CONCERTO

UNIVERSITY OF CAMBRIDGE
The Psychometrics Centre

Introduction to Item Response Theory

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Goals

- Build your own basic tests using Concerto
- General understanding of CTT, IRT and CAT concepts
  - No equations!

Understanding the Latent Variable

What is a construct?

- Personality scales?
  - Openness, Conscientiousness, Extraversion, Agreeableness, Neuroticism
- Intelligence scales?
  - Numerical Reasoning, Digit Span
- Ability scales?
  - Find egs. Of ability

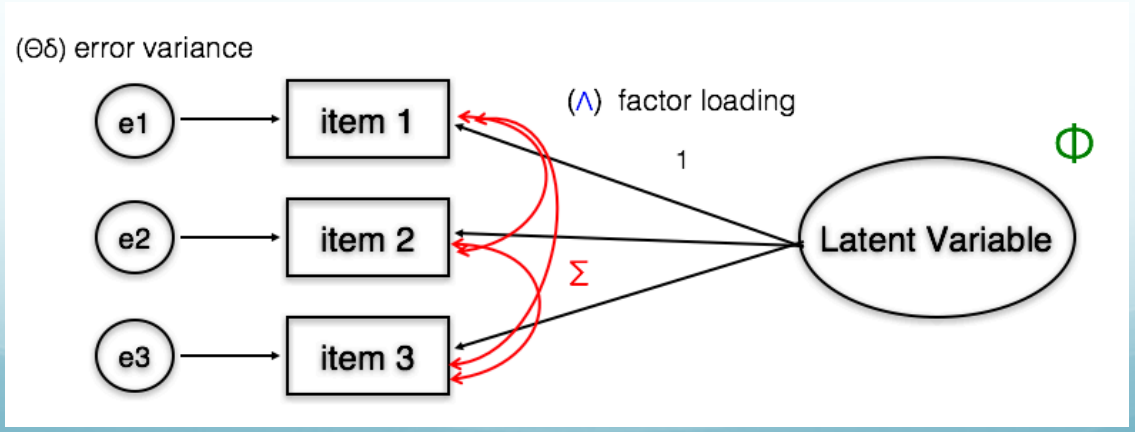
What is a construct?

- Measures and items are created in order to measure/tap a construct
- A construct is an underlying phenomenon within a scale - referred to as the **latent variable (LV)**
  - *Latent:* not directly observable
  - *Variables:* aspects of it such as strength or magnitude, change
  - Magnitude of the LV measured by a scale at the time and place of measurement is the **true score**

Latent variable as the cause of item values

- LV is regarded as a *cause* of the item score
  - i.e. Strength of LV (its true score) causes items to take on a certain score.

- Cannot directly assess the true score
  - Therefore look at correlations between the items measuring the same construct
  - Invoke the LV as cause of these correlations
  - Infer how strongly each item correlated with LV



Classical Measurement Assumptions

$$X = T + e$$

- *X = observed score*
- *T = true score*
- *e = error*

Classical Measurement Assumptions

1. Items' means unaffected by error if have a large number of respondents
2. One item's error is *not* correlated with another item's error
3. Error terms are *not* correlated with the true score of the latent variable

Classical Test Theory

- **Observed Test Score = True Score + random error**
- Item difficulty and discrimination
- Reliability
- Limitations:
  - Single reliability value for the entire test and all participants
  - Scores are item dependent
  - Item stats are sample dependent
  - Bias towards average difficulty in test construction

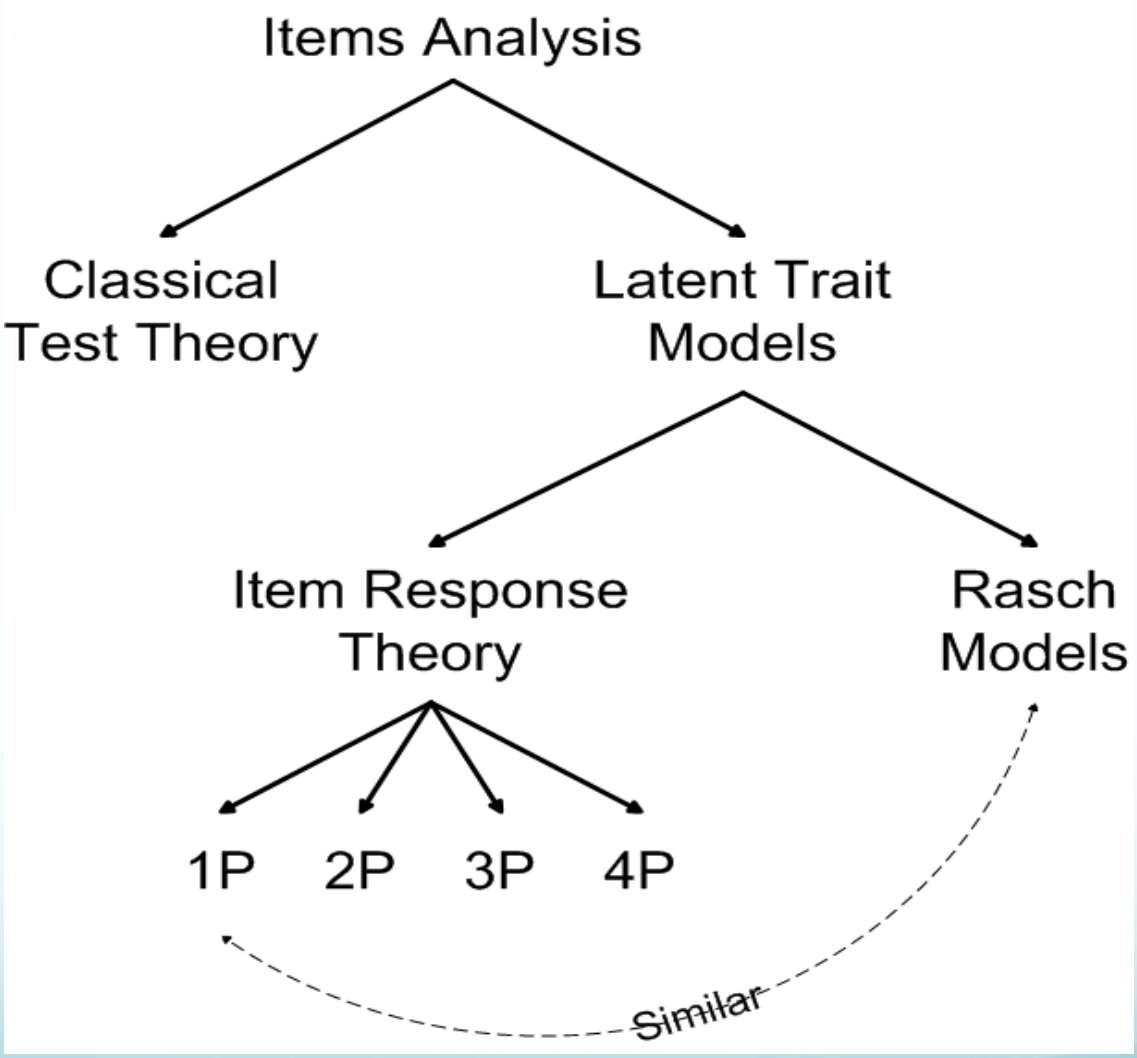
Classical Test Theory vs. Latent Trait Models

- Test level is the basis for CTT (not the item).
- While the statistics outcome are often extended to similar students taking a similar test; they only really apply to *those* students taking *that* test (norm based)
- Latent trait models go further and investigate the underlying traits which are producing the test performance.

Introduction to IRT

Some materials and examples come from the ESRC RDI in Applied Psychometrics run by:

Anna Brown (University of Cambridge)
Jan Böhnke (University of Trier)
Tim Croudace (University of Cambridge)

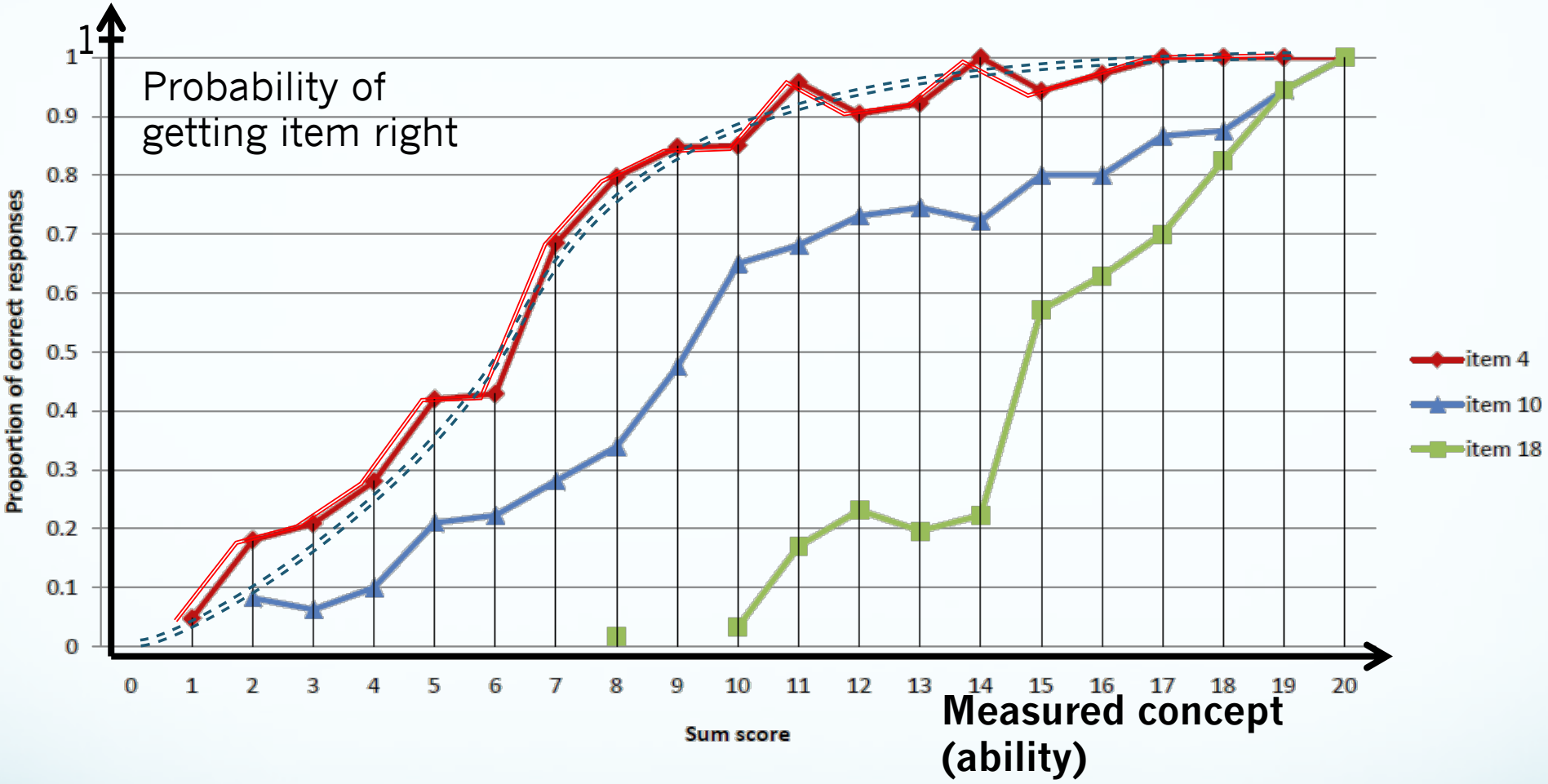

Items Analysis
Classical Test Theory
Latent Trait Models
Item Response Theory
Rasch Models
1P
2P
3P
4P
Similar

Item Response Theory

- Item Response Theory (IRT) – refers to a family of latent trait models.
- They are used to establish psychometric properties of items and scales
- IRT has many advantages over CTT that have brought IRT into more frequent use

3 Basics Components of IRT

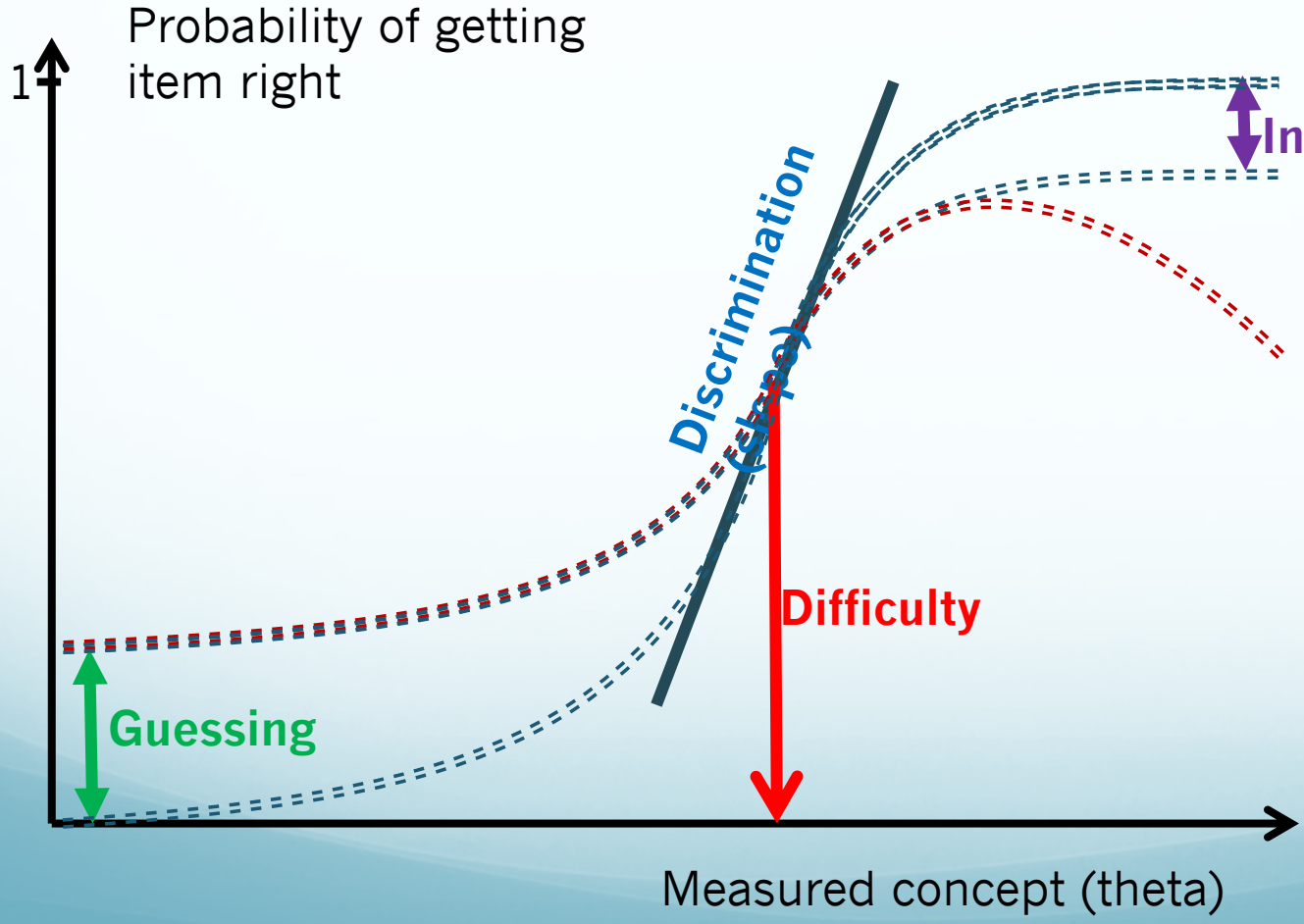
- Item Response Function (IRF) – Mathematical function that relates the latent trait to the probability of endorsing an item
- Item Information Function – an indication of item quality; an item's ability to differentiate among respondents
- Invariance – position on the latent trait can be estimated by any items with known IRFs. The ICCs are sample independent (If you spilt by groups, the item parameters should be the same).



Ratio of correct responses to items on different level of total score

Item Response Function

Binary items

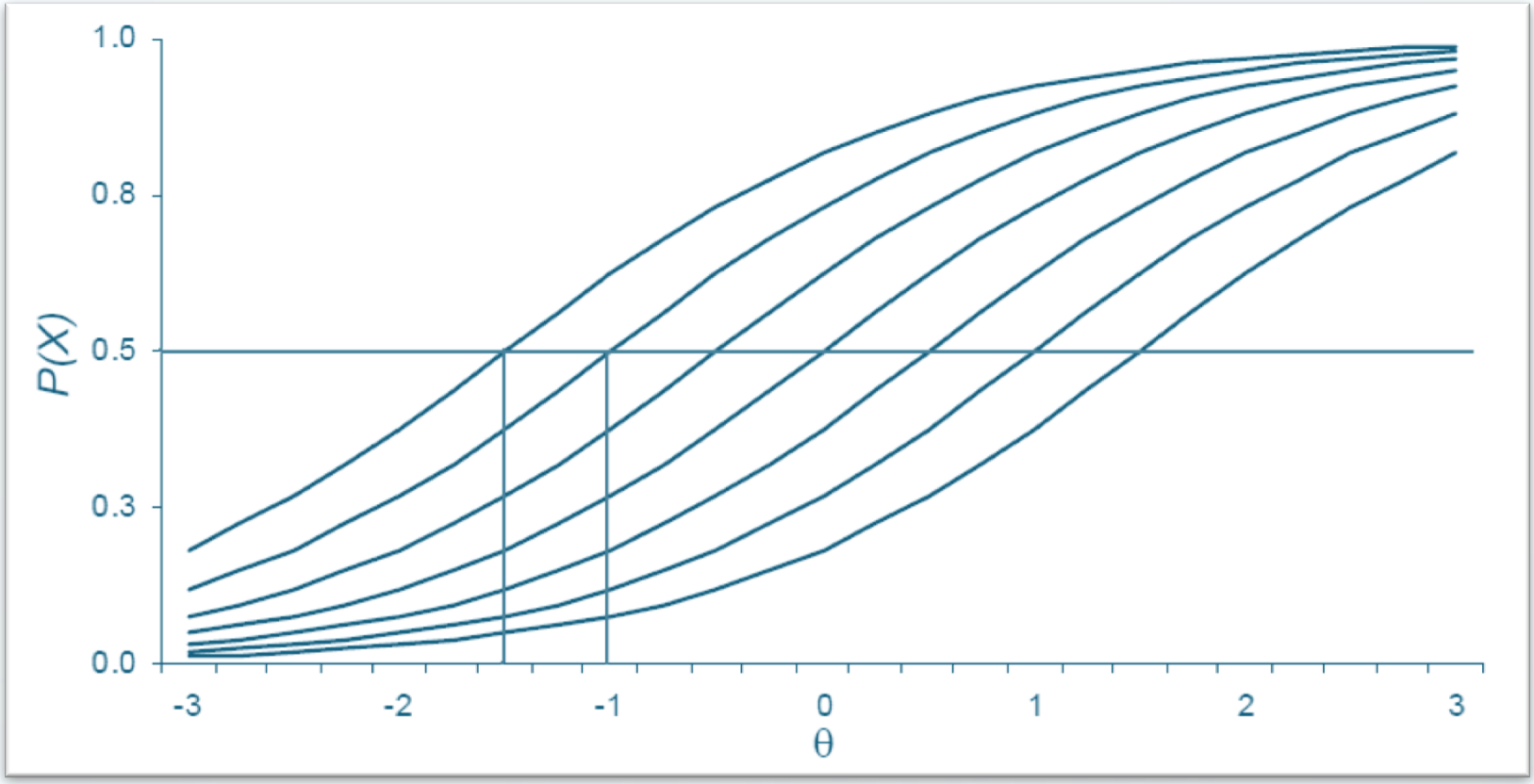


Parameters:

- **Difficulty**
- **Discrimination**
- **Guessing**
- **Inattention**

Models:

- **1 Parameter**
- **2 Parameter**
- **3 Parameter**
- **4 Parameter**
- **unfolding**



7 items of varying difficulty (b)

One-Parameter Logistic Model/Rasch Model (1PL)

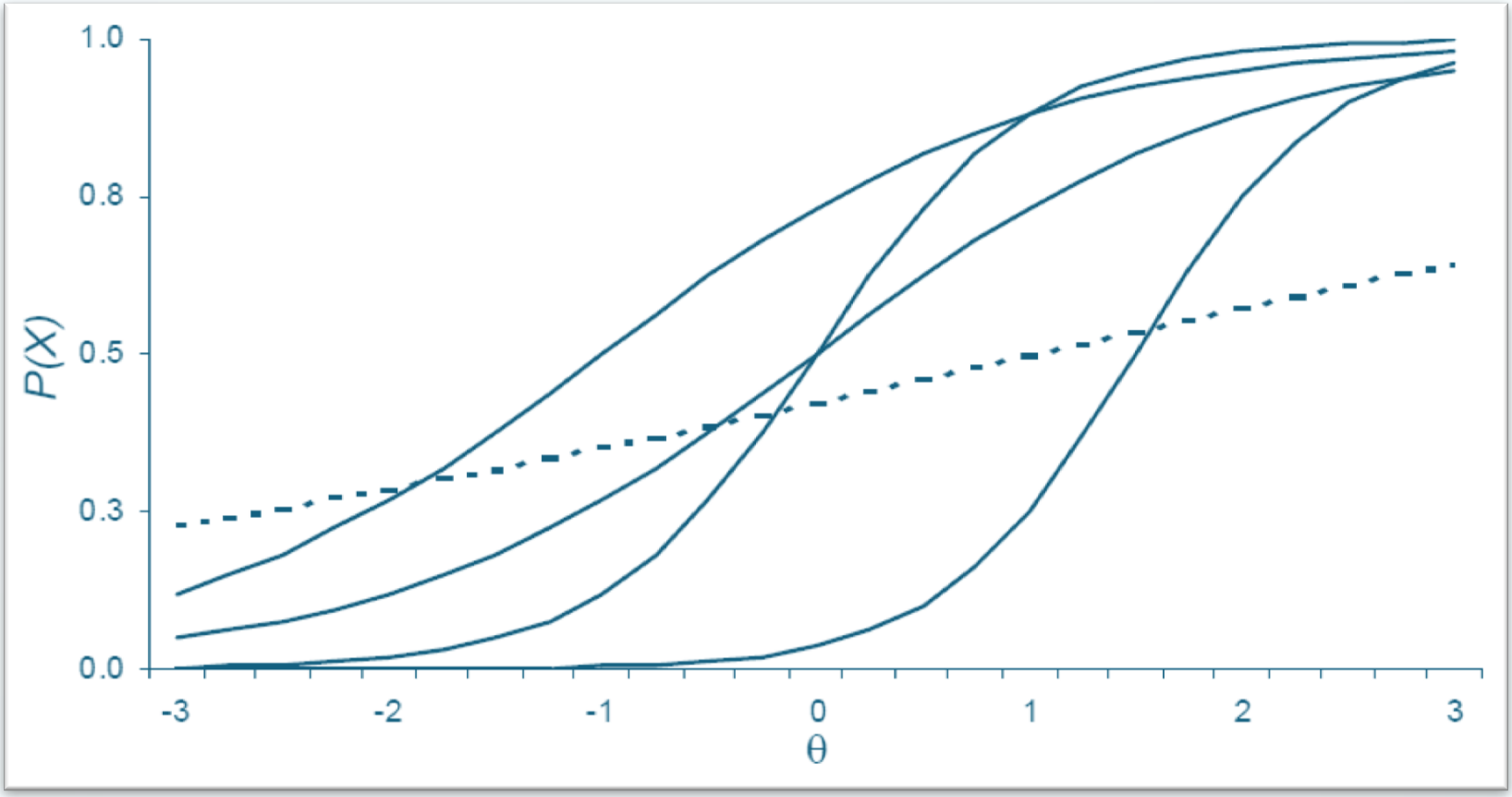
Rasch vs IRT philosophy
(roughly)

- Rasch –Model primacy. Data should fit the model

- IRT – Data primacy. Model should fit the data

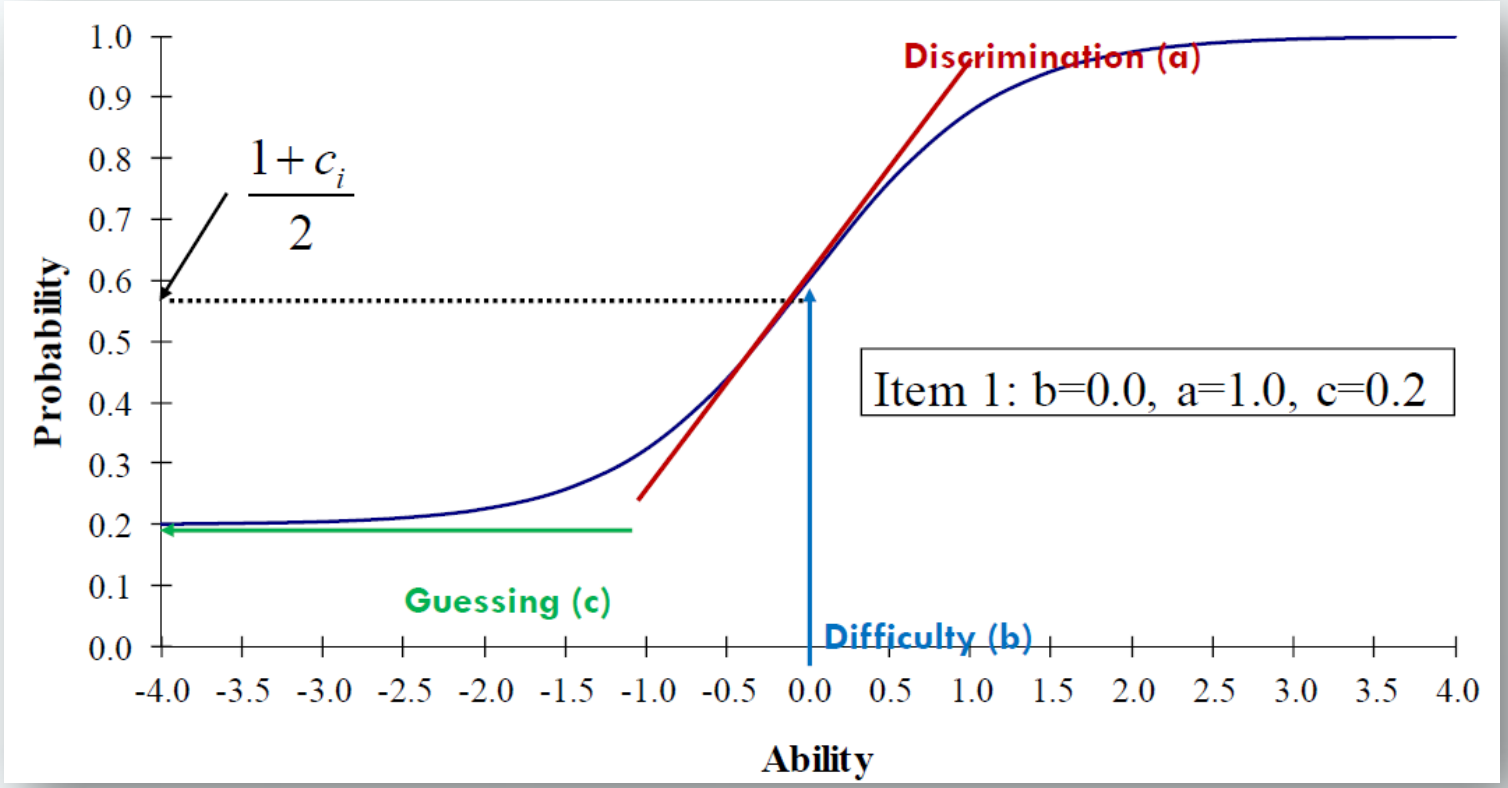
Different fields, constructs, and purposes demand different approaches.

- The jury is out as to whether this has any impact on measurement in the *real world*.



5 items of varying difficulty (b) and discrimination (a)

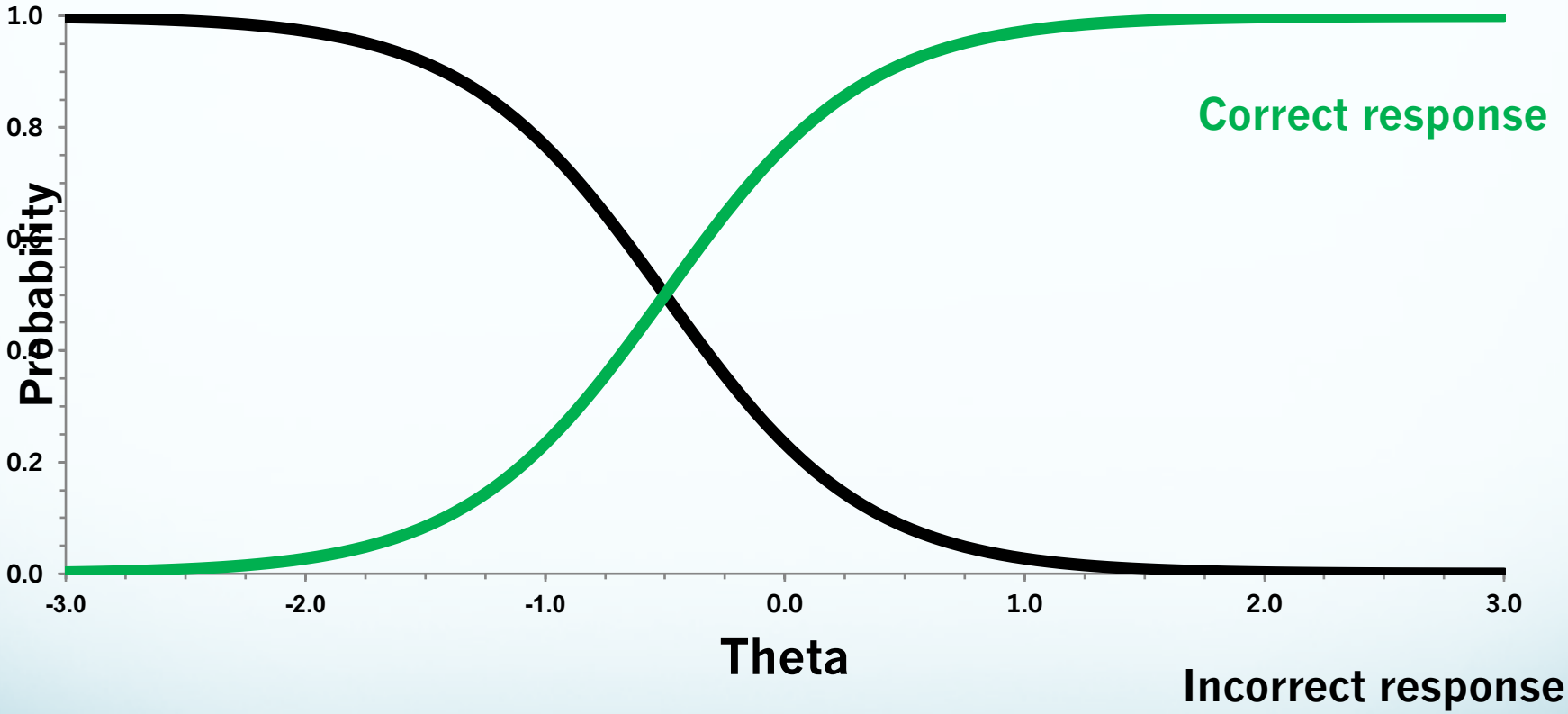
Two-Parameter Logistic Model (2PL)



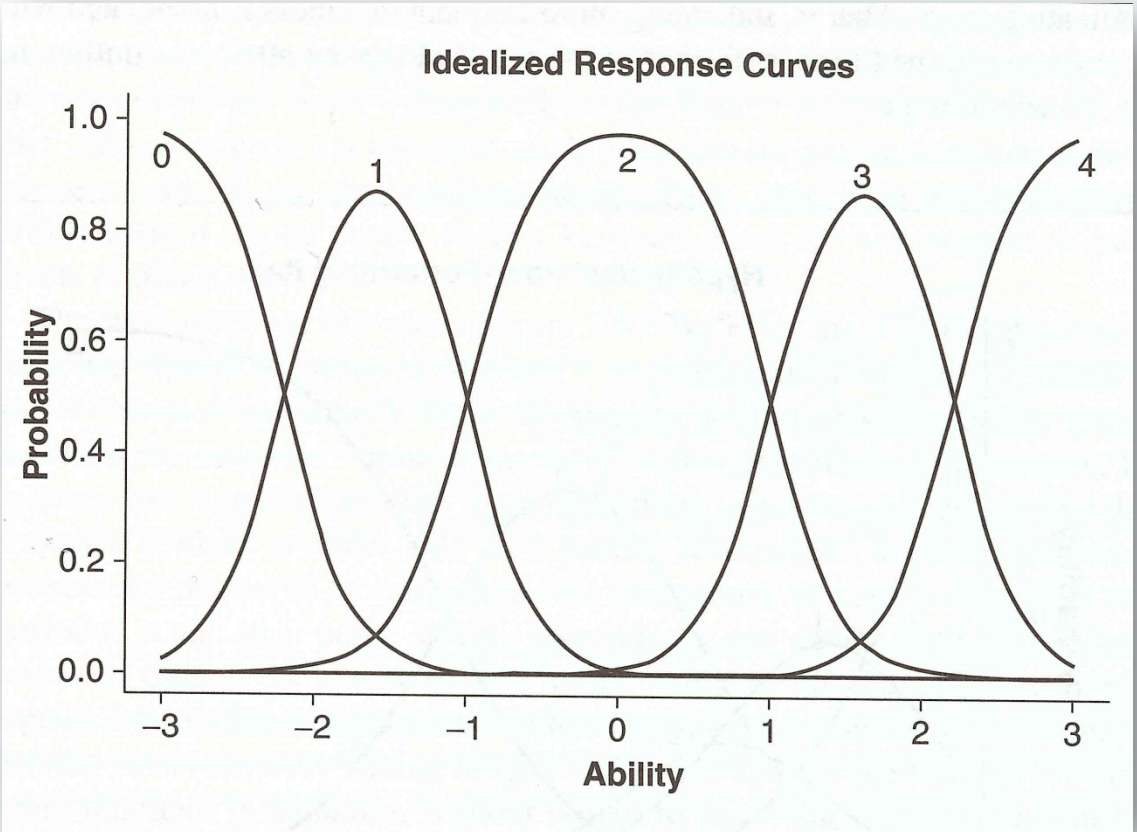
One item showing the guessing parameter (c)

Three-Parameter Model (3PL)

Probability of Correct + Probability of Incorrect = 1



Option Response Function



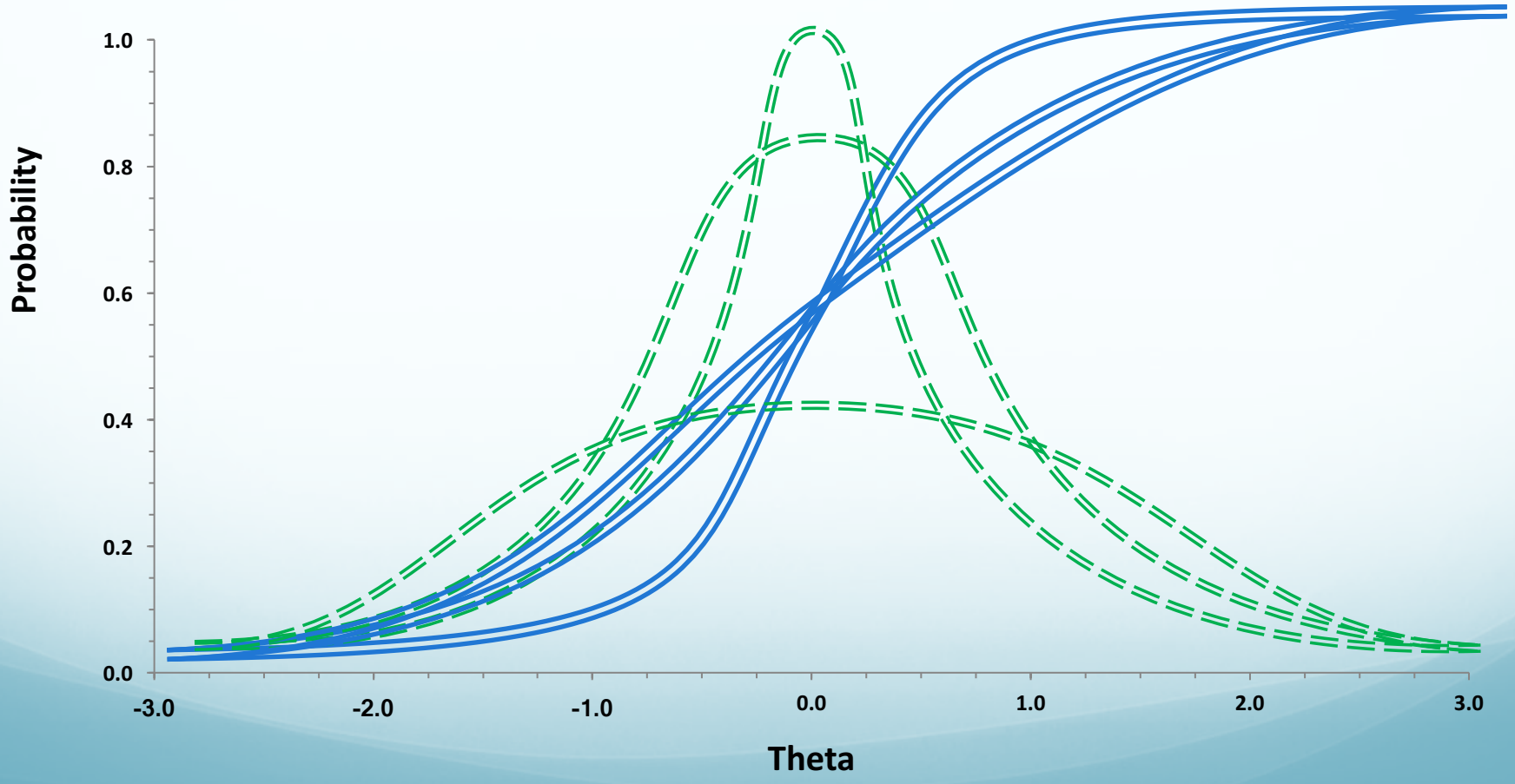
"I experience dizziness when I first wake up in the morning"
(0) "never"
(1) "rarely"
(2) "some of the time"
(3) "most of the time"
(4) "almost always"

Category Response Curves for an item representing the probability of responding in a particular category conditional on trait level

Graded Model

(example of a model with polytomous items – e.g. Likert Scales)

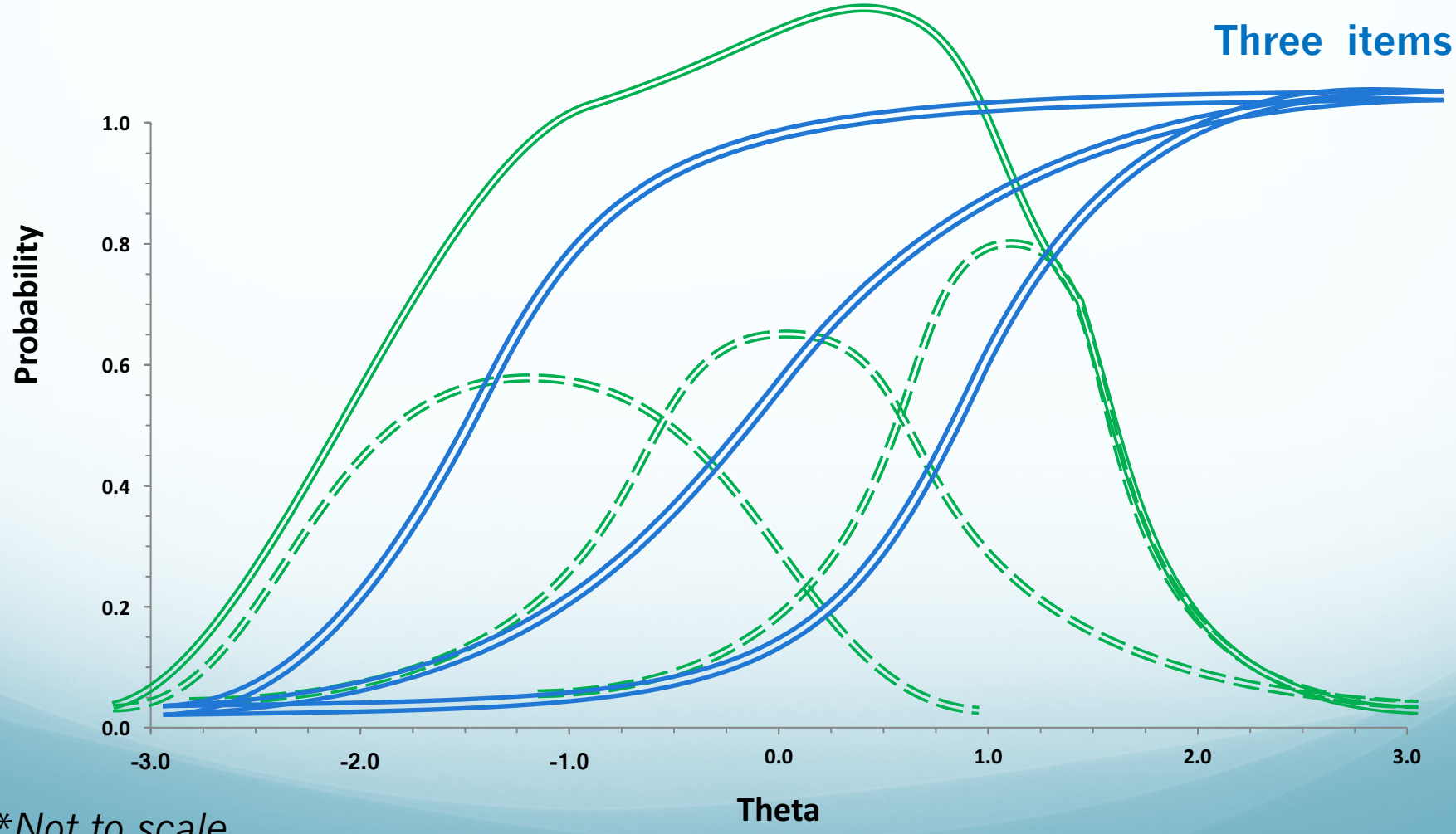
Fisher Information Function



Test Information Function

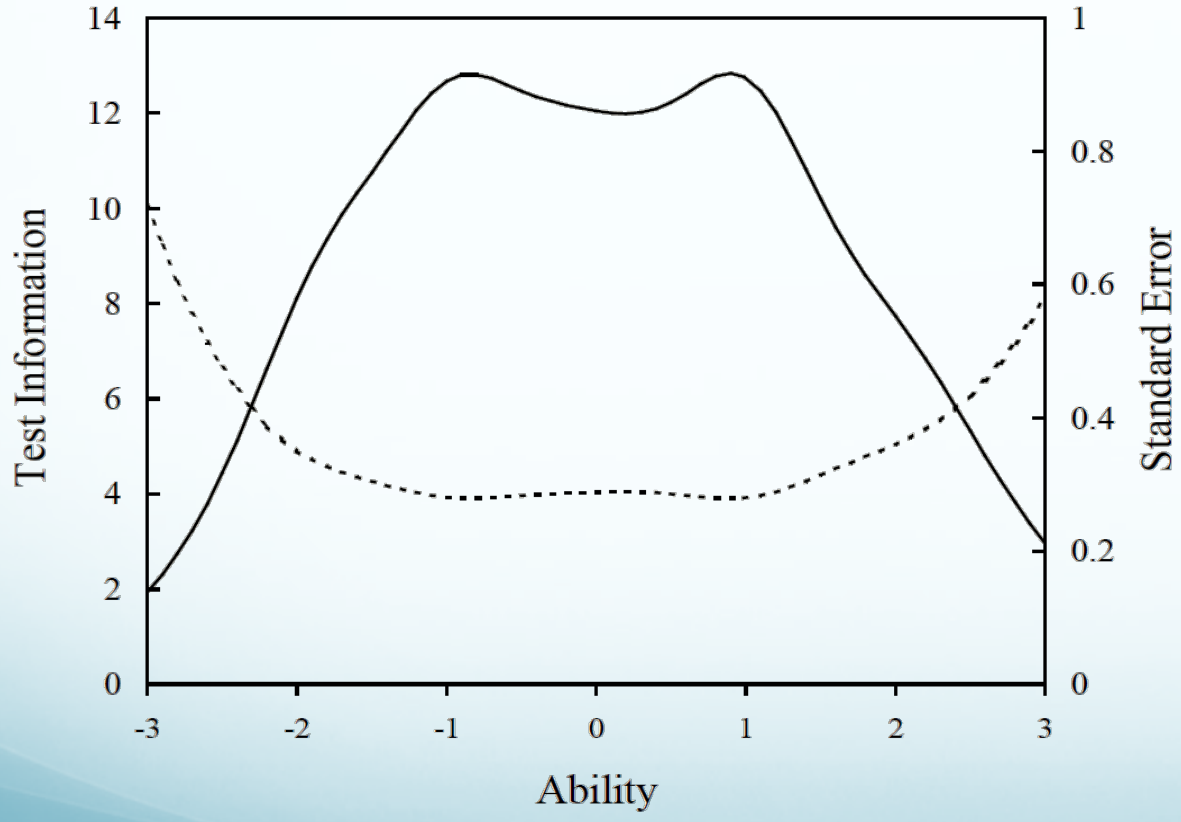
- Test Information Function (TIF) – The IIFs are additive so that we can judge the test as a whole and see at which part of the trait range it is working the best.

(Fisher) Test Information Function



*Not to scale

TIF and Standard Error (SE)

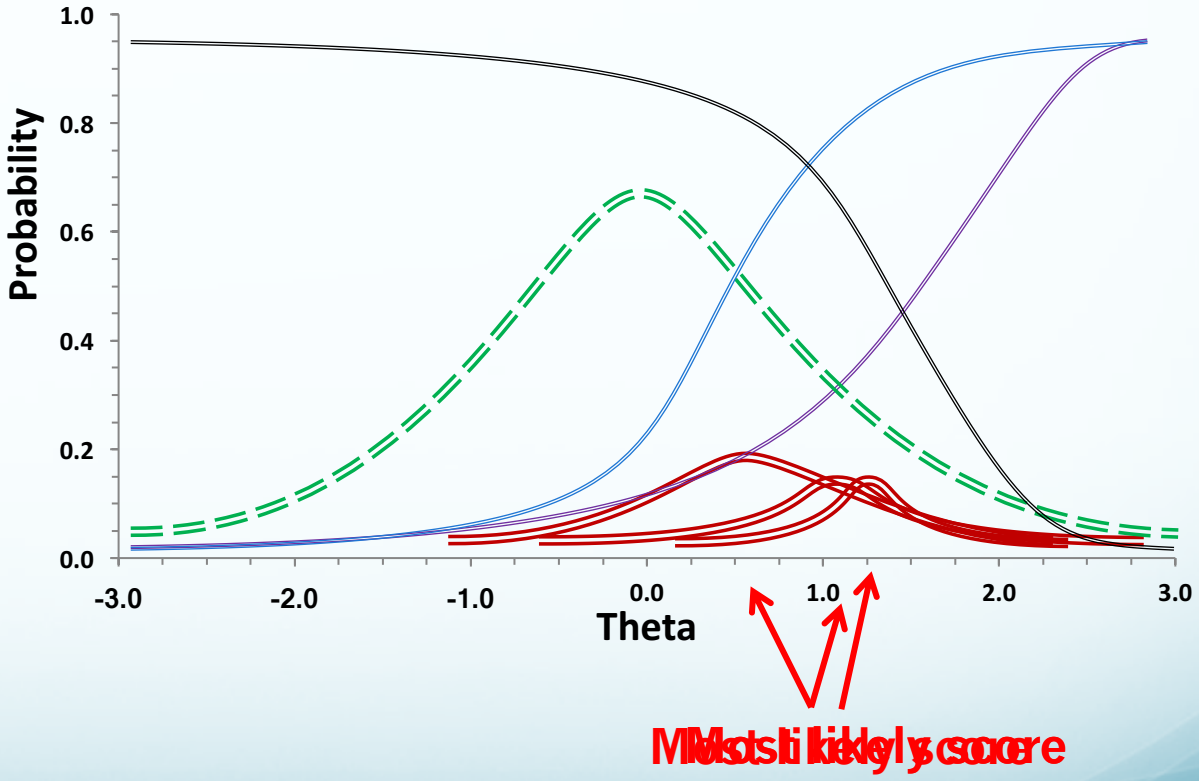


- Error of measurement inversely related to information
- Standard error (SE) is an estimate of measurement precision at a given theta

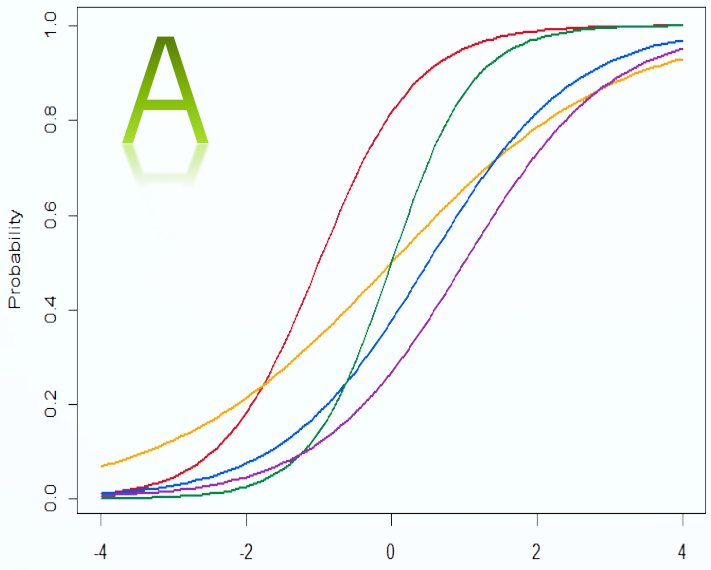
Scoring

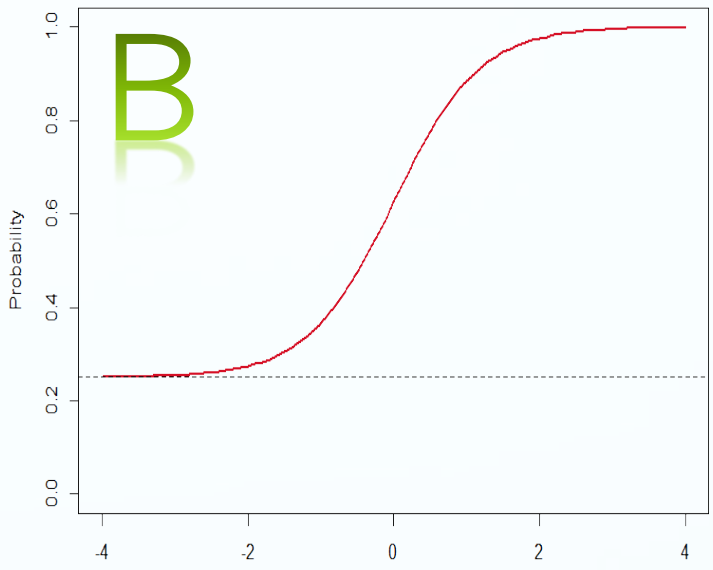
Test:

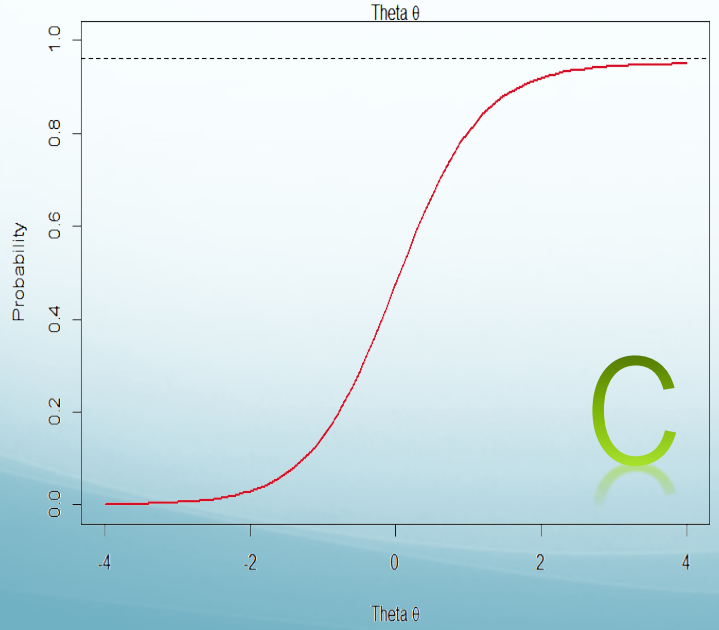
1. **Normal distribution**
2. **q1 – Correct**
3. **q2 – Correct**
4. **q3 - Incorrect**

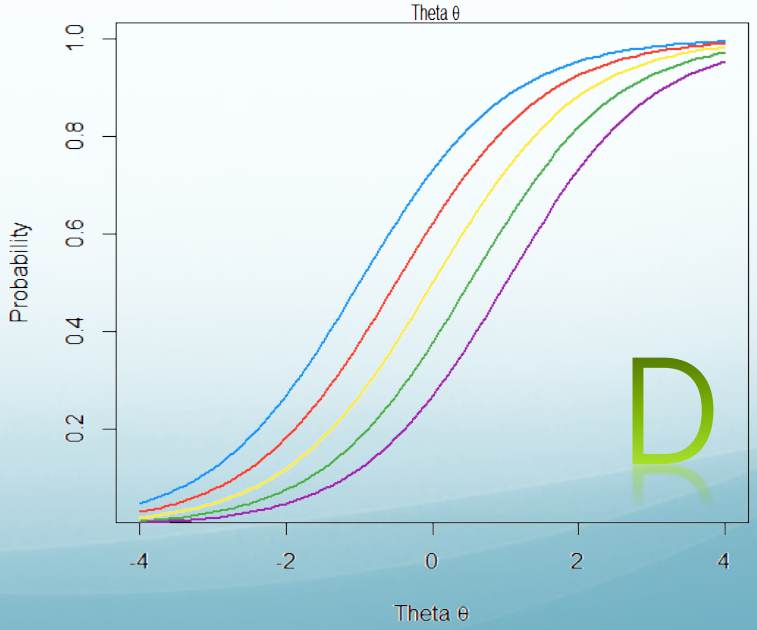


Quiz !


A
Probability
0.0
0.2
0.4
0.6
0.8
1.0
-4
-2
0
2
4


B
Probability
0.0
0.2
0.4
0.6
0.8
1.0
-4
-2
0
2
4


C
Probability
0.0
0.2
0.4
0.6
0.8
1.0
-4
-2
0
2
4
Theta θ


D
Probability
0.2
0.4
0.6
0.8
1.0
-4
-2
0
2
4
Theta θ

Quiz !

- Which of the following is not an IRT parameter?

- Unfolding
- Discrimination
- Difficulty
- Probability
- Guessing

- [HARD ONE!] Name the parameters from their shorthand

a -

b -

c -

Quiz !

- Which of the following is not an IRT parameter?

- Unfolding
- Discrimination
- Difficulty
- Probability
- Guessing

- [HARD ONE!] Name the parameters from their shorthand

a - discrimination

b – difficulty

c - guessing

IRT assumptions

- Unidimensionality – All items are assumed to measure a single common factor.
- Local independence – item responses are uncorrelated after controlling for the latent trait.
- Homogenous population – No differential item functioning (DIF) between groups.

Classical Test Theory vs. Item Response Theory

	Classical	IRT
Modelling / Interpretation	Total score	Individual items (questions)
Accuracy / Information	Same for all participants and scores	Estimated for each score / participant
Adaptivity	Virtually not possible	Possible
Score	Depends on the items	Item independent
Item Parameters	Sample dependent	Sample independent
Preferred items	Average difficulty	Any difficulty

IRT in R

ltm package

Suggested Resource:

Computerised Adaptive Testing: The State of the Art (November 2010)

Dr Philipp Doebler of the University of Munster describes the latest thinking on adaptivity in psychometric testing to an audience of psychologists.

Data we are using today:

"Mobility" Survey

- A rural subsample of 8445 women from the Bangladesh Fertility Survey of 1989 (Huqand Cleland, 1990).
- The dimension of interest is women's mobility and social freedom.
- Described in: Bartholomew, D., Steel, F., Moustaki, I. and Galbraith, J. (2002) The Analysis and Interpretation of Multivariate Data for Social Scientists. London: Chapman and Hall.
- Data is available within R software package "ltm"

“Mobility” Survey

Women were asked whether they could engage in the following activities alone (1 = yes, 0 = no):

1. Go to any part of the village/town/city.
2. Go outside the village/town/city.
3. Talk to a man you do not know.
4. Go to a cinema/cultural show.
5. Go shopping.
6. Go to a cooperative/mothers' club/other club.
7. Attend a political meeting.
8. Go to a health centre/hospital.

ltm package

```
install.packages("ltm")

require(ltm)

help(ltm)

head(Mobility)

my1pl<-rasch(Mobility)

my1pl

summary(my1pl)

plot(my1pl, type = "ICC")

plot(my1pl, type = "IIC", items=0)
```

ltm package

rasch

myrasch<-rasch(Mobility, cbind(9,1))

my2pl <- ltm(Mobility ~ z1)

anova(my1pl, my2pl)

(the smaller the better!)

Now plot ICC and IIC for 2pl model.

ltm package – scoring

```
resp<-matrix(c(1,1,1,1,0,1,0,1), nrow=1)
factor.scores(my2pl, method="EAP", resp.patterns=resp)
```

EXPLAIN: “$” addressing

theta = dataCAT$score.dat$z1

sem = dataCAT$score.dat$se.z1

```
mobIRT <- factor.scores(my2pl, resp.patterns=Mobility, method="EAP")
head(mobIRT$score.dat)
```

Compare IRT and CTT scores

```
CTT_scores <- rowSums(Mobility)

IRT_scores <- mobIRT$score.dat$z1

plot(IRT_scores, CTT_scores)


#Plot the standard error and scores

IRT_errors <- mobIRT$score.dat$se.z1

plot(IRT_scores, IRT_errors, type="p")
```

Why use Item Response Theory?

- Reliability for each examinee / latent trait level
- Modelling on the item level
- Examinee / Item parameters on the same scale (so you can compare the outcomes of two different tests measuring the same latent trait)
- Examinee / Item parameters invariance (so if you test a latent trait on a subsection of the population you can get parameter estimates for the whole population)
- Score is item independent
- Adaptive testing
- Also, test development is: cheaper and faster! (no need to re-norm every time you change an item)

Introduction to CAT

Very brief

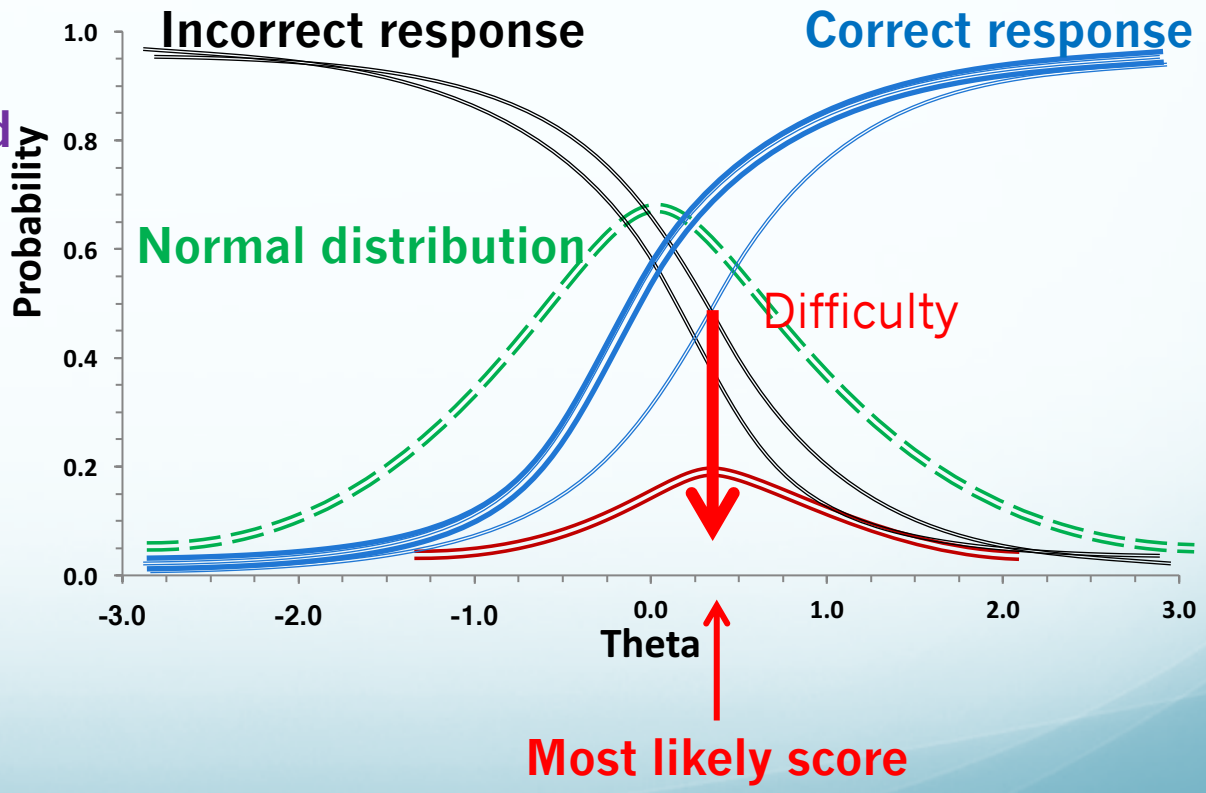
Computerized Adaptive Testing

- Standard test is likely to contain questions that are too easy and/or too difficult
- Adaptively adjusting to the level of the test to this of participant:
  - Increases the accuracy
  - Saves time / money
  - Prevents frustration

Example of CAT

Start the test:

1. **Ask first question, e.g. of medium difficulty**
2. **Correct!**
3. **Score it**
4. **Select next item with a difficulty around the most likely score (or with the max information)**
5. **And so on…. Until the stopping rule is reached**



Elements of CAT

- IRT model
- Item bank and calibration
- Starting point
- Item selection algorithm (CAT algorithm)
- Scoring-on-the-fly method
- Termination rules
- Item bank protection / overexposure
- Content Balancing

CAT Demos

http://planning.e-psychometrics.com/test/cat-demo

http://planning.e-psychometrics.com/test/cat-dna

Demos created by Aiden Loe

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