#### DISPERSION RELATIONS AND WEAK INTERACTIONS AT HIGH ENERGY\*

Presented by V. I. Zakharov Institute for Theoretical and Experimental Physics Moscow, USSR

#### I. Introduction

This talk is devoted to a dispersion approach to weak interactions at high energy. There are two kinds of problems which will be touched upon:

1. Asymptotic bounds on the total cross section.

2. Higher-order weak interactions at finite energy.

The first problem arises from the fact that for weak interactions at high energy masslessparticle (neutrino) exchange may be essential while most of the results of the theory of dispersion relations are valid only in the case of finite masses. In particular, the derivation of the Froissart bound depends on this assumption. The proper generalization of these results still represents a challenge to the theorists. Some asymptotic bounds obtained so far will be discussed at the end of the talk.

As for the application of dispersion relations to higher-order weak interactions, the hopes are high now that they will never be needed for this purpose. Indeed, if pertubative-type models of weak interactions are valid, dispersion relations for weak interactions will play a subordinate, if any, role.

Experimentally, however, there is no single piece of information which compels us to accept these models. So far weak processes are well described by the lowest-order interaction. For conventional theories it is difficult to incorporate this simple picture both at low and high energies: if one tries pertubative calculations with cut-off or introduces new particles higher-order corrections are large unless the cut-off or masses of new particles are small.

If, however, the cross section of weak interaction continues to grow with energy and no new particles are produced, we will presumably be forced to say that there is some mystery (symmetry) about coupling constants which makes them small and universal. The unitarity condition and dispersion relations will then suggest themselves as a model-independent approach to weak interactions.

In the applications discussed below dispersion relations with two subtractions are mostly used. The four-fermion coupling constants are considered here to be subtraction terms, any hope of calculating them being abandoned. The emphasis is made instead on terms of next order in energy which are assumed to be dispersive.

#### II. Pomeranchuk's Relation

Dispersion relations were first applied to weak interactions at high energy by Pomeranchuk.<sup>4</sup> He realized that the dispersion approach provides us with a model-independent connection between the low- and high-energy behavior of the amplitudes of weak interactions.

<sup>\*</sup>Report of work of A. D. Dolgov, L. B. Okun, and V. I. Zakharov.

Indeed, let us imagine that starting from some energy  $s_0$  the total cross section of weak interactions becomes large. For simplicity we assume also that it is equal to a constant  $\sigma_0$  at higher energies (see Fig. 1). In what way would it affect the amplitude at low energy? To answer this question let us calculate the dispersion contribution  $A^{(2)}$  coming from s' >  $s_0$ . It is easy to find that for t = 0

$$\frac{\mathbf{A}^{(2)}}{\mathbf{A}^{(1)}} \sim \frac{s\sigma_0}{s_0(4\sqrt{2}\mathbf{G})}, \qquad (1)$$

where  $A^{(1)}$  stands for amplitude of pointlike four-fermion interaction and is equal to  $4\sqrt{2}G$ .

Through some positivity condition it can be shown that this correction to the lowest-order amplitude due to the contribution of  $s^1 > s_0$  cannot be cancelled out by other pieces of the dispersion integral so that Eq. (1) gives a lower bound on the correction. Pomeranchuk also found the correction to  $dA^{(1)}/dt$  and showed it is more sensitive to the value of  $\sigma_0$ .

### III. At What Energy Can Weak Interactions Become Strong?

Equation (1) was used by Pomeranchuk to answer just this question. Up to now no sign of the presence of the  $A^{(2)}$  term has been found. If we turn to consideration of  $\nu p \rightarrow \nu p$  scattering, it is safe to say that for  $s \leq 10 \text{ GeV}^2$  the ratio of the amplitudes  $A(\nu p \rightarrow \nu p)/A(\nu n \rightarrow \mu p)$  is less than or equal to unity. From Eq. (1) we learn then that the total cross section of weak interaction can be comparable to the total cross section of strong interaction  $\left(\sigma_0 \sim m_N^{-2}\right)$  only at an energy squared

$$s_0 \gtrsim 10^5 \text{ GeV}^2$$
. (2)

In paper<sup>1</sup> it was assumed that  $A^{(2)}/A^{(1)} \leq 1$  up to  $s \sim G^{-1}$  and was concluded that

$$s_0 \gtrsim (Gm_N^2)^{-2} GeV^2 = 10^{10} GeV^2.$$
 (3)

Up to now we used dispersion relations for forward scattering to estimate  $A^{(2)}$ . Let us turn now to the discussion of the t-dependence of the amplitude and for simplicity let us consider first the case of lepton-lepton scattering. Then, in the lowest-order weak scattering proceeds via a single partial wave and the t-dependence of the amplitude is very smooth. This is not true for the dispersive correction  $A^{(2)}$ . Indeed, this correction arises from the dispersion contribution of high energies and reflects the structure of the amplitude at these energies. We assumed that the cross section at high energy is large; this implies a sharp t-dependence of the amplitude. As far as we assume the cross section to be a constant at s' > s<sub>0</sub>, it is natural to expect that the tdependence factorizes out and for some region of t can be approximated by an exponential function. In this way we come to the conclusion that

$$\frac{A^{(2)}(t)}{A^{(2)}(t=0)} \sim \exp\left(\frac{-t}{t_{eff}}\right), \quad -t_{eff} \sim \sigma_0^{-1}.$$
 (4)

Since existing experimental data on lepton-lepton scattering are very poor, Eq. (4) does not help directly to improve the bound (2) on  $s_0$  obtained above. However, Eq. (4) shows that at small energies there should exist some sort of halo with radius of order  $\sqrt{s_0^{-1}}$ . If one considers it to be

unacceptable from a theoretical point of view, then so can be pushed to infinity to cancel the contribution of high energies by a factor  $s/s_0$  [see Eq. (1)].

A somewhat more conservative point of view is to allow the long-range forces introduced by  $A^{(2)}$  to be comparable in strength with long-range forces arising from hadron exchange in higher orders in weak interactions. It still restricts possible value of sa severely. We would like, however, not to use additional theoretical assumptions and stick to bound (2).

For lepton-hadron scattering the situation is even more complicated since it is not clear how one can distinguish between the damping factor (4) and the usual form factor.

 $\frac{IV. Model of Strongly Interacting W-Bosons}{Appelquist and Goldman<sup>2</sup> have observed that the amplitude of elastic scattering will become$ rather large at NAL energies if W-bosons have strong pairwise interaction with hadrons (see also Biorken's Lecture in  $Erevan^3$ ).

Indeed, in this case the cross section increases promptly once production of real W-bosons is possible (see Fig. 2). Roughly speaking, we have:

$$s_0 \sim m_W^2$$
,  $\sigma \sim g^2 \sigma_{strong} \equiv G m_W^2 \sigma_{strong}'$  (5)

where g is the semiweak coupling constant. Then, Eq. (1) gives

$$\frac{A^{(2)}}{A^{(1)}} \sim \frac{a\sigma_{strong}}{4\sqrt{2}}$$
(6)

which is independent of the mass of W-boson (as far as  $Gm_W^2 \le 1$ ) and is rather large. A more accurate calculation of the graph (Fig. 2) reduces the estimate (6) somewhat, but still for  $E_{1} \approx 50$ GeV  $\sigma(\nu p \rightarrow \nu p)$  and  $\sigma(\nu n \rightarrow \mu p)$  are comparable if  $\sigma_{strong} \sim i$  mb.

Dispersion relations were also used to pose the problem of damping higher-order effects in this model (see Ref. 3).

#### V. General Form of Amplitude of Weak Interactions at Low Energy

For a more detailed discussion we need now better understanding of the structure of  $A^{(2)}$ . The problem is to describe the corrections in a model-independent way without refering to any dynamical calculations. An example of such a description is provided by an amplitude of nonrelativistic scattering at low energy

$$A \sim \frac{1}{\frac{1}{a} + i\sqrt{E}},$$
 (7)

where E is the energy and a is the scattering length.

Somewhat similar formulas can be obtained for weak interactions at low energy. The difference is that several GeV (perhaps even 100 GeV) is still a "low" energy for weak interactions.

It is reasonable therefore to consider the interaction of massless particles to simplify the formulae. In the case of elastic  $e^{+}e^{-}$  scattering we have in the second order<sup>4, 5</sup>

$$A(ee - ee) = \frac{2G^2 s}{3\pi^2} \left( t \ln \frac{-t}{\Lambda} + u \ln \frac{-u}{\Lambda} \right) + \text{possible contact term},$$
(8)

where  $\ln \Lambda$  is some parameter; t and u are the energy squared in annihilation channels.

Equation (8) is a general one in the same sense as Eq. (7) is: it keeps all the terms of the second order in s, t and satisfies the unitarity condition in this approximation. For sufficiently low s it is surely valid provided that there are no neutral weak currents which give a contribution to the unitarity sum comparable with that of the  $v\overline{v}$  intermediate state (see Fig. 3).

Terms of the third order in s, t can be described in the same way<sup>4, 5</sup> with more parameters entering the game. For the terms of higher order the calculation has not been tried yet.

#### VI. Dispersion Sum Rule for $\ln \Lambda$

Expanding a dispersion relation with two subtractions in powers of s and comparing the result with Eq. (8) one readily obtains

$$\ln \frac{\Lambda}{\dot{g}} = 6\pi \int_{\ddot{g}}^{\infty} \frac{\sigma^{+}(s)}{(Gs)^{2}} ds + 6\pi \int_{0}^{\infty} \frac{ds}{(Gs)^{2}} \left[ \sigma^{+}(s) - \sigma_{\text{theor}}^{+}(s) \right], \qquad (9)$$

where

$$\sigma^{+}(s) = \left[\sigma_{\text{tot}}^{\text{weak}}(e^{+}e^{-}) + \sigma_{\text{tot}}^{\text{weak}}(e^{-}e^{-})\right], \ \sigma_{\text{theor}}^{+} = \frac{G^{2}s}{6\pi}, \ \text{$\tilde{s}$ is arbitrary}$$

and the dependence of  $\ln \Lambda$  on  $\tilde{s}$  is just superficial.

This sum rule gives the parameter  $\ln \Lambda$  which represents the cut-off in pertubative calculations in terms of integral over the total cross section. It is a generalization of Eq. (1) which keeps the contribution of intermediate energies.

Any violation of Eq. (9) would imply the violation of dispersion relations with two subtractions. Because the integrand of the first term in rhs of Eq. (9)--the only one where integration extends to infinity--is positive such a violation, if it exists, has a chance to be established at finite energies.

Terms of higher order in the expansion of the amplitude at low energies can also be expressed in terms of some dispersive integrals. In particular, the coefficients of an expansion in powers of t are given by

$$\int \frac{\mathrm{d}^{n} A(s,t)}{\mathrm{d}t^{n}} \bigg|_{t=0} \frac{\mathrm{d}s}{s^{2}}$$
(10)

which by virtue of the inequality<sup>6</sup>

$$\frac{d^{n}A(s,t)}{dt^{n}} \ge Const \cdot s\sigma_{tot}^{n+1}$$
(11)

depends most crucially on the total cross section at high energy. In an explicit form such representations were obtained for terms of the third order. In higher orders the problem is to isolate the singularity of the amplitude at t = 0 connected with massless particle exchange so as to make the derivative in t meaningful.

#### VII. Dispersion Sum Rule for Fermi Constant G

Up to now the constant G was treated as a subtraction constant. If there exist dispersion relations with one subtraction then the constant G can be represented as an integral of the difference of total cross sections of particle and antiparticle interactions:  $^{4, 5}$ 

$$G = \frac{1}{\pi\sqrt{2}} \int_{0}^{\infty} \frac{ds}{s} \left[ \sigma_{\overline{v}e}^{\text{tot}}(s) - \sigma_{ve}^{\text{tot}}(s) \right].$$
(12)

It is worth noting that even if this dispersion integral is convergent it is not excluded that some constant should be added to the rhs of Eq. (12). The absence of this constant is an additional assumption needed to derive representation (12).

According to paper,<sup>7</sup> a dispersion sum rule for the constant G can be obtained even in the case of two subtractions. To this end one should consider the dispersion relation for the modulus of amplitude and its phase. According to paper<sup>7</sup> there are no zeros in the upper half plane if dispersion relations with two subtractions are valid and masses of particles are kept zero. Then there are no arbitrary constants in the dispersion representations for modulus and phase. As a result the following sum rule arises:

$$\int_{0}^{\infty} \frac{\mathrm{d}\mathbf{s}}{\mathbf{s}^{2}} \ln \left| \frac{A_{\nu} \mathbf{e}^{A} \overline{\mathbf{v}} \mathbf{e}}{32 G^{2} \mathbf{s}^{2}} \right| = -\frac{\sqrt{2}}{12} G \tag{13}$$

which becomes an inequality if the number of subtractions is larger than two.

## VIII. Weak Interaction of Colliding Beams with Energy 10<sup>2</sup>-10<sup>3</sup> GeV

If the Fermi coupling constant G provides the only energy scale inherent to weak interactions, experimental investigation of weak processes at energies s  $\sim G^{-1}$  will become imperative. By virtue of the unitarity condition these are the energies where higher-order corrections should be noticeable.

There exist some plans for constructing colliding beams with energy - 100 GeV to probe weak interactions. These are for lepton-lepton beams in Erevan and Novosibirsk and for proton-proton beams at Brookhaven.

The formulae obtained above may be useful to expose general features of such experiments and, later, to provide a framework for analyzing the results.<sup>4</sup>

Let us consider the simplest case of  $e^+e^-$  elastic scattering. It is easy to guess that the second-order weak and electromagnetic amplitudes become comparable to each other at large momentum transfer if

$$f_1^2(s) \sim \alpha$$
 where  $f_1(s) \equiv \frac{Gs}{6\pi\sqrt{2}}$ , (14)

 $f_1$  being the partial-wave amplitude of  $e^+e^- \rightarrow v\overline{v}$  annihilation (in the lowest order only the partial wave with j=1 is different from zero).

This guess can be checked by calculating the imaginary part (see Fig. 3) which is uniquely determined in terms of constant G. By retaining the imaginary part only one obtains a lower bound on the weak cross section. It turns out that this lower bound equals the electromagnetic differential cross section for  $\theta = 90^{\circ}$  at an energy of 225 GeV, the corresponding cross section being  $2 \cdot 10^{-37} \text{ cm}^2/\text{sr.}$ 

The measurement of the real part which is not predicted would provide us with the knowledge of some integral of the total cross section [see Eq. (9)]. Higher-order corrections are presumably smaller by the factor  $f_1 \sim \sqrt{a}$ . Thus, for such energies some kind of pertubation treatment could be applicable and for the whole range of energies and angles the scattering amplitude is expected to be described by a single parameter  $\ln \Lambda$ .

A more detailed presentation of the same problems can be found in Ref. 4. In particular, the last paper listed in this reference deals with electromagnetic corrections of higher order.

#### IX. Long-Range Forces and Weak Interactions at High Energies

Up to now dispersion relations with two subtractions at  $t \le 0$  were used. We are going to discuss now the validity of this assumption. The problem, as was already mentioned in the introduction, is that long-range forces arising from massless-particle exchange may result in rapidly growing total cross sections and invalidate dispersion relations.

It is quite clear that in general when massless-particle exchange is taken into account it is impossible to obtain any bound on the cross section. It is sufficient to say that photon exchange results in an infinite cross section. However, in the case of weak interactions arguments can be presented in favor of dispersion relations with two subtractions. The idea is that in the case of weak interactions the long-range forces are not so important because they arise from exchange of two spin 1/2 particles (neutrinos).

Just to show how this idea can work let us start with a very crude consideration. The amplitude corresponding to the simplest graph with exchange of massless particles (see Fig. 4) is proportional to

$$A \sim G^2$$
 st ln t. (15)

The partial-wave amplitudes corresponding to this expression for large j are given by

$$f_j \sim G^2 s^2 j^{-4}$$
. (16)

For large enough j one could hope that this calculation is sensible. For smaller j,  $f_j$  is larger than unity according to Eq. (16) and the calculation is senseless. For such j we use only the unitarity condition  $f_j \leq 1$ . As a result the partial-wave amplitudes are given by the curve on Fig. 5. It is clear that the cross section is of the order

$$\sigma \sim \mathbf{R}^2 \sim \mathbf{j}_0^2 / \mathbf{s}, \tag{17}$$

where  $j_0$  stands for such j that  $f_i \sim 1$  according to Eq. (16). Finally, we obtain

and, thus, the cross section is rather small, despite the long-range effects.

# X. Asymptotic Bound on the Total Cross Section: $\sigma(s) < s^{1/3}$

By formalizing the consideration of the preceding section it is possible to obtain the bound quoted in the title of the section. To give an idea of the derivation let us briefly outline one of the proofs of the Froissart bound in the case of strong interactions.

We assume that there exist dispersion relations with a finite number of subtractions for  $t \le 0$ (in the case of massless-particle exchange this assumption is still awaiting for approval or disapproval from axiomatic field theory).

For the sake of definiteness we start from dispersion relations with two subtractions, which, rather symbolically, can be written as

ReA(s,t) = ReA(0,t) + sReA'(0,t) + 
$$\frac{s^2}{\pi} \int \frac{ds' \operatorname{ImA}(s',t)}{(s'-s) s'^2} + \operatorname{left-cut term.}$$
 (19)

Let us now differentiate this relation with respect to t at t=0. It can be shown that the order of integrating over  $s^{t}$  and differentiating in t may be interchanged and we come to an integral of  $d^{n}A(s,t)/dt^{n}|_{t=0}$ . By virtue of relation (11) this derivative is bounded from below by  $so^{n+1}(s)$ . Thus, one comes to the conclusion that the integral of any power of the total cross section is convergent. This rules out a cross section growing as any power of s. To establish the (lns)<sup>2</sup> factor in the Froissart bound a more refined consideration is required, but hereafter we omit the lns factors.

So far strong interactions were considered. Where does this proof fail in the case of weak interactions? The answer to this question is that for weak interactions the amplitude is nonanalytic at t = 0 because of massless-particle exchange and some derivatives just do not exist.

The singularity at t=0 is rather mild, however. The simplest graph discussed in the preceeding section contains t lnt but it depends on s linearly and is absorbed into the subtraction term in the dispersion relations in s. For the graphs depending nontrivially on s it can be shown that the singular part of the amplitude is proportional to  $t^2$ , so that the second derivative exists (in neglect of lnt terms). Assuming that the same is true for the total amplitude we come to the asymptotic bound  $\sigma$  (s) < s<sup>1/3</sup>.

To summarize, the bound

follows from two assumptions:

a) there exist dispersion relations with finite number of subtractions for  $t \leq 0$ .

b) the singularity of the total amplitude at t = 0 is given by singularities of separate Feynman graphs.

#### XI. Two-Particle Exchange

If one believes that long-range forces arising from two massless-particle exchange in tchannel are most important some further progress can be reached. The point is that in this case the singularity of the amplitude can be studied in more detail by means of the Mandelstam representation for the double spectral function

$$\rho(B, t) = \int \frac{dz_1 dz_2}{\sqrt{z^2 + z_1^2 + z_2^2 - 2z_1 z_2 z_{-1}}} \left[ A_{B_1} A_{B_2} + A_{U_1} A_{U_2} \right]$$
(20)  
$$z = \frac{2B}{t} + 1, \ z_{1,2} = \frac{2B_{1,2}}{t} + 1,$$

where  $A_g$  and  $A_u$  are imaginary parts of the amplitude in the s- and u-channel,  $s_1$  and  $s_2$  are the energies of the upper and lower blocks of the graph of Fig. 6.

Possible values of  $s_1 \cdot s_2$  are constrained by the condition  $s_{1,2} \le s$ ,  $4s_1 \cdot s_2 \le st$ . If  $s \cdot t$  is very large then  $s_1$  and  $s_2$  are large. If  $s \cdot t$  is small then at least  $s_1$  or  $s_2$  is also small. In the former case  $A_s$  can be replaced by its asymptotic value, while in the latter case at least  $A_{s_1}$  or  $A_{s_2}$  is described by low-energy representation (8).

<sup>2</sup> As the imaginary part is proportional to  $s^2$  at small energies  $\rho(s, t)$  is proportional to  $t^2$  for small  $s \cdot t$ , in agreement with general remarks made in the preceeding section about the character of the singularity at small t.

For large values of s • t the answer may be represented in the form

$$\rho(\mathbf{s},\mathbf{t}) \sim (\mathbf{s}\mathbf{t})^{\alpha+1} \int_0^{\sqrt{\mathbf{s}/\mathbf{t}}} \int_0^{1/4\mathbf{y}} \frac{\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{y}\mathbf{x}^{\alpha}\mathbf{y}^{\alpha}}{\sqrt{1-4\mathbf{x}\mathbf{y}}}, \qquad (21)$$

where it was assumed that asymptotically  $A_g = s^{\alpha+1}$ . Equation (21) was first obtained by Rajaraman<sup>9</sup> (let us notice, however, that the upper limits of integration over x, y in Ref. 9 were erroneously put to be equal to infinity).

The integral over x, y in Eq. (21) contains a logarithmic factor but it is not essential for future analysis. What is essential is that the ratio of  $\rho(s, t)$  and A<sub>g</sub> contains a factor  $t^{\alpha+1}$  if A<sub>g</sub> is asymptotically proportional to  $s^{\alpha+1}$ .

# XII. Asymptotic Bounds on the Total Cross Section: $s^{-1} \le \sigma(s) \le s^{0}$

If one assumes that the two-particle intermediate state dominates the unitarity condition at small t and that there exist dispersion relations in s for positive t, some arguments in favor of the bounds quoted above can be given. The lower bound, as noticed by Anseim and Gribov, <sup>10</sup> is virtually contained in a paper of Gribov and Pomeranchuk (1962). <sup>11</sup> The upper bound was obtained first by Rajaraman<sup>10</sup> and discussed in Ref. 12.

In both cases Eq. (21) is used and at small t the imaginary part  $A_{g}(t, s)$  is replaced by its optical value.

Then, if the cross section is falling faster than  $s^{-1}$  the imaginary part in t of  $A_s$  contains according to (21) factor  $t^{-\epsilon}$  ( $\epsilon > 0$ ) as compared with  $A_s$  itself and this is inconsistent for  $t \to 0$ . In this way the lower bound arises.

To present the argument against growing a cross section we should notice first that if the cross section is growing as some power of s then the effective value of t should fall at least as the same power of s. Otherwise, the elastic cross section is larger than the total cross section. Indeed,

$$\sigma_{ee} \sim \frac{\left| \text{ImA}(s, t=0) \right|^2}{s^2} t_{eff'} \sigma_{ee} < \sigma_{tot} \sim s^{\alpha'}$$
(22)

and

$$t_{eff} \lesssim s^{-\alpha}$$
.

With this information in hand we see that  $\rho(s, t)$  is small as compared with  $A_s$  for  $t - t_{eff'}$ and it is plausible that it cannot feed the growing cross section.

To realize this idea Rajaraman calculated first the potential as a function of  $A_g$  and then, in the eikonal representation,  $A_g$  as a function of the potential. The result of the calculation is a selfconsistency condition. This condition cannot be satisfied unless  $\sigma(s)$  is not growing asymptotically. The weak point of this derivation is that the potential is determined from  $\rho(s, t)$  through a dispersion integral in s at fixed t. However, for s tending to infinity any finite t becomes much larger than  $t_{eff}$  which falls as some power of s. For such s replacement of  $A_s$  by its optical value to calculate  $\rho(s, t)$  is not justified and, strictly speaking, there is no selfconsistency condition.

This problem was studied in detail in paper<sup>12</sup> and it was found that this difficulty can be overcome and shown that up to possible logarithmic factors the cross section is bounded by a constant. The most essential assumptions are the use of dispersion relations in s for positive t and dominance of two-particle intermediate states in the t-channel unitarity sum up to  $t(s) - s^{\epsilon} [1/\sigma(s)]$ ,  $\sigma(s) - s^{\alpha}$ ,  $\epsilon$  and  $\alpha$  being positive numbers. It is worth emphasizing once more that to obtain this bound much stronger assumptions are needed than those used to derive the bound  $\sigma(s) < s^{1/3}$ .

#### References

<sup>1</sup>Ya. Pomeranchuk, unpublished (1966) and Yad. Fiz. <u>11</u>, 852 (1970) [Sov. J. Nucl. Phys. <u>11</u>, 477 (1970)]. The paper was written by V. N. Gribov and V. M. Shekhter with participation of I. M. Shmushkevich and published postmortem. This paper was also reviewed by I. Yu. Kobzarev in the Proceedings of the Kiev Conference on High Energy Physics (Naukova Dumka, Kiev, 1972), p. 557.

<sup>2</sup>T. Appelquist and T. Goldman, Harvard University preprint (1972).

- <sup>3</sup>J. D. Bjorken, Lecture given at the International School on High Energy Physics in Erevan (1971).
  <sup>4</sup>A. D. Dolgov, L. B. Okun, and V. I. Zakharov, Nucl. Phys. <u>B37</u>, 493 (1972); Yad. Fiz. <u>14</u>, 1044, 1247 (1971); Lecture given at the International School on High Energy Physics in Erevan (1971), to be published; Proceedings of the American Physical Society Meeting in Rochester (1971); Nucl. Phys. B41, 197 (1972).
- <sup>5</sup>T. Appelquist and J. D. Bjorken, Phys. Rev. D4, 3726 (1971).

<sup>6</sup>S. W. MacDowell and A. Martin, Phys. Rev. 135, B960 (1964).

- <sup>7</sup>L. Lucaszuk, Report #1385/VII/PH, Warsaw (1972).
- <sup>8</sup>A. D. Dolgov, L. B. Okun, and V. I. Zakharov, Phys. Letters <u>37B</u>, 298 (1971); Yad. Fiz. <u>15</u>, 808 (1972).
- <sup>9</sup>R. Rajaraman, Phys. Rev. 178, 2211, 2221 (1969).

- <sup>10</sup>A. A. Anselm and V. N. Gribov, preprint, A. F. Ioffe Physico-Technical Institute, Leningrad (1972).
- <sup>11</sup>V. N. Gribov and I. Ya. Pomeranchuk, Zh. Teor. i Exp. Fiz. <u>43</u>, 1970 (1962) [Sov. Phys. -JETP <u>16</u>, 1387 (1963)].

<sup>12</sup>V. N. Gribov, A. D. Dolgov, L. B. Okun, and V. I. Zakharov, Nucl. Phys., to be published.



Fig. 1.







Fig. 3.



Fig. 4.



Fig. 5.



Fig. 6.