CLASS NOTES ON

ELECTRICAL MEASUREMENTS & INSTRUMENTATION

FOR

7th SEMESTER OF

ELECTRICAL ENGINEERING & EEE (B.TECH PROGRAMME)

DEPARTMENT OF ELECTRICAL ENGINEERING

MEASURING INSTRUMENTS

1.1 Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories.

Instrument

Absolute Instrument **Secondary Instrument**

1.2 Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer.

1.3 Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.

Secondary instruments

Indicating instruments **Recording** **Integrating** **Electromechanically Indicating instruments**

1.3.1 Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer indication gives the magnitude of measuring quantity.

1.3.2 Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

1.3.3 Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time.

1.3.4 Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary. They are

(a) Deflecting force

(b) Controlling force

(c)Damping force

1.4 Deflecting force

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

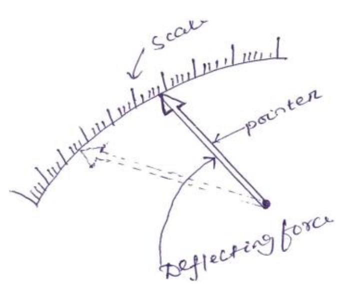

Fig. 1.1 Pointer scale

1.4.1 Magnitude effect

When a current passes through the coil (Fig.1.2), it produces a imaginary bar magnet. When a soft-iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

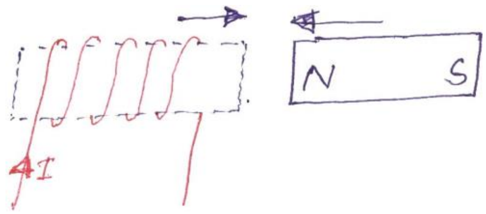

Fig. 1.2

If two soft iron pieces are place near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

1.4.2 Force between a permanent magnet and a current carrying coil

When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet and a force is produced between them. This principle is utilized in the moving coil type instrument.

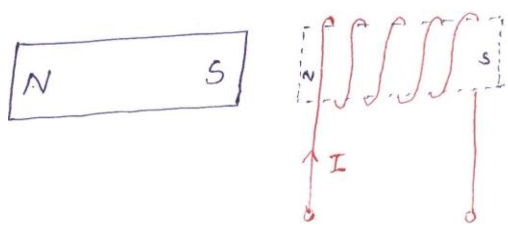

Fig. 1.3

1.4.3 Force between two current carrying coil

When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electrodynamometer type instrument.

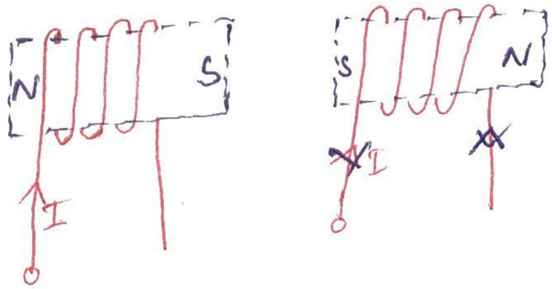

Fig. 1.4

1.5 Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c \tag{1.1}$$

1.5.1 Spring control

Two springs are attached on either end of spindle (Fig. 1.5).The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection θ.

$$T_C \propto \theta \tag{1.2}$$

The deflecting torque produced T_d proportional to 'I'. When $T_C = T_d$, the pointer will come to a steady position. Therefore

$$\theta \propto I \tag{1.3}$$

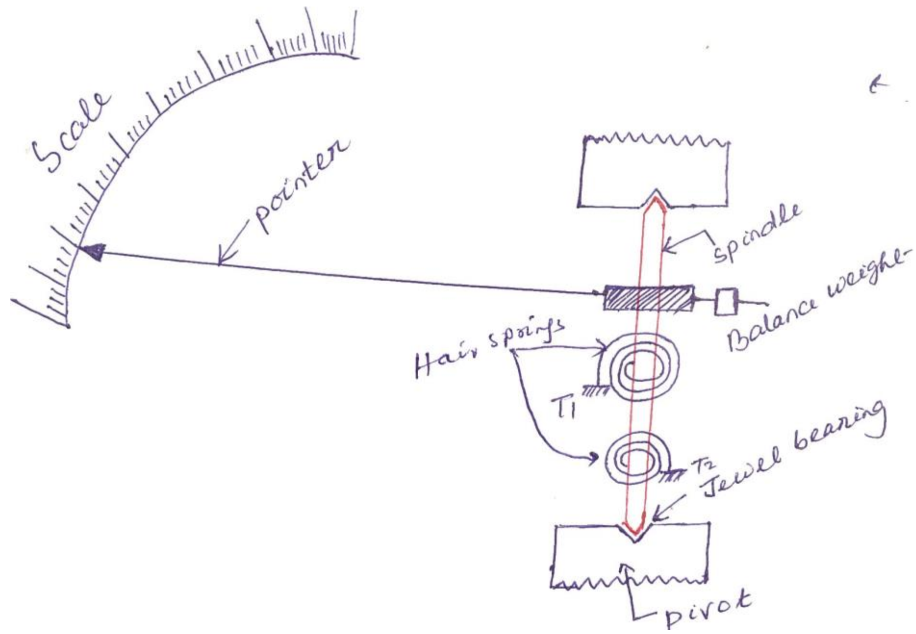

Fig. 1.5

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

1.6 Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation is quickly, a damping force is necessary. This force is produced by different systems.

(a) Air friction damping
(b) Fluid friction damping
(c) Eddy current damping

1.6.1 Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.

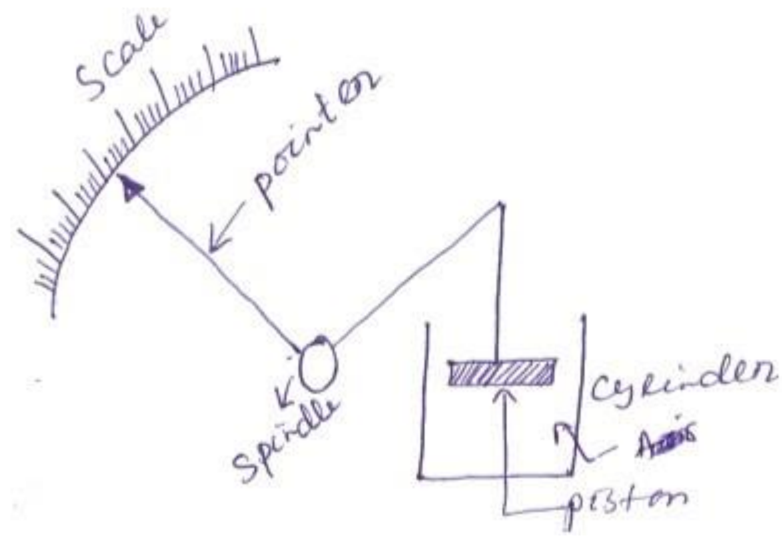

Fig. 1.6

If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

1.6.2 Eddy current damping

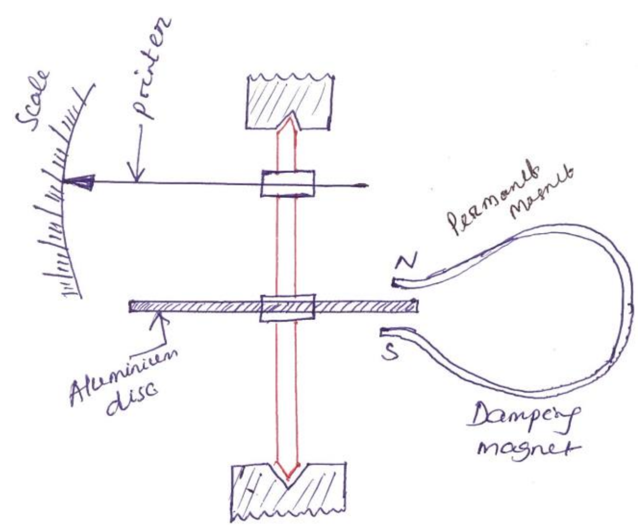

Fig. 1.6 Disc type

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz’s law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

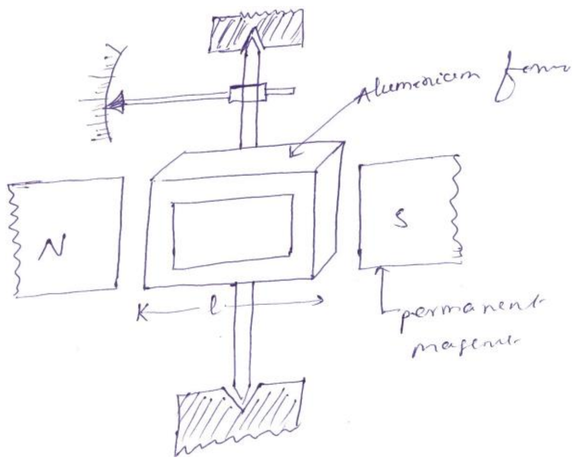

Fig. 1.6 Rectangular type

1.7 Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

Construction: A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminum former.

Control: Spring control is used.

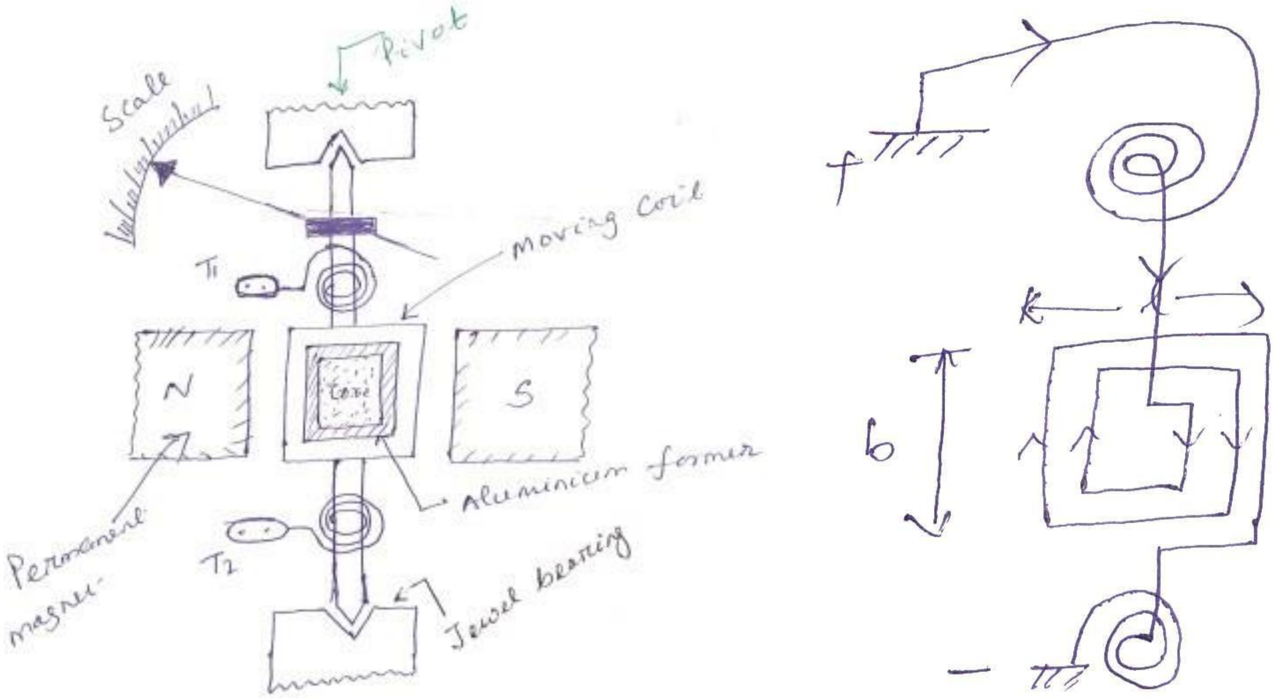

Fig. 1.7

Principle of operation

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.

If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC

Let T_d =deflecting torque

T_C = controlling torque

θ = angle of deflection

K=spring constant

b=width of the coil

l=height of the coil or length of coil

N=No. of turns

I=current

B=Flux density

A=area of the coil

The force produced in the coil is given by

$F = BIL \sin\theta$ (1.4)

When $\theta = 90^\circ$

For N turns, $F = NBIL$ (1.5)

Torque produced $T_d = F \times \perp_r$ distance (1.6)

$T_d = NBIL \times b = BINA$ (1.7)

$T_d = BANI$ (1.8)

$T_d \propto I$ (1.9)

<u>Advantages</u>

- ✓ Torque/weight is high
- ✓ Power consumption is less
- ✓ Scale is uniform
- ✓ Damping is very effective
- ✓ Since operating field is very strong, the effect of stray field is negligible
- ✓ Range of instrument can be extended

<u>Disadvantages</u>

- ✓ Use only for D.C.
- ✓ Cost is high
- ✓ Error is produced due to ageing effect of PMMC
- ✓ Friction and temperature error are present

1.7.1 Extension of range of PMMC instrument

Case-I: Shunt

A low shunt resistance connected in parrel with the ammeter to extent the range of current. Large current can be measured using low current rated ammeter by using a shunt.

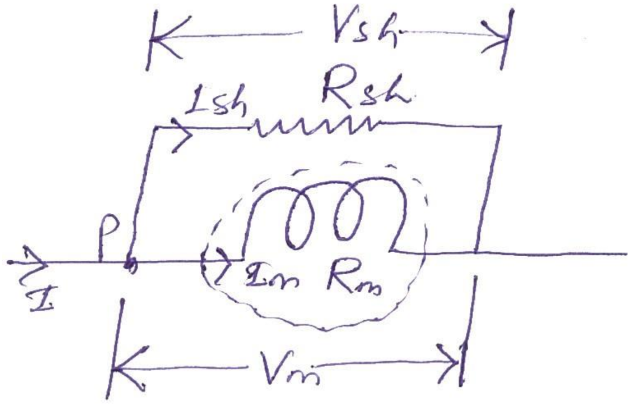

Fig. 1.8

Let R_m =Resistance of meter

R_{sh} =Resistance of shunt

I_m = Current through meter

I_{sh} =current through shunt

I= current to be measure

$$\therefore V_m = V_{sh} \tag{1.10}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m} \tag{1.11}$$

Apply KCL at 'P' $I = I_m + I_{sh}$ (1.12)

Eq^n (1.12) ÷ by I_m

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m} \tag{1.13}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \tag{1.14}$$

$$\therefore\ I = I_m\left(1 + \frac{R_m}{R_{sh}}\right) \tag{1.15}$$

$\left(1 + \frac{R_m}{R_{sh}}\right)$ is called multiplication factor

Shunt resistance is made of manganin. This has least thermoelectric emf. The change is resistance, due to change in temperature is negligible.

Case (II): Multiplier

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter of small rating with a multiplier.

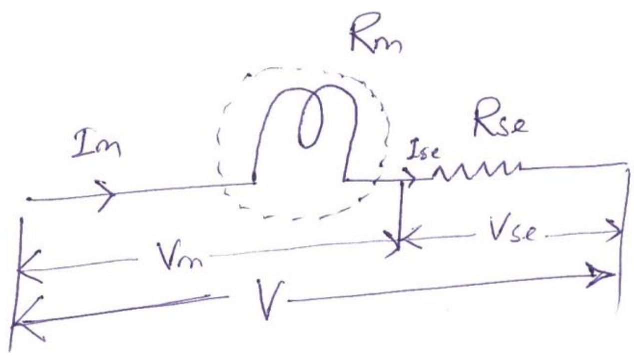

Fig. 1.9

Let R_m =resistance of meter

R_{se} =resistance of multiplier

V_m =Voltage across meter

V_{se} = Voltage across series resistance

V= voltage to be measured

$$I_m = I_{se} \tag{1.16}$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}} \tag{1.17}$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m} \tag{1.18}$$

Apply KVL, $V = V_m + V_{se}$ (1.19)

Eq^n (1.19) $\div V_m$

$$\frac{V}{V_m} = 1 + \frac{V_{se}}{V_m} = \left(1 + \frac{R_{se}}{R_m}\right) \quad (1.20)$$

$$\therefore V = V_m\left(1 + \frac{R_{se}}{R_m}\right) \quad (1.21)$$

$\left(1 + \frac{R_{se}}{R_m}\right) \rightarrow$ Multiplication factor

1.8 Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

1.8.1 Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10). The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

Torque developed by M.I

Let 'θ' be the deflection corresponding to a current of 'i' amp

Let the current increases by di, the corresponding deflection is '$\theta + d\theta$'

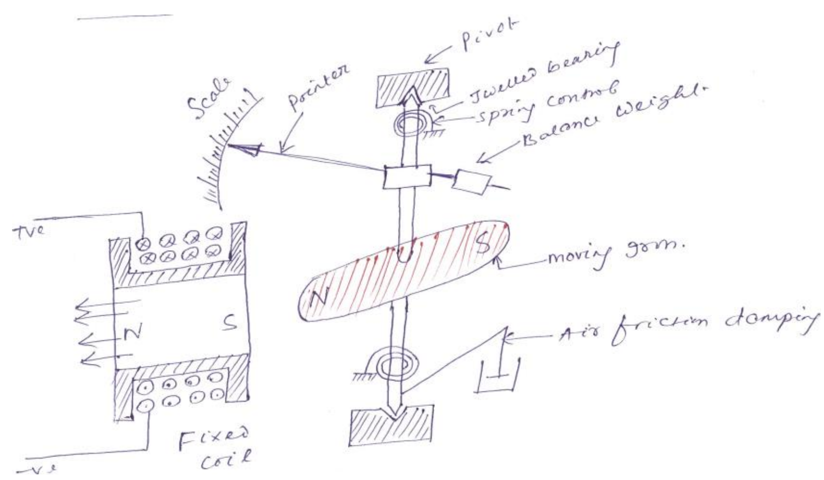

Fig. 1.10

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ‘L+dL’. The current change by ‘di’ is dt seconds.

Let the emf induced in the coil be ‘e’ volt.

$$e = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt} \tag{1.22}$$

Multiplying by ‘idt’ in equation (1.22)

$$e \times idt = L\frac{di}{dt} \times idt + i\frac{dL}{dt} \times idt \tag{1.23}$$

$$e \times idt = Lidi + i^2 dL \tag{1.24}$$

Eq^n (1.24) gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection.

Fig. 1.11

Change in energy stored=Final energy-initial energy stored

$$=\frac{1}{2}(L+dL)(i+di)^2-\frac{1}{2}Li^2$$

$$=\frac{1}{2}\{(L+dL)(i^2+di^2+2idi)-Li^2\}$$

$$=\frac{1}{2}\{(L+dL)(i^2+2idi)-Li^2\}$$

$$=\frac{1}{2}\{Li^2+2Lidi+i^2dL+2ididL-Li^2\}$$

$$=\frac{1}{2}\{2Lidi+i^2dL\}$$

$$=Lidi+\frac{1}{2}i^2dL \tag{1.25}$$

Mechanical work to move the pointer by $d\theta$

$$=T_d\,d\theta \tag{1.26}$$

By law of conservation of energy,
Electrical energy supplied=Increase in stored energy+ mechanical work done.

Input energy= Energy stored + Mechanical energy

$$Lidi+i^2dL=Lidi+\frac{1}{2}i^2dL+T_d\,d\theta \tag{1.27}$$

$$\frac{1}{2}i^2dL=T_d\,d\theta \tag{1.28}$$

$$T_d=\frac{1}{2}i^2\frac{dL}{d\theta} \tag{1.29}$$

At steady state condition $T_d = T_C$

$$\frac{1}{2}i^2\frac{dL}{d\theta}=K\theta \tag{1.30}$$

$$\theta=\frac{1}{2K}i^2\frac{dL}{d\theta} \tag{1.31}$$

$$\theta\propto i^2 \tag{1.32}$$

When the instruments measure AC, $\theta\propto i^2_{rms}$

Scale of the instrument is non uniform.

Advantages

- ✓ MI can be used in AC and DC
- ✓ It is cheap
- ✓ Supply is given to a fixed coil, not in moving coil.
- ✓ Simple construction
- ✓ Less friction error.

Disadvantages

- ✓ It suffers from eddy current and hysteresis error
- ✓ Scale is not uniform
- ✓ It consumed more power
- ✓ Calibration is different for AC and DC operation

1.8.2 Repulsion type moving iron instrument

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.

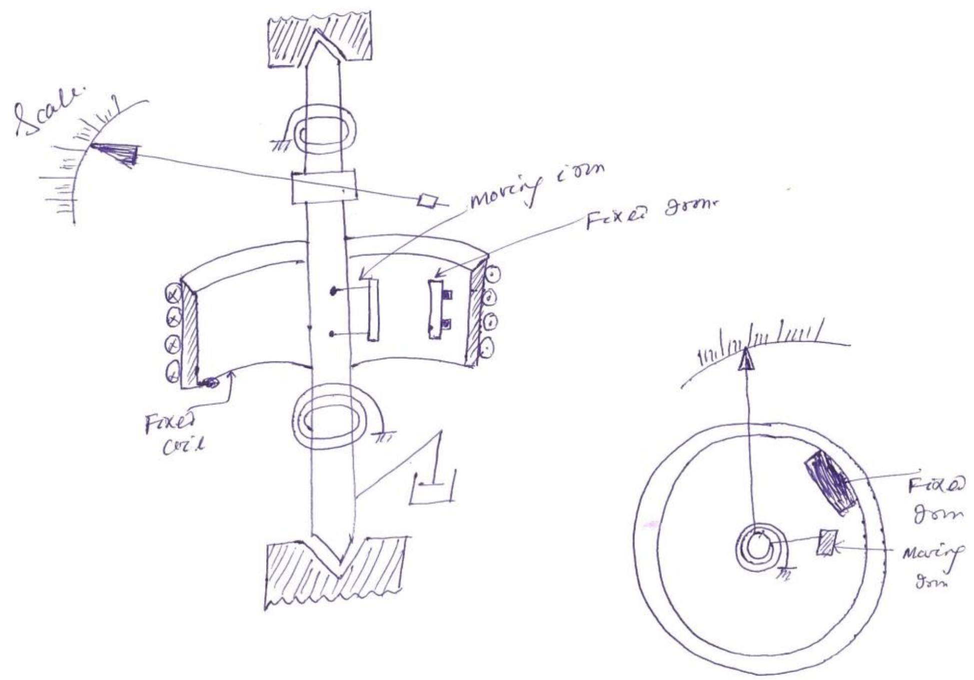

Fig. 1.12

1.9 Dynamometer (or) Electromagnetic moving coil instrument (EMMC)

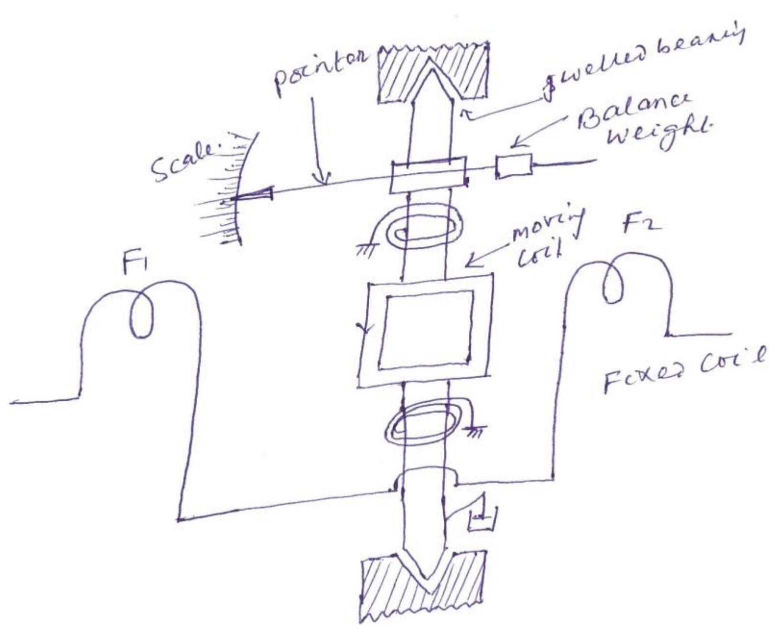

Fig. 1.13

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

Construction: A fixed coil is divided in to two equal half. The moving coil is placed between the two half of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non metallic former the moving coil is wounded.

Control: Spring control is used.

Damping: Air friction damping is used.

Principle of operation:

When the current flows through the fixed coil, it produced a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i_1 and i_2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC

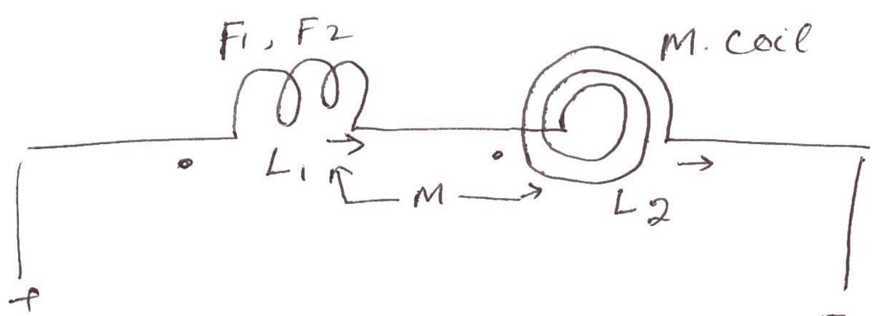

Fig. 1.14

Let

L_1=Self inductance of fixed coil

L_2= Self inductance of moving coil

M=mutual inductance between fixed coil and moving coil

i_1=current through fixed coil

i_2=current through moving coil

Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M \tag{1.33}$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta} \tag{1.34}$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta}(L_1 + L_2 + 2M) \tag{1.35}$$

The value of L_1 and L_2 are independent of 'θ' but 'M' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta} \tag{1.36}$$

$$T_d = i^2 \frac{dM}{d\theta} \tag{1.37}$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \tag{1.38}$$

$$T_C = T_d \tag{1.39}$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta} \tag{1.40}$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil.

1.9.1 Extension of EMMC instrument

Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are designed such that the resistance of each branch is same.

Therefore

$I_1 = I_2 = I$

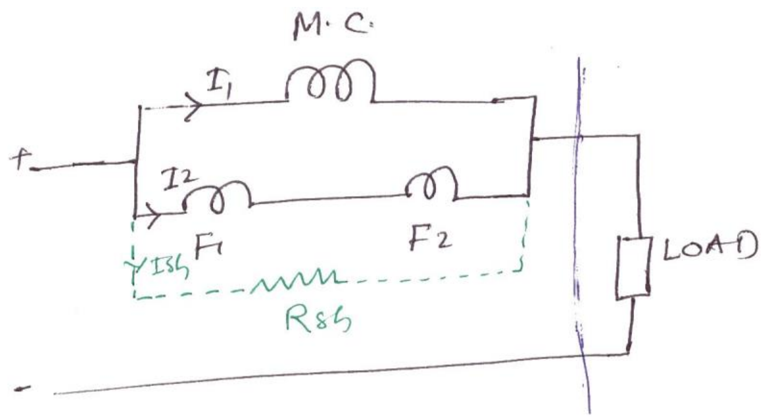

Fig. 1.15

To extend the range of current a shunt may be connected in parallel with the meter. The value R_{sh} is designed such that equal current flows through moving coil and fixed coil.

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \qquad (1.41)$$

Or $$\therefore T_d = I^2 \frac{dM}{d\theta} \qquad (1.42)$$

$$T_C = K\theta \qquad (1.43)$$

$$\theta = \frac{I^2}{K} \frac{dM}{d\theta} \qquad (1.44)$$

$$\therefore \theta \propto I^2 \text{ (Scale is not uniform)} \qquad (1.45)$$

Case-II Voltmeter connection

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be connected in series to extent the range of voltmeter.

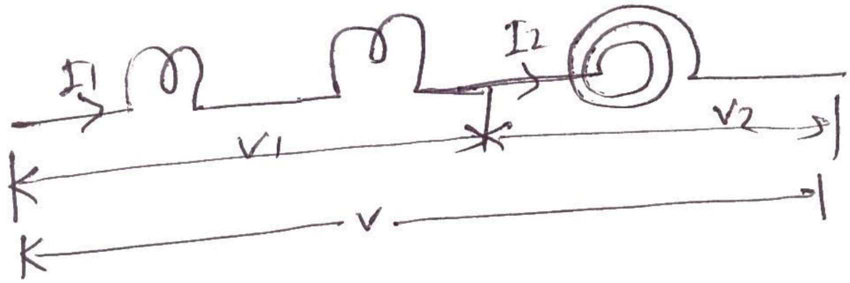

Fig. 1.16

$$I_1 = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2} \tag{1.46}$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta} \tag{1.47}$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta} \tag{1.48}$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta} \tag{1.49}$$

$$T_d \propto V^2 \tag{1.50}$$

$$\therefore \theta \propto V^2 \quad \text{(Scale in not uniform)} \tag{1.51}$$

Case-III As w**attmeter**

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

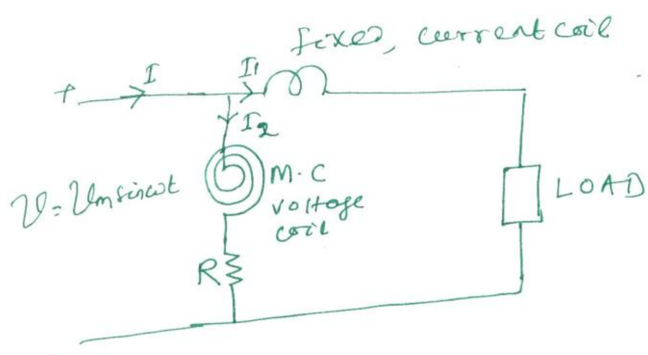

Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance 'R'. The current,

$$I_2 = \frac{v_m \sin wt}{R} \tag{1.52}$$

Let the phase difference between the currents I_1 and I_2 is ϕ

$$I_1 = I_m \sin(wt - \phi) \tag{1.53}$$

$$T_d = I_1 I_2 \frac{dM}{d\theta} \tag{1.54}$$

$$T_d = I_m \sin(wt - \phi) \times \frac{V_m \sin wt}{R} \frac{dM}{d\theta} \tag{1.55}$$

$$T_d = \frac{1}{R} (I_m V_m \sin wt \sin(wt - \phi)) \frac{dM}{d\theta} \tag{1.56}$$

$$T_d = \frac{1}{R} I_m V_m \sin wt.\sin(wt - \phi) \frac{dM}{d\theta} \tag{1.57}$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\Pi} \int_0^{2\Pi} T_d \times d(wt) \tag{1.58}$$

$$(T_d)_{avg} = \frac{1}{2\Pi} \int_0^{2\Pi} \frac{1}{R} \times I_m V_m \sin wt.\sin(wt - \phi) \frac{dM}{d\theta} \times d(wt) \tag{1.59}$$

$$(T_d)_{avg} = \frac{V_m I_m}{2 \times 2\Pi} \times \frac{1}{R} \times \frac{dM}{d\theta} \left[\int \{\cos\phi - \cos(2wt - \phi)\} dwt \right] \tag{1.60}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta} \left[\int_0^{2\Pi} \cos\phi.dwt - \int_0^{2\Pi} \cos(2wt - \phi).dwt \right] \tag{1.61}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta} \left[\cos\phi [wt]_0^{2\Pi} \right] \tag{1.62}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta} [\cos\phi(2\Pi - 0)] \tag{1.63}$$

$$(T_d)_{avg} = \frac{V_m I_m}{2} \times \frac{1}{R} \times \frac{dM}{d\theta} \times \cos\phi \tag{1.64}$$

$$(T_d)_{avg} = V_{rms} \times I_{rms} \times \cos\phi \times \frac{1}{R} \times \frac{dM}{d\theta} \tag{1.65}$$

$$(T_d)_{avg} \propto KVI\cos\phi \qquad (1.66)$$

$$T_C \propto \theta \qquad (1.67)$$

$$\theta \propto KVI\cos\phi \qquad (1.68)$$

$$\theta \propto VI\cos\phi \qquad (1.69)$$

Advantages

- ✓ It can be used for voltmeter, ammeter and wattmeter
- ✓ Hysteresis error is nill
- ✓ Eddy current error is nill
- ✓ Damping is effective
- ✓ It can be measure correctively and accurately the rms value of the voltage

Disadvantages

- ✓ Scale is not uniform
- ✓ Power consumption is high(because of high resistance)
- ✓ Cost is more
- ✓ Error is produced due to frequency, temperature and stray field.
- ✓ Torque/weight is low.(Because field strength is very low)

Errors in PMMC

- ✓ The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.
- ✓ The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.
- ✓ When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

1.10 Difference between attraction and repulsion type instrument

An attraction type instrument will usually have a lower inductance, compare to repulsion type instrument. But in other hand, repulsion type instruments are more suitable for economical production in manufacture and nearly uniform scale is more easily obtained. They are therefore much more common than attraction type.

1.11 Characteristics of meter

1.11.1 Full scale deflection current(I_{FSD})

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between 2 μA to 30mA.

1.11.2 Resistance of the coil(R_m)

This is ohmic resistance of the moving coil. It is due to ρ, L and A. For an ammeter this should be as small as possible.

1.11.3 Sensitivity of the meter(S)

$$S = \frac{1}{I_{FSD}}(\Omega / volt), \uparrow S = \frac{Z\uparrow}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in Ω/volt. When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading affect is negligible. It is also defend as one over full scale deflection current.

1.12 Error in M.I instrument

1.12.1 Temperature error

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganin, so that the resistance is almost constant.

1.12.2 Hysteresis error

Due to hysteresis affect the reading of the instrument will not be correct. When the current is decreasing, the flux produced will not decrease suddenly. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

1.12.3 Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

1.12.4 Stray field error

Since the operating field is weak, the effect of stray field is more. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

1.12.5 Frequency error

When the frequency changes the reactance of the coil changes.

$$Z = \sqrt{(R_m + R_S)^2 + X_L^2} \tag{1.70}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R_m + R_S)^2 + X_L^2}} \tag{1.71}$$

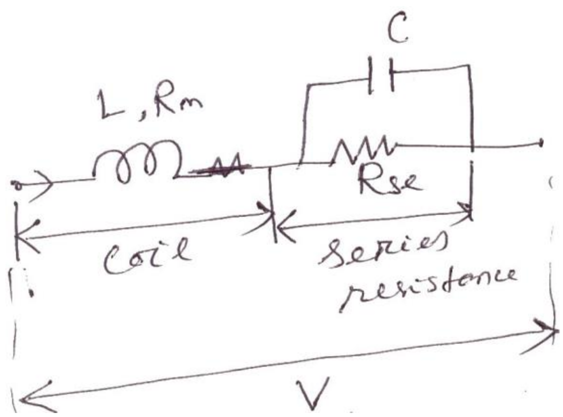

Fig. 1.18

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance, ($X_L - X_C$) is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$C = 0.41 \frac{L}{(R_S)^2} \tag{1.72}$$

1.13 Electrostatic instrument

In multi cellular construction several vans and quadrants are provided. The voltage is to be measured is applied between the vanes and quadrant. The force of attraction between the vanes and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage.

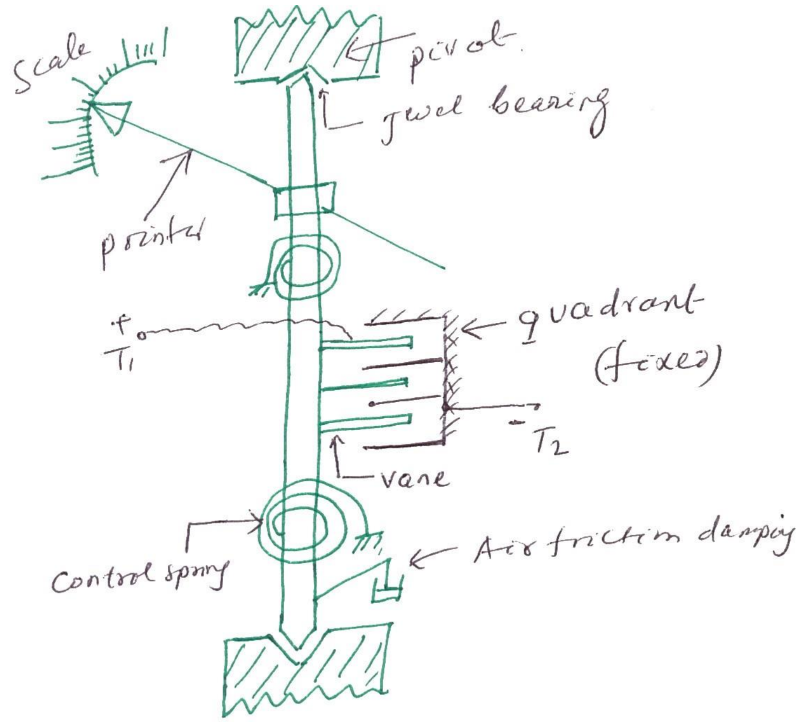

Fig. 1.19

<u>Torque develop by electrostatic instrument</u>

V=Voltage applied between vane and quadrant

C=capacitance between vane and quadrant

Energy stored= $\frac{1}{2}CV^2$ (1.73)

Let ' θ' be the deflection corresponding to a voltage V.

Let the voltage increases by dv, the corresponding deflection is' $\theta + d\theta$'

When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{dC}{dt}V + C\frac{dV}{dt} \quad (1.74)$$

$V \times dt$ multiply on both side of equation (1.74)

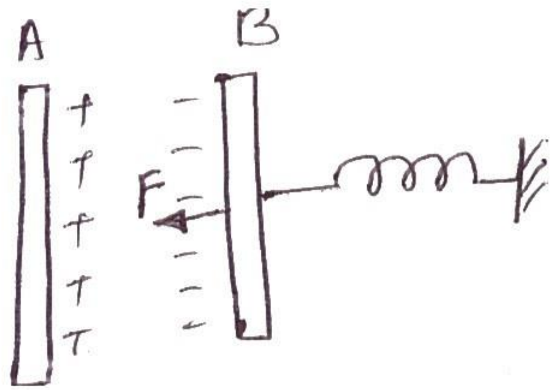

Fig. 1.20

$$Vidt = \frac{dC}{dt}V^2dt + CV\frac{dV}{dt}dt \tag{1.75}$$

$$Vidt = V^2dC + CVdV \tag{1.76}$$

Change in stored energy= $\frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$ (1.77)

$$= \frac{1}{2}\left[(C + dC)V^2 + dV^2 + 2VdV\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left[CV^2 + CdV^2 + 2CVdV + V^2dC + dCdV^2 + 2VdVdC\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}V^2dC + CVdV$$

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + F \times rd\theta \tag{1.78}$$

$$T_d \times d\theta = \frac{1}{2}V^2dC \tag{1.79}$$

$$T_d = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right) \tag{1.80}$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right) \tag{1.81}$$

$$\theta = \frac{1}{2K}V^2\left(\frac{dC}{d\theta}\right) \tag{1.82}$$

Advantages

- ✓ It is used in both AC and DC.
- ✓ There is no frequency error.
- ✓ There is no hysteresis error.
- ✓ There is no stray magnetic field error. Because the instrument works on electrostatic principle.
- ✓ It is used for high voltage
- ✓ Power consumption is negligible.

Disadvantages

- ✓ Scale is not uniform
- ✓ Large in size
- ✓ Cost is more

1.14 Multi range Ammeter

When the switch is connected to position (1), the supplied current I_1

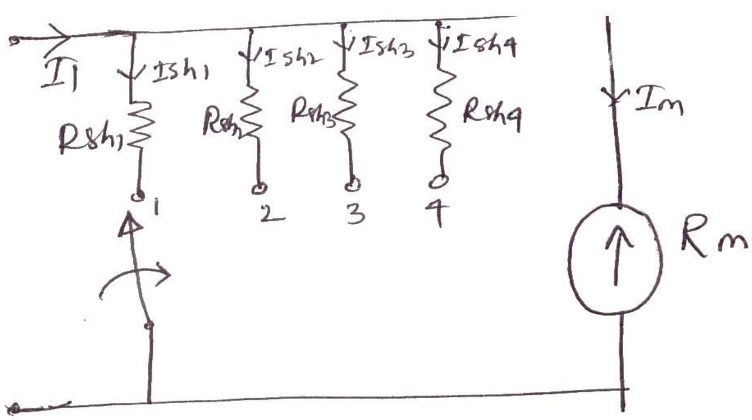

Fig. 1.21

$$I_{sh1}R_{sh1} = I_m R_m \tag{1.83}$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m} \tag{1.84}$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, \quad R_{sh1} = \frac{R_m}{m_1 - 1}, \quad m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, \quad m_2 = \frac{I_2}{I_m} \tag{1.85}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, \quad m_3 = \frac{I_3}{I_m} \tag{1.86}$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, \quad m_4 = \frac{I_4}{I_m} \tag{1.87}$$

1.15 Ayrton shunt

$$R_1 = R_{sh1} - R_{sh2} \tag{1.88}$$

$$R_2 = R_{sh2} - R_{sh3} \tag{1.89}$$

$$R_3 = R_{sh3} - R_{sh4} \tag{1.90}$$

$$R_4 = R_{sh4} \tag{1.91}$$

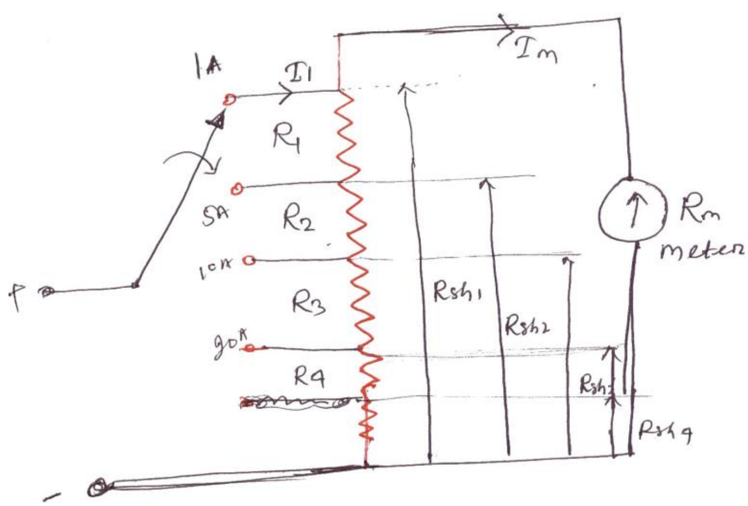

Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Aryton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

1.16 Multi range D.C. voltmeter

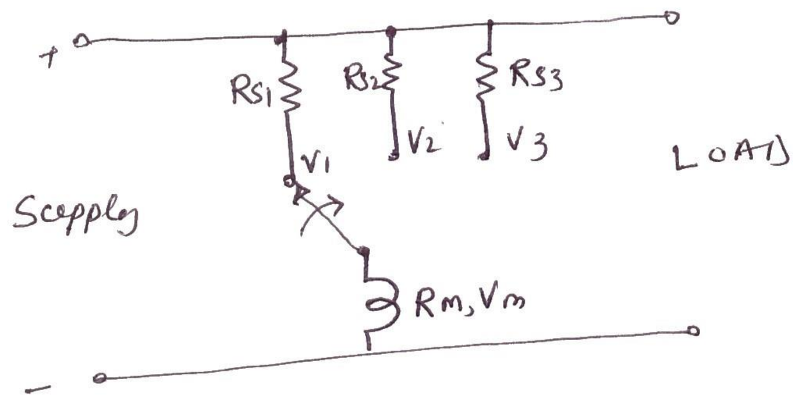

Fig. 1.23

$$R_{s1} = R_m\,(m_1 - 1)$$
$$R_{s2} = R_m\,(m_2 - 1) \qquad (1.92)$$
$$R_{s3} = R_m\,(m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m},\; m_2 = \frac{V_2}{V_m},\; m_3 = \frac{V_3}{V_m} \qquad (1.93)$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required**.**

1.17 Potential divider arrangement

The resistance R_1, R_2, R_3 and R_4 is connected in series to obtained the ranges V_1, V_2, V_3 and V_4

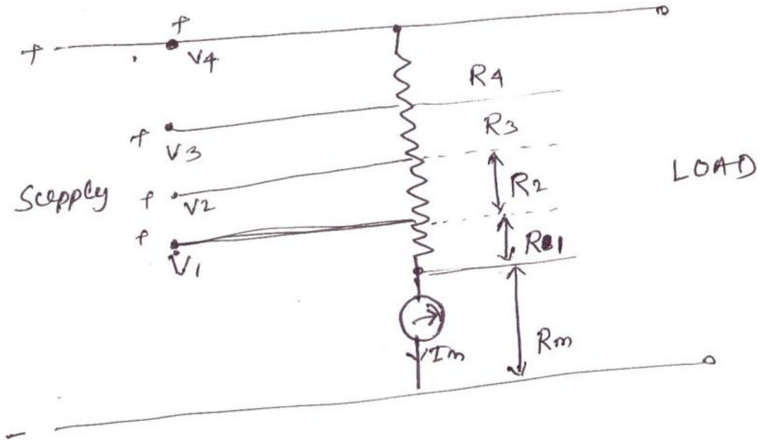

Fig. 1.24

Consider for voltage V$_1$, $(R_1 + R_m)I_m = V_1$

$$\therefore\ R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{\left(\frac{V_m}{R_m}\right)} - R_m = \left(\frac{V_1}{V_m}\right)R_m - R_m \qquad (1.94)$$

$$R_1 = (m_1 - 1)R_m \qquad (1.95)$$

For V$_2$, $(R_2 + R_1 + R_m)I_m = V_2 \Rightarrow R_2 = \frac{V_2}{I_m} - R_1 - R_m$ (1.96)

$$R_2 = \frac{V_2}{\left(\frac{V_m}{R_m}\right)} - (m_1 - 1)R_m - R_m \qquad (1.97)$$

$$R_2 = m_2 R_m - R_m - (m_1 - 1)R_m$$

$$= R_m(m_2 - 1 - m_1 + 1) \qquad (1.98)$$

$$R_2 = (m_2 - m_1)R_m \qquad (1.99)$$

For V$_3$ $(R_3 + R_2 + R_1 + R_m)I_m = V_3$

$$R_3 = \frac{V_3}{I_m} - R_2 - R_1 - R_m$$

$$= \frac{V_3}{V_m}R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$= m_3 R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_3 = (m_3 - m_2)R_m$$

For V_4 $\quad (R_4 + R_3 + R_2 + R_1 + R_m)I_m = V_4$

$$R_4 = \frac{V_4}{I_m} - R_3 - R_2 - R_1 - R_m$$

$$= \left(\frac{V_4}{V_m}\right)R_m - (m_3 - m_2)R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_4 = R_m\left[m_4 - m_3 + m_2 - m_2 + m_1 - m_1 + 1 - 1\right]$$

$$R_4 = (m_4 - m_3)R_m$$

Example: 1.1

A PMMC ammeter has the following specification

Coil dimension are 1cm$\times$ 1cm. Spring constant is 0.15 $\times 10^{-6}\, N-m/rad$, Flux density is $1.5 \times 10^{-3}\, wb/m^2$ .Determine the no. of turns required to produce a deflection of 90^0 when a current 2mA flows through the coil.

Solution:

At steady state **condition** $T_d = T_C$

$BANI = K\theta$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

A=$1\times10^{-4}\, m^2$

K= $0.15 \times 10^{-6} \frac{N-m}{rad}$

B=$1.5\times10^{-3} wb/m^2$

I= $2 \times 10^{-3} A$

$\theta = 90^\circ = \frac{\Pi}{2} rad$

N=785 ans.

Example: 1.2

The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV.The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

Solution:

Case-1

V_m =400mV

$I_m = 20mA$

I=200A

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20\Omega$$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}}\right)$$

$$200 = 20 \times 10^{-3} \left[1 + \frac{20}{R_{sh}}\right]$$

$R_{sh} = 2 \times 10^{-3}\,\Omega$

Case-II

V=1000V

$$V = V_m \left(1 + \frac{R_{se}}{R_m}\right)$$

$$4000 = 400 \times 10^{-3} \left(1 + \frac{R_{se}}{20}\right)$$

$R_{se} = 49.98k\Omega$

Example: 1.3

A 150 v moving iron voltmeter is intended for 50HZ, has a resistance of 3kΩ. Find the series resistance required to extent the range of instrument to 300v. If the 300V instrument is used to measure a d.c. voltage of 200V. Find the voltage across the meter?

Solution:

$R_m = 3k\Omega$,$V_m = 150V$,$V = 300V$

$$V = V_m\left(1 + \frac{R_{se}}{R_m}\right)$$

$$300 = 150\left(1 + \frac{R_{se}}{3}\right) \Rightarrow R_{se} = 3k\Omega$$

Case-II $$V = V_m\left(1 + \frac{R_{se}}{R_m}\right)$$

$$200 = V_m\left(1 + \frac{3}{3}\right)$$

$$\therefore V_m = 100V$$ Ans

Example: 1.4

What is the value of series resistance to be used to extent '0'to 200V range of 20,000Ω/volt voltmeter to 0 to 2000 volt?

Solution:

$$V_{se} = V - V = 1800$$

$$I_{FSD} = \frac{1}{20000} = \frac{1}{Sensitivity}$$

$$V_{se} = R_{se} \times i_{FSD} \Rightarrow R_{se} = 36M\Omega$$ ans.

Example: 1.5

A moving coil instrument whose resistance is 25Ω gives a full scale deflection with a current of 1mA. This instrument is to be used with a manganin shunt, to extent its range to 100mA. Calculate the error caused by a 10^0C rise in temperature when:

(a) Copper moving coil is connected directly across the manganin shunt.

(b) A 75 ohm manganin resistance is used in series with the instrument moving coil.

The temperature co-efficient of copper is $0.004/^0$C and that of manganin is 0.00015^0/C.

Solution:

Case-1

$$I_m = 1mA$$

$$R_m = 25\Omega$$

I=100mA

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{25}{R_{sh}}\right) \Rightarrow \frac{25}{R_{sh}} = 99$$

$$\Rightarrow R_{sh} = \frac{25}{99} = 0.2525\Omega$$

Instrument resistance for 10^0C rise in temperature, $R_{mt} = 25(1 + 0.004\times10)$

$$R_t = R_o\,(1+\rho_t \times t)$$

$$R_{m/t=10^\circ} = 26\Omega$$

Shunt resistance for 10^0C, rise in temperature

$$R_{sh/t=10^\circ} = 0.2525(1 + 0.00015\times10) = 0.2529\Omega$$

Current through the meter for 100mA in the main circuit for 10^0C rise in temperature

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)\Bigg|_{t=10^\circ C}$$

$$100 = I_{mt}\left(1+\frac{26}{0.2529}\right)$$

$$I_m\big|_{t=10} = 0.963mA$$

But normal meter current=1mA

Error due to rise in temperature=(0.963-1)*100=-3.7%

Case-b As voltmeter

Total resistance in the meter circuit= $R_m + R_{sh} = 25 + 75 = 100\Omega$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{100}{R_{sh}}\right)$$

$$R_{sh} = \frac{100}{100-1} = 1.01\Omega$$

Resistance of the instrument circuit for 10^0C rise in temperature

$$R_m|_{t=10} = 25(1 + 0.004 \times 10) + 75(1 + 0.00015 \times 10) = 101.11\Omega$$

Shunt resistance for 10^0C rise in temperature

$$R_{sh}|_{t=10} = 1.01(1 + 0.00015 \times 10) = 1.0115\Omega$$

$$I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)$$

$$100 = I_m\left(1 + \frac{101.11}{1.0115}\right)$$

$$I_m\Big|_{t=10^\circ} = 0.9905mA$$

Error =(0.9905-1)*100=-0.95%

Example: 1.6

The coil of a 600V M.I meter has an inductance of 1 henery. It gives correct reading at 50HZ and requires 100mA. For its full scale deflection, what is % error in the meter when connected to 200V D.C. by comparing with 200V A.C?

Solution:

$V_m = 600V$, $I_m = 100mA$

Case-I A.C.

$$Z_m = \frac{V_m}{I_m} = \frac{600}{0.1} = 6000\Omega$$

$$X_L = 2\Pi fL = 314\Omega$$

$$R_m = \sqrt{Z_m^2 - X_L^2} = \sqrt{(6000)^2 - (314)^2} = 5990\Omega$$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{200}{6000} = 33.33mA$$

Case-II D.C

$$I_{DC} = \frac{V_{DC}}{R_m} = \frac{200}{5990} = 33.39mA$$

Error= $\dfrac{I_{DC} - I_{AC}}{I_{AC}} \times 100 = \dfrac{33.39 - 33.33}{33.33} \times 100 = 0.18\%$

Example: 1.7

A 250V M.I. voltmeter has coil resistance of 500Ω, coil inductance 0f 1.04 H and series resistance of 2kΩ. The meter reads correctively at 250V D.C. What will be the value of capacitance to be used for shunting the series resistance to make the meter read correctly at 50HZ? What is the reading of voltmeter on A.C. without capacitance?

Solution: $C = 0.41 \dfrac{L}{(R_S)^2}$

$$= 0.41 \times \frac{\square 1.04}{(2 \times 10^3)^2} = 0.1 \mu F$$

For A.C $Z = \sqrt{(R_m + R_{Se})^2 + X_L^2}$

$$Z = \sqrt{(500 + 2000)^2 + (314)^2} = 2520\Omega$$

With D.C

$R_{total} = 2500\Omega$

For $2500\Omega \rightarrow 250$V

$$1\Omega \rightarrow \frac{250}{2500}$$

$$2520\Omega \rightarrow \frac{250}{2500} \times 2520 = 248V$$

Example: 1.8

The relationship between inductance of moving iron ammeter, the current and the position of pointer is as follows:

Reading (A)	1.2	1.4	1.6	1.8
Deflection (degree)	36.5	49.5	61.5	74.5
Inductance (μH)	575.2	576.5	577.8	578.8

Calculate the deflecting torque and the spring constant when the current is 1.5A?

Solution:

For current I=1.5A, θ=55.5 degree=0.96865 rad

$$\frac{dL}{d\theta} = \frac{577.65 - 576.5}{60 - 49.5} = 0.11\mu H / \deg ree = 6.3\mu H / rad$$

Deflecting torque , $T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} = \frac{1}{2} (1.5)^2 \times 6.3 \times 10^{-6} = 7.09 \times 10^{-6} N - m$

Spring constant, $K = \frac{T_d}{\theta} = \frac{7.09 \times 10^{-6}}{0.968} = 7.319 \times 10^{-6} \frac{N - m}{rad}$

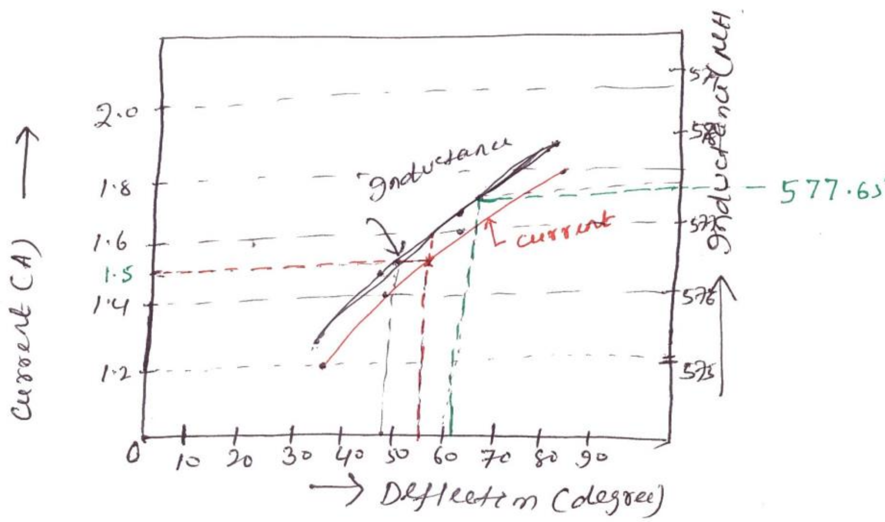

Fig. 1.25

Example: 1.9

For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection θ as $M = -6 \cos(\theta + 30^\circ) mH$ .Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60^0.

Solution:

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6 \cos(\theta + 30^\circ)$$

$$\frac{dM}{d\theta} = 6 \sin(\theta + 30) mH$$

$$\frac{dM}{d\theta}\Big|_{\theta=60} = 6 \sin 90 = 6 mH / \deg$$

$$T_d = I^2 \frac{dM}{d\theta} = (50 \times 10^{-3})^2 \times 6 \times 10^{-3} = 15 \times 10^{-6} N - m$$

Example: 1.10

The inductance of a moving iron ammeter with a full scale deflection of 90^0 at 1.5A, is given by the expression $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$, where θ is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

Solution:

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\left.\frac{dL}{d\theta}\right|_{\theta=90^\circ} = 40 - 8\theta - 3\theta^2 \mu H / rad$$

$$\left.\frac{dL}{d\theta}\right|_{\theta=90^\circ} = 40 - 8 \times \frac{\Pi}{2} - 3\left(\frac{\Pi}{2}\right)^2 \mu H / rad = 20 \mu H / rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\frac{\Pi}{2} = \frac{1}{2} \frac{(1.5)^2}{K} \times 20 \times 10^{-6}$$

K=Spring constant=$14.32 \times 10^{-6} N - m / rad$

For I=1A, $\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$

$$\therefore \theta = \frac{1}{2} \times \frac{(1)^2}{14.32 \times 10^{-6}} \left(40 - 8\theta - 3\theta^2\right)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^\circ$$

Example: 1.11

The inductance of a moving iron instrument is given by $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$, where θ is the deflection in radian from zero position. The spring constant is $12 \times 10^{-6} N - m / rad$. Estimate the deflection for a current of 5A.

Solution:

$$\frac{dL}{d\theta} = (5 - 2\theta)\frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} (5 - 2\theta) \times 10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^\circ$$

Example: 1.12

The following figure gives the relation between deflection and inductance of a moving iron instrument.

Deflection (degree)	20	30	40	50	60	70	80	90
Inductance (μH)	335	345	355.5	366.5	376.5	385	391.2	396.5

Find the current and the torque to give a deflection of (a) 30^0 (b) 80^0 . Given that control spring constant is $0.4 \times 10^{-6} N - m / \deg ree$

Solution:

$$\theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

(a) For $\theta = 30^\circ$

The curve is linear

$$\therefore \left(\frac{dL}{d\theta}\right)_{\theta=30} = \frac{355.5 - 335}{40 - 20} = 1.075 \mu H / \deg ree = 58.7 \mu H / rad$$

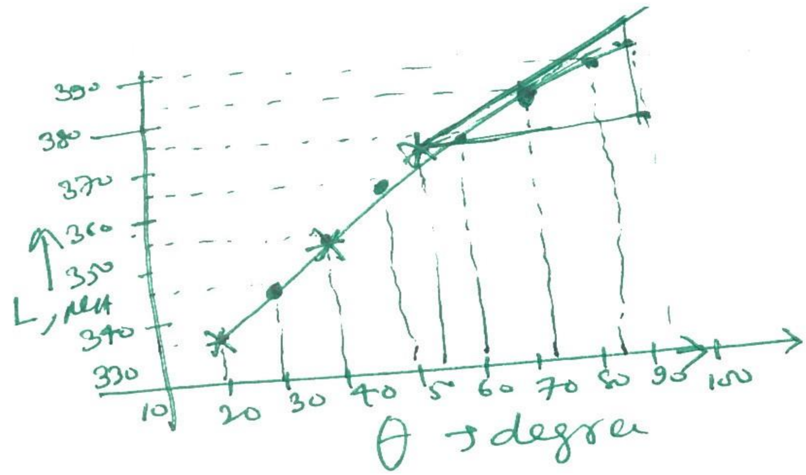

Fig. 1.26

Example: 1.13

In an electrostatic voltmeter the full scale deflection is obtained when the moving plate turns through 90^0. The torsional constant is $10\times10^{-6}\,N-m/rad$. The relation between the angle of deflection and capacitance between the fixed and moving plates is given by

Deflection (degree)	0	10	20	30	40	50	60	70	80	90
Capacitance (PF)	81.4	121	156	189.2	220	246	272	294	316	334

Find the voltage applied to the instrument when the deflection is 90^0?

Solution:

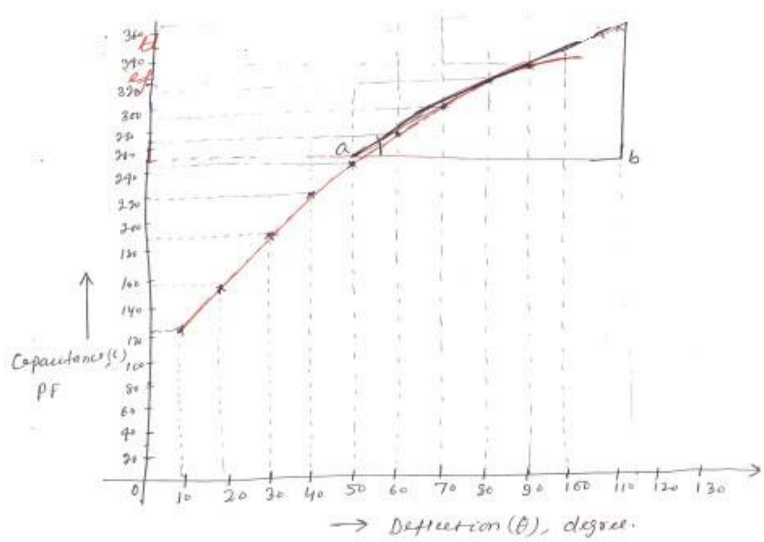

Fig. 1.27

$$\frac{dC}{d\theta} = \tan\theta = \frac{bc}{ab} = \frac{370 - 250}{110 - 44} = 1.82PF / \deg ree = 104.2PF / rad$$

Spring constant $K = 10 \times 10^{-6} \frac{N - m}{rad} = 0.1745 \times 10^{-6} N - m / \deg ree$

$$\theta = \frac{1}{2K} V^2 \left(\frac{dC}{d\theta}\right) \Rightarrow V = \sqrt{\frac{2K\theta}{\frac{dC}{d\theta}}}$$

$$V = \sqrt{\frac{2 \times 0.1745 \times 10^{-6} \times 90}{104.2 \times 10^{-12}}} = 549 volt$$

Example: 1.14

Design a multi range d.c. mille ammeter using a basic movement with an internal resistance $R_m = 50\Omega$ and a full scale deflection current $I_m = 1mA$. The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

Solution:

Case-I 0-10mA

Multiplying power $m = \frac{I}{I_m} = \frac{10}{1} = 10$

$\therefore$ Shunt resistance $R_{sh1} = \frac{R_m}{m - 1} = \frac{50}{10 - 1} = 5.55\Omega$

Case-II 0-50mA

$$m = \frac{50}{1} = 50$$

$$R_{sh2} = \frac{R_m}{m - 1} = \frac{50}{50 - 1} = 1.03\Omega$$

Case-III 0-100mA, $m = \frac{100}{1} = 100\Omega$

$$R_{sh3} = \frac{R_m}{m - 1} = \frac{50}{100 - 1} = 0.506\Omega$$

Case-IV 0-500mA**,** $m = \frac{500}{1} = 500\Omega$

$$R_{sh4} = \frac{R_m}{m - 1} = \frac{50}{500 - 1} = 0.1\Omega$$

Example: 1.15

A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of 120^0, when a potential difference of 100mV is applied across it. The moving coil has dimension of 30mm*25mm and is wounded with 100 turns. The control spring constant is $0.375\times10^{-6} N-m/\text{degree}$. Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper=$1.7\times10^{-8}\Omega m$.

Solution:

Data given

$V_m = 100mV$

$R_m = 20\Omega$

$\theta = 120^\circ$

N=100

$K = 0.375\times10^{-6} N-m/\text{deg}\,ree$

$R_C = 30\% of R_m$

$\rho = 1.7\times10^{-8}\,\Omega m$

$$I_m = \frac{V_m}{R_m} = 5\times10^{-3} A$$

$T_d = BANI\ , T_C = K\theta = 0.375\times10^{-6}\times120 = 45\times10^{-6} N-m$

$$B = \frac{T_d}{ANI} = \frac{45\times10^{-6}}{30\times25\times10^{-6}\times100\times5\times10^{-3}} = 0.12wb/m^2$$

$R_C = 0.3\times20 = 6\Omega$

Length of mean turn path =2(a+b) =2(55)=110mm

$$R_C = N\left(\frac{\rho l}{A}\right)$$

$$A = \frac{N\times\rho\times(l_t)}{R_C} = \frac{100\times1.7\times10^{-8}\times110\times10^{-3}}{6}$$

$$= 3.116 \times 10^{-8}\, m^2$$
$$= 31.16 \times 10^{-3}\, mm^2$$
$$A = \frac{\Pi}{4} d^2 \Rightarrow d = 0.2mm$$

Example: 1.16

A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

(1) The shunt resistance for a full scale deflection corresponding to 100A

(2) The resistance for full scale reading with 1000V.

Calculate the power dissipation in each case?

Solution:

Data given

$$I_m = 10mA$$
$$V_m = 100mV$$
$$I = 100\, A$$
$$I = I_m \left(1 + \frac{R_m}{R_{sh}}\right)$$
$$100 = 10 \times 10^{-3} \left(1 + \frac{10}{R_{sh}}\right)$$
$$R_{sh} = 1.001 \times 10^{-3}\, \Omega$$
$$R_{se} = ??, V = 1000V$$
$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$
$$V = V_m \left(1 + \frac{R_{se}}{R_m}\right)$$
$$1000 = 100 \times 10^{-3} \left(1 + \frac{R_{se}}{10}\right)$$
$$\therefore R_{se} = 99.99K\Omega$$

Example: 1.17

Design an Aryton shunt to provide an ammeter with current ranges of 1A,5A,10A and 20A. A basic meter with an internal resistance of 50w and a full scale deflection current of 1mA is to be used.

Solution: Data given

$$I_m = 1\times10^{-3}\,A \quad R_m = 50\Omega$$

$$I_1 = 1A \quad I_2 = 5A \quad I_3 = 10\,A \quad I_4 = 20\,A$$

$$m_1 = \frac{I_1}{I_m} = 1000\,A$$

$$m_2 = \frac{I_2}{I_m} = 5000\,A$$

$$m_3 = \frac{I_3}{I_m} = 10000\,A$$

$$m_4 = \frac{I_4}{I_m} = 20000\,A$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{50}{1000-1} = 0.05\Omega$$

$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{50}{5000-1} = 0.01\Omega$$

$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000-1} = 0.005\Omega$$

$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{50}{20000-1} = 0.0025\Omega$$

$\therefore$ The resistances of the various section of the universal shunt are

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04\Omega$$

$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005\Omega$$

$$R_3 = R_{sh3} - R_{sh4} = 0.005 - 0.025 = 0.0025\Omega$$

$$R_4 = R_{sh4} = 0.0025\Omega$$

Example: 1.18

A basic d' Arsonval meter movement with an internal resistance $R_m = 100\Omega$ and a full scale current of $I_m = 1mA$ is to be converted in to a multi range d.c. voltmeter with ranges of 0-10V, 0-50V, 0-250V, 0-500V. Find the values of various resistances using the potential divider arrangement.

Solution:

Data given

$R_m = 100\Omega$

$I_m = 1mA$

$V_m = I_m \times R_m$

$V_m = 100 \times 1 \times 10^{-3}$

$V_m = 100mV$

$$m_1 = \frac{V_1}{V_m} = \frac{10}{100 \times 10^{-3}} = 100$$

$$m_2 = \frac{V_2}{V_m} = \frac{50}{100 \times 10^{-3}} = 500$$

$$m_3 = \frac{V_3}{V_m} = \frac{250}{100 \times 10^{-3}} = 2500$$

$$m_4 = \frac{V_4}{V_m} = \frac{500}{100 \times 10^{-3}} = 5000$$

$R_1 = (m_1 - 1)R_m = (100 - 1) \times 100 = 9900\Omega$

$R_2 = (m_2 - m_1)R_m = (500 - 100) \times 100 = 40K\Omega$

$R_3 = (m_3 - m_2)R_m = (2500 - 500) \times 100 = 200K\Omega$

$R_4 = (m_4 - m_3)R_m = (5000 - 2500) \times 100 = 250K\Omega$

AC BRIDGES

2.1 General form of A.C. bridge

AC bridge are similar to D.C. bridge in topology(way of connecting).It consists of four arm AB,BC,CD and DA .Generally the impedance to be measured is connected between ‘A’ and ‘B’. A detector is connected between ‘B’ and ’D’. The detector is used as null deflection instrument. Some of the arms are variable element. By varying these elements, the potential values at ‘B’ and ‘D’ can be made equal. This is called balancing of the bridge.

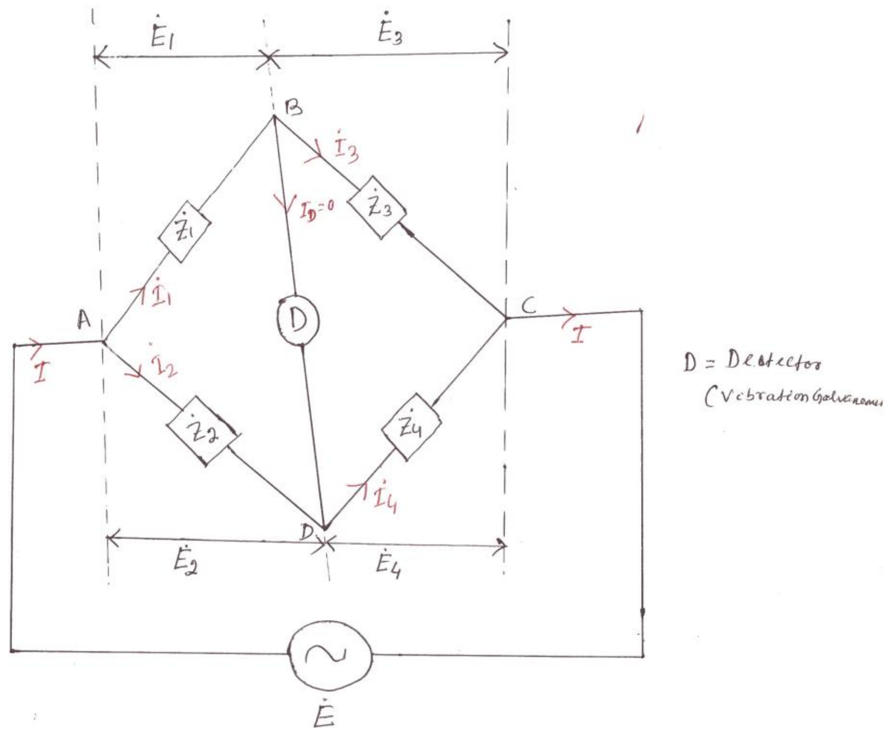

Fig. 2.1 General form of A.C. bridge

At the balance condition, the current through detector is zero.

$$\therefore \dot{I}_1 = \dot{I}_3$$

$$\dot{I}_2 = \dot{I}_4$$

$$\therefore \frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{I}_3}{\dot{I}_4} \qquad (2.1)$$

At balance condition,

Voltage drop across 'AB'=voltage drop across 'AD'.

$$\dot{E}_1 = \dot{E}_2$$

$$\therefore \dot{I}_1 \dot{Z}_1 = \dot{I}_2 \dot{Z}_2 \qquad (2.2)$$

Similarly, Voltage drop across 'BC'=voltage drop across 'DC'

$$\dot{E}_3 = \dot{E}_4$$

$$\therefore \dot{I}_3 \dot{Z}_3 = \dot{I}_4 \dot{Z}_4 \qquad (2.3)$$

From Eqn. (2.2), we have $\therefore \dfrac{\dot{I}_1}{\dot{I}_2} = \dfrac{\dot{Z}_2}{\dot{Z}_1}$ (2.4)

From Eqn. (2.3), we have $\therefore \dfrac{\dot{I}_3}{\dot{I}_4} = \dfrac{\dot{Z}_4}{\dot{Z}_3}$ (2.5)

From equation -2.1, it can be seen that, equation -2.4 and equation-2.5 are equal.

$$\therefore \frac{\dot{Z}_2}{\dot{Z}_1} = \frac{\dot{Z}_4}{\dot{Z}_3}$$

$$\therefore \dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$$

Products of impedances of opposite arms are equal.

$$\therefore |Z_1|\angle\theta_1 |Z_4|\angle\theta_4 = |Z_2|\angle\theta_2 |Z_3|\angle\theta_3$$

$$\Rightarrow |Z_1 Z_4|\angle\theta_1 + \theta_4 = |Z_2 Z_3|\angle\theta_2 + \theta_3$$

$$|Z_1 Z_4| = |Z_2 Z_3|$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

* For balance condition, magnitude on either side must be equal.

* Angle on either side must be equal.

Summary

For balance condition,

- $\dot{I}_1 = \dot{I}_3 , \dot{I}_2 = \dot{I}_4$
- $|Z_1 Z_4| = |Z_2 Z_3|$
- $\theta_1 + \theta_4 = \theta_2 + \theta_3$
- $\dot{E}_1 = \dot{E}_2$ & $\dot{E}_3 = \dot{E}_4$

2.2 Types of detector

The following types of instruments are used as detector in A.C. bridge.

- Vibration galvanometer
- Head phones (speaker)
- Tuned amplifier

2.2.1 Vibration galvanometer

Between the point 'B' and 'D' a vibration galvanometer is connected to indicate the bridge balance condition. This A.C. galvanometer which works on the principle of resonance. The A.C. galvanometer shows a dot, if the bridge is unbalanced.

2.2.2 Head phones

Two speakers are connected in parallel in this system. If the bridge is unbalanced, the speaker produced more sound energy. If the bridge is balanced, the speaker do not produced any sound energy.

2.2.3 Tuned amplifier

If the bridge is unbalanced the output of tuned amplifier is high. If the bridge is balanced, output of amplifier is zero.

2.3 Measurements of inductance

2.3.1 Maxwell's inductance bridge

The choke for which R_1 and L_1 have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

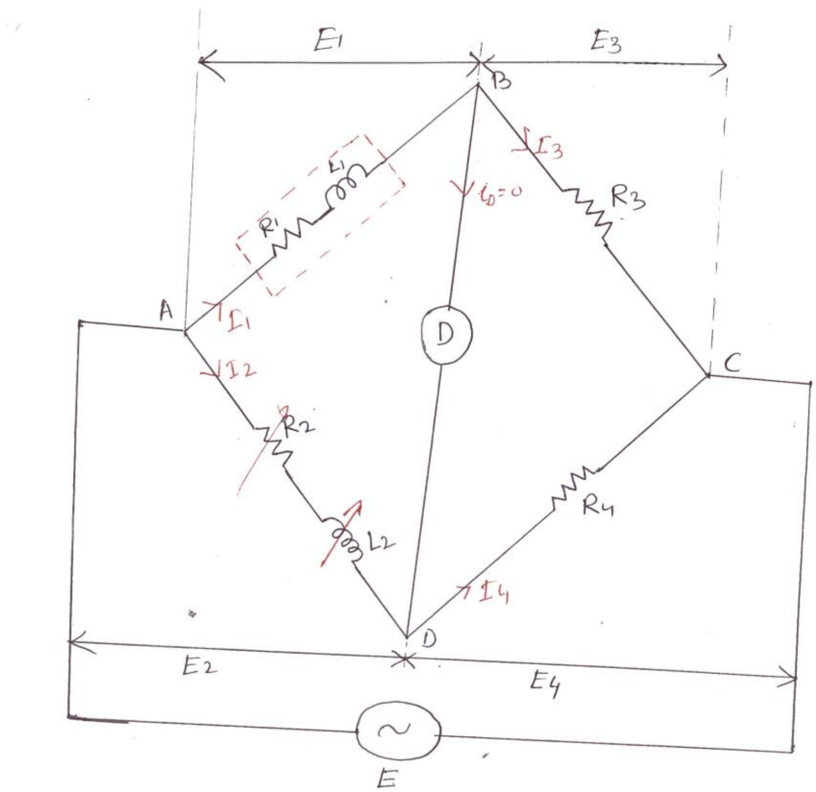

Fig. 2.2 Maxwell's inductance bridge

L_2 is adjusted, until the detector indicates zero current.

Let R_1= unknown resistance

L_1= unknown inductance of the choke.

L_2= known standard inductance

R_1,R_2,R_4= known resistances.

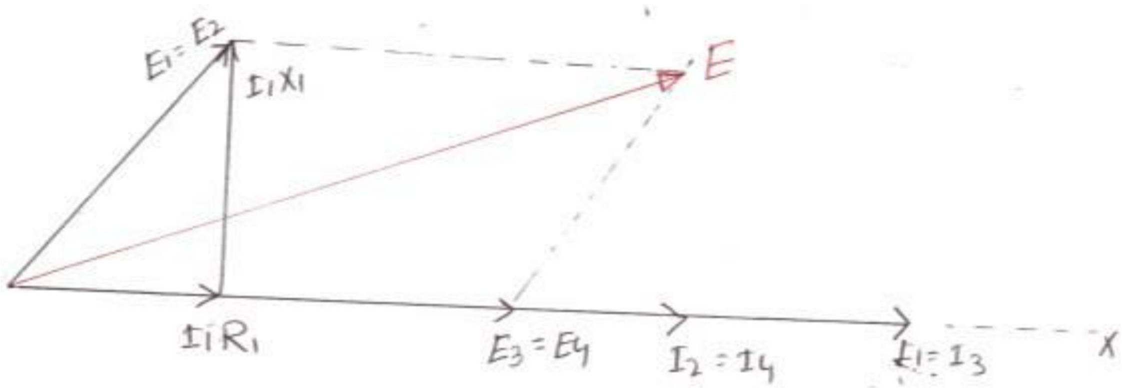

Fig 2.3 Phasor diagram of Maxwell's inductance bridge

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + jwL_1)R_4 = (R_2 + jwL_2)R_3$$

$$R_1R_4 + jwL_1R_4 = R_2 R_3 + jwL_2 R_3$$

Comparing real part,

$$R_1R_4 = R_2 R_3$$

$$\therefore R_1 = \frac{R_2 R_3}{R_4} \tag{2.6}$$

Comparing the imaginary parts,

$$wL_1R_4 = wL_2 R_3$$

$$L_1 = \frac{L_2 R_3}{R_4} \tag{2.7}$$

Q-factor of choke, $Q = \frac{WL_1}{R_1} = \frac{WL_2 R_3R_4}{R_4R_2R_3}$

$$Q = \frac{WL_2}{R_2} \tag{2.8}$$

Advantages

- ✓ Expression for R_1 and L_1 are simple.
- ✓ Equations area simple
- ✓ They do not depend on the frequency (as w is cancelled)
- ✓ R_1 and L_1 are independent of each other.

Disadvantages

- ✓ Variable inductor is costly.
- ✓ Variable inductor is bulky.

2.3.2 Maxwell's inductance capacitance bridge

Unknown inductance is measured by comparing it with standard capacitance. In this bridge, balance condition is achieved by varying 'C_4'.

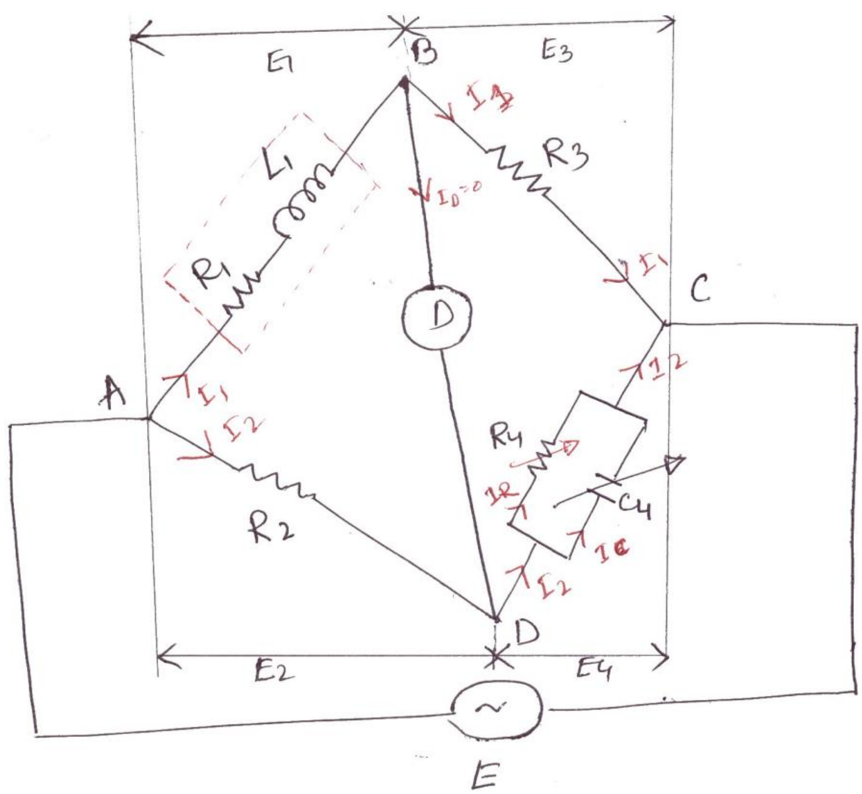

Fig 2.4 Maxwell's inductance capacitance bridge

At balance condition, $Z_1Z_4=Z_3Z_2$ (2.9)

$$Z_4 = R_4 \parallel \frac{1}{jwC_4} = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}}$$

$$Z_4 = \frac{R_4}{jwR_4C_4 + 1} = \frac{R_4}{1 + jwR_4C_4} \quad (2.10)$$

$\therefore$ Substituting the value of Z_4 from eqn. (2.10) in eqn. (2.9) we get

$$(R_1 + jwL_1) \times \frac{R_4}{1 + jwR_4C_4} = R_2 R_3$$

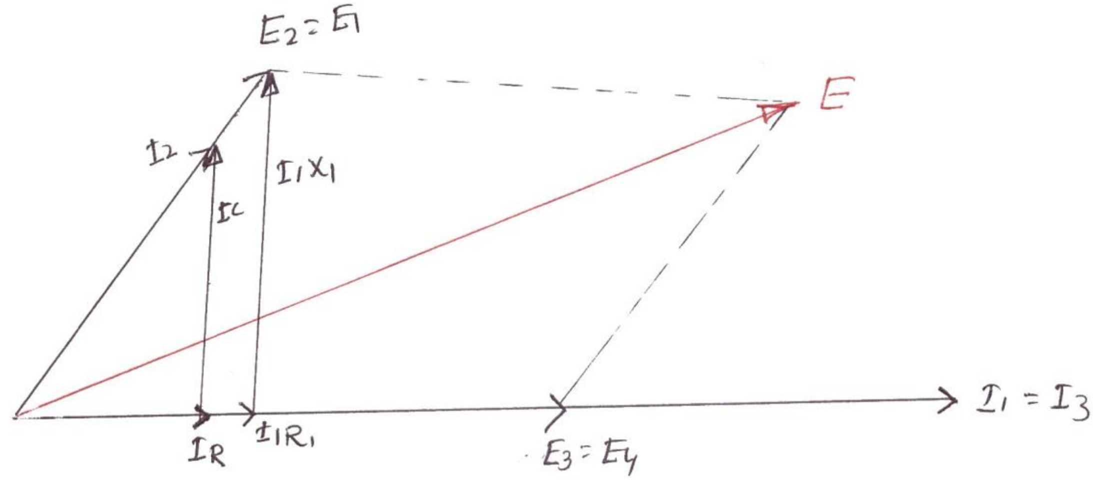

Fig 2.5 Phasor diagram of Maxwell's inductance capacitance bridge

$$(R_1 + jwL_1)R_4 = R_2 R_3 (1 + jwR_4C_4)$$

$$R_1R_4 + jwL_1R_4 = R_2 R_3 + jwC_4 R_4 R_2 R_3$$

Comparing real parts,

$$R_1R_4 = R_2 R_3$$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4} \tag{2.11}$$

Comparing imaginary part,

$$wL_1 R_4 = wC_4 R_4 R_2 R_3$$

$$L_1 = C_4 R_2 R_3 \tag{2.12}$$

Q-factor of choke,

$$Q = \frac{WL_1}{R_1} = w \times C_4 R_2 R_3 \times \frac{R_4}{R_2 R_3}$$

$$Q = wC_4 R_4 \tag{2.13}$$

Advantages

- ✓ Equation of L_1 and R_1 are simple.
- ✓ They are independent of frequency.
- ✓ They are independent of each other.
- ✓ Standard capacitor is much smaller in size than standard inductor.

Disadvantages

- ✓ Standard variable capacitance is costly.
- ✓ It can be used for measurements of Q-factor in the ranges of 1 to 10.
- ✓ It cannot be used for measurements of choke with Q-factors more than 10.

  We know that $Q = wC_4R_4$

For measuring chokes with higher value of Q-factor, the value of C_4 and R_4 should be higher. Higher values of standard resistance are very expensive. Therefore this bridge cannot be used for higher value of Q-factor measurements.

2.3.3 Hay's bridge

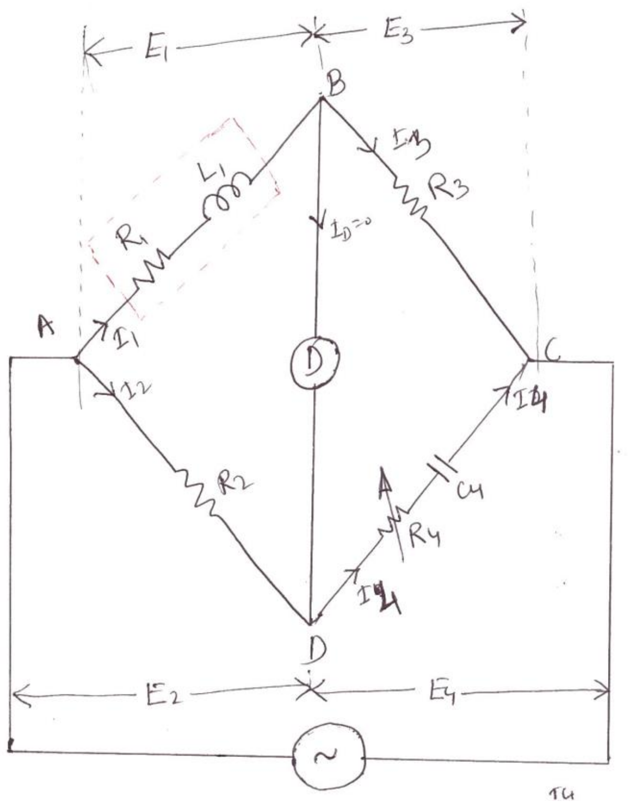

Fig 2.6 Hay's bridge

- $\dot{E}_1 = I_1 R_1 + jI_1 X_1$
- $\dot{E} = \dot{E}_1 + \dot{E}_3$
- $\dot{E}_4 = \dot{I}_4 R_4 + \dfrac{I_4}{jwC_4}$
- $\dot{E}_3 = I_3 R_3$

$$Z_4 = R_4 + \frac{1}{jwC_4} = \frac{1 + jwR_4C_4}{jwC_4}$$

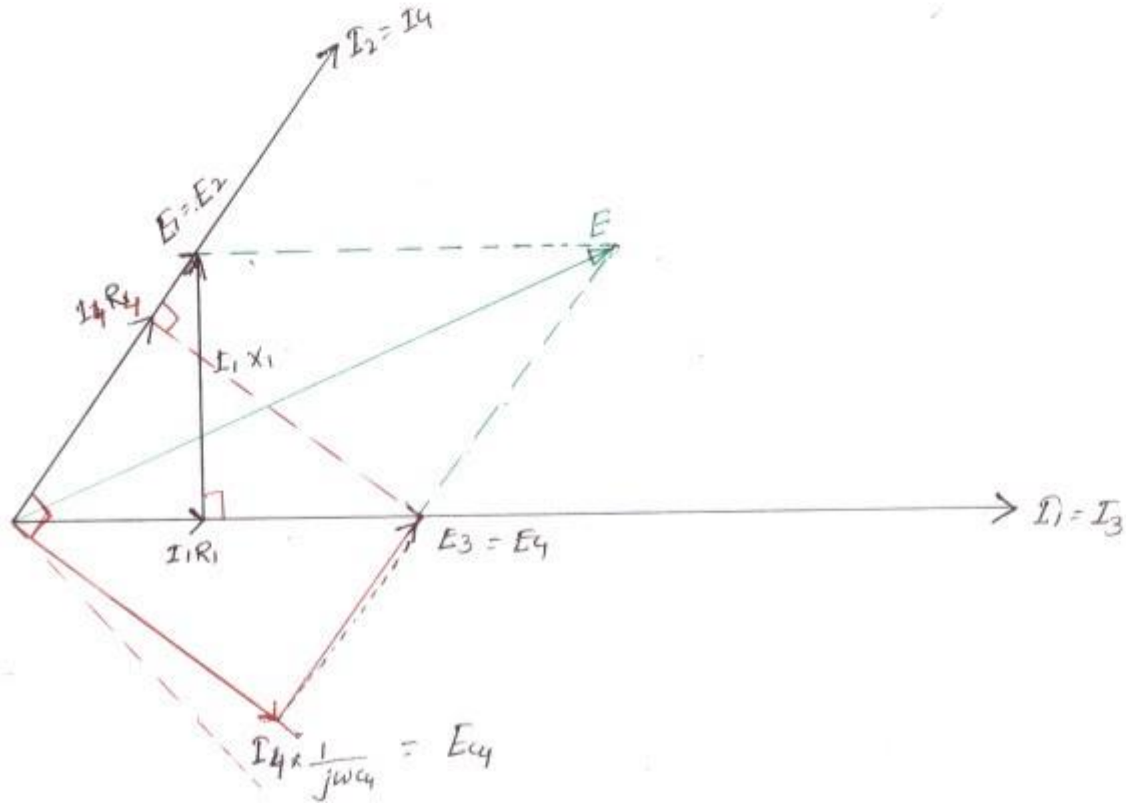

Fig 2.7 Phasor diagram of Hay's bridge

At balance condition, $Z_1Z_4=Z_3Z_2$

$$(R_1 + jwL_1)\left(\frac{1 + jwR_4C_4}{jwC_4}\right) = R_2R_3$$

$$(R_1 + jwL_1)(1 + jwR_4C_4) = jwR_2C_4R_3$$

$$R_1 + jwC_4R_4R_1 + jwL_1 + j^2w^2L_1C_4R_4 = jwC_4R_2R_3$$

$$(R_1 - w^2L_1C_4R_4) + j(wC_4R_4R_1 + wL_1) = jwC_4R_2R_3$$

Comparing the real term,

$$R_1 - w^2L_1C_4R_4 = 0$$

$$R_1 = w^2L_1C_4R_4 \tag{2.14}$$

Comparing the imaginary terms,

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4\,R_4\,R_1 + L_1 = C_4\,R_2\,R_3$$

$$L_1 = C_4R_2R_3 - C_4R_4R_1 \tag{2.15}$$

Substituting the value of R_1 fro eqn. 2.14 into eqn. 2.15, we have,

$$L_1 = C_4\,R_2\,R_3 - C_4\,R_4 \times w^2\,L_1C_4\,R_4$$

$$L_1 = C_4\,R_2\,R_3 - w^2\,L_1C_4{}^2\,R_4{}^2$$

$$L_1(1+ w^2\,L_1C_4{}^2\,R_4{}^2\,) = C_4\,R_2\,R_3$$

$$L_1 = \frac{C_4R_2R_3}{1+ w^2L_1C_4{}^2\,R_4{}^2} \tag{2.16}$$

Substituting the value of L_1 in eqn. 2.14 , we have

$$R_1 = \frac{w^2C_4{}^2\,R_2\,R_3R_4}{1+ w^2C_4{}^2\,R_4{}^2} \tag{2.17}$$

$$Q = \frac{wL_1}{R_1} = \frac{w\times\,C_4\,R_2\,R_3}{1+ w^2C_4{}^2R_4{}^2} \times \frac{1+ w^2C_4{}^2\,R_4{}^2}{w^2C_4{}^2\,R_4\,R_2\,R_3}$$

$$Q = \frac{\square 1}{wC_4\,R_4} \tag{2.18}$$

Advantages

- ✓ Fixed capacitor is cheaper than variable capacitor.
- ✓ This bridge is best suitable for measuring high value of Q-factor.

Disadvantages

- ✓ Equations of L1and R1 are complicated.
- ✓ Measurements of R_1 and L_1 require the value of frequency.
- ✓ This bridge cannot be used for measuring low Q- factor.

2.3.4 Owen's bridge

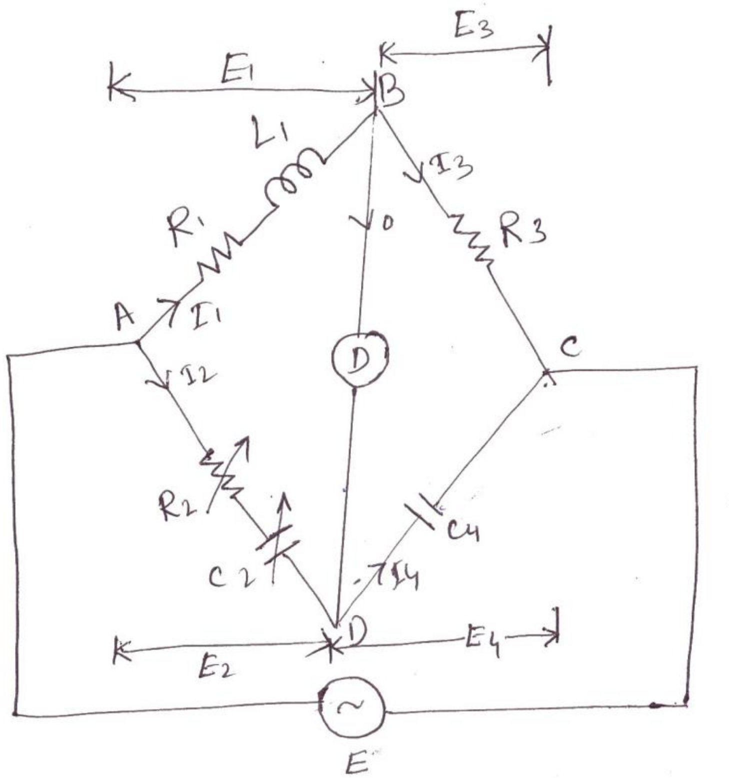

Fig 2.8 Owen's bridge

- ➢ $E_1 = I_1R_1 + jI_1X_1$
- ➢ I_4 leads E_4 by 90^0

- $\dot{E} = \dot{E}_1 + \dot{E}_3$
- $\dot{E}_2 = I_2 R_2 + \dfrac{I_2}{jwC_2}$

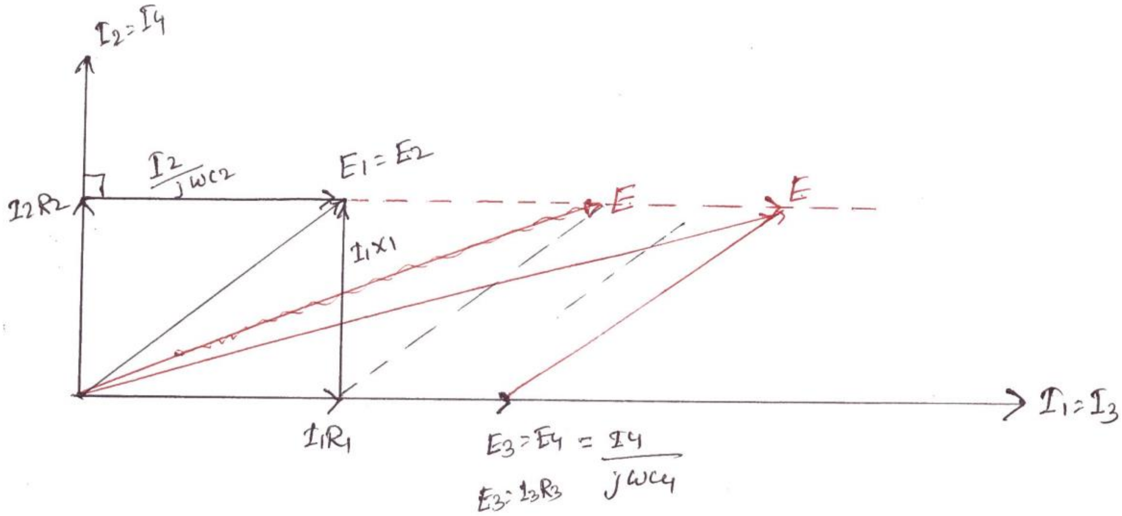

Fig 2.9 Phasor diagram of Owen's bridge

Balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$Z_2 = R_2 + \frac{1}{jwC_2} = \frac{jwC_2 R_2 + 1}{jwC_2}$$

$$\therefore (R_1 + jwL_1) \times \frac{1}{jwC_4} = \frac{(1 + jwR_2C_2) \times R_3}{jwC_2}$$

$$C_2 (R_1 + jwL_1) = R_3C_4 (1 + jwR_2C_2)$$

$$R_1C_2 + jwL_1C_2 = R_3C_4 + jwR_2C_2 R_3C_4$$

Comparing real terms,

$$R_1C_2 = R_3C_4$$

$$R_1 = \frac{R_3 C_4}{C_2}$$

Comparing imaginary terms,

$$wL_1 C_2 = wR_2 C_2 R_3 C_4$$

$$L_1 = R_2 R_3 C_4$$

Q- factor = $\dfrac{WL_1}{R_1} = \dfrac{wR_2 R_3 C_4 C_2}{R_3 C_4}$

$$Q = wR_2 C_2$$

Advantages

- ✓ Expression for R_1 and L_1 are simple.
- ✓ R_1 and L_1 are independent of Frequency.

Disadvantages

- ✓ The Circuits used two capacitors.
- ✓ Variable capacitor is costly.
- ✓ Q-factor range is restricted.

2.3.5 Anderson's bridge

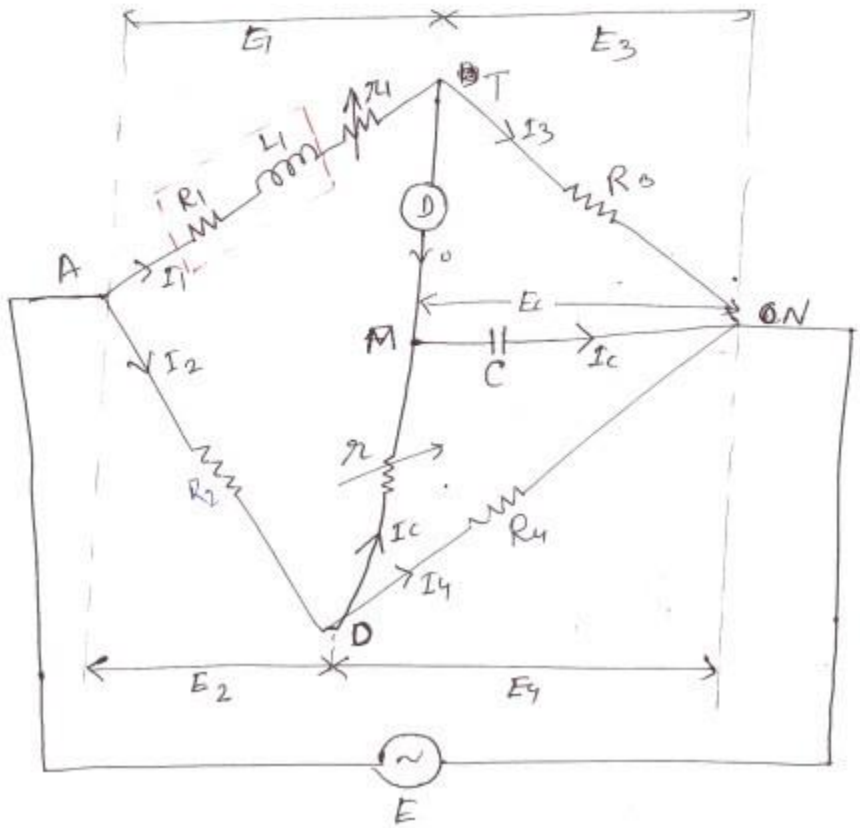

Fig 2.10 Anderson's bridge

- $\dot{E}_1 = I_1(R_1 + r_1) + jI_1X_1$
- $E_3 = E_C$
- $\dot{E}_4 = \dot{I}_C r + E_C$
- $I_2 = I_4 + I_C$
- $\bar{E}_2 + \bar{E}_4 = \bar{E}$
- $\bar{E}_1 + \bar{E}_3 = \bar{E}$

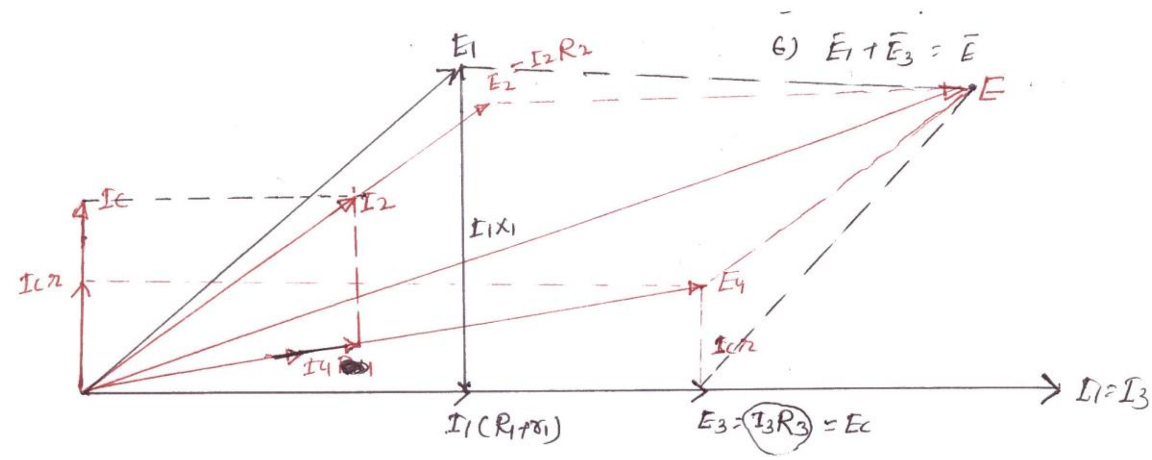

Fig 2.11 Phasor diagram of Anderson's bridge

Step-1 Take I_1 as references vector .Draw $I_1R_1^1$ in phase with I_1

$R_1^1 = (R_1 + r_1)$, I_1X_1 is $\perp_r$ to $I_1R_1^1$

$E_1 = I_1R_1^1 + jI_1X_1$

Step-2 $I_1 = I_3$, E_3 is in phase with I_3, From the circuit ,

$E_3 = E_C$, I_C leads E_C by 90^0

Step-3 $E_4 = I_C r + E_C$

Step-4 Draw I_4 in phase with E_4 , By KCL , $\bar{I}_2 = \bar{I}_4 + \bar{I}_C$

Step-5 Draw E_2 in phase with I_2

Step-6 By KVL , $\bar{E}_1 + \bar{E}_3 = \bar{E}$ or $\bar{E}_2 + \bar{E}_4 = \bar{E}$

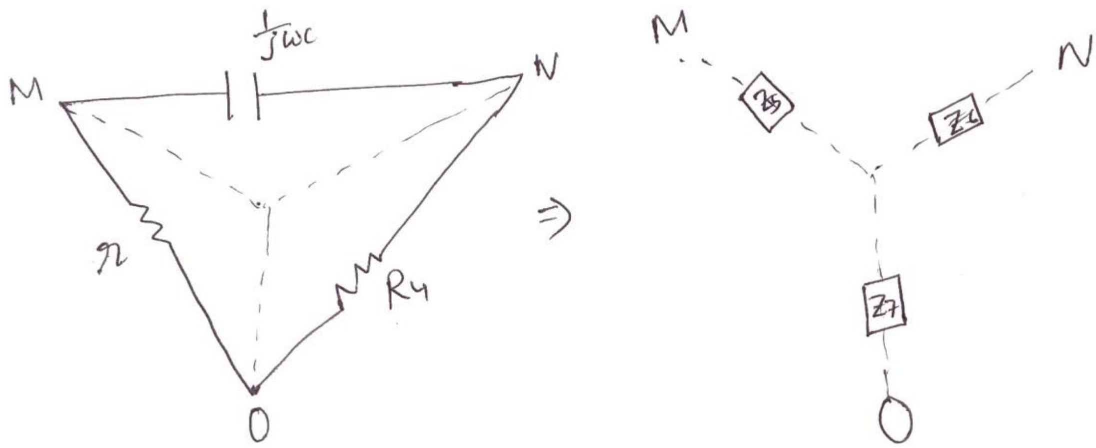

Fig 2.12 Equivalent delta to star conversion for the loop MON

$$Z_7 = \frac{R_4 \times r}{R_4 + r + \frac{1}{jwc}} = \frac{jwCR_4 r}{1 + jwC\,(R_4 + r)}$$

$$Z_6 = \frac{R_4 \times \frac{1}{jwC}}{R_4 + r + \frac{1}{jwc}} = \frac{R_4}{1 + jwC(R_4 + r)}$$

$$(R_1^1 + jwL_1) \times \frac{R_4}{1 + jwC(R_4 + r)} = R_3\left(R_2 + \frac{jwCR_4 r}{1 + jwC(R_4 + r)}\right)$$

$$\Rightarrow \frac{(R_1^1 + jwL_1)R_4}{1 + jwC(R_4 + r)} = R_3\left[\frac{R_2(1 + jwC(R_4 + r)) + jwCrR_4}{1 + jwC(R_4 + r)}\right]$$

$$\Rightarrow R_1^1 R_4 + jwL_1 R_4 = R_2 R_3 + jCwR_2 R_3 (r + R_4) + jwCrR_4 R_3$$

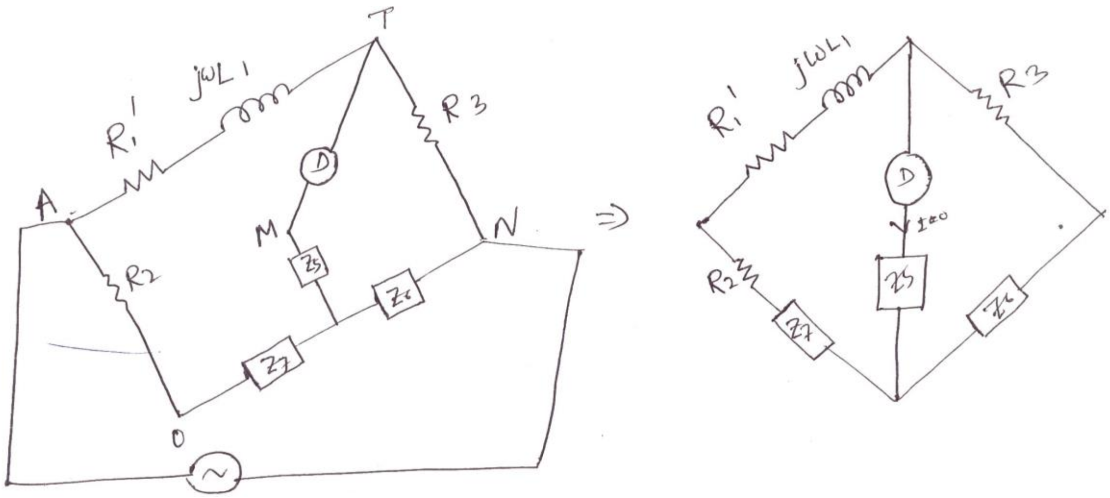

Fig 2.13 Simplified diagram of Anderson's bridge

Comparing real term,

$$R_1^1 R_4 = R_2 R_3$$

$$(R_1 + r_1)R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

Comparing the imaginary term,

$$wL_1R_4 = wCR_2 R_3 (r + R_4) + wcrR_3R_4$$

$$L_1 = \frac{R_2 R_3 C}{R_4}(r + R_4) + R_3 rC$$

$$L_1 = R_3 C\left[\frac{R_2}{R_4}(r + R_4) + r\right]$$

Advantages

- ✓ Variable capacitor is not required.
- ✓ Inductance can be measured accurately.
- ✓ R_1 and L_1 are independent of frequency.
- ✓ Accuracy is better than other bridges.

Disadvantages

- ✓ Expression for R_1 and L_1 are complicated.
- ✓ This is not in the standard form A.C. bridge.

2.4 Measurement of capacitance and loss angle. (Dissipation factor)

2.4.1 Dissipation factors (D)

A practical capacitor is represented as the series combination of small resistance and ideal capacitance.

From the vector diagram, it can be seen that the angle between voltage and current is slightly less than 90^0. The angle ‘ δ ’ is called loss angle.

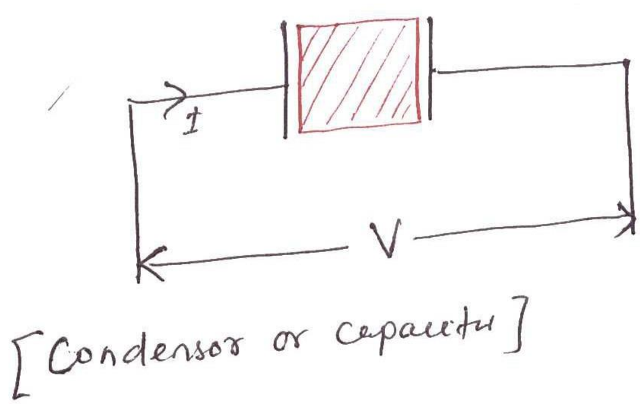

Fig 2.14 Condensor or capacitor

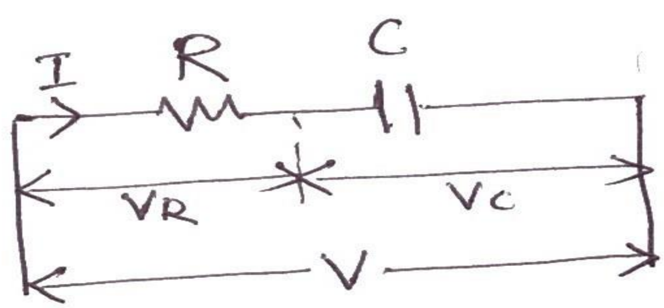

Fig 2.15 Representation of a practical capacitor

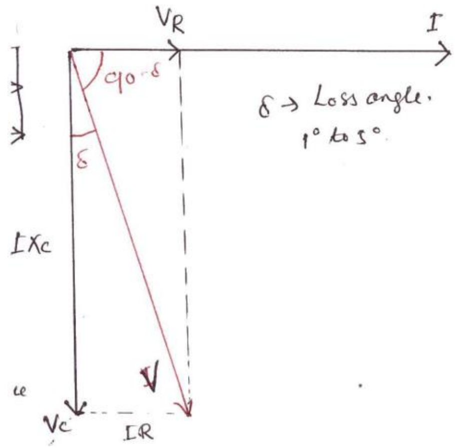

Fig 2.16 Vector diagram for a practical capacitor

A dissipation factor is defined as 'tan δ'.

$$\therefore \tan\,\delta = \frac{IR}{IX_C} = \frac{R}{X_C} = wCR$$

$$D = wCR$$

$$D = \frac{1}{Q}$$

$$D = \tan\delta = \frac{\sin\delta}{\cos\delta} \cong \frac{\delta}{1}$$ For small value of 'δ' in radians

$D \cong \delta \cong$ Loss Angle ('δ' must be in radian)

2.4.2 Desauty's Bridge

C_1= Unknown capacitance

At balance condition,

$$\frac{1}{jwC_1} \times R_4 = \frac{1}{jwC_2} \times R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow C_1 = \frac{R_4 C_2}{R_3}$$

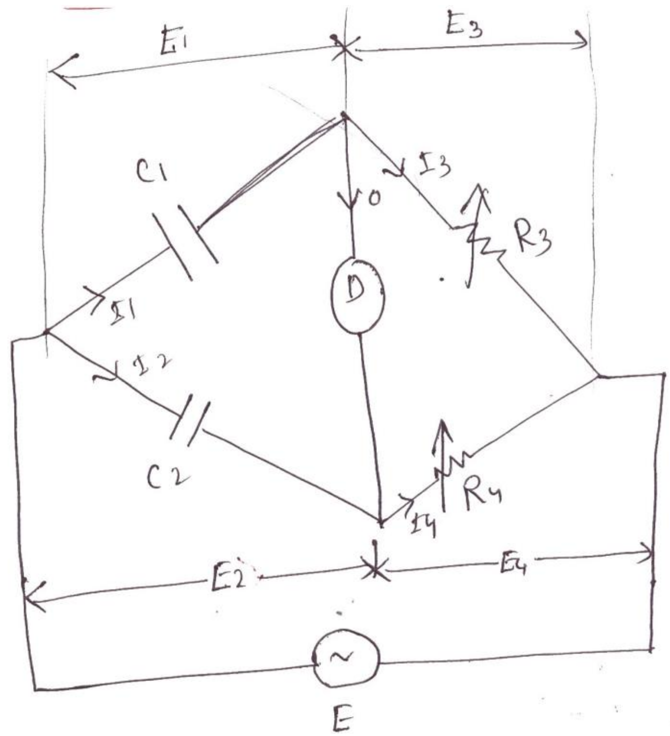

Fig 2.17 Desauty's bridge

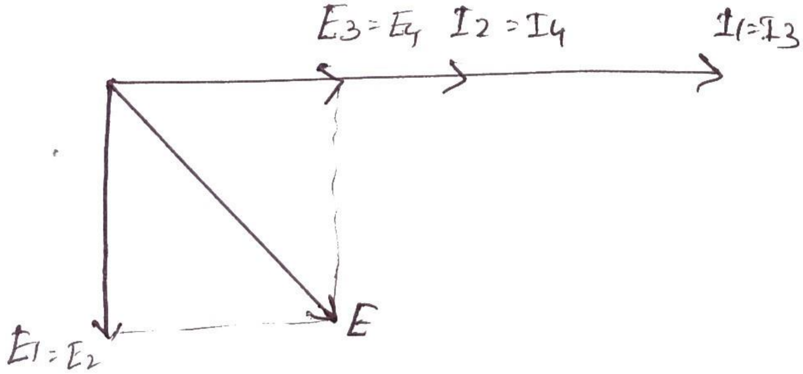

Fig 2.18 Phasor diagram of Desauty's bridge

2.4.3 Modified desauty's bridge

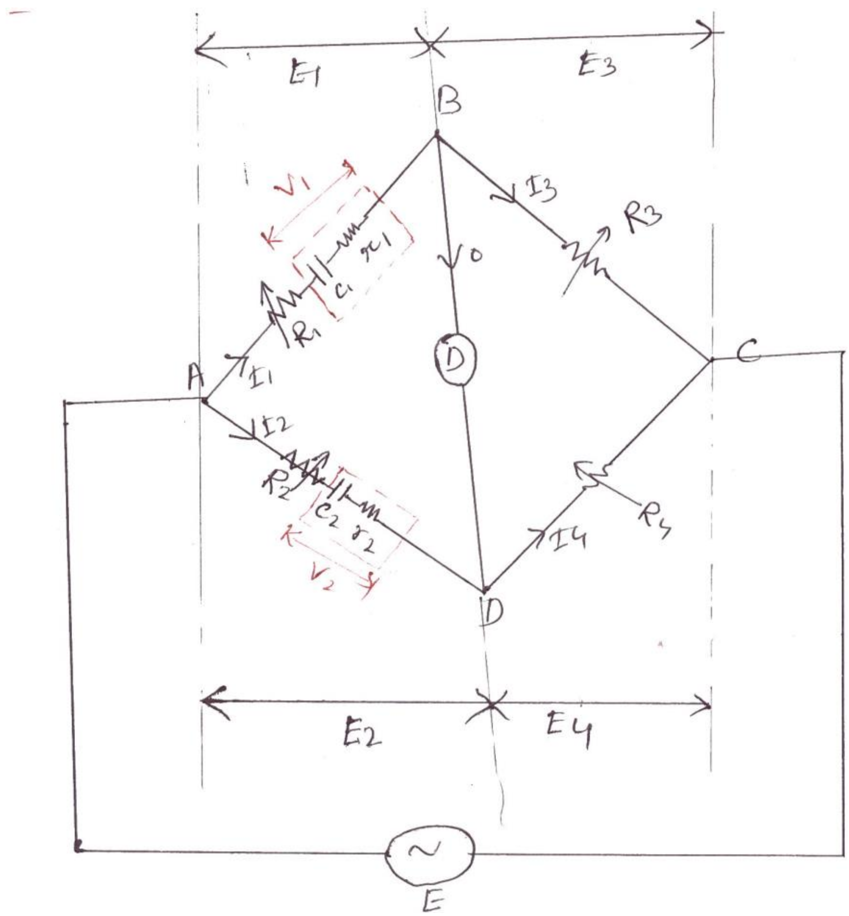

Fig 2.19 Modified Desauty's bridge

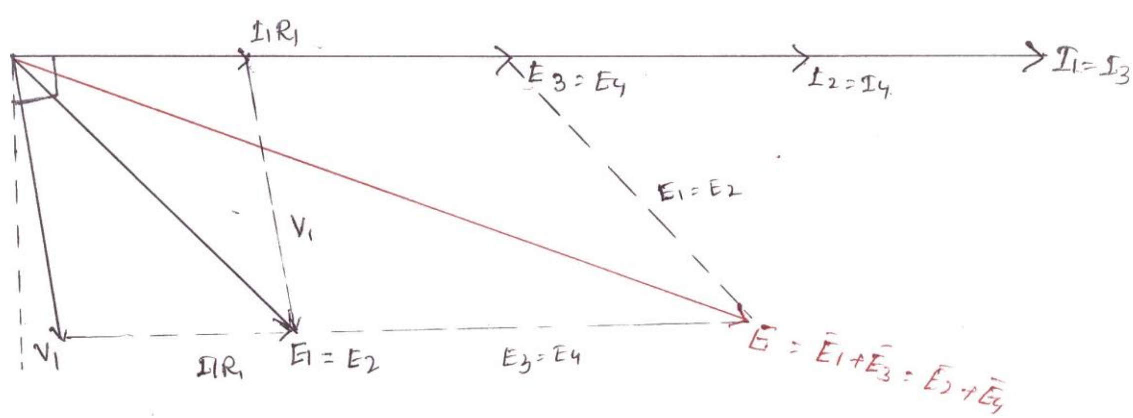

Fig 2.20 Phasor diagram of Modified Desauty's bridge

$$R_1^1 = (R_1 + r_1)$$

$$R_2^1 = (R_2 + r_2)$$

At balance condition, $(R_1^1 + \frac{1}{jwC_1})R_4 = R_3(R_2^1 + \frac{1}{jwC_2})$

$$R_1^1 R_4 + \frac{R_4}{jwC_1} = R_3 R_2^1 + \frac{R_3}{jwC_2})$$

Comparing the real term, $R_1^1 R_4 = R_3 R_2^1$

$$R_1^1 = \frac{R_3 R_2^1}{R_4}$$

$$R + r_1 = \frac{(R_2 + r_2)R_3}{R_4}$$

Comparing imaginary term,

$$\frac{R_4}{wC_1} = \frac{R_3}{wC_2}$$

$$C_1 = \frac{R_4 C_2}{R_3}$$

Dissipation factor $D = wC_1r_1$

Advantages

- ✓ r_1 and c_1 are independent of frequency.
- ✓ They are independent of each other.
- ✓ Source need not be pure sine wave.

2.4.4 Schering bridge

$$E_1 = I_1 r_1 - jI_1 X_4$$

$C_2 = C_4$= Standard capacitor (Internal resistance=0)

C_4= Variable capacitance.

C_1= Unknown capacitance.

r_1= Unknown series equivalent resistance of the capacitor.

R_3=R_4= Known resistor.

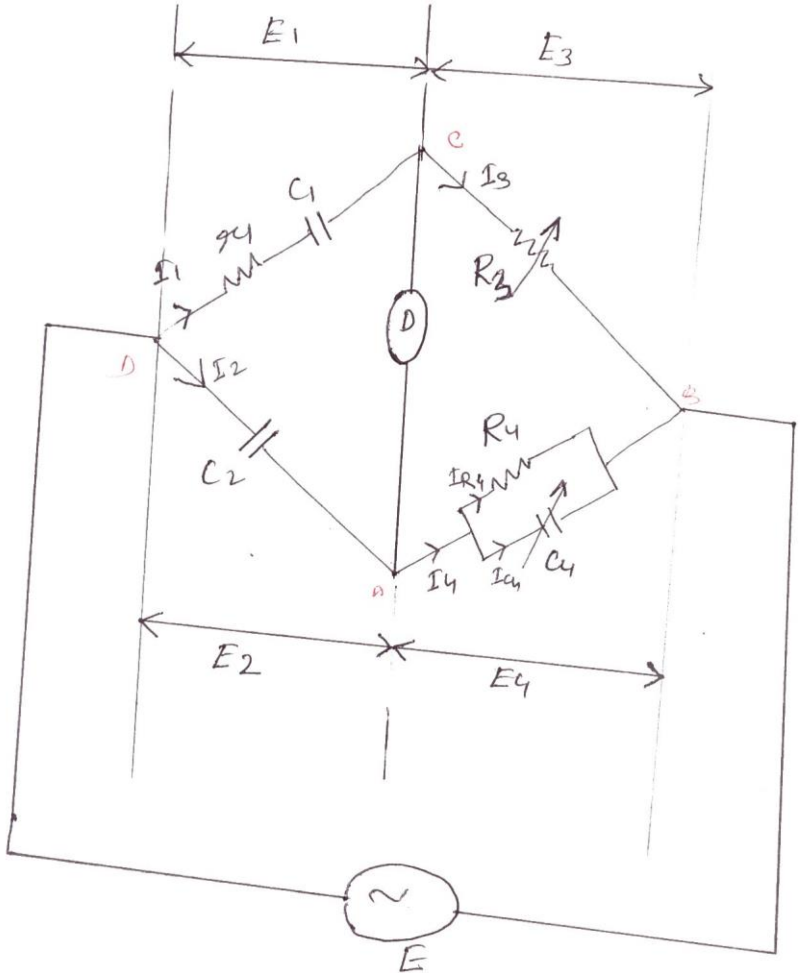

Fig 2.21 Schering bridge

$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

$$Z_4 = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}} = \frac{R_4}{1 + jwC_4 R_4}$$

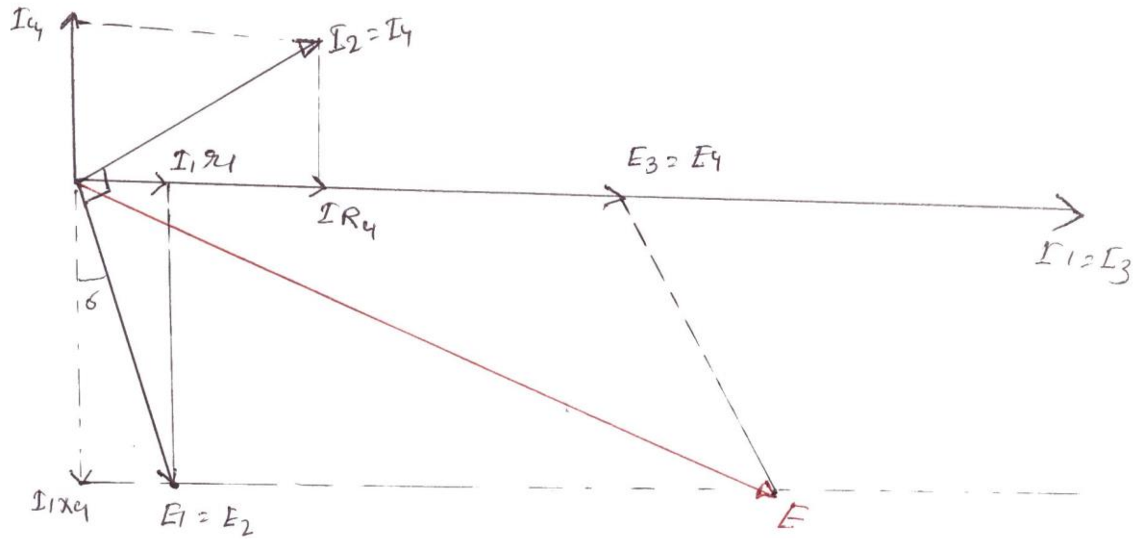

Fig 2.22 Phasor diagram of Schering bridge

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{1+jwC_1r_1}{jwC_1} \times \frac{R_4}{1+jwC_4R_4} = \frac{R_3}{jwC_2}$$

$$(1+jwC_1r_1)R_4C_2 = R_3C_1(1+jwC_4r_4)$$

$$R_2C_2 + jwC_1r_1R_4C_2 = R_3C_1 + jwC_4R_4R_3C_1$$

Comparing the real part,

$$\therefore C_1 = \frac{R_4C_2}{R_3}$$

Comparing the imaginary part,

$$wC_1r_1R_4C_2 = wC_4R_3R_4C_1$$

$$r_1 = \frac{C_4R_3}{C_2}$$

Dissipation factor of capacitor,

$$D = wC_1 r_1 = w \times \frac{R_4 C_2}{R_3} \times \frac{C_4 R_3}{C_2}$$

$$\therefore D = wC_4 R_4$$

Advantages

- ✓ In this type of bridge, the value of capacitance can be measured accurately.
- ✓ It can measure capacitance value over a wide range.
- ✓ It can measure dissipation factor accurately.

Disadvantages

- ✓ It requires two capacitors.
- ✓ Variable standard capacitor is costly.

2.5 Measurements of frequency

2.5.1 Wein's bridge

Wein's bridge is popularly used for measurements of frequency of frequency. In this bridge, the value of all parameters are known. The source whose frequency has to measure is connected as shown in the figure.

$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

$$Z_2 = \frac{R_2}{1 + jwC_2 R_2}$$

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{jwC_1 r_1 + 1}{jwC_1} \times R_4 = \frac{R_2}{1 + jwC_2 R_2} \times R_3$$

$$(1 + jwC_1 r_1)(1 + jwC_2 R_2) R_4 = R_2 R_3 \times jwC_1$$

$$\left[1 + jwC_2 R_2 + jwC_1 r_1 - w^2 C_1 C_2 r_1 R_2\right] = jwC_1 \frac{R_2 R_3}{R_4}$$

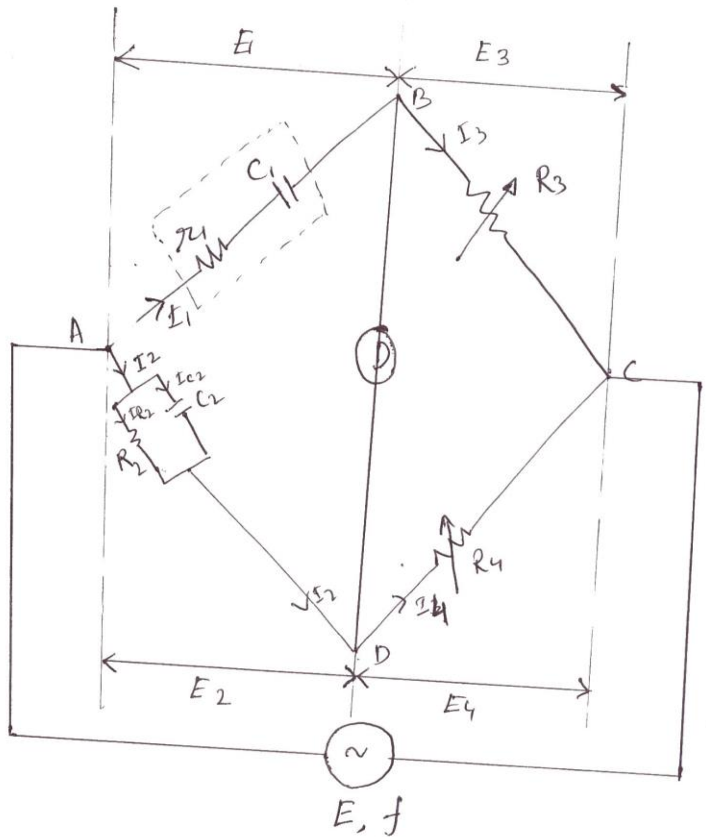

Fig 2.23 Wein's bridge

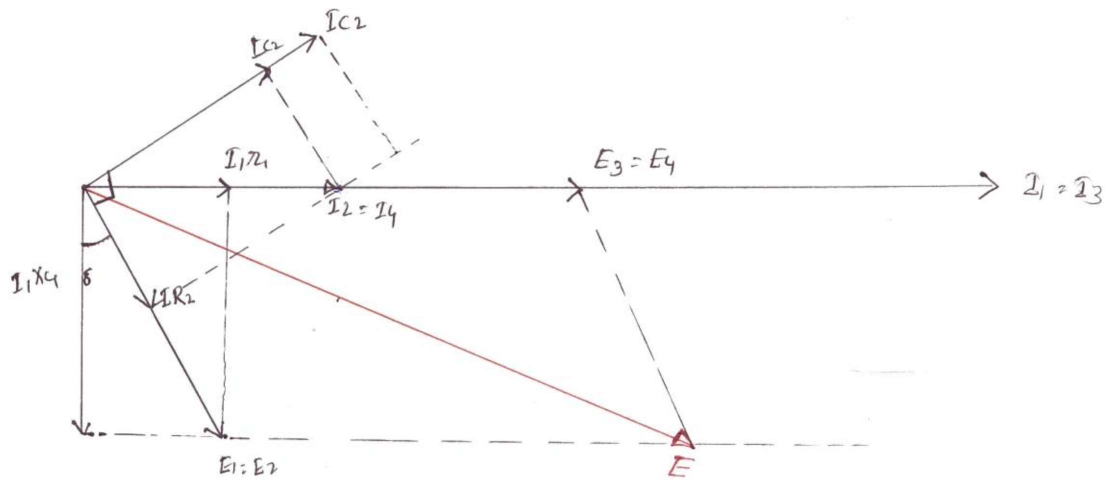

Fig 2.24 Phasor diagram of Wein's bridge

Comparing real term,

$$1 - w^2 C_1 C_2 r_1 R_2 = 0$$

$$w^2 C_1 C_2 r_1 R_2 = 1$$

$$w^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$w = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}}, \; f = \frac{1}{2\Pi\sqrt{C_1 C_2 r_1 R_2}}$$

NOTE

The above bridge can be used for measurements of capacitance. In such case, r_1 and C_1 are unknown and frequency is known. By equating real terms, we will get R_1 and C_1. Similarly by equating imaginary term, we will get another equation in terms of r_1 and C_1. It is only used for measurements of Audio frequency.

A.F=20 HZ to 20 KHZ

R.F=>> 20 KHZ

Comparing imaginary term,

$$wC_2 R_2 + wC_1 r_1 = wC_1 \frac{R_2 R_3}{R_4}$$

$$C_2 R_2 + C_1 r_1 = \frac{C_1 R_2 R_3}{R_4} \quad \text{.........} (2.19)$$

$$C_1 = \frac{1}{w^2 C_2 r_1 R_2}$$

Substituting in eqn. (2.19), we have

$$C_2 R_2 + \frac{r_1}{w^2 C_2 r_1 R_2} = \frac{R_2 R_3}{R_4} C_1$$

Multiplying $\frac{R_4}{R_2 R_3}$ in both sides, we have

$$C_2 R_2 \times \frac{R_4}{R_2 R_3} + \frac{1}{w^2 C_2 R_2} \times \frac{R_4}{R_2 R_3} = C_1$$

$$C_1 = \frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2^2 R_2^2 R_3}$$

$$w^2 C_1 r_1 C_2 R_2 = 1$$

$$r_1 = \frac{1}{w^2 C_2 R_2 C_1} = \frac{1}{w^2 C_2 R_2 \left[\frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2^2 R_2^2 R_3} \right]}$$

$$= \frac{1}{\left[\frac{w^2 C_2^2 R_2 R_4}{R_3} + \frac{R_4}{R_2 R_3} \right]}$$

$$\therefore r_1 = \frac{1}{\frac{R_4}{R_3} \left[w^2 C_2^2 R_2 + \frac{1}{R_2} \right]}$$

$$\therefore r_1 = \frac{R_3}{R_4} \left[\frac{1}{\left(w^2 C_2^2 R_2 + \frac{1}{R_2} \right)} \right]$$

2.5.2 High Voltage Schering Bridge

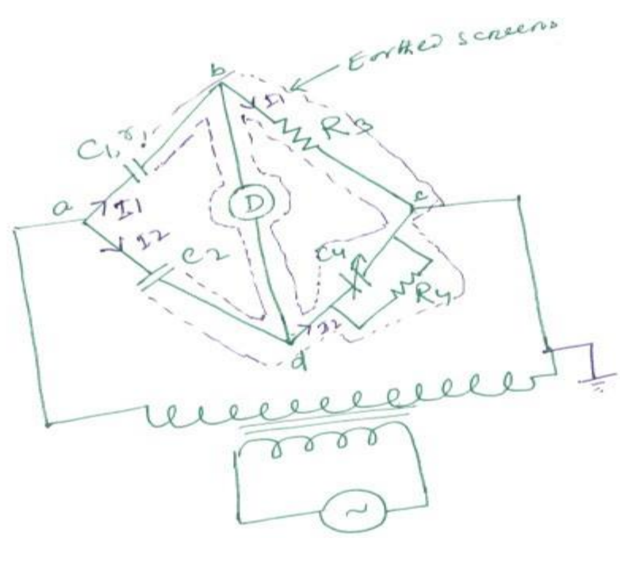

Fig 2.25 High Voltage Schering bridge

(1) The high voltage supply is obtained from a transformer usually at 50 HZ.

2.6 Wagner earthing device:

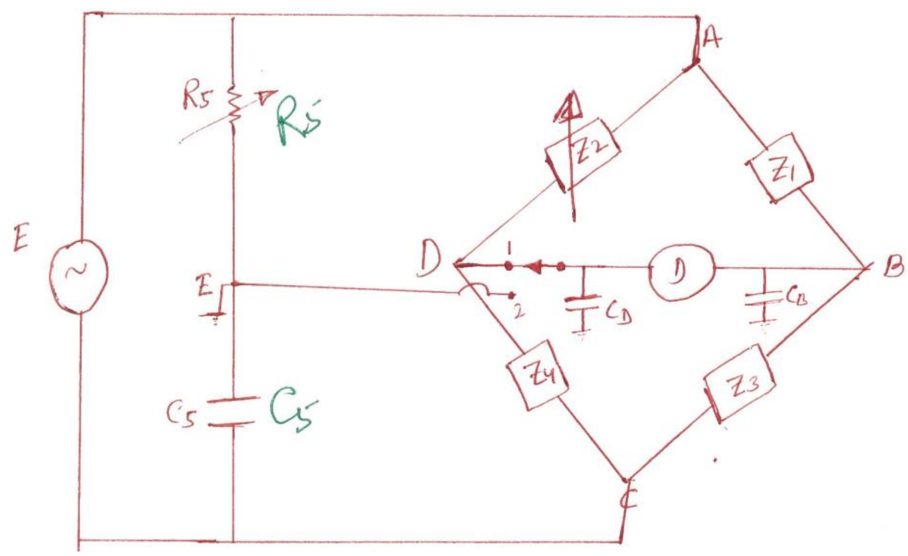

Fig 2.26 Wagner Earthing device

Wagner earthing consists of ‘R’ and ‘C’ in series. The stray capacitance at node ‘B’ and ‘D’ are C_B, C_D respectively. These Stray capacitances produced error in the measurements of ‘L’ and ‘C’. These error will predominant at high frequency. The error due to this capacitance can be eliminated using wagner earthing arm.

Close the change over switch to the position (1) and obtained balanced. Now change the switch to position (2) and obtained balance. This process has to repeat until balance is achieved in both the position. In this condition the potential difference across each capacitor is zero. Current drawn by this is zero. Therefore they do not have any effect on the measurements.

What are the sources of error in the bridge measurements?

- ✓ Error due to stray capacitance and inductance.
- ✓ Due to external field.
- ✓ Leakage error: poor insulation between various parts of bridge can produced this error.
- ✓ Eddy current error.
- ✓ Frequency error.

- ✓ Waveform error (due to harmonics)
- ✓ Residual error: small inductance and small capacitance of the resistor produce this error.

Precaution

- ✓ The load inductance is eliminated by twisting the connecting the connecting lead.
- ✓ In the case of capacitive bridge, the connecting lead are kept apart.($\because C = \frac{A \in 0 \in r}{d}$)
- ✓ In the case of inductive bridge, the various arm are magnetically screen.
- ✓ In the case of capacitive bridge, the various arm are electro statically screen to reduced the stray capacitance between various arm.
- ✓ To avoid the problem of spike, an inter bridge transformer is used in between the source and bridge.
- ✓ The stray capacitance between the ends of detector to the ground, cause difficulty in balancing as well as error in measurements. To avoid this problem, we use wagner earthing device.

2.7 Ballastic galvanometer

This is a sophisticated instrument. This works on the principle of PMMC meter. The only difference is the type of suspension is used for this meter. Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphorous bronze wire is used for suspension.

When the D.C. voltage is applied to the terminals of moving coil, current flows through it. When a current carrying coil kept in the magnetic field, produced by permanent magnet, it experiences a force. The coil deflects and mirror deflects. The light spot on the glass scale also move. This deflection is proportional to the current through the coil.

$i = \frac{Q}{t}$, $Q = it = \int idt$

$\theta \propto Q$, deflection $\propto$ Charge

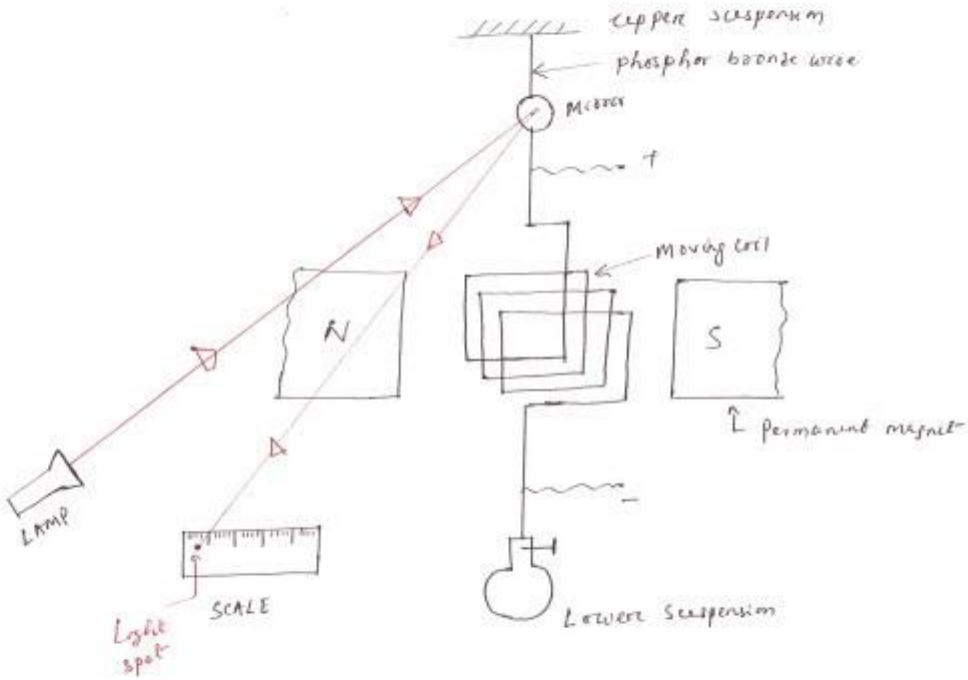

Fig 2.27 Ballastic galvanometer

2.8 Measurements of flux and flux density (Method of reversal)

D.C. voltage is applied to the electromagnet through a variable resistance R_1 and a reversing switch. The voltage applied to the toroid can be reversed by changing the switch from position 2 to position ‘1’. Let the switch be in position ‘2’ initially. A constant current flows through the toroid and a constant flux is established in the core of the magnet.

A search coil of few turns is provided on the toroid. The B.G. is connected to the search coil through a current limiting resistance. When it is required to measure the flux, the switch is changed from position ‘2’ to position ‘1’. Hence the flux reduced to zero and it starts increasing in the reverse direction. The flux goes from + ϕ to - ϕ, in time ‘t’ second. An emf is induced in the search coil, science the flux changes with time. This emf circulates a current through R_2 and B.G. The meter deflects. The switch is normally closed. It is opened when it is required to take the reading.

2.8.1 Plotting the BH curve

The curve drawn with the current on the X-axis and the flux on the Y-axis, is called magnetization characteristics. The shape of B-H curve is similar to shape of magnetization characteristics. The residual magnetism present in the specimen can be removed as follows.

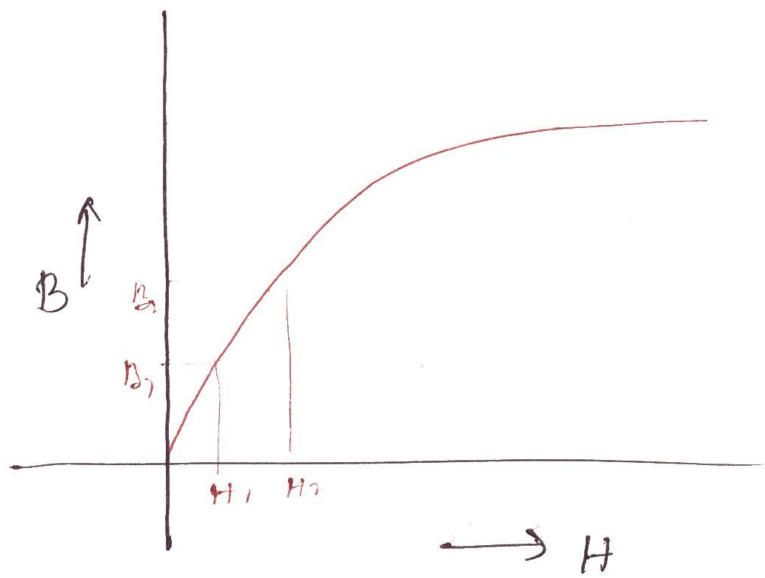

Fig 2.28 BH curve

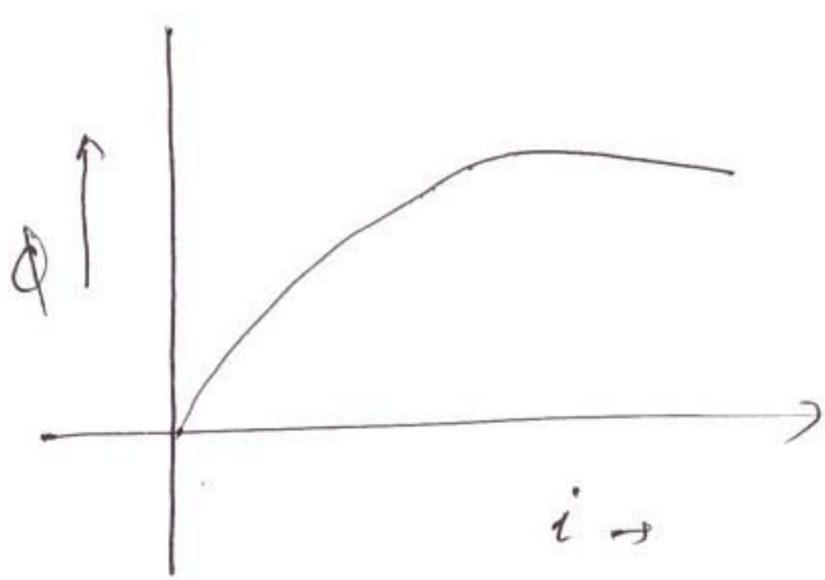

Fig 2.29 Magnetization characteristics

Close the switch ‘S_2’ to protect the galvanometer, from high current. Change the switch S1 from position ‘1’ to ‘2’ and vice versa for several times.

To start with the resistance ‘R_1’ is kept at maximum resistance position. For a particular value of current, the deflection of B.G. is noted. This process is repeated for various value of current. For each deflection flux can be calculated.($B = \frac{\phi}{A}$)

Magnetic field intensity value for various current can be calculated.().The B-H curve can be plotted by using the value of ‘B’ and ‘H’.

2.8.2 Measurements of iron loss:

Let R_P= pressure coil resistance

R_S = resistance of coil S1

E= voltage reading= Voltage induced in S_2

I= current in the pressure coil

V_P= Voltage applied to wattmeter pressure coil.

W= reading of wattmeter corresponding voltage V

W_1= reading of wattmeter corresponding voltage E

$$W \rightarrow V \qquad \frac{W_1}{W} = \frac{E}{V} \Rightarrow W_1 = \frac{E \times W}{V}$$

$$W_1 \rightarrow E_P$$

W1=Total loss=Iron loss+ Cupper loss.

The above circuit is similar to no load test of transformer.

In the case of no load test the reading of wattmeter is approximately equal to iron loss. Iron loss depends on the emf induced in the winding. Science emf is directly proportional to flux. The voltage applied to the pressure coil is V. The corresponding of wattmeter is ‘W’. The iron loss corresponding E is $E = \frac{WE}{V}$. The reading of the wattmeter includes the losses in the pressure coil and copper loss of the winding S1. These loses have to be subtracted to get the actual iron loss.

2.9 Galvanometers

D-Arsonval Galvanometer

Vibration Galvanometer

Ballistic C

2.9.1 D-arsonval galvanometer (d.c. galvanometer)

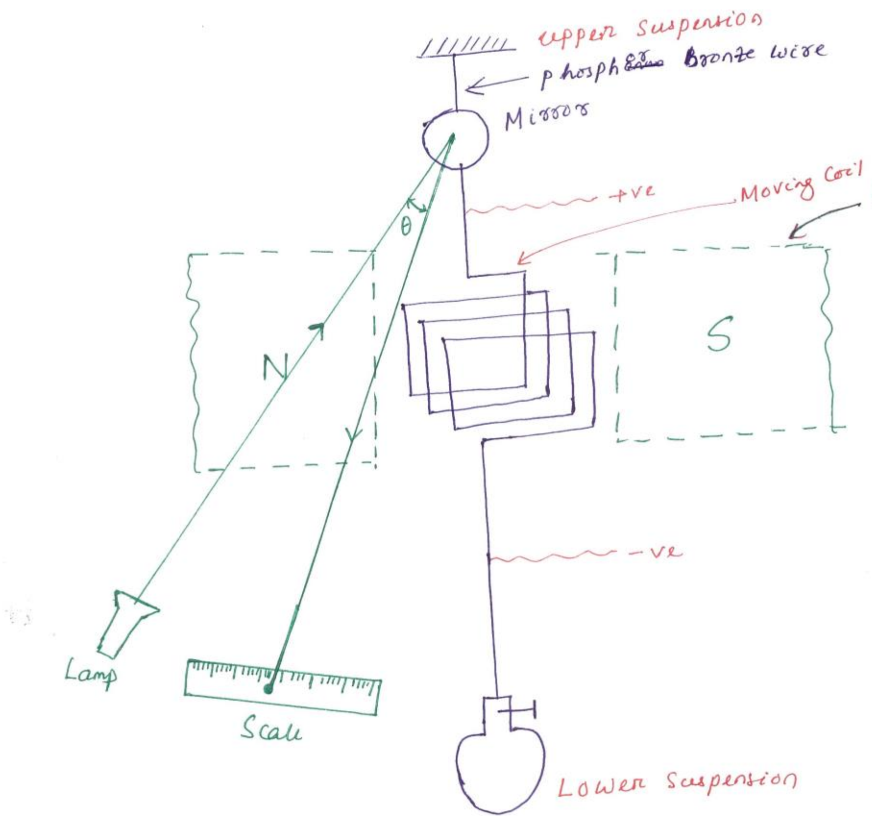

Fig 2.30 D-Arsonval Galvanometer

Galvanometer is a special type of ammeter used for measuring A or mA. This is a sophisticated instruments. This works on the principle of PMMC meter. The only difference is the type of suspension used for this meter. It uses a sophisticated suspension called taut suspension, so that moving system has negligible weight.

Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphors bronze is used for suspension.

When D.C. voltage is applied to the terminal of moving coil, current flows through it. When current carrying coil is kept in the magnetic field produced by P.M. , it experiences a force. The light spot on the glass scale also move. This deflection is proportional to the current through the coil. This instrument can be used only with D.C. like PMMC meter.

The deflecting Torque,

T_D=BINA

T_D=GI, Where G=BAN

$T_C=K_S\theta=S\theta$

At balance, $T_C=T_D \Rightarrow S\theta=GI$

$$\therefore \theta=\frac{GI}{S}$$

Where G= Displacements constant of Galvanometer

S=Spring constant

2.9.2 Vibration Galvanometer (A.C. Galvanometer)

The construction of this galvanometer is similar to the PMMC instrument except for the moving system. The moving coil is suspended using two ivory bridge pieces. The tension of the system can be varied by rotating the screw provided at the top suspension. The natural frequency can be varied by varying the tension wire of the screw or varying the distance between ivory bridge piece.

When A.C. current is passed through coil an alternating torque or vibration is produced. This vibration is maximum if the natural frequency of moving system coincide with supply frequency. Vibration is maximum, science resonance takes place. When the coil is vibrating , the mirror oscillates and the dot moves back and front. This appears as a line on the glass scale. Vibration galvanometer is used for null deflection of a dot appears on the scale. If the bridge is unbalanced, a line appears on the scale

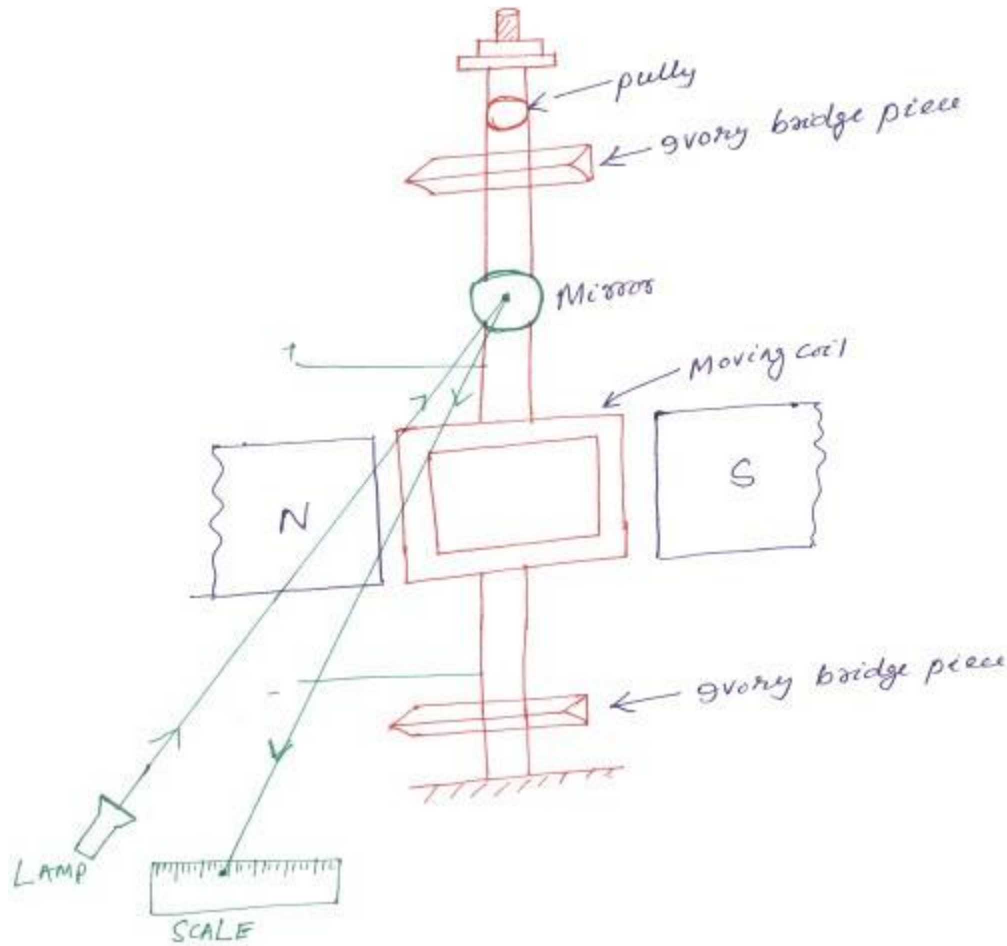

Fig 2.31 Vibration Galvanometer

Example 2.2-In a low- Voltage Schering bridge designed for the measurement of permittivity, the branch 'ab' consists of two electrodes between which the specimen under test may be inserted, arm 'bc' is a non-reactive resistor R_3 in parallel with a standard capacitor C_3, arm CD is a non-reactive resistor R_4 in parallel with a standard capacitor C_4, arm 'da' is a standard air capacitor of capacitance C_2. Without the specimen between the electrode, balance is obtained with following values , C_3=C_4=120 pF, C_2=150 pF, R_3=R_4=5000Ω.With the specimen inserted, these values become C_3=200 pF,C_4=1000 pF,C_2=900 pF and R_3=R_4=5000Ω. In such test w=5000 rad/sec. Find the relative permittivity of the specimen?

Sol: Relative permittivity(ε_r) = $\dfrac{\text{capacitance measured with given medium}}{\text{capacitance measured with air medium}}$

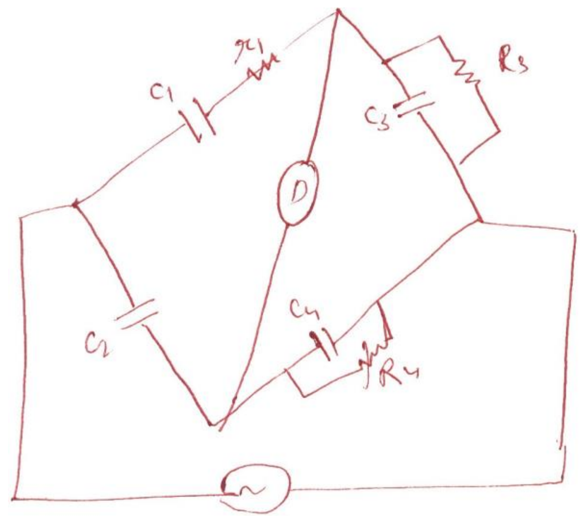

Fig 2.32 Schering bridge

$$C_1 = C_2 \left(\frac{R_4}{R_3}\right)$$

Let capacitance value C_0, when without specimen dielectric.

Let the capacitance value C_S when with the specimen dielectric.

$$C_0 = C_2 \left(\frac{R_4}{R_3}\right) = 150 \times \frac{5000}{5000} = 150\ pF$$

$$C_S = C_2 \left(\frac{R_4}{R_3}\right) = 900 \times \frac{5000}{5000} = 900\ pF$$

$$\varepsilon_r = \frac{C_S}{C_0} = \frac{900}{150} = 6$$

Example 2.3- A specimen of iron stamping weighting 10 kg and having a area of 16.8 cm^2 is tested by an episten square. Each of the two winding S_1 and S_2 have 515 turns. A.C. voltage of 50 HZ frequency is given to the primary. The current in the primary is 0.35 A. A voltmeter connected to S_2 indicates 250 V. Resistance of S_1 and S_2 each equal to 40 Ω. Resistance of pressure coil is 80 kΩ. Calculate maximum flux density in the specimen and iron loss/kg if the wattmeter indicates 80 watt?

Sol^{n}- $E = 4.44 f \phi_m N$

$$B_m = \frac{\square E}{4.44 fAN} = 1.3 wb/m^2$$

Iron loss= $W(1+\frac{R_S}{R_P}) - \frac{E^2}{(R_S + R_P)}$

$$= 80(1+\frac{40}{80\times 10^3}) - \frac{250^2}{(40+80\times 10^3)} = 79.26 watt$$

Iron loss/ kg=79.26/10=7.926 w/kg.