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CHARACTERISTICS OF ALFVÉN SURFACE WAVES ALONG MOVING CYLINDRICAL PLASMA COLUMNS

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Abstract—The dispersive and stability characteristics of Alfvén Surface Waves (ASW) along the boundary of a moving cylindrical plasma column, surrounded by a stationary medium embedded in a parallel magnetic field is studied. The nature of the symmetric and asymmetric modes on the interface parameters is also discussed.

1. INTRODUCTION

IT HAS been well established that a static boundary with discontinuities in magnetic field and density always support surface waves (HASEGAWA and UBEROI, 1982; WENTZEL, 1979; UBEROI and SOMASUNDARAM, 1980, 1982*a, b*; SOMASUNDARAM and UBEROI, 1982; EDWIN and ROBERTS, 1982; RAE and ROBERTS, 1981). Hence in a moving boundary, in addition to the growing or unstable modes, the stable surface waves should also exist. The existence of such surface modes have been reported in the recent past (GERONICOLAS, 1977; TREHAN and SINGH, 1978; SATYA NARAYANAN and SOMASUNDARAM, 1982, 1985).

There have been numerous studies on surface waves. For example, HOLLWEG (1982) studied surface waves on Solar Wind Tangential Discontinuities. He showed that discontinuities in the magnetic field direction can support MHD surface waves resembling the Alfvén Wave with minor differences in their properties. Other works on Alfvén waves and hydromagnetic surface waves include BELCHER and DAVIS (1971), DOBROWOLNY (1977), BURLAGA (1971), DAILY (1973), IONSON (1978), ROBERTS (1981).

Kelvin-Helmholtz instabilities arise mainly due to the relative motions of the plasma media. In such configurations, generally, there is either a discontinuity in the magnetic field or in the density or in both. MIURA and PRITCHET (1982) and LEE *et al.* (1981) have studied the KH instability in sheared magnetohydrodynamic flows.

In our earlier work (SATYA NARAYANAN and SOMASUNDARAM, 1985) we have studied the Alfvén Surface Waves (ASW) along the boundaries of an incompressible, moving plasma slab, surrounded by a static medium both of which are embedded in a parallel magnetic field. Since the surface waves are two dimensional in nature, the geometry of the plasma column can affect the propagation characteristics of the surface waves.

In this paper we study the ASW along the boundary of a moving cylindrical plasma column, surrounded by a stationary column embedded in a parallel magnetic

field. A sharp density discontinuity exists between the moving and stationary medium. There is also a discontinuity in the magnitude of the magnetic field at the boundary. Two branches of ASW, upper and lower, are found to exist in this configuration. However, their existence depends on the value of the basic flow velocity of the moving column compared to the Alfvén wave velocity. In each branch, the symmetric and asymmetric modes are studied for different values of the density and magnetic field discontinuity.

The study of stability curves for these modes reveal that, for a given basic flow, a stable surface wave becomes unstable or vice versa when the thickness of the moving plasma column is changed. Hence, an unstable mode along a cylindrical column of varying radius might cause the generation of a stable surface wave.

2. DISPERSION RELATION

The linearized equations of motion governing the electromagnetic and hydrodynamic properties of an incompressible, infinitely conducting and moving plasma fluid are given by

$$\frac{\partial \bar{b}}{\partial t} + U \frac{\partial \bar{b}}{\partial z} = B_{o1} \frac{\partial \bar{v}}{\partial z}, \tag{1}$$

$$\rho_{o1} \left(\frac{\partial \bar{v}}{\partial t} + U \frac{\partial \bar{v}}{\partial z} \right) = -\nabla \left(p + \frac{\bar{B}_{o1} \cdot \bar{b}}{4\pi} \right) + \frac{\bar{B}_{o1}}{4\pi} \frac{\partial \bar{b}}{\partial z}, \tag{2}$$

$$\nabla \cdot \bar{b} = 0, \tag{3}$$

where \bar{v} , p and \bar{b} are the perturbed fluid velocity, pressure and magnetic field, respectively, while ρ_{o1} , B_{o1} are the unperturbed density and magnetic field, respectively in medium 1 (see Fig. 1), U is the basic velocity of the moving column.

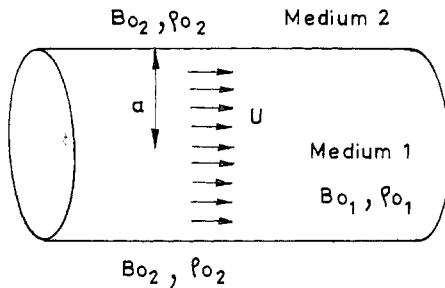


FIG. 1.—The geometry.

Taking the divergence of equation (2) and manipulating with the other equations results in

$$\nabla^2 = 0, \tag{4}$$

where $\tilde{p} = p + (\bar{B}_{o1} \cdot \bar{b}/4\pi)$ and ∇^2 is the Laplace operator. For perturbations of the form

$$f(r, \varphi, z, t) = f(r) \exp [i(kz + m\varphi - \omega t)],$$

solution of equation (4) can be written as

$$\tilde{p}_1 = AI_m(kr), \quad r < a, \tag{5}$$

where $I_m(kr)$ is the modified Bessel function of order m , ‘ a ’ is the radius of the moving plasma medium 1, and A is an arbitrary constant.

For the stationary medium $2(r > a)$, a set of equations similar to equations (1)–(4) can be written with $U = 0$. The solution of the total pressure field can be written in a similar way as

$$\tilde{p}_2 = BK_m(kr), \quad r > a \tag{6}$$

where $K_m(kr)$ is the modified Bessel function of the second kind and of order m . B is an arbitrary constant.

Applying the matching conditions, namely, the total pressure and normal component of velocity are continuous at the interfaces, we obtain the dispersion relation as

$$\frac{\rho_{o2} I'_m(ka) K_m(ka)}{\rho_{o1} K'_m(ka) I_m(ka)} (\omega^2 - k^2 V_{A2}^2) = (kU - \omega)^2 - k^2 V_{A1}^2, \tag{7}$$

where ρ_{o2} is the unperturbed density of the medium 2 and $V_{A1,2} = B_{o1,2} / \sqrt{4\pi\rho_{o1,2}}$ are the bulk of Alfvén wave velocities in the media 1 and 2, respectively. The dash denotes derivative with respect to the argument.

Defining

$$F_m(ka) = -\frac{I'_m(ka) K_m(ka)}{K'_m(ka) I_m(ka)}, \tag{8}$$

the dispersion relation (7) can be rearranged to give

$$\omega^2 [\eta F_m(ka) + 1] - \omega^2 kU + (U^2 - V_{A1}^2) k^2 - \eta F_m(ka) k^2 V_{A2}^2 = 0, \tag{9}$$

where $\eta = \rho_{o2} / \rho_{o1}$.

The non-dimensional phase velocity of the ASW for the cylindrical geometry is given by

$$\frac{\omega}{kV_{A1}} = \frac{V \pm \{ [1 + \eta F_m(ka)] [1 + \beta^2 F_m(ka)] - V^2 \eta F_m(ka) \}^{1/2}}{[1 + \eta F_m(ka)]}, \tag{10}$$

where $\beta = B_{o2} / B_{o1}$ and $V = U / V_{A1}$ is the non-dimensional velocity characterizing the basic flow.

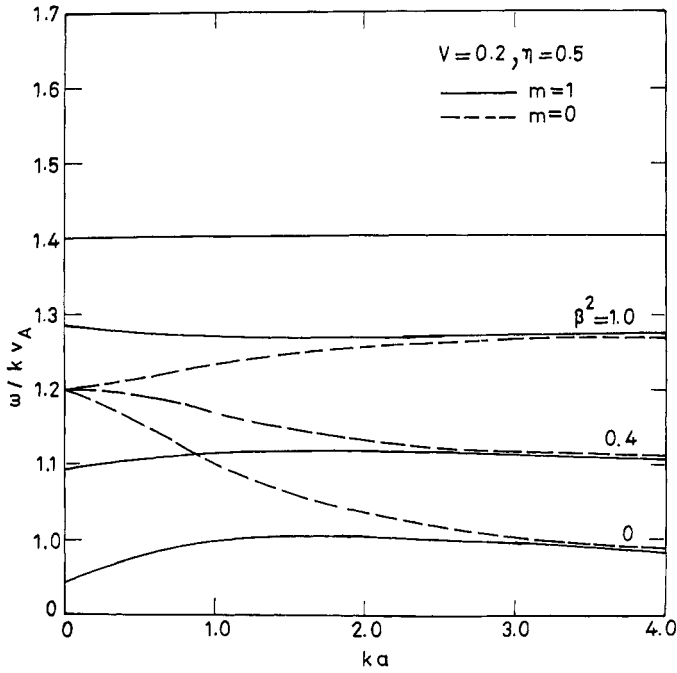


FIG. 2.—Dispersion curves of symmetric (-----) and asymmetric (——) modes for $V = 0.2$, $\eta = 0.5$ and different values of β^2 .

3. DISCUSSION

Before analysing the dispersion relation (10), we consider a few limiting cases.

For the symmetric mode ($m = 0$) in the limit $ka \rightarrow 0$, $F_0(ka) \rightarrow 0$ (UBEROI and SOMASUNDARAM, 1980), so that equation (10) reduces to

$$\frac{\omega}{kV_{A1}} = V \pm 1. \tag{11}$$

Equation (11) shows that for values of $V \leq 1$, there exists only one positive value for ω/kV_{A1} , i.e. there is only one surface mode for the basic velocity $U \leq V_{A1}$ (we discard the negative value for ω/kV_{A1}). For $V > 1$ ($U > V_{A1}$), there are two branches of surface waves, upper (+ve sign) and lower (-ve sign).

In the limit $ka \rightarrow \infty$, $F_0(ka) \rightarrow 1$ so that equation (10) reduces to

$$\frac{\omega}{kV_{A1}} = \frac{V \pm [(1 + \eta)(1 + \beta^2) - V^2\eta]^{1/2}}{(1 + \eta)}. \tag{12}$$

For the asymmetric mode ($m = 1$), both for $ka \rightarrow 0$ and $ka \rightarrow \infty$, $F_1(ka) \rightarrow 1$ (UBEROI and SOMASUNDARAM, 1980). Therefore equation (10) reduces to equation (12). Equation (12) shows that the existence of a surface wave is determined by the value $V^2\eta$, i.e. by the basic flow velocity and the magnitude of discontinuity in the density.

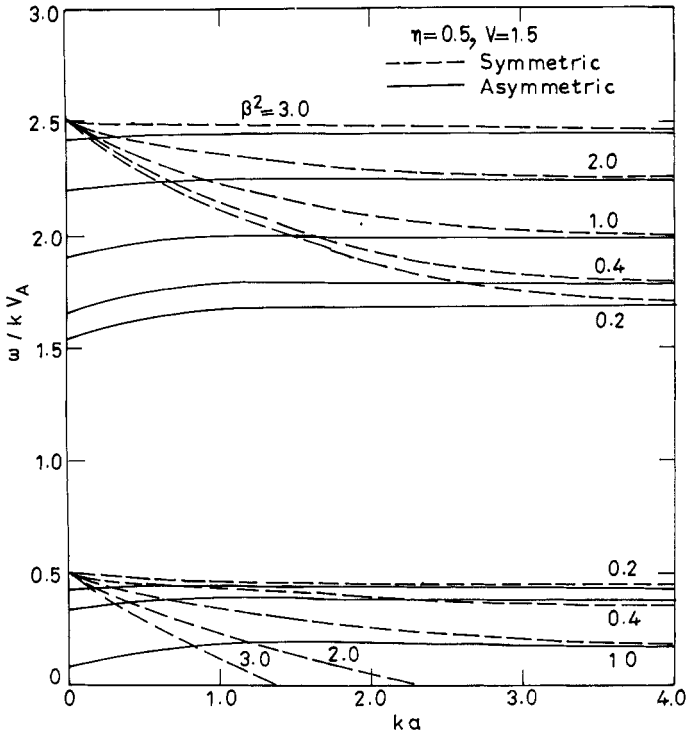


FIG. 3.—Same as in Fig. 2, but for $V = 1.5, \eta = 0.5$.

When $V = 0$, equation (12) reduces to

$$\frac{\omega}{kV_{A1}} = \left[\frac{1 + \beta^2}{1 + \eta} \right]^{1/2}, \tag{13}$$

as obtained by HASEGAWA and UBEROI (1982).

When the surrounding plasma column is stationary, i.e. when $V = 0$, equation (10) reduces to

$$\frac{\omega}{kV_{A1}} = \frac{\{[1 + \eta F_m(ka)][1 + \beta^2 F_m(ka)]\}^{1/2}}{1 + \eta F_m(ka)}. \tag{14}$$

Equation (14) is the dispersion relation for ASW propagating along a cylindrical plasma column and has been studied in detail by UBEROI and SOMASUNDARAM (1980).

The dispersion curve obtained from the equation (10) for $V = 0.2$ and $\eta = 0.5$ for different values of β are shown in Fig. (2), for symmetric ($m = 0; \dots$) and asymmetric ($m = 1; \dots$) modes. The curves represent the upper branch only, for the lower branch does not exist in this case. It is clear that the effect of finite thickness is significant only for $ka \lesssim 4$. For $ka \gtrsim 4$, both modes are indistinct and their phase

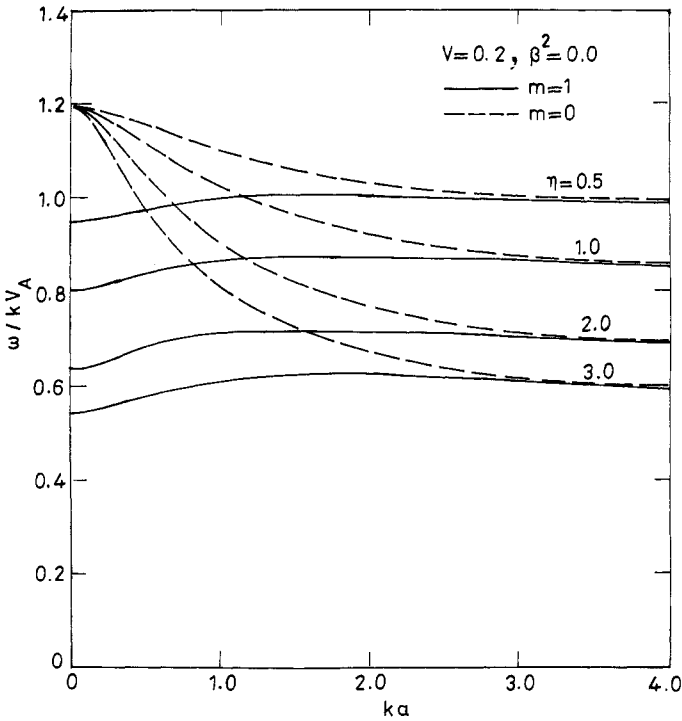


FIG. 4.—Same as in Fig. 2, but for $V = 0.2, \beta^2 = 0.0$ and different values of η .

velocities are also constant. For $ka \lesssim a$, the symmetric modes are more sensitive to the thickness of the plasma column than the asymmetric modes.

The effect of the basic flow velocity on the ASW characteristics are shown in Fig. (3) for $\eta = 0.5, V = 1.5$ and different values of β^2 . It is interesting to note that the lower branch which was absent for low flow velocity ($V = 0.2$) appears for high flow velocity ($V = 1.5$). The symmetric modes suffer upper cut-off in k , while the asymmetric modes suffer lower cut-off (not shown in Fig. 3) in k . This is qualitatively the same as that for a slab geometry (SATYA NARAYANAN and SOMASUNDARAM, 1985). However, the effect of magnetic field discontinuity is more on the ASW along a plasma slab than on the ASW along a cylindrical plasma under identical conditions. In other words, the propagation window in k is wider for ASW along a cylindrical column than along the slab plasma. As the magnitude field discontinuity given by β increases, the upper cut-off value in k for the symmetric mode increases.

The dispersion curves, when the surrounding static medium-2 is field free ($\beta = 0$) and $V = 0.2$ for different values of η (the magnitude of the density discontinuity) are shown in Fig. 4. For the values chosen, only the upper branch of ASW exists.

An increase in the density ρ_{o2} of the static medium reduces the phase velocities of both the symmetric and asymmetric modes.

The stability curves for $\eta = 0.5$ and different values of β^2 are given in Fig. 5. These curves are obtained for those values of the phase speeds such that the dispersion relation has only real roots. In other words, the modes are neutral in nature.

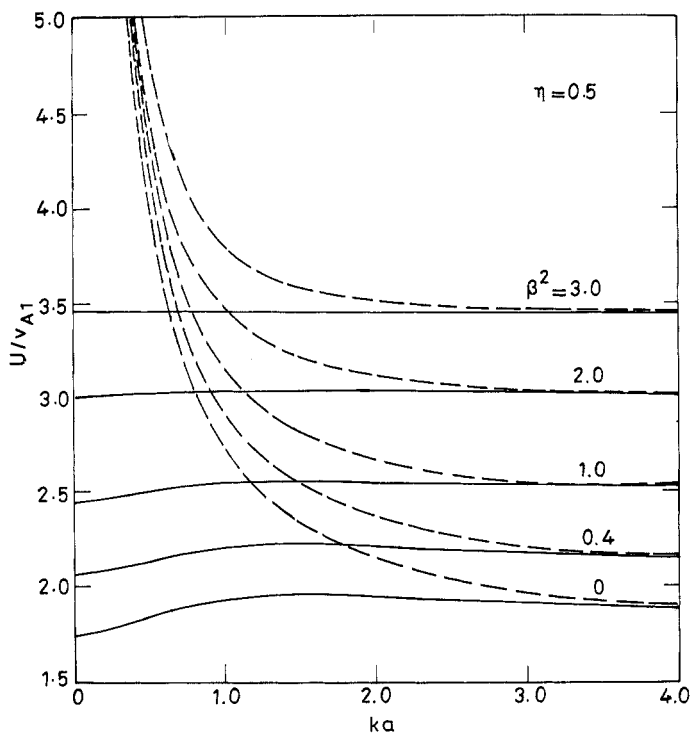


FIG. 5.—Stability curves of Alfvén Surface Waves for symmetric (-----) and asymmetric (——) modes for $\eta = 0.5$ and different values of β^2 . The region lying above the neutral curve is unstable while below is stable for each of the mode corresponding to different values of β^2 .

The stability curves divide the region into stable and unstable regions. The region above the neutral curve is unstable while the region below is stable. The stability of both the symmetric and asymmetric modes depend on the interface parameters η and β^2 as can be seen from Fig. 5. For a fixed value of η and different values of β^2 , the region of stability keeps on changing.

The stability curves for the symmetric mode is very similar to the symmetric mode of the slab plasma (SATYA NARAYANAN and SOMASUNDARAM, 1985). However, the asymmetric mode has a few interesting features. The stable and unstable regions of this mode, for any given value of β^2 , go from stable to unstable ones and then back to stable regions as ka increases. This effect is pronounced for low values of β^2 . This feature is due to the curvature effect. As a result, a stable surface wave might become unstable when the thickness of the plasma column increases.

4. CONCLUSION

A moving plasma column surrounded by a stationary plasma embedded in parallel magnetic fields can support two branches of Alfvén Surface Waves, upper and lower branches depending on the basic flow velocity of the moving column. The upper branch exists for all values of the basic flow velocity. The lower branch exists only when the basic flow velocity is greater than the bulk Alfvén velocity in the moving

column. As the magnitude of the magnetic field discontinuity increases, both the modes of this branch suffer upper and lower cut-off in k . However, compared to a slab plasma, the propagation window for ASW is larger for cylindrical column. Finally the stability results reveal that the stable surface waves might become unstable as the thickness of the plasma column is increased.

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