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# A PROGRAMMING SYSTEM ON MAXIMUM ENTROPY SPECTRAL ANALYSIS

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# C O N T E N T S

# A PROGRAMMING SYSTEM ON MAXIMUM ENTROPY SPECTRAL ANALYSIS

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## ABSTRACT

This note describes the data adoptive maximum entropy method of spectral analysis originally due to Burg. Four ways of calculating the spectral estimates are presented. A general programming system has been developed in FORTRAN IV and tested on IBM 360/44. The system can be used to calculate power spectrum using any one of the four methods. The system is provided to determine the optimum Prediction Error Filter (PEF), using either 'FPE' criterion or 'AIC' criterion or 'CAT' criterion. The system is designed to provide exact location of peaks alongwith its amplitude by calculating the spectral density at more number of frequency points in an iterative way, such that the ordinate of the peak is within some small percentage of the neighbouring ordinates, at each of the approximate peaks originally calculated. The system calculates spectral areas if desired. Control card information is provided. The input and output for a sample problem is supplied.

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Key Words:
1) Power Spectrum 2) Maximum Entropy Method 3) Burg's method
4) Levinson Recurrence relations 5) Time Series Analysis
6) Speech Processing

1. *Introduction:-*

*This note describes the maximum entropy method of analysing a time series data. A brief mathematical description of the method is presented in Section 2. A FORTRAN IV routine has been developed to do the mathematical calculations, and has been tested on IBM 360/44. Section 3 contains system details while the procedure for using this programme has been given in Section 4. A sample problem alongwith its input and output are provided in Section 5.*

2. *Mathematical Description:-*

*Maximum entropy method determines the spectral density function P(f) by maximizing the entropy density*

$$E = 1/4\, f_n \int_{-f_n}^{f_n} LOG\ P(f)\, df \qquad ..(1)$$

*Subject to the following M+1 constraints*

$$R_k = \int_{-f_n}^{f_n} P(f)\ EXP\ (i2\pi fkh)\, df \text{ for } k=0 \text{ to } M \qquad ..(2)$$

*where 'h' is the interval between two observations,* $f_n = 1/(2h)$ *is the myquist frequency and* $R_k$ *is the auto correlation of lag K. Solving the equations (1) and (2) (for details of derivation see Smylie et al, 1973) one obtains*

for P(f) the following expression

$$P(f) = \frac{P_{M+1}}{2f_n \left| 1 + \sum_{K=1}^{M} {}^{M}A_K \; EXP\,(-i2\pi fkh) \right|^2} \qquad ..(3)$$

Where ${}^{M}A_K$ for K=1 to M are the prediction error filter (PEF) coefficients and $P_{M+1}$ is the output power of the M+1 point PEF. The coefficients ${}^{M}A_K$ and $P_{M+1}$ satisfy the following matrix equation:

$$\begin{bmatrix} r_0 & r_1 & \cdots & r_M \\ r_1 & r_0 & \cdots & r_M \\ \vdots & \vdots & \cdots & \vdots \\ r_M & r_M & \cdots & r_0 \end{bmatrix} \begin{bmatrix} 1 \\ {}^{M}A_1 \\ \vdots \\ {}^{M}A_M \end{bmatrix} = \begin{bmatrix} P_{M+1} \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \qquad ..(4)$$

The matrix equation (4) can be solved to obtain the values ${}^{M}A_K$ for K = 1 to M and $P_{M+1}$ which in turn can be used to determine the power spectrum from equation (3) at any value of the frequency f. However there are many ways of solving equation (4) and they are described briefly below.

Method 1:-

Estimate $R_K$ the auto correlation corresponding to lag K, for K=0 to M from the given data $X_t$ where t = 1 to N and N denotes the number of data points. Substitute the auto correlations in the matrix equation (4). As equation (4) is

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a set of M+1 equations in M+1 unknowns, the unknown parameters $M^AK$ for K=1 to M and $P_{M+1}$ can be obtained by inverting the auto correlation matrix and premultiplying the RHS vector with the inverse matrix.

Thus this procedure requires (1) the length M of the PEF (lag) to be used (2) an independent estimate of the auto correlation function $R_K$ for K=1 to M (3) if the power spectrum is to be calculated with a PEF of M+1, then $R_K$ has to be calculated for K=M+1 to M+1 and a matrix of size M+1+1 has to be inverted again. When 1 takes a negative value, though additional calculation of $R_K$ is not involved, matrix inversion has to be carried out. Method 2 aims at removing some of these drawbacks.

Method 2:-

The coefficients $M^AK$ for K=1 to M and $P_{M+1}$ in equation (4) can be obtained using Levinson's recursion relations. The recursive formulae express the coefficients of a higher order matrix equation in terms of the coefficients of a matrix of order less than 1. Thus the coefficients $M^AK$ and $P_{M+1}$ contained in the matrix equation of order M+1 can be obtained from the coefficients $M-1^AK$ and $P_M$ in the matrix equation of order M, (for details see Levinson, 1947) where the suffix on the left of $M-1^AK$ indicate the order of the matrix-1, and the suffix in $P_M$ indicate the order of the matrix equation.

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The recursive relations, connecting the coefficients $M^{\Delta}K$ and $P_{M+1}$ with $M-1^{\Delta}K$ and $P_M$ are given by

$$M^{\Delta}K = M-1^{\Delta}K + (M^{\Delta}M)\ (M-1^{\Delta}M-K), \text{ for } K=1 \text{ to } M-1 \quad ..(5)$$

$$P_{M+1} = P_M\ (1- M^{\Delta 2}M) \quad ..(6)$$

From equations (5) and (6) the coefficients $M^{\Delta}K$ for $K=1$ to $M-1$ and $P_{M+1}$ can be obtained provided $M^{\Delta}M$ is known, as the other coefficients $M-1^{\Delta}K$ and $P_M$ are available from matrix equation of order $M$. The subsequent methods differ in calculating this coefficient. The Levinson method (Method 2) determines thiscoefficient using the formula

$$M^{\Delta}M = -\sum_{K=0}^{M-1} (R_{M-K})\ (M-1^{\Delta}K)/P_M \quad ..(7)$$

and as such requires an independent estimate of $R_K$ for $K=0$ to $M$. Method 3 described below aims at removing the requirement of an independent estimate of $R_K$. Initially we have for $M=0$.

$$P_1 = \sum_{t=1}^{N} X_t^2/N \quad ..(8)$$

$$M^{\Delta}0 = 1 \text{ for all } M \quad 0 \quad ..(9)$$

$^{\Delta}1$ and $P_2$ can be obtained from (7) and (6) respectively in that order. Now to obtain the values for $M=2$ to $M$ use the recurrence relations (7), (5) and (6) in that order.

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Method 3:-

This method due to Burg (1967, 1968) uses a recursive procedure of estimating simultaneously $R_K$ for K=0to M alongwith $P_{M+1}$ and $M^A K$ for K=1 to M from the data. The procedure determines the PEF by minimizing the average power obtained by running the filter in both forward and backward directions. The method replaces equation (7) by the following equation

$$M^A M = -2 \sum_{t=1}^{N-M} (M^F t)\,(M^B t) \Big/ \sum_{t=1}^{N-M} (M^{F2} t + M^{B2} t) \qquad ..(10)$$

$M^F t$ and $M^B t$ in equation (10) satisfy the following recurrence relations (For details see Anderson, 1974).

$$M^F t = M\text{-}1^F t + (M\text{-}1^A M\text{-}1)\,(M\text{-}1^B t) \text{ for } t=1 \text{ to } N\text{-}M \qquad ..(11)$$

and

$$M^B t = M\text{-}1^B t+1 + (M\text{-}1^A M\text{-}1)\,(M\text{-}1^F t+1), \text{ for } t=1 \text{ to } N\text{-}M \qquad ..(12)$$

Initially we have for M=0

$$0^F t = X_t, \text{ for } t = 1 \text{ to } N \qquad ..(13)$$

$$0^B t = X_t, \text{ for } t = 1 \text{ to } N \qquad ..(14)$$

$1^F t$, $1^B t$ can be obtained from (11) and (12) with $0^A 0=0$, $1^A 1$ and $P_2$ can be obtained from (10) and (6) respectively in that order. Now to obtain the values for M=2 to M use the recurrence relations (11), (12), (10), (5) and (6) in that order.

Method 4:-

This method known as the geometric mean method (for details see Itakuva et al, 1971) which is generally used in

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speech analysis replaces equation (10) by the following equation

$$M^{A}_{M} = -\sum_{t=1}^{N-M} (M^{F}t)\,(M^{B}t) \Big/ \left(\sum_{t=1}^{N-M} M^{F}{}^{2}t \cdot \sum_{t=1}^{N-M} M^{B2}t\right)^{1/2} \quad ..(15)$$

$M^{A}1$ is obtained and $P_2$ are now obtained from (15) and (6) respectively in that order. Recurrence relations (11), (12), (15), (5) and (6) are used in that order to get the values for $M=2$ to $M$.

The method of derivation remains similar if $R_K$ is replaced by $C_K$ where $C_K$ is the auto covariance function of lag K. The auto correlation $R_K$ introduced above is not defined in the usual statistical way, but defined as the average of the products $X_t$ with $X_{t+K}$ for $t = 1$ to $N-K$. For other information pertaining to (1) determination of M (2) the number of frequency points at which power spectrum to be estimated etc. see section 3 details of the system.

3. Details of the system:-

The system contains a small main program which calls the 'BERCSR' subroutine which contains a number of program control cards described latter. The system can be used to calculate the power spectrum using any one of the 4 methods described above by choosing proper control parameters. The subroutine 'BERCSR' calls in turn 3 subroutines.

*The subroutine 'FNSPEC' calculates in an iterative way the exact location of the peak by doubling each time the number of points at which power spectrum is estimated, till the side power values are within E% of the power value at a peak. This is a special feature of the package. E has to be supplied by the user. The default option for E is 1.0. This procedure is carried out for each suspected peak. However, this procedure can be suppressed by a control parameter.*

*The subroutine 'SITCSR' calculates the spectral areas using Simpson's integration procedure which is another special feature of the system. The spectral areas are more meaningful for presenting the results and the total area under the spectral curve can be used as one of the criteria for testing whether any frequencies are missed or to test to a certain extent whether the series is stationary or non stationary. The procedure for calculating spectral areas takes considerable computer time and should be used with caution. This procedure again can be suppressed by a control parameter.*

*The subroutine 'PLOTCS' graphs the power spectral density against frequency (or periodicity). A control parameter controls whether to plot spectral density or log spectral density. Plotting can be suppressed by a control parameter.*

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The length 'M' of the PEF can be supplied by the user. The system provides an optimum 'M' on request. Using this M it calculates the power spectrum which is another special feature of the package. For determination of optimum 'M' the user can opt for one of the following 3 ways:

1) Final prediction error (FPE) creteria by Akaike (1969, 1970)

The FPE for a filter of order M+1 is given by

$$FPE\ (M+1) = (N+M+1)\ P_{M+1} / (N-M-1)$$

2) Information Theoretic Criterion (AIC) by Akaike (1974)

The AIC for a filter of order M+1 is given by

$$AIC\ (M+1) = LOG\ (P_{M+1}) + 2M/N$$

3) Auto regressive transfer function criterion (CAT) by Parzen (1976). The CAT for a filter of order M+1 is given by

$$CAT\ (M+1) = 1/N \sum_{K=1}^{M} \frac{N-K}{N\ P_{K+1}} - \frac{N-M}{N\ P_{M+1}}$$

The program recolours the spectrum if trend is removed from the original data using a moving average procedure.

The main program simply calls the subroutine 'BERCSR' and the whole programming logic including input/output and control card information is contained in this subroutine. The main program is just used to give an object dimension to the program.

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4. Control Card Information:-

The program calculates the MEM Power Spectrum for a given time series data. The computations are done in double precision.

Input:

The input to the program contains a maximum of 4 control cards.

Control card 1:-

The control card reads a total of 24 parameters in 14I4, 9I1, 5X, F10.4 format. By default these parameters assume zero. The parameter values are to be punched right justified.

Parameter 1: -

This parameter specifies the number of data points. The maximum number being 1000.

Parameter 2:-

This parameter reads the starting prediction error filter (PEF) from which calculation of power spectrum will be done.

Parameter 3:-

This parameter supplies the input unit from which data is read. By default card reader, that is 5.

Parameter 4:-

This is a decisive parameter and reads

either the value 0 or 1 or 2. If the parameter value is 1 standardization of the data will be done. If the value taken by this parameter is 2 then only mean of the series is subtracted from each observation.

Parameter 5:-

This parameter specifies the number of times the calculation of power spectrum is to be repeated using different lags (PEF's), If the value taken by this parameter is 1, then control card 3 will be present.

Parameter 6:-

This parameter controls the plotting of the power spectrum. If the value is 0 graph will be plotted by calling the subroutine 'PLOTCS'.

Parameter 7:-

This parameter specifies the number of frequency points at which spectrum is to be calculated.

Parameter 8:-

This parameter determines whether to plot log of the spectral density or spectral density. If the value of the parameter is 1 the former will be executed.

Parameter 9:-

This parameter specifies the order of the moving average used for removing trend from a given time series data so that

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the power spectrum is recoloured in order to get the true power spectrum.

Parameter 10:-

This parameter specifies the number of frequency points at which spectral density is to be calculated.

Parameter 11:-

This parameter specifies the increment to be given to the number of frequency points specified by parameter 10. The power spectrum is recalculated at the new number of frequency points.

Parameter 12:-

This parameter gives the maximum number of frequency points using which power spectrum is calculated. In other words this parameter indicates when to stop incrementing the value of parameter 10 by value of parameter 11.

Parameter 13:-

This parameter specifies the increment to be given to the lag (PEF) specified by parameter 2 using which (new PEF) power spectrum is calculated with the original values of the parameters 10, 11 and 12.

Parameter 14:-

This parameter supplies the maximum lag (PEF) exceeding which calculation of power spectrum will be ceased. In other terms an increment specified by parameter 13 will be given to the lag (PEF) till the updated lag (PEF) does not exceed this parameter value.

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Parameter 15:-

This is a decisive parameter. If the parameter takes the value 0, lag (PEF) will be determined automatically using FPE criterion by the program. In this case it is obvious that the value of the 2nd parameter is dummy. If the value of this parameter is 1 lag (PEF) will not be determined by the program.

Parameter 16:-

This is again a decisive parameter, and if the parameter takes the value 0, exact location of the peak is determined by the program calling the subroutine (FNSPEC). If the value is 1 it is not determined.

Parameter 17:-

This is another decisive parameter. If it takes a value 1 spectral areas are calculated by the program calling 'SITCSR'. If the parameter takes the value 0 it does not calculate. As the calculation of spectral areas takes longer computer time, this parameter should be used sparingly.

Parameter 18:-

This parameter is used whether to print the original data or not. If the value of this parameter is 1 the program prints the data.

Parameter 19:-

This is a decisive parameter and takes either 0 or 1 or 2. If the supplied value is 1 then lag (PEF)will be determined by the program using 'AIC' criterion. If the supplied value is 2 lag (PEF) will be determined by the program using 'CAT' criterion. If value is 0 it will not be determined.

Parameter 20:-

This is again a decisive parameter and takes either 0 or 1. If the given value is 1 then output corresponding to intermcdiate stages will be printed (for example $_{M}A_{K}$, $P_{M+1}$ etc. otherwise not.

Parameter 21:-

This is another decisive parameter and takes either 0 or 1 or 2 or 3. If the value taken by this parameter is 0 then Burg's procedure is used for cstimating the power spectrum. If the value taken is 1, then Levinson's recursive procedure is used, if the value taken is 2 geometric method is used and if the value taken is 3 then direct method that is method 1 is use

Parameter 22:-

This is a decisive parameter and takes either 0 or 1. I the value taken by this parameter is 0 then unbiased auto-covariance or auto correlation is used in methods 1 and 2, on the other hand if the supplied value is 1 then biased but consistent estimate is used.

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Parameter 23:-

This is another decisive parameter and takes either 0 or 1 or 2. If the value taken by this parameter is 0 then auto correlation $R_K$ is calculated and used in the sense described in section 2, if the value taken is 1 then auto covariance is used instead of $R_K$ and if the supplied value is 2 then auto correlation $R_K$ is calculated in the usual statistical way and used in methods 1 and 2.

Parameter 24:-

This parameter specifies the E value required in 'FNSPEC' subroutine. By default this takes a value of 1.0.

Control Card 2:-

This control card reads the object time format in which data is punched.

Control Card 3:-

This control card will be present if the value taken by parameter 5 of control card No.1 is greater than 1. This control card reads the lags (PEF'S) for which power spectrum to be calculated in 2014 format using each lag (PEF) power spectrum will be calculated.

Control Card 4:-

This card is meant for giving a name to the data set that is going to be processed.

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5. Sample Problem:-

Calculate the MEM Power Spectra using Burg's recursive relation to the following data having the following requirements. The data is read from cards, the number of data points being 54. Standardize the data before processing calculate the spectral density at 20 frequency points. Determine the optimum PEF using FPE criterion. Determine the exact peaks. Suppose data is punched in 26 F3.0. The title of the problem is 'DATA KENDALL, V-3 MARRIAGE RATE IN ENGLAND 1843-1896.

Data:-

-6, 1, 12, 10, -6, -8, -6, 3, 4, 7, 11, 3, -8, -2, -3, -7, 3, 4, -5, -7, 1, 6, 8, 9, -2, -8, -10, -7, 0, 8, 12, 7, 5, 4, -3, -6, -12, -5, 0, 5, 7, 3, -4, -8, -6, -5, 1, 6, 6, 2, -6, -5, -6, 1.

Input:-

Control card :-

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 2 | 2 | Column number |
| 34 | 6 | 4 | 7 | 8 | |
| 54 | 1 | 1 | 2 | 0 | Information punched |

Control card 2:-

1 . . . . . Column number

(26F3.0) Information punched

Control card 4:-

1 . . . . .

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DATA KENDALL, V-3 MARRIAGE RATE IN ENGLAND 1843-1846.

After these control cards the data is provided which is punched in 26F3.0.

Output:-

Spectral Density:-

0.2486, 0.2733, 0.3693, 0.6558, 1.8421, 10.0952, 3.2924, 0.9949, 0.5357, 0.3958, 0.3601, 0.3722, 0.3833, 0.3254, 0.2186, 0.1359, 0.0886, 0.0634, 0.0501, 0.0436

Exact Peaks:

1, periodicity 7.71 years; spectral density 11.212

2, periodicity 3.33 years; spectral density 0.3833

5 A. How to execute the program ?

JOB card

// EXEC SPBERCSR

Control Card 1

Control card 2

Control card 4

Data

/*

/*

/&

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