMCs are interconnected and pilot entered data which is entered at one CDU is automatically transferred to the other one. In other words, one FMC provides the master function and the other the slave function. The pilots may select their own EHSI display (full or expanded VOR, full or expanded ILS, map or plan) regardless of what is displayed on the other EHSI.

Independent. The first stage of degradation of the system occurs when a disparity is sensed between the outputs of the two FMCs. Now each CDU/FMC works **independently** of the other and the pilots are left to identify the serviceable system. Each CDU will supply its own EHSI, however now the pictures on each of the EHSIs (assuming that they are in the same mode with the same range option selected) will differ.

Single. The next stage of degradation of the system when one FMC or CDU fails altogether. You are now down to a **single system operation,** however both EHSIs can be driven from the same FMC/ CDU providing only that both pilots select the same mode and range setting.

Back-Up Navigation. Finally, should both FMC/CDUs fail, the pilots are left with blank EHSIs and the prospect of limited use of the FMS. Navigation is achieved by manually tuning en route and approach aids which are subsequently displayed on a conventional RMI and analogue DME readout.

19. The control of the different FMS modes is described in the Aircraft Operation Manual (AOM)

Lateral Navigation Guidance

20. The FMC calculates the great circle tracks and distances between successive waypoints in the active flight plan. These are the track lines which are shown on the EHSI map display. The active flight plan includes the SID, the STAR and any relevant holding patterns. Under normal circumstances **(managed guidance)** the FMC will command the autopilot to maintain the defined track (at a particular altitude and speed). With the aircraft flown manually **(selected guidance)** the FMC commands the human pilot to maintain a particular value of a parameter (heading, speed etc.) by making selections on the Flight Control Panel (FCP). **At any time the pilot can take control of the lateral navigation** of the aircraft by going into heading mode. The FMC will automatically revert to heading mode whenever LNAV capture parameters are out of limits or when, for example, a waypoint is reached and no route is defined beyond that point.

Vertical Navigation Guidance

21. Providing that the pilot does not modify the climb profile, the FMC will command a climb with thrust at the airspeed limit associated with the departure airfield until above the speed limit altitude or flight level. Thereafter the climb will continue at climb thrust and **economy speed** to the demanded cruise level. Where altitude/level constraints are imposed by the SID (cross point X at/at or below/at or above a given altitude or flight level), these constraints will be shown on the EHSI map and plan displays. The aircraft will comply with these constraints providing that the FMC remains in the **fully managed mode.** In the event that ATC impose an altitude constraint, this can be entered by the pilot as a vertical revision to the waypoint to which the constraint applies. If, during the climb, the FMC senses that the aircraft will not be able to comply with the constraint due to an insufficient rate of climb, the pilot will be warned. The FMC will capture any altitude which is selected and armed on the Mode Control Panel (MCP)/Flight Control Unit (FCU).

22. During the cruise, **economy speed** will be used until the top of descent point.

23. The top of descent point is computed by the FMC, as it were, from touchdown backwards. The FMC has knowledge of the aerodrome elevation, and the QNH is manually entered by the pilots. The exact vertical distance from the cruise level to touch down is therefore known. Flight level or altitude constraints, as defined by the STAR and the approach procedure are stored in the navigational data base, and the descent profile is computed to account for these constraints. The descent will normally be computed such that, wherever possible, the engines will be at idle power (which is fuel efficient). The descent will be computed at **economy speed** down to the point where the STAR imposes a maximum speed constraint, and thereafter at speeds which will enable the slats/flaps/landing gear to be extended at the appropriate points. Wind velocities for the descent can be manually entered by the pilots in order to refine the computation.

24. Typical VNAV climb, cruise and descent profiles for a B757 are illustrated at Figure 24-4 and Figure 24-5.

FIGURE 24-4
Typical VNAV Climb / Cruise Profile

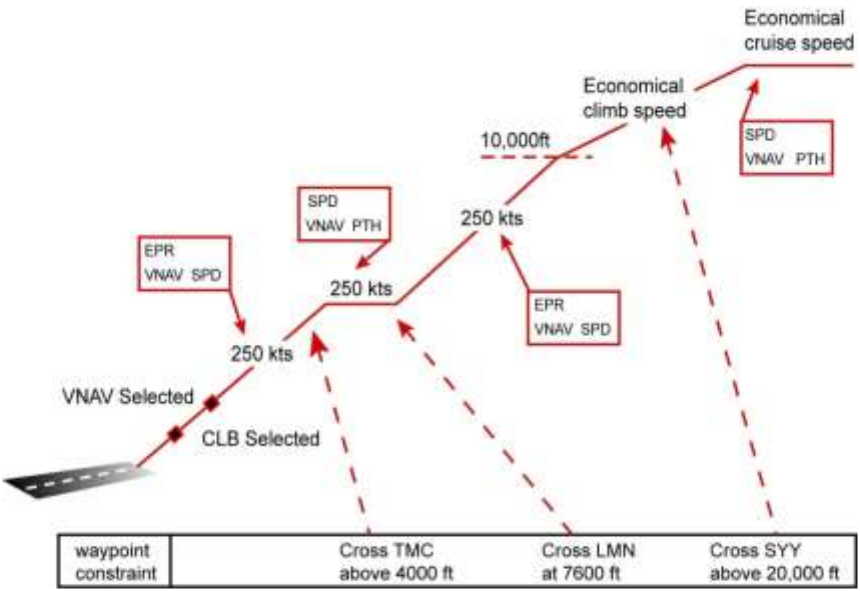
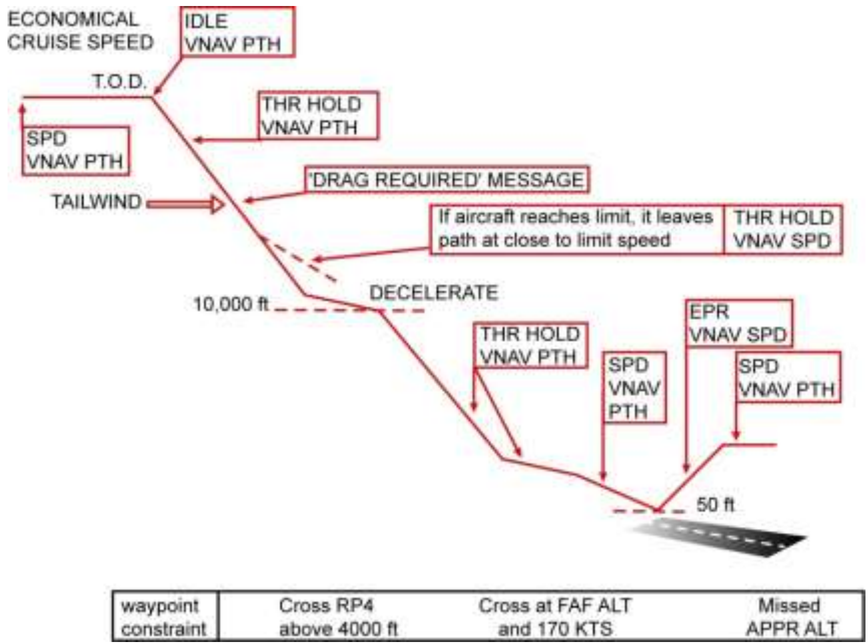


FIGURE 24-5
Typical VNAV Descent Profile



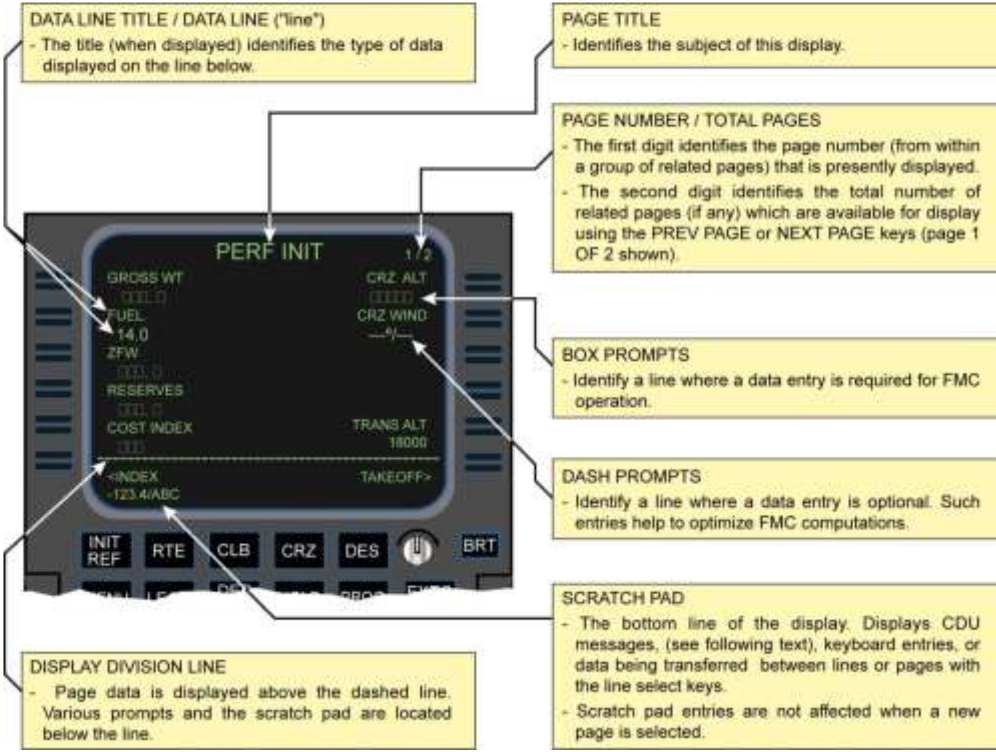
The Use of the MCDU

25. The remainder of this chapter describes in general terms, the use of the MCDU and gives brief details of some of the displayed information that is available to the operator.

MCDU Page Layout - Information Blocks

26. Figure 24-6 gives details of a typical page layout on an MCDU.

FIGURE 24-6
MCDU
Information Blocks



MCDU Messages

27. There are two categories of MCDU messages (see Scratch Pad in Figure 24-6). **Alerting Messages** have the highest priority and identify a condition which must be acknowledged and corrected by the crew before further FMC-guided flight is advisable or possible. **Advisory Messages** have lower priority and inform the crew of MCDU entry errors or system status.

28. The generation of any message causes the white MCDU MSG light to illuminate. Alerting messages also illuminate the amber FMC Alert Light on each pilot's instrument panel.

29. If the Scratch Pad is empty, any message is displayed immediately when generated. Some messages will displace an existing Scratch Pad entry and are also displayed immediately when generated. Other messages will not be displayed until the Scratch Pad has been cleared; however, the MSG light will still be illuminated. A new entry in the Scratch Pad overrides any displayed message. Messages caused by MCDU entry errors are displayed only on the associated MCDU; other messages are displayed on both MCDUs.

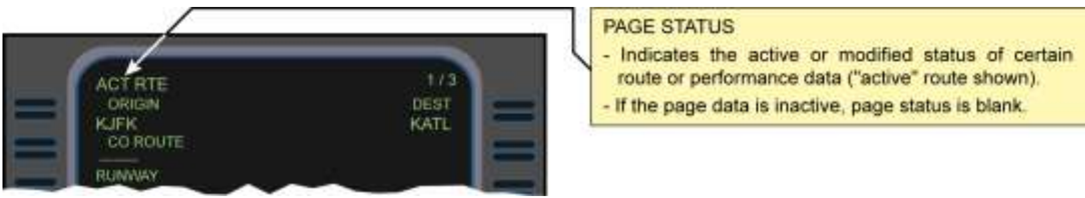
30. When multiple messages have been generated, they will be 'stacked' for display in priority sequence, or in the order of their occurrence if of the same priority. As each message is cleared, the next message in the stack is displayed. Most messages are cleared with the CLR key on the MCDU, or by correcting the condition. Other messages are cleared by changing the displayed page; this will delete the entry which caused the message.

31. The FMC/MCDU is designed to automatically preserve the most capable modes of navigation and guidance that can be maintained with the equipment and navigation aids available. If an error or system failure results in reduced capability (downmoding), then the FMC may generate a crew message for display in the MCDU Scratch Pad. (If other system inputs to the FMC should fail, affected MCDU Displays are blanked to prevent the display of misleading or erroneous data. For example, loss of the total fuel input causes all performance-related data to be blank).

Page Status

32. Figure 24-7 shows how page status is indicated on the MCDU.

FIGURE 24-7
MCDU Page Status Indications



Function and Mode Keys

33. Figure 24-8 and Figure 24-9 outline the purpose of various function and mode keys.

FIGURE 24-8
MCDU Function and Mode Keys

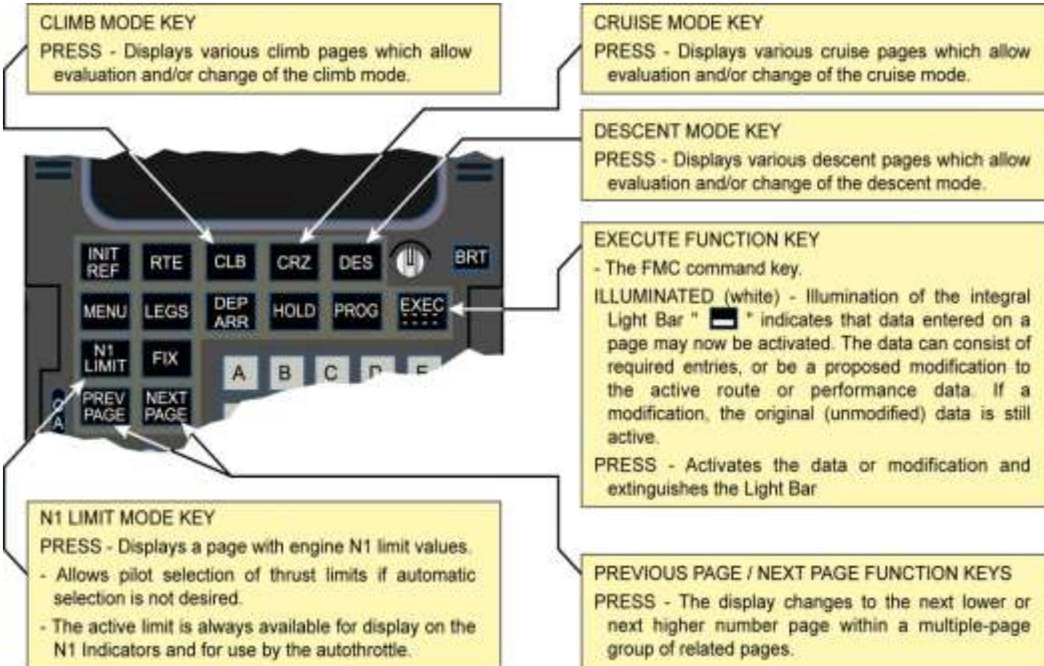
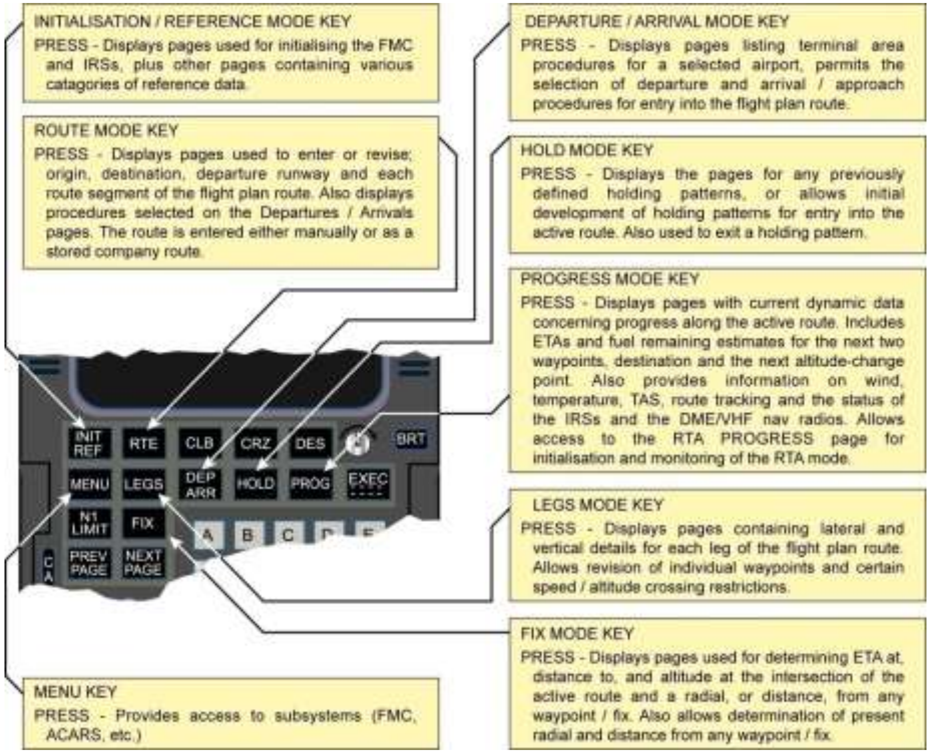


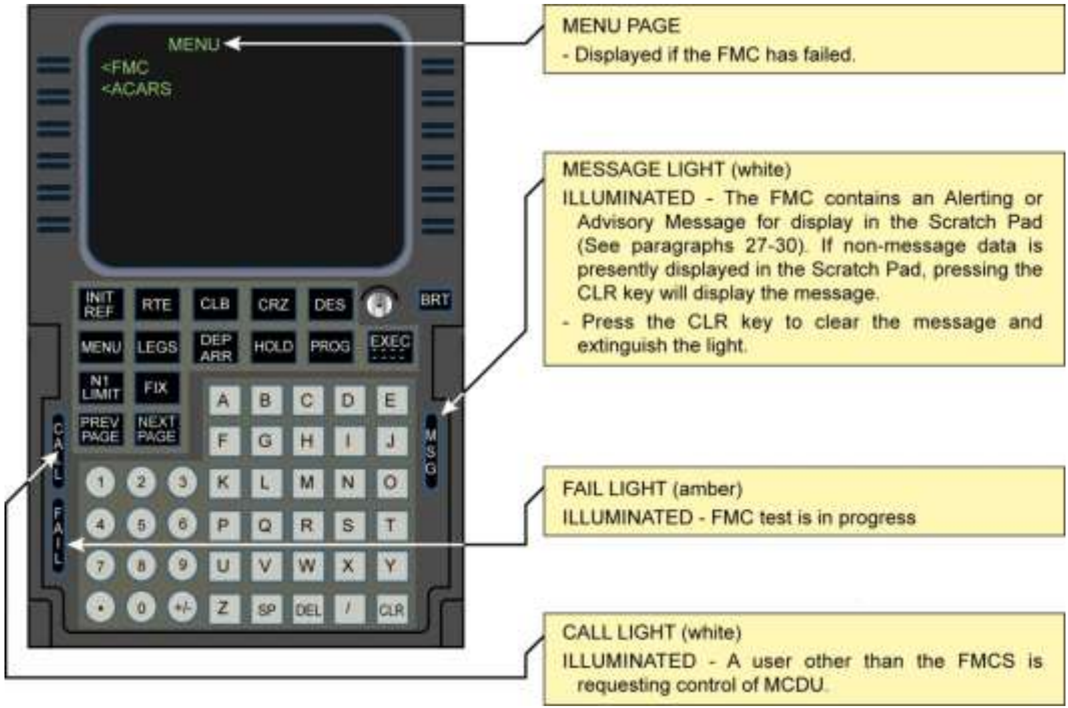
FIGURE 24-9
MCDU Mode Keys (cont'd)



MCDU Warning Lights

34. Figure 24-10 outlines the purpose of the various MCDU warning lights.

FIGURE 24-10
MCDU Warning Lights



MCDU Page Sequencing Following Power Application

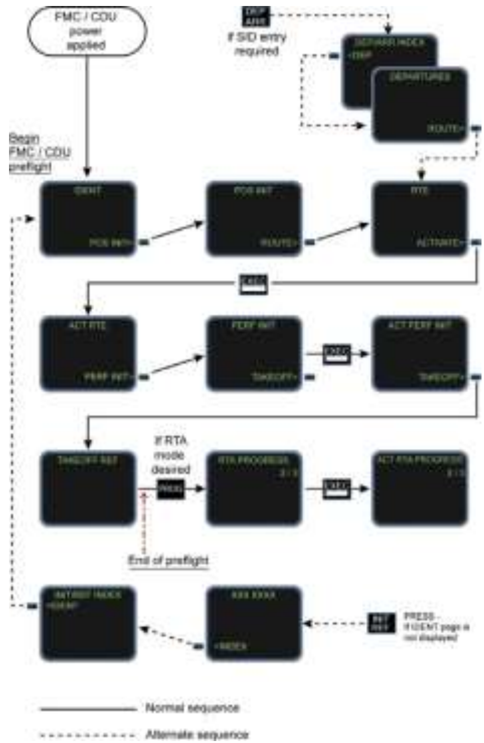
35. Although the FMC contains many displayable pages, proper page selection and execution are not difficult. Automatic display of some pages by phase of flight, as well as access prompts on many displays, provide assistance with the proper sequence of steps to initialise, activate, and fly the desired flight plan.

36. For example, Figure 24-11 shows how the FMC guides the crew through the required pages of a normal preflight. Upon initial power application, the IDENT page normally appears. If the IDENT page does not appear, then it may be accessed via the INIT/REF INDEX page, as shown at the bottom of the diagram. After checking the displayed data, line select key 6R is pressed in order to display the next logical page, POS INIT. The crew continues with each page, checking and entering data as required, then line selecting 6R, until preflight is complete.

37. If a Standard Instrument Departure (SID) must be entered into the route, press the DEP/ARR mode key for access to the DEPARTURES page. Following selection of the SID, line selection of 6R returns the display to the RTE page.

38. When the EXEC key illuminates, it must be pressed before continuing in order to activate the entered data. The RTE page requires line selection of the ACTIVATE prompt before the EXEC key will illuminate. This two-step procedure protects the crew from inadvertent activation of unintended data.

FIGURE 24-11
Preflight MCDU
Page Sequencing



Self Assessed Exercise No. 13

QUESTION:

QUESTION 1.

The product of the first integration of the output from the N-S accelerometer is:

QUESTION 2.

The product of a double integration of the east-west accelerometer is:

QUESTION 3.

With reference to a north-referenced inertial navigation system, gyrocompassing is:

QUESTION 4.

The corrections fed to the platform gimbal motors of a north-referenced inertial navigation system, during the align mode, use inputs from:

QUESTION 5.

The computer of a north-referenced INS, in flight, provides compensation for:

QUESTION 6.

In a stable platform, north-referenced INS, the mean groundspeed is 550kt. After 21.1mins. the indicated groundspeed is overreading by 5kt. The indicated groundspeed after 63.3mins. is:

QUESTION 7.

In an Inertial Reference System the gyros are strapdown/non-strapdown, and the accelerometers are strapdown/non-strapdown. (Delete as appropriate)

QUESTION 8.

In an INS which is Schuler tuned, the largest unbounded errors are due to:

QUESTION 9.

A stable north-referenced INS is able to display computed wind. The inputs used for this calculation are:

QUESTION 10.

At the end of a 6 hour 24 minute flight an aircraft is parked on a ramp where the coordinates are S36°52.0' E152°17.0'. The INS readout is S37°08.0' E151°52.0'. The radial error rate for the flight is:

QUESTION 11.

An INS-equipped aircraft flies from 56°N 20°W (waypoint 3) to 56°N 30°W (waypoint 4).

The initial track at waypoint 3 is:

QUESTION 12.

An INS-equipped aircraft flies from 56°N 20°W (waypoint 3) to 56°N 30°W (waypoint 4).

With constant drift during flight the aircrafts heading will decrease by:

QUESTION 13.

See FIGURE 122 in the Reference Book. – This diagram shows the situation after an INS equipped aircraft has passed waypoint 1 and is tracking along the dashed line marked TK. With HDG/DA selected, the DA readout will be:

QUESTION 14.

See FIGURE 122 in the Reference Book. - This diagram shows the situation after an INS equipped aircraft has passed waypoint 1 and is tracking along the dashed line marked TK. If track change, 0, 2, are inserted at the aircrafts present position (i.e. to make the present position WPT 0), the DSR TK readout will be:

QUESTION 15.

During the cruise, which information channels does the FMC provide to the AFDS:

QUESTION 16.

When a box prompt appears on the MCDU page, is the data input mandatory or optional:

QUESTION 17.

Where do scratch pad entries appear on the MCDU page layout:

QUESTION 18.

What is the normal period of validity of the navigation database?

QUESTION 19.

In an aircraft fitted with multi-inertial systems, what two positions does the FMC position correction vector joint together?

QUESTION 20.

FMC systems are normally duplicated in large commercial aircraft. What is the normal mode of operation for using such systems?

QUESTION 21.

What does the term managed guidance mean, in connection with FMC operation:

QUESTION 22.

What are the two categories of message that can be displayed on an MCDU?

QUESTION 23.

What is the purpose of the DEP/ARR key on the MCDU?

QUESTION 24.

Upon initial power application, what page is normally displayed on the MCDU:

QUESTION 25.

The initialisation of the FMC via the MCDU keyboard/screen requires the operator to enter/check data on each separate screen display before proceeding to the next page. Which is the main key that is pressed to cycle from page to page.

ANSWERS:

ANSWER 1.

The product of the first integration from the N-S accelerometer is N-S velocity/speed along the local meridian.

ANSWER 2.

The second integration of the east-west accelerometer is speed integrated with respect to time, which gives distance east-west. Distance east-west (nm) is also departure, or distance along the parallel of latitude.

ANSWER 3.

Gyrocompassing is the process by which the platform is aligned with true north, using earth gravity, in the align mode.

ANSWER 4.

During the align mode a north-referenced inertial navigation system uses inputs from the latitude setting, the accelerometers and the east gyro.

ANSWER 5.

The computer of a north referenced INS, in flight, provides corrections for earth rotation, transport wander and coriolis.

ANSWER 6.

After 63.3 mins the groundspeed will be under reading by 5kts (i.e. 545kts).

ANSWER 7.

In an IRS the gyros and accelerometers are both strapdown.

ANSWER 8.

The largest unbounded errors are due to the real wander of the platform gyroscopes.

ANSWER 9.

The inputs used in the computation of wind velocity are Heading (true), Track (true), Groundspeed, TAS.

ANSWER 10.

See FIGURE 117 in the Reference Book

ANSWER 11.

See FIGURE 118 in the Reference Book

ANSWER 12.

See FIGURE 119 in the Reference Book

Heading will decrease by 2 x Conversion angle (ca)

= 2 x 4.145°

= 8.29°

ANSWER 13.

See FIGURE 120 in the Reference Book

ANSWER 14.

See FIGURE 121 in the Reference Book

ANSWER 15.

During the cruise the FMC provides both LNAV and VNAV information to the AFDS.

ANSWER 16.

In the case of a box prompt, data input is mandatory.

ANSWER 17.

Scratch pad entries appear at the bottom of the screen, below the display division line.

ANSWER 18.

The information in the navigation database is normally valid for a 28-day period.

ANSWER 19.

The vector goes between the 'mean' inertial position and the FMC displayed position.

ANSWER 20.

Duplicate FMC systems are normally operated in dual mode.

ANSWER 21.

Managed guidance means that the FMC will command the autopilot to maintain the defined track (at a particular altitude and speed).

ANSWER 22.

The two categories of message that can be displayed on an MCDU are Alerting and Advisory Messages.

ANSWER 23.

The DEP/ARR key permits selection of departure and arrival/approach procedures for entry into the flight plan route.

ANSWER 24.

When power is initially applied to an MCDU the IDENT page normally appears.

ANSWER 25.

The initialisation process requires the user to cycle from page to page on the MCDU by using line select key 6R.

The Compass Swing

Co-efficient A

Co-efficient B

Co-efficient C

Deviation on Any Heading

Calculation of Sine and Cosine Values

Compass Swinging

The Correcting Swing

Removing Co-efficients A, B and C

Adjusting Direct Reading Compasses

Residual Deviation

The Calibration Swing

Mathematically Derived Residual Deviations

Permissible Levels of Residual Deviation

The Compass Swing

1. **Purpose.** The purpose of a compass swing is to calculate and if necessary, correct for deviation of the compass system.

2. **Deviation.** The following points about deviation should be noted:

   (a) Deviation is the angular difference between magnetic heading and compass heading.

   (b) Deviation is caused by the effect of the magnetic or magnetised elements of the aircraft itself upon the compass magnets (in a direct reading compass) or upon the detector unit (in a gyro magnetic compass). Deviation may also be caused by mis-alignment of the direct reading compass lubber line or of the gyro compass detector with the aircraft's fore and aft axis.

   (c) Deviation is not constant. It changes with change of heading and with change of magnetic latitude.

   (d) Deviation is said to be westerly (or negative) if compass north lies to the west of magnetic north, see Figure 25-1. In this event the compass heading will be greater than the magnetic heading (deviation west compass heading best). Alternatively deviation is said to be easterly (or positive) if compass north lies to the east of magnetic north, see Figure 25-2. In this event the compass heading will be less than the magnetic heading (deviation east compass heading least).

(e) Deviation is mathematically expressed as the correction that is required to convert the compass reading to the correct magnetic value (with easterly deviation the compass reading is less than the magnetic heading and therefore the deviation is given a positive value).

FIGURE 25-1
Effect of Westerly Deviation

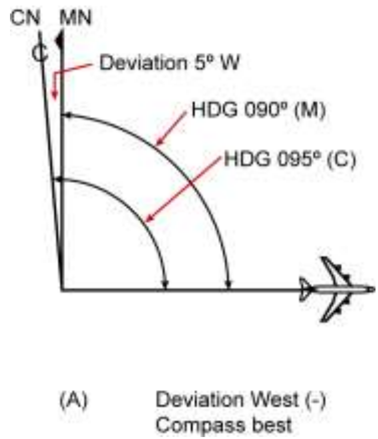
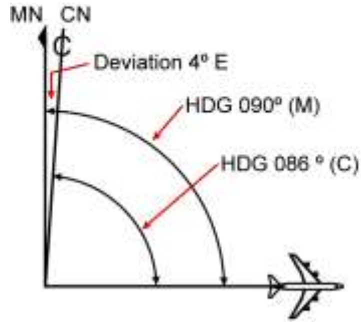


FIGURE 25-2
Effect of Easterly Deviation



(B) Deviation East (+)
Compass least

3. **Components of total deviation.** The total deviation present in an aircraft compass system are resolved into several 'coefficients'. Coefficient A is the error created usually (but not always) by a misalignment of the compass datum or 'lubber' line or detector unit. Coefficient B is created by a magnetic influence in the fore and aft axis. Coefficient C is created by a magnetic influence in the athwartships axis. The objective of the compass swing is to isolate the value of each of these coefficients and if necessary, correct for them.

Co-efficient A

4. For a direct reading compass to correctly indicate magnetic heading two criteria must be realised:

(a) The aircraft must exert no magnetic influence on the compass magnets.

(b) The lubber line of the compass must be correctly aligned with the aircraft fore and aft axis.

5. With a gyro magnetic compass again the aircraft must exert no magnetic influence on the sensing element (the detector unit), and now the detector unit (rather than the lubber line) must be correctly aligned with the aircraft fore and aft axis.

6. The effect of a misaligned lubber line in a direct reading compass is illustrated at Figure 25-3.

FIGURE 25-3
Effect of Misaligned Lubber Line

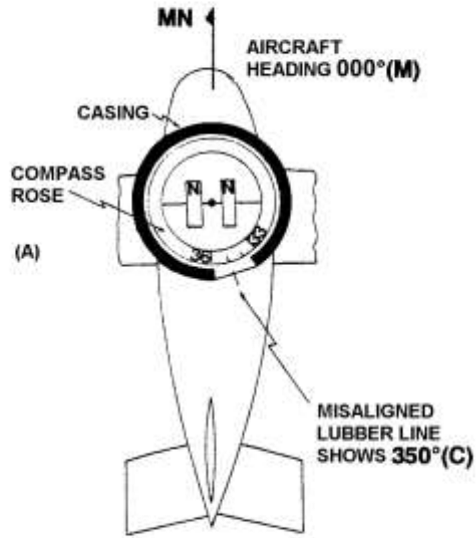
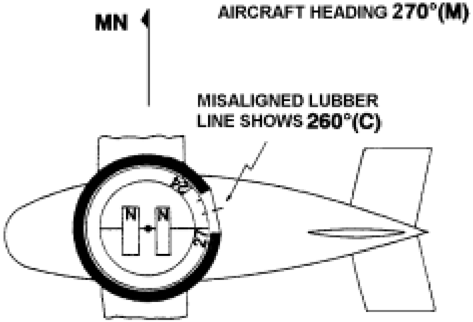


FIGURE 25-4
Effect of Misaligned Lubber Line

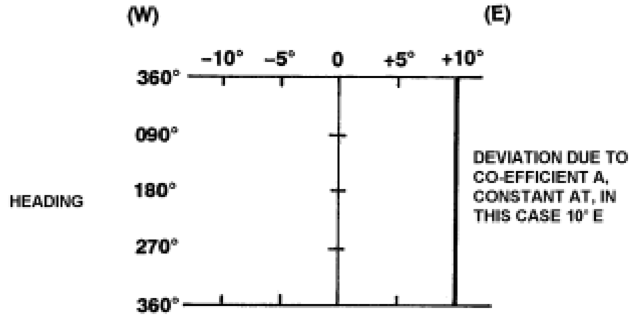


7. In Figure 25-3 the aircraft is heading 000°(M). The aircraft is assumed to have no deviating magnetic fields to affect the compass, and therefore the magnets within the compass are pointing to magnetic north and are aligned with the aircraft fore and aft axis. Unfortunately the lubber line is misaligned with respect to the aircraft fore and aft axis and so the compass, in this case, is reading 350°.

8. Figure 25-4 shows the same aircraft on a heading of 270°(M), and the compass is now reading 260°. The compass is still in error by 10° and is still under-reading. In this case a constant deviation of 10° east (+) exists on all headings and at all latitudes because of the misalignment of the lubber line.

9. The deviation caused by co-efficient A in the above example may be graphically illustrated as shown at Figure 25-5.

FIGURE 25-5 Graphical Representation of a Coefficient A of +10°



Co-efficient B

10. Assume for the moment that the sum of all the magnetic influences within the aircraft is represented by a single bar magnet lying along the aircraft fore and aft axis. The deviating influence of this mythical bar magnet would depend upon the aircraft's heading.

11. Figure 25-6(a) shows such a bar magnet with its south-seeking end in the nose of the aircraft. The aircraft is heading 000°(M) and all other deviating factors (co-efficients A and C) are ignored. The bar magnet which represents the aircraft's own magnetic field is lying parallel to the sensing magnets within the direct reading compass and therefore no deviation is evident on this heading.

12. Figure 25-6(b) shows the same aircraft on a heading of 090°(M). Remembering that **unlike poles attract whilst like poles repel** it can be seen that the compass magnets are partially attracted to the magnetism in the nose of the aircraft and are no longer pointing to the correct direction of magnetic north.

13. Figure 25-6(c) shows the aircraft on a heading of 180°(M) and again the bar magnet representing the aircraft's magnetic field causes no deviation at the compass.

14. Figure 25-6(d) shows the aircraft on a heading of 270°(M), and now compass north lies to the left of magnetic north.

FIGURE 25-6
Coefficient +B

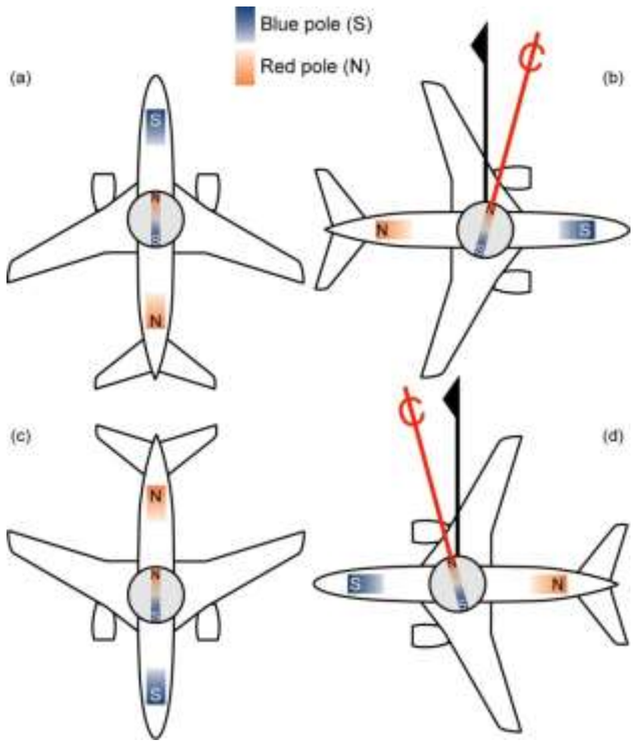
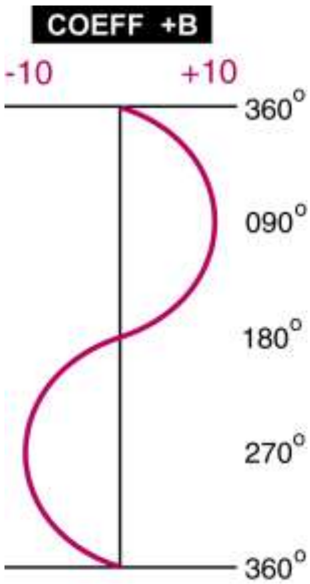


FIGURE 25-7
Graphical Presentation of Deviation due to Coefficient +B



15. Figure 25-7 shows the graphical representation of the deviation due to a co-efficient +B on all headings. The maximum deviation in this case is assumed to be 10°. **Deviation due to co-efficient B varies as a function of the sine of the aircraft's magnetic heading.** Because it is co-efficient +B which is considered, the sine relationship means that the value of deviation is maximum, and positive, on east and maximum, but negative, on west. Were we to repeat the process for a co-efficient -B (with the north-seeking end of our mythical bar magnet in the nose of the aircraft) the deviation would be maximum, but negative, on east and maximum, and positive, on west.

16. The formula which equates the deviation due to a co-efficient B to the value (and sign) of that co-efficient is:

The deviation due to co-efficient B = Co-efficient B x sine heading

Co-efficient C

17. Assume for the moment that the sum of all the magnetic influences within the aircraft is represented by a single bar magnet lying along the aircraft lateral axis. The deviating influence of this mythical bar magnet would again depend upon the aircraft's heading.

18. Figure 25-8(a) shows such a bar magnet with its south-seeking end in the starboard wing of the aircraft. The aircraft is heading 360°(M) and all other deviating factors (co-efficients A and B) are ignored. The bar magnet which represents the aircraft's own magnetic field is lying at 90° to the Earth's field and therefore deviation is at a maximum.

19. Figure 25-8(b) shows the same aircraft on a heading of 090°(M). Now the aircraft's magnetic field lies parallel to the Earth's magnetic field and there is no deviation.

20. Figure 25-8(c) shows that the deviation on 180°(M) is again at a maximum but now in the opposite direction to that suffered on north.

21. Figure 25-8(d) shows that the deviation on 270°(M) is again zero.

22. Figure 25-10 shows the graphical representation of the deviation due to a co-efficient +C on all headings. The maximum deviation in this case is assumed to be 10°. **Deviation due to co-efficient C varies as a function of the cosine of the aircraft's magnetic heading.** Because it is co-efficient +C which is considered the cosine relationship means that the value of deviation is maximum, and positive, on north and maximum, but negative, on south. Were we to repeat the process for a co-efficient -C (with the north-seeking end of our mythical bar magnet in the starboard wing of the aircraft) the deviation would be negative on north and positive on south.

FIGURE 25-8
Coefficient + C

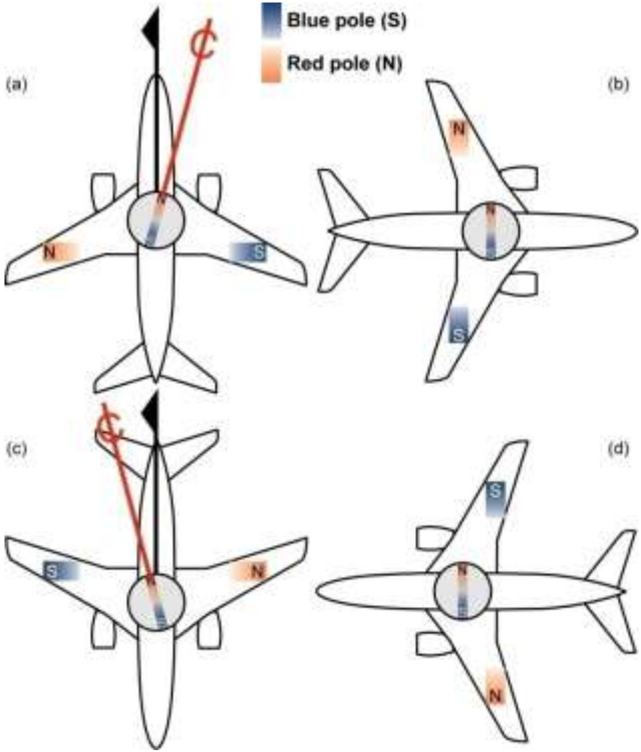
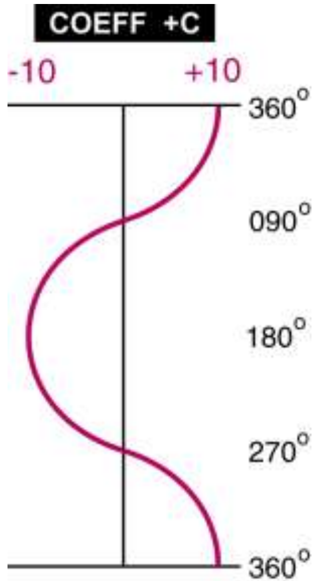


FIGURE 25-9
Graphical Presentation of Deviation due to Coefficient +C



23. The formula which equates the deviation due to a co-efficient C to the value (and sign) of that co-efficient is:

Deviation due to co-efficient C = co-efficient C x cosine heading

Deviation on Any Heading

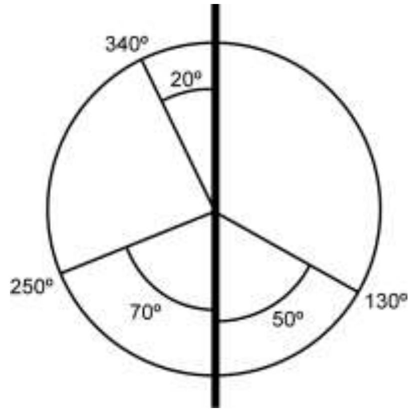
24. From the foregoing it is evident that the total deviation on any heading is a combination of co-efficients A, B and C. The formula for calculating the total deviation on any heading is:

Deviation on any heading = A+ (B x sin heading) + (C x cos heading)

Calculation of Sine and Cosine Values

25. Since the use of scientific calculators is not permitted in the examination, it is necessary to calculate the sign and magnitude of the sine or cosine of any angle between 0° and 360°. The natural value tables which are issued during the examination give values between 0° and 90° and therefore the first step is to calculate the angle to be used, and for this it is necessary to consider Figure 25-10

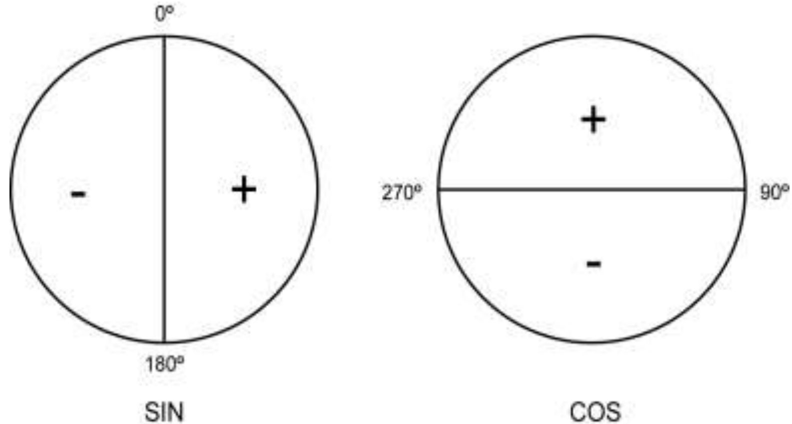
FIGURE 25-10
Sine/Cosine Angles



26. The angle required is found by comparing the angle given and the nearest vertical reference line. The angle between the two is the required angle. For example, if the angle given is 130° then the angle whose sine and cosine value has the same numeric value is 50°. If the angle given is 250°, then the sine or cosine value will be numerically the same as 70°. For 340°, the required angle is 20°.

27. Having found the necessary angle which will be used in the tables, it is now necessary to establish whether the sine or cosine is positive or negative, and for this it is necessary to refer to Figure 25-11.

FIGURE 25-11
Sine/Cosine Signs



28. At Figure 25-11 we see that the sine values of angles between 0° and 180° are positive, and between 180° and 360° are negative. Similarly we see that cosine values of angles between 270° and 90° are positive, and between 90° and 270° are negative.

29. Armed with Figure 25-10 and Figure 25-11 we can establish the correct value and sign of the sine or cosine of any angle.

EXAMPLE 25-1

EXAMPLE

Determine the natural sine and cosine values of 157°, 223° and 356°.

For sine 157° use 23° and the value is positive	=	+0.391
For cosine 157° use 23° and the value is negative	=	-0.920
For sine 223° use 43° and the value is negative	=	-0.682
For cosine 223° use 43° and the value is negative	=	-0.731
For sine 356° use 4° and the value is negative	=	-0.070
For cosine 356° use 4° and the value is positive	=	+0.998

30. One final piece of mathematics revision before we continue. Remember that, if you multiply a positive value by another positive value, the product is positive; if you multiply a positive value by a negative value, the product is negative; and if you multiply a negative value by another negative value the result is positive.

EXAMPLE 25-2

EXAMPLE

Given that co-efficient A is -0.5°, co-efficient B is -1.5° and co-efficient C is +0.5°, determine the total deviation on a heading of 253°(C).

Deviation on 253°	=	A + (B.sin 253°) + (C.cos 253°)
Sin 253°	=	-0.956
Cos 253°	=	-0.292
Deviation on 253°	=	-0.5 + (-1.5 x -0.956) + (+0.5 x -0.292)
	=	-0.5 + (+1.43) + (-0.14)
	=	+ 0.79

EXAMPLE 25-3

EXAMPLE

Given that co-efficient A is +1°, co-efficient B is -1° and co-efficient C is +2°, determine the total deviation on a heading of 240°(C).

SOLUTION

Deviation on 240°	=	A + (B.sin 240°) + (C.cos 240°)
Sin 240°	=	-0.866
Cos 240°	=	-0.5
Deviation on 240°	=	+1 + (-1 x -0.866) + (+2 x -0.5)
	=	+1 + (+0.866) + (-1)
	=	+ 0.866

Now for a slightly different type of question.

EXAMPLE 25-4

EXAMPLE

Given that co-efficient A is -2°, what would be the signs of co-efficients B and C, given that the heading on which the maximum deviation occurs is 330°(C)?

SOLUTION

Deviation on any heading = A+(B X Sin heading)+C Cos heading

The maximum deviation will occur on a heading where the signs of the deviations caused by co-efficients A, B and C are all the same. Co-efficient A is -2° and therefore the deviation caused by co-efficient A is negative on all headings.

The deviation caused by co-efficient B must be negative on 330°. In order to achieve this, co-efficient B must be a positive value since the sine of 330° is -0.5, and a positive value multiplied by a negative value gives a negative product.

The deviation caused by co-efficient C must also be negative on 330°. In order to achieve this, co-efficient C must be a negative value since the cosine of 330° is +0.866, and again a positive value multiplied by a negative value gives a negative product.

Compass Swinging

31. In order to minimise the compass deviation it is necessary to keep the aircraft's own magnetic field as small as possible. This is considered at the design stage and influences the choice of materials used, and the design and location of electrical equipment.

32. Having minimised the deviating effect of the aircraft on the compass at the design stage, it is periodically necessary to **compass swing** the aircraft. During this procedure the magnitude and direction of the remaining deviations are measured on various headings. These deviations are then reduced by producing magnetic fields within the compass which are hopefully equal in magnitude but opposite in polarity to the aircraft's own magnetic fields.

33. Compass swings are carried out in a surveyed area which is relatively clear of external magnetic influences such as might be caused by underground electric cables. During the compass swing, normal flying conditions are simulated as far as possible, with the engines running and all electrical services switched on.

34. Compass swings should be carried out on the following occasions:

(a) On installation of the compass.

(b) Periodically as specified in the relevant BCAR (British Civil Airworthiness Requirements).

(c) Whenever the accuracy of the compass is in doubt.

(d) When the compass has been subjected to shock (such as a heavy landing).

(e) Following a lightning strike.

(f) If the aircraft has been left standing on one heading for a long period of time.

(g) Following a move to a new magnetic latitude.

(h) Following any significant modification, repair or replacement of a component containing significant amounts of magnetic materials (such as an engine change).

(i) Following any significant addition of, or modification to, electrical or radio/ navigation systems.

(j) Prior to flight, following the loading or off-loading of a cargo containing significant amounts of ferro-magnetic materials.

35. There are many procedures for measuring the magnitude of the co-efficients affecting an aircraft compass. Most of these techniques involve an accurate **datum compass** (such as a Medium Landing Compass), the reading of which is compared with the reading of the aircraft compass on various headings.

36. The Medium Landing Compass is a small portable bearing compass, having a high degree of accuracy, which can be used for ground calibration of aircraft compasses. It is designed to use on the ground and is fitted with a tripod and a bubble level. The horizontal compass card is read through a prism which forms part of the sighting head.

37. If the datum compass is to give an accurate reading with reference to magnetic north it must itself be free from any deviating magnetic fields. The person who is aligning the datum compass and taking the readings should therefore remove all metal objects from his person before the swing. If you are so involved, and normally wear a trouser belt with a metal buckle, think ahead!

38. In order to compare the reading of the datum compass with that of the aircraft compass(es) the datum compass is **sighted** along the aircraft's fore and aft axis. With larger aircraft this is normally achieved by suspending **sighting rods** vertically beneath the aircraft. At a distance of a least 50 metres from the aircraft the **hair lines** of the datum compass are aligned with the sighting rods. The reading of the datum compass is then noted.

39. There are many ways of completing a compass swing, for the purpose of this syllabus we need to consider only a simple four point correcting swing.

The Correcting Swing

40. The swing is commenced on a cardinal heading, it does not matter which cardinal. We will start on a heading of east, that is to say a compass heading which is fairly close to 090°, it does not have to be exact. In this case the next comparison between aircraft and datum compass would be made on south, and the next on west. It is then necessary to stop, calculate the value of co-efficient B, and adjust the aircraft compass if necessary.

41. Returning to Figure 25-7 should convince the reader that deviation caused solely by co-efficient B is a maximum on headings of east and west, and is of equal magnitude and opposite sign on these headings.

42. Co-efficient C will not affect the readings on headings of east and west (see Figure 25-9). Co-efficient A is effectively eliminated from the formulae for co-efficients B and C since it will cause equal deviation on all headings (Figure 25-5).

43. The formula for calculating co-efficient B is:

$$\text{Co-efficient B} = \frac{\text{Deviation on east} \quad \text{Deviation on west}}{2}$$

44. Let us now put figures to this swing and see just how simple the procedure really is:

Aircraft Compass	Datum Compass
089°	091°
182°	179°
272°	268°

45. From the above figures the deviation is calculated. Appreciate that the aircraft compass readings are in effect compass headings (subject to deviation) and that the datum compass readings are in effect magnetic headings (the datum compass is remote from the aircraft and is therefore free from deviation). Remember that, if the aircraft compass heading is least deviation is east (+) and that, if the aircraft compass heading is best deviation is west (-).

Aircraft Compass	Datum Compass	Deviation
089°	091°	+2°
182°	179°	-3°
272°	268°	-4°

46. Using the formula:

$$\text{Co-efficient B} = \frac{(+2^\circ) - (-4^\circ)}{2}$$

$$= \frac{+2^\circ + 4^\circ}{2}$$

$$= \frac{+6^\circ}{2}$$

$$= +3^\circ$$

47. It may be necessary to correct for co-efficient B at this stage (if it is outside limits), and this procedure will be covered shortly.

48. Continuing with the swing the aircraft compass and datum compass readings are now taken on a heading which is close to north.

49. The logic which gave us our simple formula for co-efficient B will also give the following simple formula for co-efficient C:

$$\text{Co-efficient C} = \frac{\text{Deviation on north - Deviation on south}}{2}$$

50. Continuing with the swing:

Aircraft Compass	Datum Compass	Deviation
089°	091°	+2°
182°	179°	-3°
272°	268°	-4°
358°	359°	+1°

51. Using the formula:

$$\text{Co-efficient C} = \frac{\text{Dev N} - \text{Dev S}}{2}$$

$$= \frac{+1^\circ - (-3)^\circ}{2}$$

$$= \frac{+1^\circ + 3^\circ}{2}$$

$$= \frac{+4^\circ}{2}$$

$$= +2^\circ$$

52. Again a correction for co-efficient C could be made here if required.

53. The formula for calculating co-efficient A is:

$$\text{Co-efficient A} = \frac{\text{Sum of observed deviations}}{\text{The number of observations}}$$

54. In this case the sum of the deviations [(+2) + (-3) + (–4) + (+1)] is -4°.

$$\text{Co-efficient A} = \frac{-4^\circ}{4}$$

$$= -1^\circ$$

55. If necessary, co-efficient A can be removed at this stage, the method will be discussed shortly.

56. In the event that any of the three co-efficients are outside limits and have been removed, a further four point correcting swing will be required and hopefully the co-efficients will now be within limits.

EXAMPLE 25-5

EXAMPLE

Using the following readings which were obtained during the swing of a direct reading compass, determine the values of co-efficients A, B and C.

Aircraft Compass	Datum Compass
359°	000°
090°	086°
180°	175°
269°	273°

SOLUTION

Aircraft Compass	Datum Compass	Deviation
359°	000°	+1°
090°	086°	-4°
180°	175°	-5°
269°	273°	+4°

$$\text{Co-efficient A} = \frac{\text{Sum of observed deviations}}{\text{The number of observations}}$$

In this case the sum of the deviations [(+1) + (-4) + (–5) + (+4)] is -4°.

Co-efficient A
$$= \frac{-4^o}{4}$$
$$= -1^o$$

Co-efficient B
$$= \frac{\text{Dev E} - \text{Dev W}}{2}$$
$$= \frac{(-4^o) + (+4)^o}{2}$$
$$= \frac{-8^o}{2}$$
$$= -4^o$$

Co-efficient C
$$= \frac{\text{Dev N} - \text{Dev S}}{2}$$
$$= \frac{+1^o - (-5)^o}{2}$$
$$= \frac{+6^o}{2}$$
$$= +3^o$$

Removing Co-efficients A, B and C

57. The principles for removing co-efficients are the same, regardless of whether it is a direct reading compass or a gyro magnetic compass which is considered.

58. Co-efficient A results from **misalignment.** Co-efficient A errors are therefore removed by re-alignment of the lubber line in the direct reading compass, or the detector unit in the gyro magnetic system.

59. Errors due to co-efficients B and C are minimised by deliberately introducing magnetic fields which have an equal but opposite effect to that of the aircraft's own magnetic fields. This is achieved by means of **scissor magnets** in direct reading compasses and electro magnets in gyro slaved compasses.

Adjusting Direct Reading Compasses

60. It is the E type compass which is discussed in the following paragraphs. The compass is fitted to its mountings using slotted channels which enable the entire compass to be rotated once the retaining screws are loosened (using a non-magnetic screwdriver).

61. Co-efficient A can be removed on any heading, since it has the same deviating effect on all headings.

62. If it is necessary to remove a positive (easterly) co-efficient A the compass is physically rotated in a clockwise direction by the required number of degrees. This will cause the compass reading to increase.

EXAMPLE 25-6

EXAMPLE

A direct reading compass is found to have a co-efficient A of +2°, and this is to be removed on a compass heading of 329°. What should the compass read after compensation, and how is compensation achieved?

SOLUTION

Co-efficient A = +2° (east), the compass is therefore under-reading before compensation. It is necessary to **increase** the compass reading to 331° (329° + 2°) and this is achieved by loosening the retaining screws (using non-magnetic tools) and rotating the body of the compass in a clockwise direction until 331° appears under the lubber line. The retaining screws are then secured without disturbing the reading.

63. Conversely, to remove a negative (westerly) co-efficient A the compass is rotated in an anti-clockwise direction, and this causes the compass reading to decrease.

64. To remove co-efficients B and C scissor magnets are adjusted using grub screws located under a cover on the instrument face. These scissor magnets are attached to the compass casing and therefore change position relative to the sensing magnets as the aircraft alters heading. Their effect upon the compass reading will therefore depend on the aircraft heading, in much the same way as the aircraft magnetic fields causing deviations B and C.

65. Figure 25-12 shows an aircraft with co-efficient +C represented by a magnet with its south-seeking end in the starboard wing. At Figure 25-12 the scissor magnets used to compensate for co-efficient C are in their neutral position.

66. These two small scissor magnets have equal pole strengths and when set in the neutral position they exert no influence on the pendulously suspended sensing magnets of the compass.

67. Figure 25-13 shows the same aircraft, but now the scissor magnets have been adjusted to compensate for deviations caused by the co-efficient +C.

FIGURE 25-12
Compass Scissors Magnets

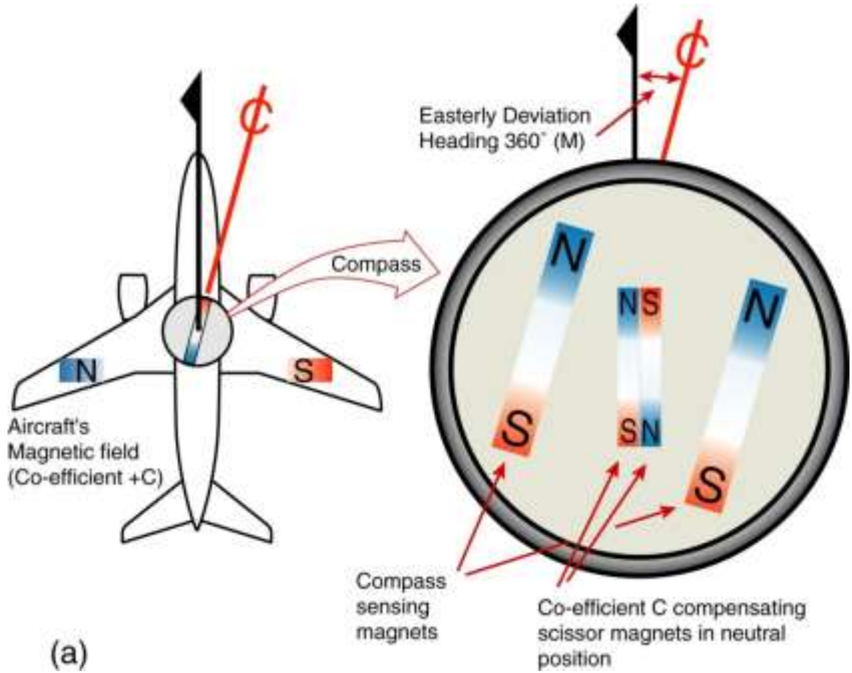
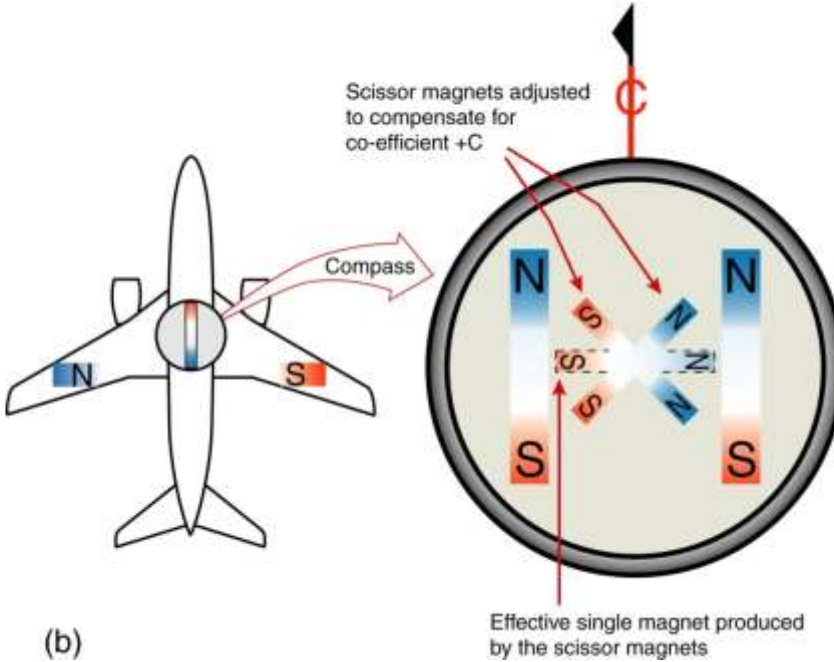
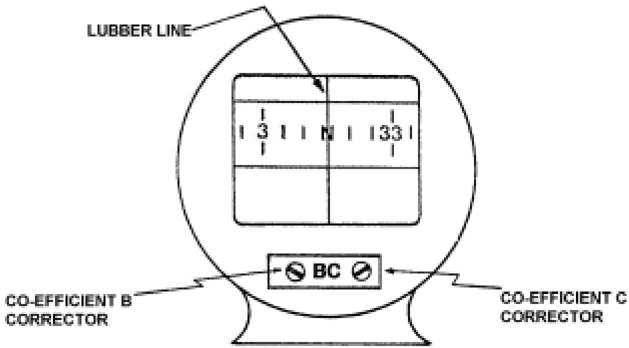


FIGURE 25-13
Compass Scissor Magnets after Adjustment for Co-efficient +C



68. Figure 25-14 shows the grub screws on an E type direct reading compass, the cover has been removed. It is necessary to work with non-magnetic tools when making the adjustments.

FIGURE 25-14
Coefficient B & C Correctors



69. Compensation is achieved by turning the appropriate grub screw until the required heading is shown beneath the lubber line. In order to avoid turning the grub screw the wrong way remember that **negative** co-efficients are removed by turning the appropriate grub screw **clockwise**, whilst **positive** co-efficients are removed by turning the appropriate grub screw **anti-clockwise.**

70. Co-efficient B is removed on headings of east or west, whilst co-efficient C is removed on headings of north or south. It is on these headings that the deviation caused by the appropriate co-efficient will be at a maximum. That is to say that the deviation caused by the co-efficient will be equal in magnitude to the co-efficient itself.

71. When compensating for co-efficient B on an easterly heading obey the sign of the co-efficient when calculating the compass reading after compensation.

72. When compensating for co-efficient B on a westerly heading the sign of the co-efficient is reversed when calculating the compass reading after compensation (the sine of 270° is negative).

EXAMPLE 25-7

EXAMPLE

A direct reading compass is found to have a co-efficient B of +3°, and this is to be removed on an easterly heading. Before compensation the compass is reading 089°. What should the compass read after compensation, and how is this compensation achieved?

SOLUTION

The compass should be made to read 092° (089° + 3°), and this is achieved by turning the **B** grub screw in an anti-clockwise direction.

EXAMPLE 25-8

EXAMPLE

A direct reading compass is found to have a co-efficient B of +4°, and this is to be removed on a westerly heading. Before compensation the compass is reading 272°. What should the compass read after compensation, and how is this compensation achieved?

SOLUTION

The compass should be made to read 268° (272° - 4°), and this is achieved by turning the B grub screw in an anti-clockwise direction.

NOTE:

Note that the grub screw is turned in an anti-clockwise direction since, although the correction is subtractive, the deviation being compensated is itself positive.

---

73. The same logic applies to compensation for co-efficient C, but now it is the C grub screw which is turned in the appropriate direction. When removing co-efficient C on a northerly heading obey the sign of the co-efficient to calculate heading after compensation. When removing co-efficient C on a southerly heading reverse the sign of the co-efficient.

74. You are not now required to know how to adjust gyro slaved compasses.

Residual Deviation

75. Unfortunately, since neither the original assessment of co-efficients, nor the compensation, will be totally accurate, small **residual deviations** will persist after compensation. It is necessary to determine the values of these remaining deviations, and to tabulate them on the **compass deviation card** which is then attached to the aircraft adjacent to the compass. It is an airworthiness requirement that the residual deviation is stated at no more than 45° intervals on a deviation card which is to be located close to the compass in the case of a direct reading compass (and close to the master compass indicator and to each remote compass indicator, in the case of a gyro slaved compass).

76. There are two options available to determine the magnitude and sign of the residual deviations in order to complete the deviation card. The first method is to conduct a calibration swing.

The Calibration Swing

77. Having completed the correcting swing(s) and ensured that the residual values of the co-efficients are (now) within limits, a calibration swing is completed. This normally requires that the deviations are observed on twelve headings 30° apart, which may include the four cardinal heading observations from the final correcting swing. The observed deviations on these twelve heading may be used in their raw state to complete the deviation card.

Mathematically Derived Residual Deviations

78. Unfortunately, when the raw data from a calibrating swing is used to determine the residual deviations for the deviation card, these observed values may themselves be subject to error. BCARs may therefore dictate that a mathematical approach is used to determine the residual deviations. For the sake of simplicity, the mathematics of residual deviation is considered only on the cardinal headings in the following paragraphs. We will tackle the problem by working step by step through the following example.

EXAMPLE 25-9

EXAMPLE

The following readings were obtained during the swing of a direct reading magnetic compass:

Aircraft Compass (compass heading)	Datum Compass (magnetic heading)
359°	004°
092°	091°
181°	184°
268°	273°

(a) Determine the values of co-efficients A, B and C.

(b) Having compensated for co-efficients A, B and C determine the values of the residual deviations remaining on the magnetic headings 004°, 091°, 184° and 273°.

(a) The first step is to determine the values of the three co-efficients.

Aircraft Compass	Datum Compass	Deviation
359°	004°	+5°
092°	091°	-1°
181°	184°	+3°
268°	273°	+5°
		+12°

$$\text{Co-efficient A} = \frac{\text{Sum of deviations}}{4} = \frac{+12°}{4} = +3°$$

$$\text{Co-efficient B} = \frac{\text{Dev E - Dev W}}{2} = \frac{(-1) - (+5)}{2} = \frac{-6}{2} = -3°$$

$$\text{Co-efficient C} = \frac{\text{Dev N - Dev S}}{2} = \frac{(+5) - (+3)}{2} = \frac{+2}{2} = +1°$$

(b) Now we need to step carefully through the calculation of the residual deviations, assuming that the co-efficients determined above have been removed. If we are insulting your intelligence by going too slowly I apologise, however it is a procedure which is best tackled methodically.

All that has happened in the table below is that the aircraft compass headings, datum compass readings and the consequent deviations have been transferred to the first three columns.

(c)

1	2	3	4	5	6	7	8
Comp Hdg	Mag Hdg	Deviation before compensation	Deviations removed			Sum of deviations removed (columns 4, 5 and 6)	Residual deviations (column 3 minus column 7)
			A	B sin hdg	C cos hdg		
359	004	+5					
092	091	-1					
181	184	+3					
268	273	+5					

The next step is to consider the sign and the magnitude of the deviations which would have been caused by the co-efficients A, B and C on the four headings, had they not been removed.

Completing column 4 presents no problem, since the co-efficient A of +3° would have resulted in a deviation of +3° on each heading.

When completing column 5 we need to consider the deviating influence that a co-efficient B of -3° would have had on the four cardinal headings, had it not been removed. On north and south this co-efficient would have exerted no deviating influence (the sine of 0° and 180° is zero). On east the deviation caused by this co-efficient would have been maximum and negative and on west maximum and positive.

When completing column 6 we need to consider the deviating influence that a co-efficient C of +1° would have had on the four cardinal headings, had it not been removed. On east and west this co-efficient would have exerted no deviating influence (the cosine of 090° and 270° is zero). On north the deviation caused by this co-efficient would have maximum and positive and on south maximum and negative.

Please note that, since all of the headings considered are very close to the cardinal points, it is acceptable to take the sine of small angles as zero, the cosine of the same angles as unity (1), the sine of angles close to 90° as unity (1), and the cosine of the same angles as zero.

The table can now be completed as far as column 6.

1	2	3	4	5	6	7	8
Comp Hdg	Mag Hdg	Deviation before compensation	Deviations removed			Sum of deviations removed (columns 4, 5 and 6)	Residual deviations (column 3 minus column 7)
			A	B sin hdg	C cos hdg		
359	004	+5	+3	0	+1		
092	091	-1	+3	-3	0		
181	184	+3	+3	0	-1		
268	273	+5	+3	+3	0		

The next step is to add algebraically the values given in columns 4, 5 and 6 for each of the headings and to enter the sum of these deviations in column 7. The values in column 7 therefore represent the total deviation which would have been suffered on each of the headings, due to co-efficients A, B and C, had these co-efficients not been removed.

1	2	3	4	5	6	7	8
Comp Hdg	Mag Hdg	Deviation before compensation	Deviations removed			Sum of deviationsremoved (columns 4, 5 & 6)	Residual deviations (column 3 minus column 7)
			A	B sin hdg	C cos hdg		
359	004	+5	+3	0	+1	+4	
092	091	-1	+3	-3	0	0	
181	184	+3	+3	0	-1	+2	
268	273	+5	+3	+3	0	+6	

Finally, by subtracting algebraically the values in column 7 from the values for the same heading in column 3, the residual deviation is established for that heading. Column 3 gives the deviation which was observed before the co-efficients were removed. Column 7 gives the mathematical summation of the deviating effects of these co-efficients. By correcting for these co-efficients we have reduced the original deviations (column 3) by the values in column 7 to give the residual deviations in column 8.

1	2	3	4	5	6	7	8
Comp Hdg	Mag Hdg	Deviation before compensation	Deviations removed			Sum of deviations removed (columns 4, 5 and 6)	Residual deviations (column 3 minus column 7)
			A	B sin hdg	C cos hdg		
359	004	+5	+3	0	+1	+4	+1°
092	091	-1	+3	-3	0	0	-1°
181	184	+3	+3	0	-1	+2	+1°
268	273	+5	+3	+3	0	+6	-1°

79. One final example to consider before we leave the problems of residual deviation behind us.

EXAMPLE 25-10

EXAMPLE

The following readings were obtained during the swing of a direct reading magnetic compass.

Compass Heading	Magnetic Heading
359°	000°
090°	086°
180°	175°
269°	273°

(a) Determine the values of co-efficients A, B and C.

(b) Determine the compass heading required, after compensation for co-efficients A, B and C, to obtain a true heading of 288° in a position where the local magnetic variation is 15°E

SOLUTION

(a)

Compass Heading	Magnetic Heading	Deviation
359°	000°	+1°
090°	086°	-4°
180°	175°	-5°
269°	273°	+4°
		-4°

Co-efficient A $= \dfrac{-4^{o}}{4}$

$= -1^{o}$

Co-efficient B $= \dfrac{(-4) - (+4)}{2}$

$= -4^{o}$

Co-efficient C $= \dfrac{(+1) - (-5)}{2}$

$= +3^{o}$

(b) The second part of this question is capable of inducing panic into some candidates when encountered in the examination because the heading given isn't a cardinal. By adjusting the given true heading for variation the magnetic heading which was used in the correcting swing is achieved - end of panic.

Heading true	=	288°
Variation	=	15°E
Heading magnetic	=	273°

1	2	3	4	5	6	7	8
Comp Hdg	Mag Hdg	Deviation before compensation	Deviations removed			Sum of deviations removed (columns 4, 5 and 6)	Residual deviations (column 3 minus column 7)
			A	B sin hdg	C cos hdg		
269	273	+4	-1	+4	0	+3	+1°

Heading magnetic	=	273°
Deviation	=	+1°
Heading compass	=	272°

Permissible Levels of Residual Deviation

80. British Civil Airworthiness Requirements (BCARs) state that after correction, the greatest deviation on any heading shall be 3° for a direct reading compass which is used as a primary heading reference and 10° for a direct reading compass which is used as a standby heading reference. The greatest deviation (after correction) on any heading which is permitted for remote reading (gyro slaved) compasses is 1°.

81. Furthermore BCARs state that the maximum permissible level of co-efficients B and C, after correction, shall not exceed 15° for direct reading compasses, and 5° for remote reading compasses.

Self Assessed Exercise No. 14

QUESTIONS:

QUESTION 1.

Variation only varies with the position of the aircraft on the Earth relative to the True and Magnetic Poles.

State the factor(s) which affect deviation

QUESTION 2.

State the 2 components of aircraft magnetism which represent the major deviating influences on a compass.

QUESTION 3.

Deviation comprises 3 coefficients: A, B, C

a. The effect of Coefficient A is at a maximum on________________headings.

b. The effect of Coefficient B is at a maximum on_______________ headings.

c. The effect of Coefficient C is at a maximum on_______________ headings.

QUESTION 4.

State the formula for total deviation on any heading.

QUESTION 5.

Given that Coefficient A = -2°, Coefficient B = +1° and Coefficient C = -2°, determine the total deviation on a heading of 120°

QUESTION 6.

List the occasions when a full compass calibrating swing is required.

QUESTION 7.

Explain how Coefficient A is compensated for following a correcting swing on a Direct Reading Compass (DRC).

QUESTION 8.

Explain how Coefficient A is compensated for following a correcting swing on a Gyro Slaved/ Remote Reading Compass.

QUESTION 9.

Explain what is meant by Compass Safe Distance.

QUESTION 10.

The following readings were obtained during a compass swing:

Magnetic (Datum) Hdg	Compass Hdg
360°	006°
090°	088°
180°	182°
270°	272°

Determine the value of coefficients A, B and C

QUESTION 11.

During a compass swing the coefficients of deviation were determined as follows:

Coeff A = +2 Coeff B = -2 Coeff C = +3 Using this information, determine the following

a. The deviation caused by these coefficients on a heading of 266°M

b. The deviation caused by these coefficients on a heading of 183°M

c. The compass reading when the aircraft is parked on a datum heading of 272°M

The correct magnetic heading when the aircraft is parked on 178°C

QUESTION 12.

The following readings were obtained during a compass swing:

Magnetic (Datum) Hdg	Compass Hdg
355°	360°
088°	091°
182°	181°
271°	268°

Using this information, determine the following:

a. The magnetic heading (after correction) when on a compass heading of 269C

b. The compass heading (after correction) when on a magnetic heading of 178°M

QUESTION 13.

During a compass swing the values of the coefficients of deviation were calculated as follows:

Coeff A = -3 Coeff B = +1 Coeff C = -4

The compass was then corrected for these values. If the deviation on west before compensation was -3°, determine the compass heading after compensation on a magnetic heading of 270°M.

ANSWERS:

ANSWER 1.

Deviation is caused by:

1. Misalignment of the lubber (DRC) or the magnetic detector unit (RRC).

2. The effect of aircraft magnetism on the magnetic assembly (DRC) or magnetic detector unit (RRC).

3. As a result, deviation is not constant and can vary with change of heading or change of magnetic latitude.

See 061-25-paragraph 2

ANSWER 2.

Horizontal Hard Iron Magnetism and Vertical Soft Iron Magnetism are the 2 major deviating influences on a compass.

See 061-25-paragraph 4 to paragraph 9

ANSWER 3.

a. Coefficient A is constant and affects all headings; therefore, it is maximum on ALL headings.

See 061-25-paragraph 4 to paragraph 9

b. Coefficient B is a measure of the deviating influence of aircraft magnetism in the longitudinal or fore and aft axis; the maximum effect will be on East/West headings.

See 061-25-paragraph 10 to paragraph 16

c. Coefficient C is a measure of the deviating influence of aircraft magnetism in the lateral or athwartships axis; the maximum effect will be on North/South headings.

See 061-25-paragraph 17 to paragraph 23

ANSWER 4.

Total Deviation on any heading

= Coeff A + (Coeff B x sin heading) + (Coeff C x cos heading).

See 061-25-paragraph 24

ANSWER 5.

Deviation on 120 = -2 + (+1 x sin 120) + (-2 x cos 120)

= -2 + 0.866 + 1

= -0.134°

ANSWER 6.

See 061-25-paragraph 34

ANSWER 7.

See 061-25-paragraph 58

ANSWER 8.

See 061-25-paragraph 58

ANSWER 9.

See 061-6-15 to 17

ANSWER 10.

compass heading (a/c)	magnetic heading (datum)	deviation before correction	deviations to be removed			sum of deviations to be removed	residual deviation
			A	B	C		
006	360	-6					
088	090	+2					
182	180	-2					
272	270	-2					
		Total = -8					

$$\text{Coeff A} = \frac{\text{Sum of Deviations}}{\text{No of Observations}} = \frac{-8}{4} = -2$$

$$\text{Coeff B} = \frac{\text{Deviation on E - Deviation on W}}{2} = \frac{+2-(-2)}{2} = \frac{+4}{2} = +2$$

$$\text{Coeff C} = \frac{\text{Deviation on N - Deviation on S}}{2} = \frac{+6-(-2)}{2} = \frac{-4}{2} = -2$$

ANSWER 11.

Deviation on any Hdg = A +(B x sin Hdg) + (C x cos Hdg)

a. On a heading of West sin 270 = -1

cos 270 = 0

Therefore, Dev'n on West (266°M) = A - B

= +2 - (-2)

= +4°

b. On a heading of South sin 180 = 0

cos 180 = -1

Therefore, Dev'n on South (183°M) = A - C

= +2 - (+3)

= -1

c. Deviation on West = A - B = +4°

i.e. 4°E

On a datum heading of 272°M

deviation 4°E (Deviation East, Compass East)

Therefore, compass hdg = 272° -4°

$= 268°C$

d. Deviation on South = A - C = -1°

i.e. 1°W

Aircraft heading = 178°

deviation = 1°W (Deviation West, Compass Best)

Therefore, correct magnetic heading = 178° -1°

= 177°(M)

ANSWER 12.

Residual Deviation = Deviation before correction - Sum of Deviation to be removed

compass heading (a/c)	magnetic heading (datum)	deviation before correction	deviations to be removed			sum of deviations to be removed	residual deviation
			A	B	C		
360	355	-5	-1		-3	-1 + (-3) = -4	
091	088	-3	-1	-3		-1 + (-3) = -4	
181	182	+1	-1		+3	-1 + 3 = +2	+1 - (+2) = -1
268	271	+3	-1	+3		-1 + 3 = +2	+3 - (+2) = +1

Total = -4

$$\text{Coeff A} = \frac{\text{Total Deviations}}{\text{No of Observations}} = \frac{-4}{4} = -1$$

$$\text{Coeff B} = \frac{\text{Deviation on E - Deviation on W}}{2} = \frac{-3 - (+3)}{2} = -3$$

(Reverse the signs on West & South)

$$\text{Coeff C} = \frac{\text{Deviation on N - Deviation on S}}{2} = \frac{-5 - (+1)}{2} = \frac{-6}{2} = -3$$

QUESTION 13.

Residual Deviation = Initial Deviation - Correction

Initial Deviation is given as -3°

Total correction (Deviation on West) = (Coeff A + (Coeff B x sin hdg) + (Coeff C x cos hdg))

= -3 + (+1 x sin 270) + (-4 x cos 270)

But, sin 270 = -1 (hence, changing the sign of Coeff B)

cos 270 = 0

Therefore, correction = -3 + (+1 x -1) + (0)

= -3 + (-1)

= -4

Therefore, Residual Deviation = -3 - (-4)

= -3 +4

= +1° (1E)

Deviation East, Compass Least

Therefore, compass heading = 270°(M) -1°E

= 269°(C)

FIGURE 17

Plotting Chart

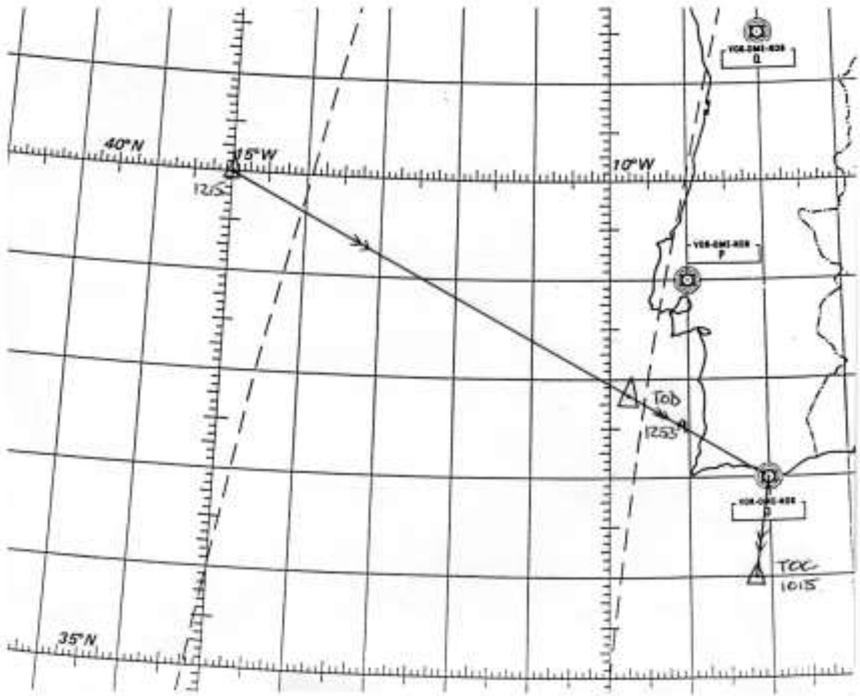


FIGURE 66

Flight Navigational Log

Time	Tk (T)	W/V	Hdg (T)	Varn	Hdg (M)		Observations	M RAS	PA OAT	TAS	G/S	Dist	Time	ETA
1125	Init 153	230/90	166	11W	177	O/H	'W' 52° N 010° W S/H 'S' 43° 25' N 004° 00' W	M0.70	FL330 −60	399	368			
1135				11W		⤢	'W' 350°(RMI) /64 nm 350−11W=339°T Plot 159°T							
1135						⊙	51° 00' N 009° 23' W							
1135	TMG 159		165				Mean W/V 233/43			400	(384	64	10.0)	
1138	153	7P	160	10W	170	A/H	// Tk				390			
1145						⤢	'U' DME 139 nm. Transfer dist 97½ nm				390			
1200				8W		⤢	'T' VOR 096(RMI) 096−8W=088 Plot 268°T							
1200						⊙	47° 56' N 007° 05' W							
1202						A/H	// Tk							
1215				8W		⤢	'T' NDB 049(RMI) 049−8W=041 Plot 221°				380			
1225				7W		⤢	'S' VOR 157(RMI) 157−7W=150 Plot 330°T							
1225						⊙	45° 33' N 005° 44' W							
1231	Mean 148	210/70	157	7W	164	△	45° 00' N 005° 21' W A/H 'S'	M0.70		400	380	111	18.5	1249.5

FIGURE 73

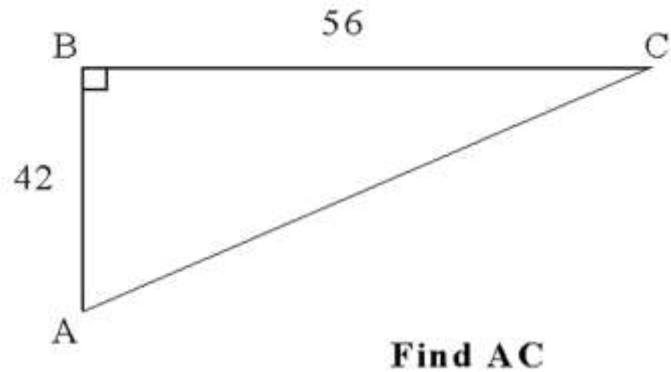


FIGURE 74

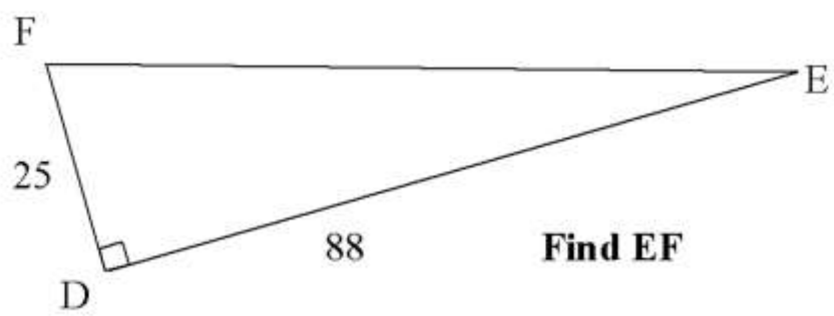


FIGURE 75

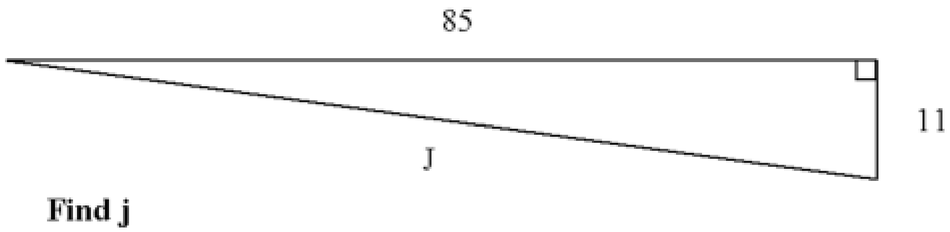


FIGURE 76

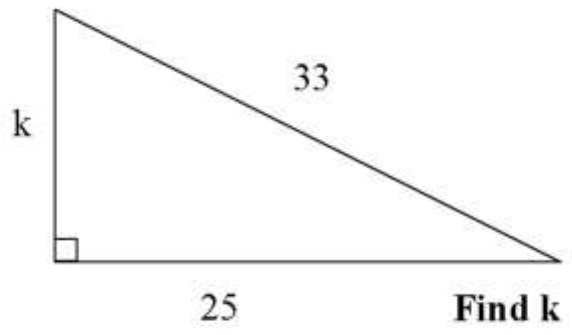


FIGURE 77

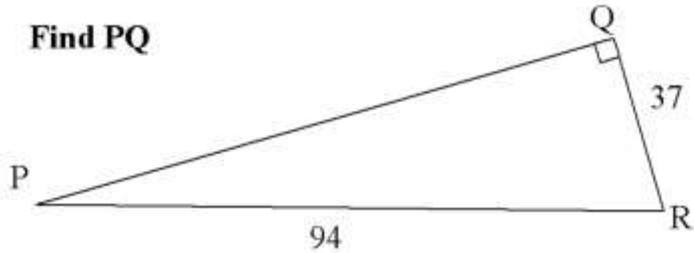


FIGURE 78

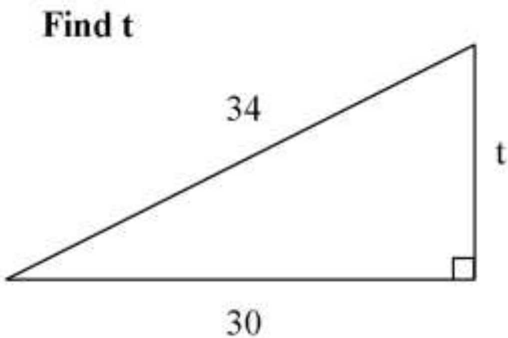
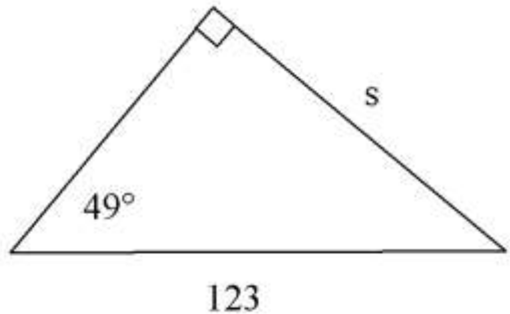
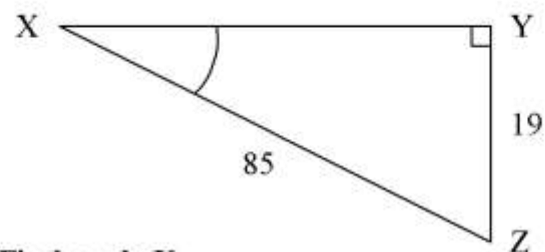


FIGURE 79



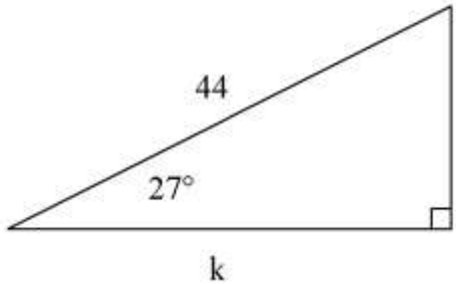
Find s

FIGURE 80



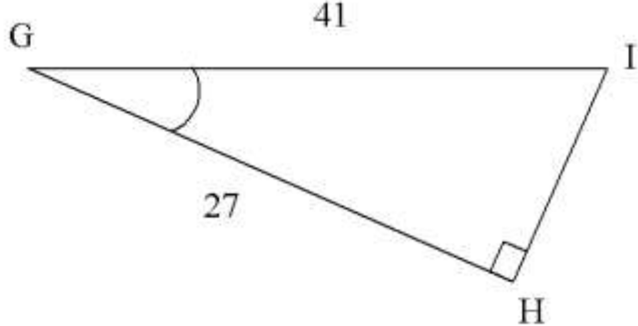
Find angle X

FIGURE 81



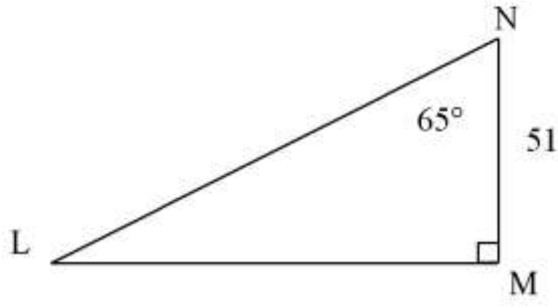
Find k

FIGURE 82



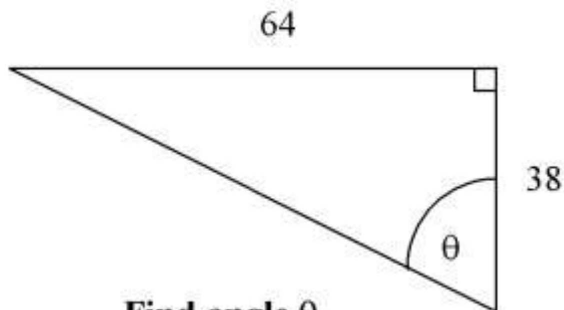
Find angle G

FIGURE 83



Find LM

FIGURE 84



Find angle θ

FIGURE 85

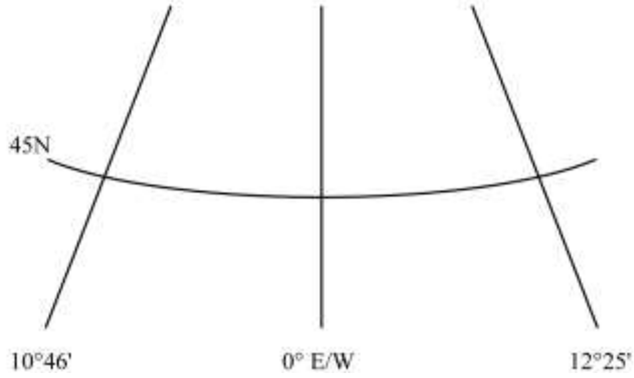


FIGURE 86

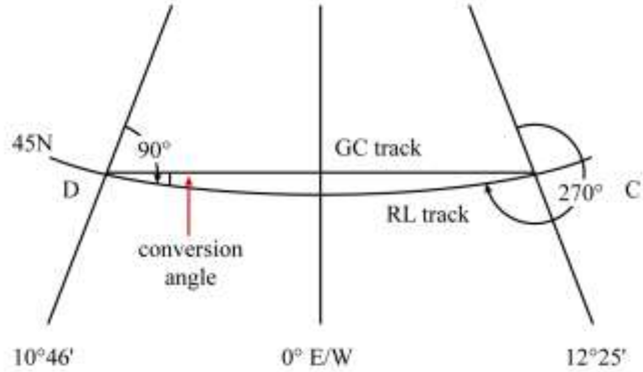


FIGURE 87

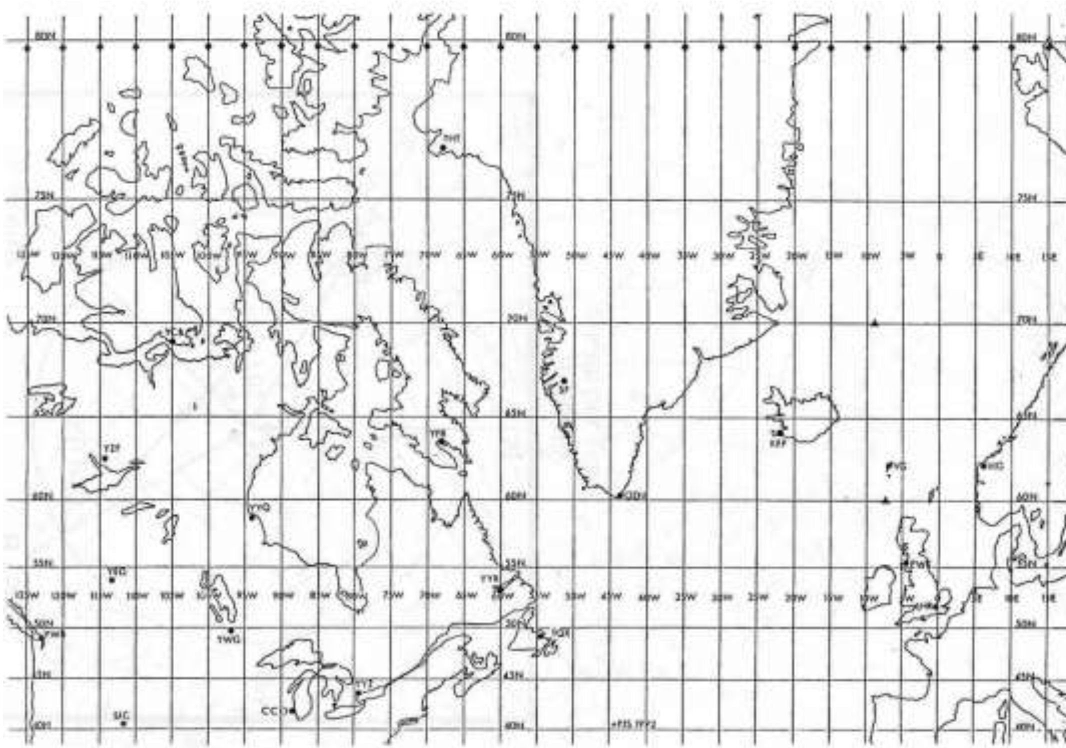


FIGURE 89

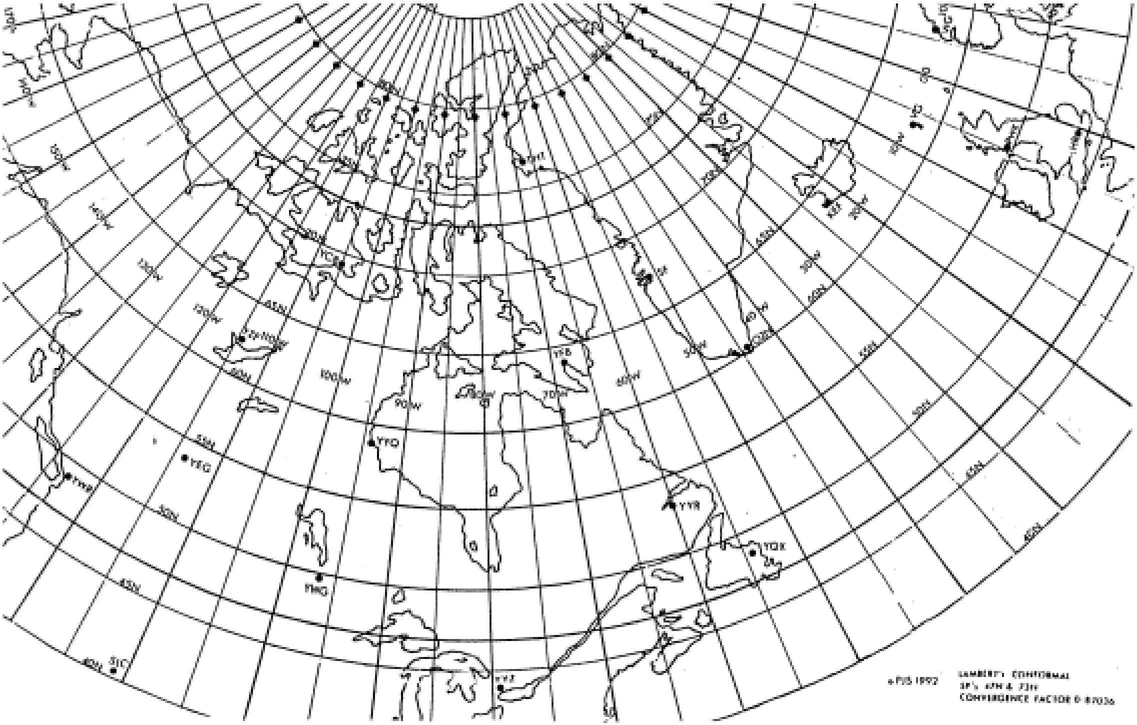

YYQ
YEG
YVR
YWG
YYR
YQX
YYZ
YCB
YFB
YZS
80N
75N
70N
65N
60N
55N
50N
45N
40N
30W
40W
50W
60W
70W
90W
100W
110W
120W
130W
© FIS 1992
LAMBERT'S CONFORMAL

FIGURE 90

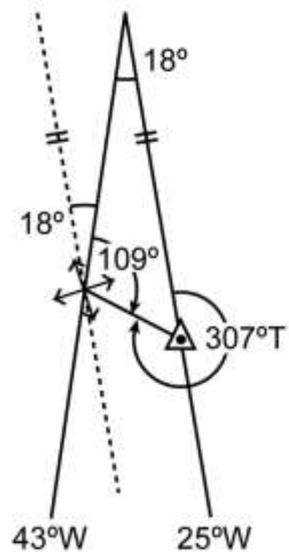
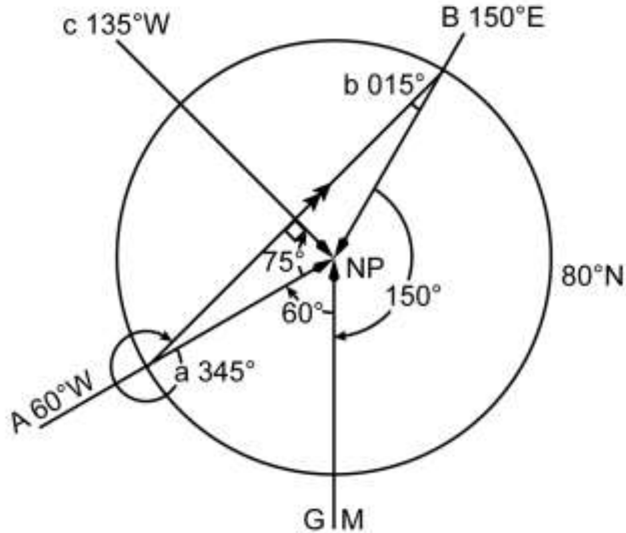


FIGURE 91



a) 345°

b) 015°

c) 135°W

FIGURE 92

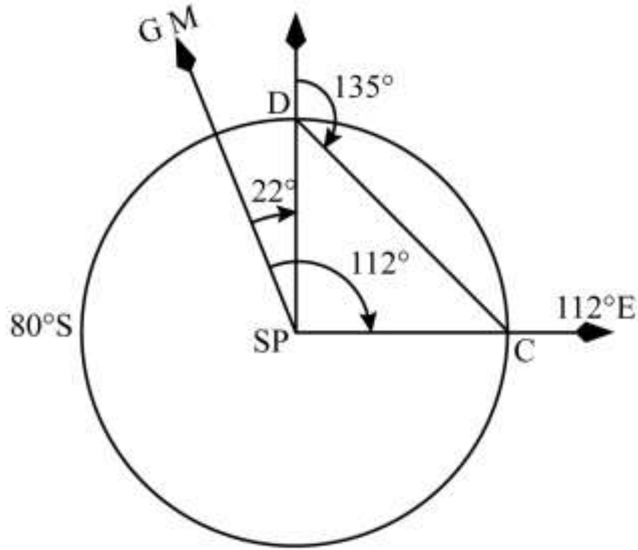


FIGURE 93

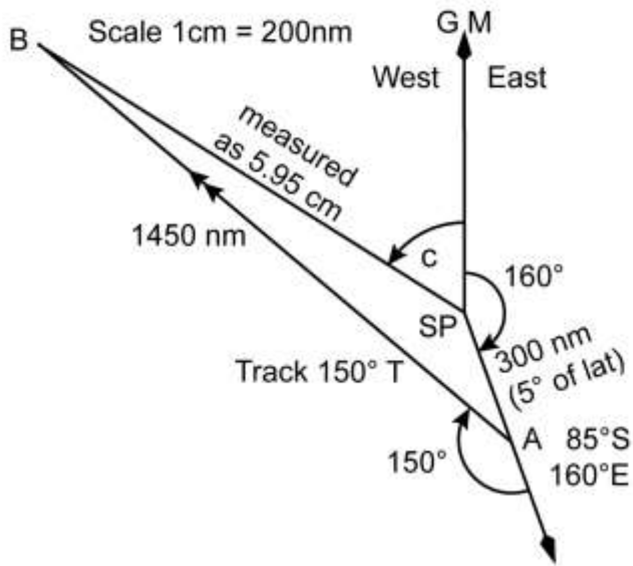

B
Scale 1cm = 200nm
G M
West
East
measured
as 5.95 cm
1450 nm
c
160°
SP
300 nm
(5° of lat)
Track 150° T
A 85°S
160°E
150°

FIGURE 94

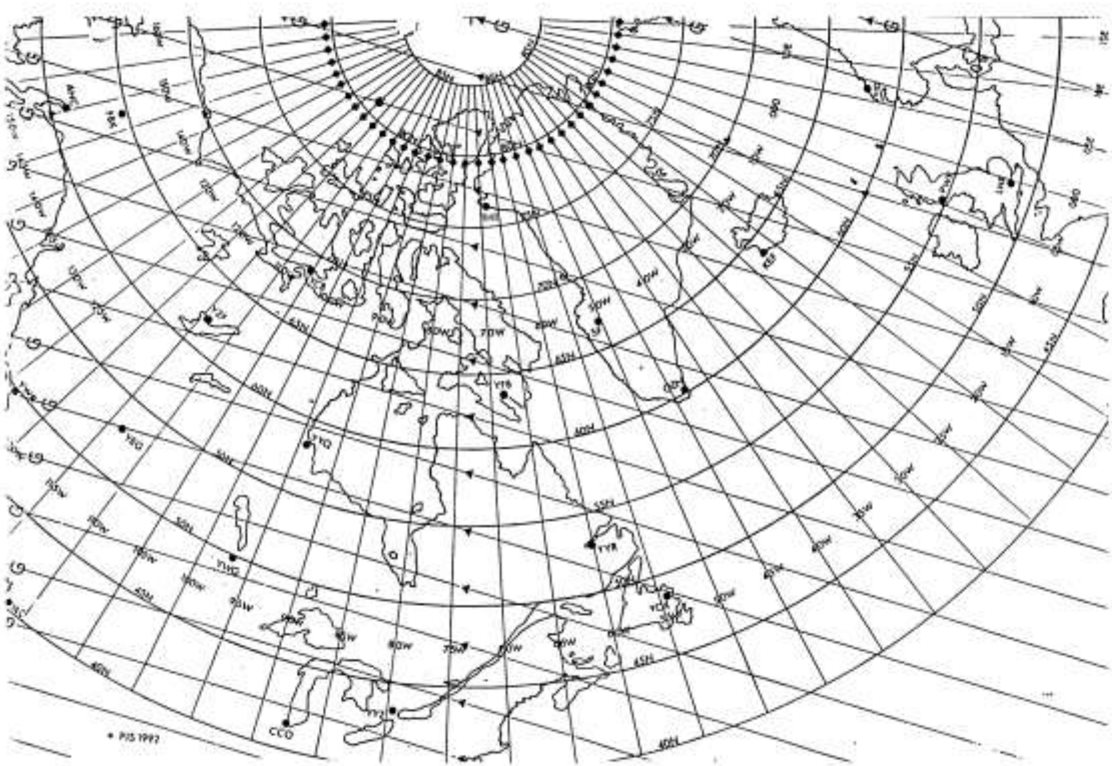


FIGURE 95

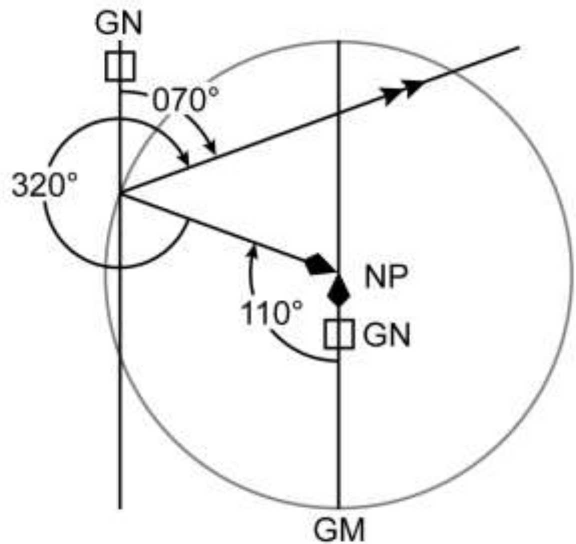


FIGURE 96

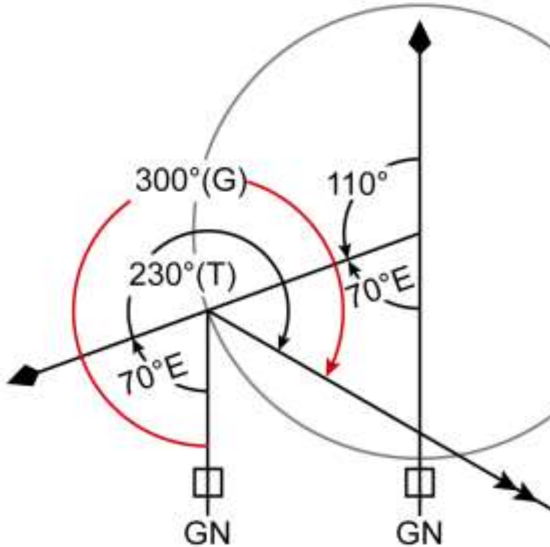


FIGURE 97

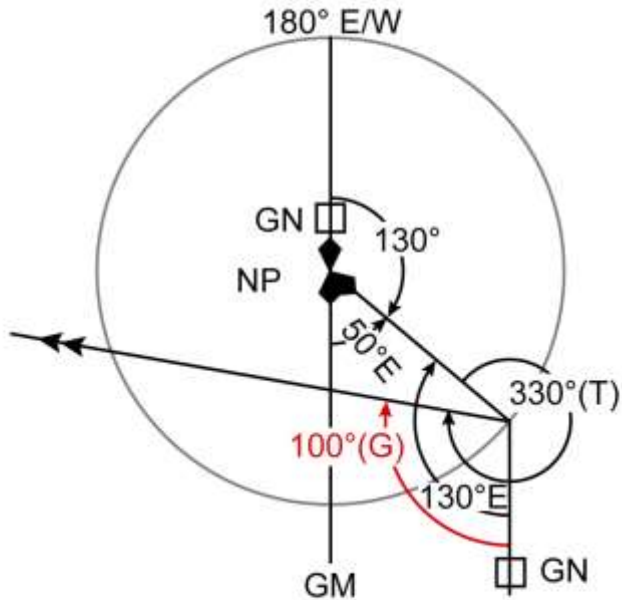


FIGURE 98

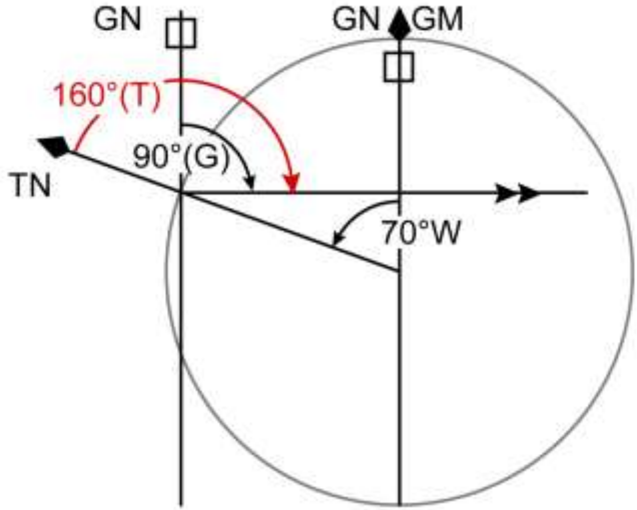


FIGURE 99

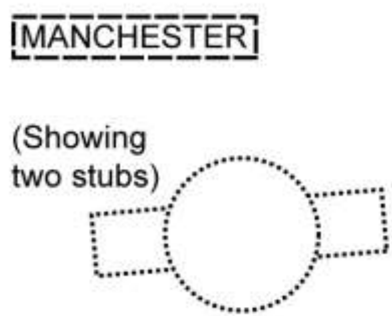


FIGURE 100

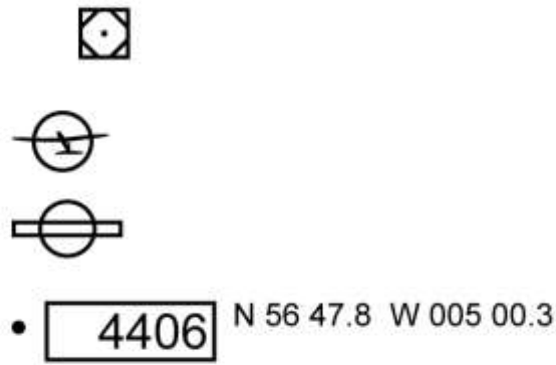


FIGURE 101

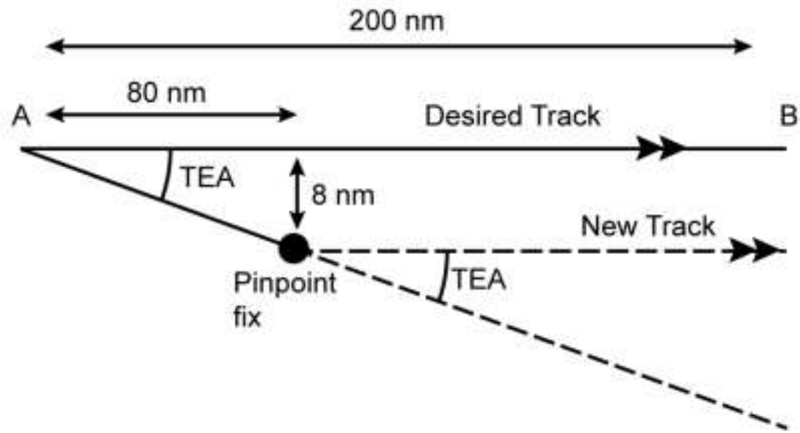

200 nm
80 nm
A
Desired Track
B
TEA
8 nm
New Track
Pinpoint
fix
TEA

FIGURE 102

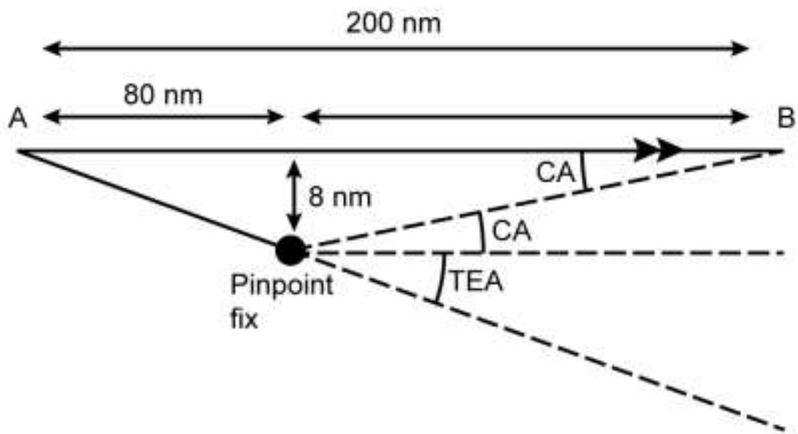


FIGURE 107

Flight Navigational Log

Time	Tk (T)	W/V	Hdg (T)	Varn	Hdg (M)		Observations	M RAS	PA OAT	TAS	G/S	Dist	Time	ETA

FIGURE 108

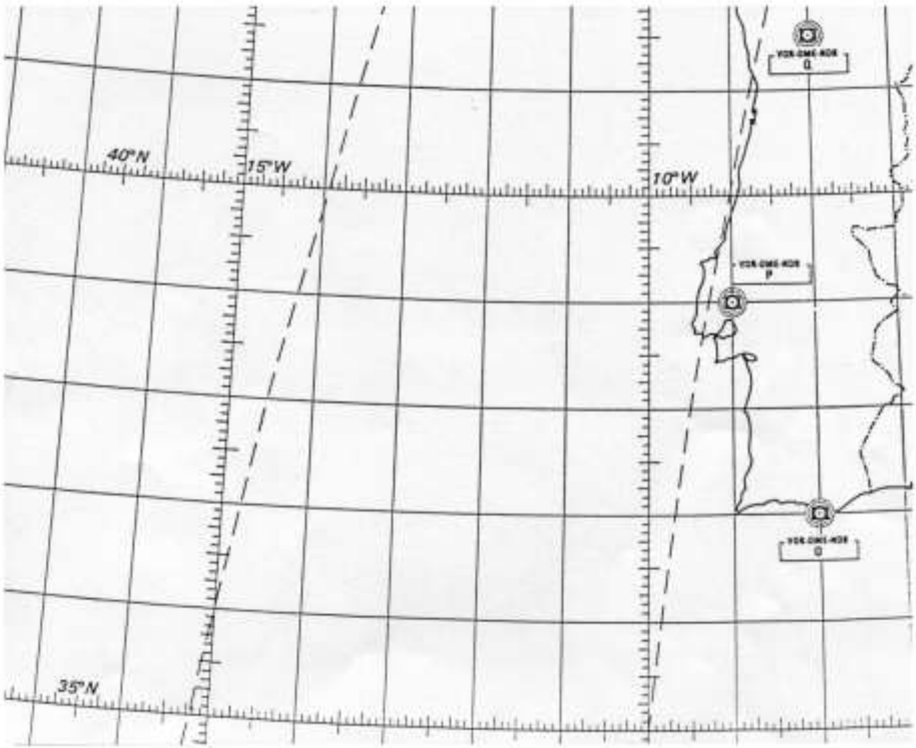


FIGURE 114

Flight Navigational Log

Time	Tk (T)	W/V	Hdg (T)	Varn	Hdg (M)		Observations	M RAS	PA OAT	TAS	G/S	Dist	Time	ETA

FIGURE 116

FIGURE 117

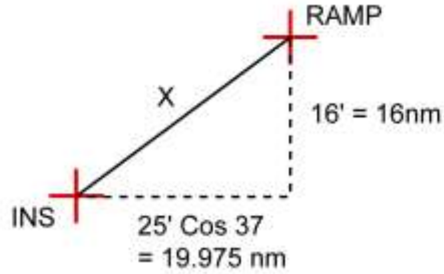


FIGURE 118

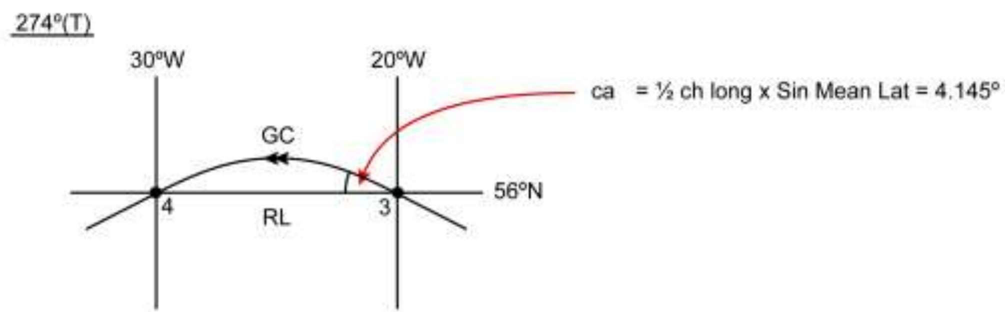


FIGURE 119

8°

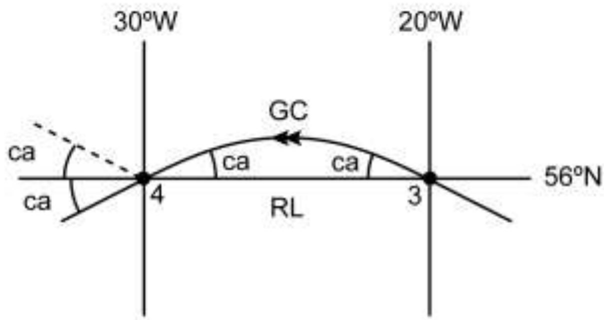


FIGURE 120

5R

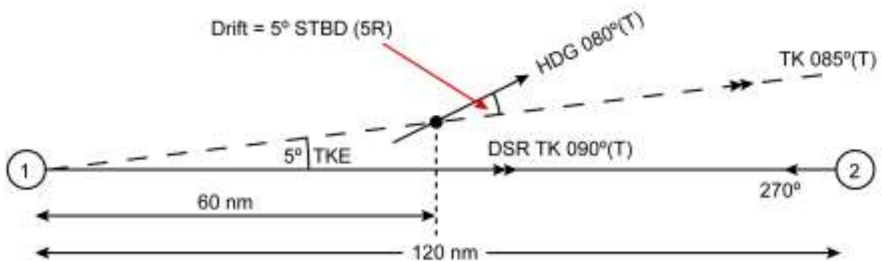


FIGURE 121

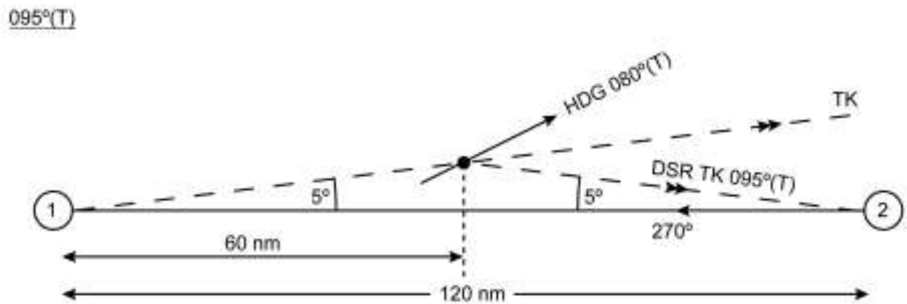


FIGURE 122

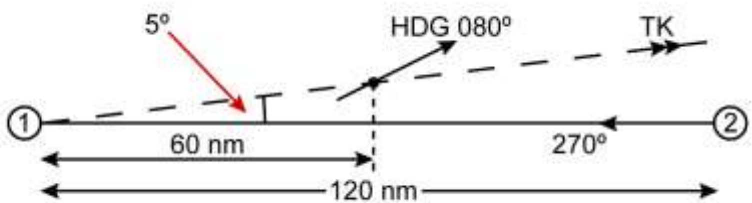


FIGURE 180

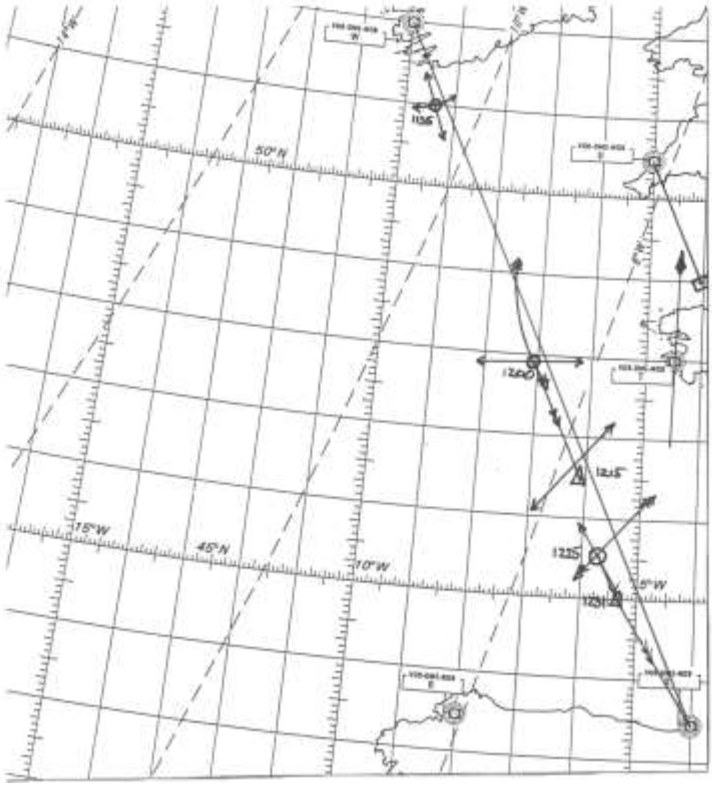


FIGURE 341

Flight Navigational Log

Time	Tk (T)	W/V	Hdg (T)	Varn	Hdg (M)		Observations	M RAS	PA OAT	TAS	G/S	Dist	Time	ETA
							1. The Climb							
1000	188	270/70	203	7W	210	O/H	'O' 37°N 008°W 4000 ft ↑ FL340	200	FL190 -30	264	246	61½	15.0	1015.0
							(climb 30000 @ 2000 ft/min=15 mins)							
							2. The Descent							
1215	Mean 119	210/60	127	9W	136		40°N 015°W S/H 'O'	0.75	FL350 -55	432	430	278 378	38.9	1253.9
1253.9	Final 121	225/50	131	8W	139		TOD A/H 'O' FL350 ↓ FL50	210	FL200 -25	286	293	98	20.0	1313.9
							(descend 30000 @ 1500 ft/min = 20 mins)							