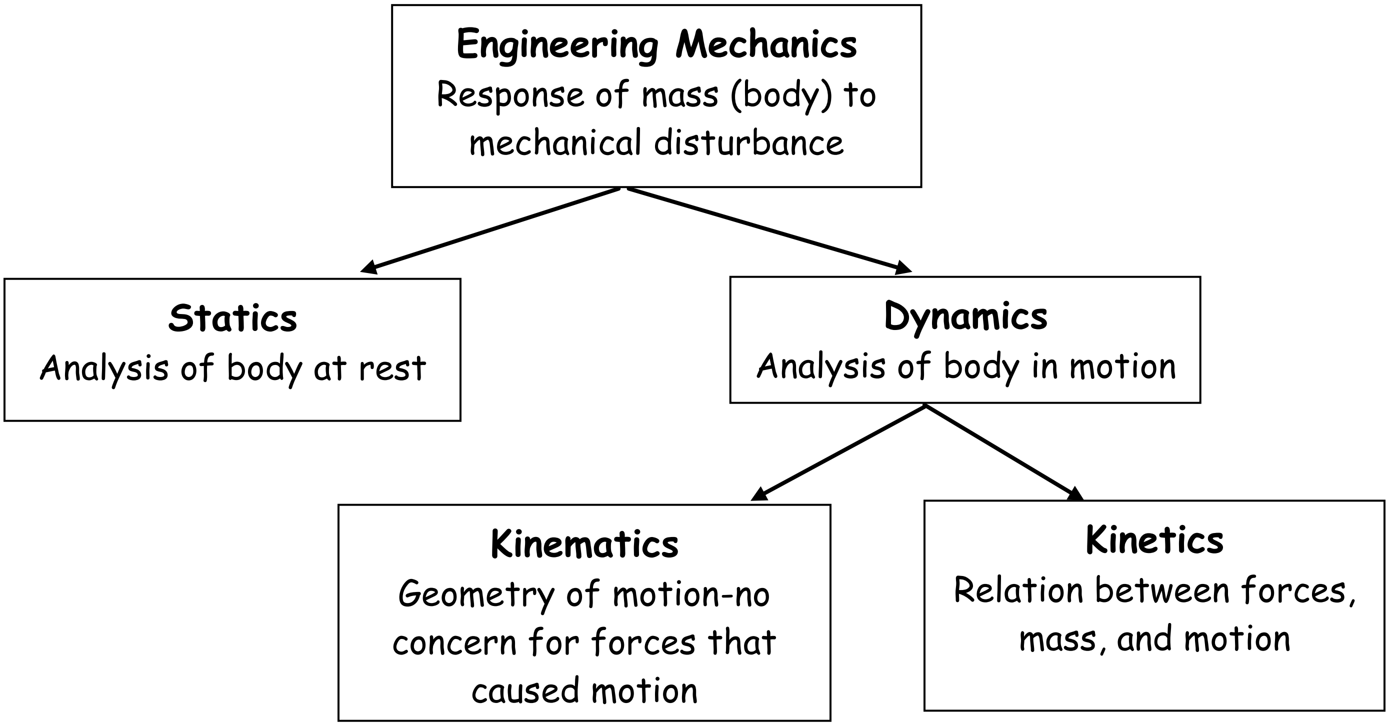
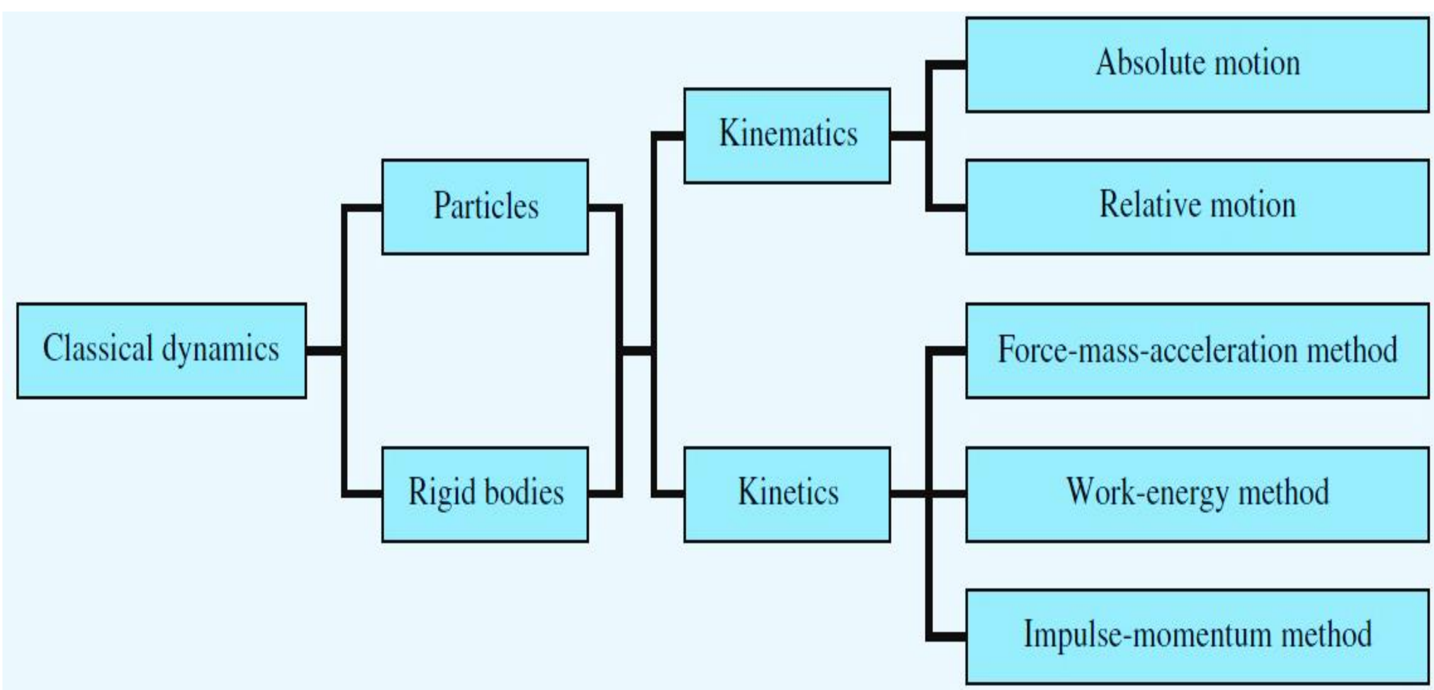

Engineering Mechanics
Response of mass (body) to mechanical disturbance
Statics
Analysis of body at rest
Dynamics
Analysis of body in motion
Kinematics
Geometry of motion-no concern for forces that caused motion
Kinetics
Relation between forces, mass, and motion


Classical dynamics
Particles
Rigid bodies
Kinematics
Kinetics
Absolute motion
Relative motion
Force-mass-acceleration method
Work-energy method
Impulse-momentum method

Basic Concepts:

A **particle*:*** is a body of negligible dimensions

Rigid Body: is that body whose changes in shape are negligible compared with its overall dimensions or with the changes in position of the body as a whole, such as rigid link, rigid disc…..etc.

Absolute motion: the motion of body in relative to another body which is at rest or to a fixed point located on this body.

Relative motion: the motion of body in relative to another moved body.

Scalar quantities: are those quantities which have *magnitude* only e.g. mass, time, volume, density ….etc.

Vector quantities: are those quantities which have *magnitude* as well as *direction* e.g. velocity, acceleration, force …..etc.

Chapter Two

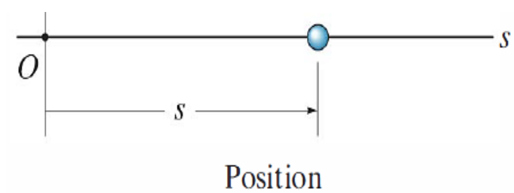
Kinematics of Particles

Kinematics: is that branch of dynamics which is responsible to study the motion of bodies without reference to the forces which are cause this motion, i.e it's relate the motion variables (displacement, velocity, acceleration) with the time

1. Rectilinear Motion:

The particle moves along a straight line path. The kinematics of a particle is described by specifying, at any given instant (the particle's position, velocity and acceleration).

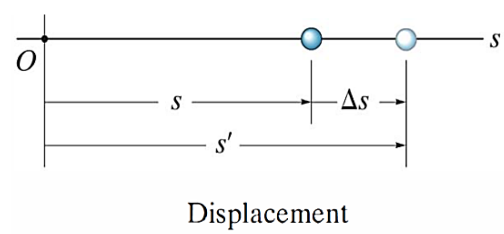
Position(s): The straight line path of a particle will be defined using *one coordinate axis* **s**.



Displacement (Δs): The displacement of the particle is defined as the change in its position

$$\Delta \boldsymbol{s} = \boldsymbol{s}' - \boldsymbol{s} \qquad (1)$$

Where:
s' : Final position of particle
s : initial position of particle



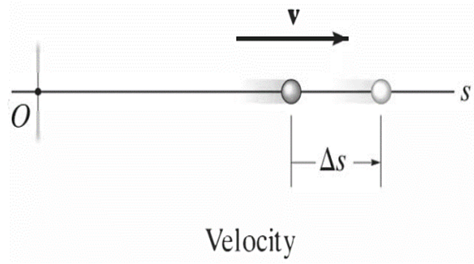
Note:
Δs is positive(+) if s' located to the right of s .
Δs is negative (-) if s' located to the left of s .

- The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels.
- The distance traveled is a *positive scalar* that represents the total length of path over which the particle travels.

Velocity (v):

average velocity, $\boldsymbol{v_{avg}} = \frac{\boldsymbol{\Delta s}}{\boldsymbol{\Delta t}}$ (2)

instantaneous velocity, $v = \lim_{\Delta t \to 0} \left(\frac{\Delta s}{\Delta t}\right)$, or



Velocity

$\left(\overset{+}{\rightarrow}\right)$ $$\boldsymbol{v = \frac{ds}{dt} = \dot{s}}$$ (3)

The magnitude of the velocity is known as the *speed.*

Average speed , $\left(\boldsymbol{v_{sp}}\right)_{\boldsymbol{avg}} = \frac{\boldsymbol{Total\ distance\ travelt\ by\ particle}}{\boldsymbol{Elapsed\ time}}$

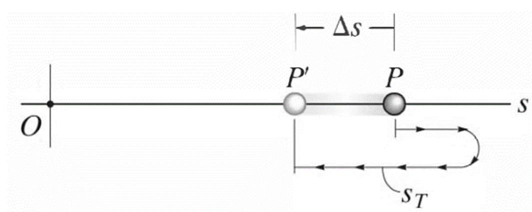
$$\boldsymbol{\left(v_{sp}\right)_{avg} = \frac{S_T}{\Delta t}}$$ (always positive) (4)

Velocity units are: **m/s** or **ft/s**

For example,

$v_{avg} = \frac{-\Delta s}{\Delta t}$ (-)

$\left(v_{sp}\right)_{avg} = \frac{s_T}{\Delta t}$ (+)



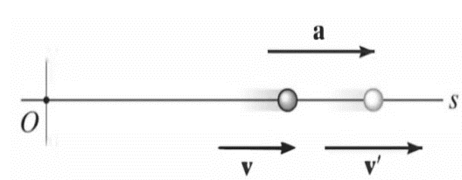
Average velocity and
Average speed

Acceleration (a):

average acceleration**,** $\boldsymbol{a_{avg}} = \frac{\boldsymbol{\Delta v}}{\boldsymbol{\Delta t}}$ (5)

$\boldsymbol{\Delta v} = \text{final vel.} - \text{intial vel.} = \boldsymbol{v' - v}$

Instantaneous acceleration, $a = lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right)$, or



$$\boldsymbol{a = \frac{dv}{dt} = \dot{v} = \frac{d^2s}{dt^2} = \ddot{s}}$$ (6)

Note: Both $\boldsymbol{a_{avg}}$ & $\boldsymbol{a}$ can be either positive or negative**.**

From equations (3) & (6) by eliminating the time (dt), we get;

$$\boxed{\boldsymbol{a\,ds = v\,dv}} \qquad (7)$$

A. Constant acceleration, ($\boldsymbol{a = constant \ \ a_c}$) :

1- $a = a_c = \frac{dv}{dt}$

$dv = a_c dt$

$\int_{v_0}^{v} dv = \int_{t_0}^{t} a_c dt$, at $t_0 = 0$, $v = v_0$

$$\therefore \quad \boxed{\boldsymbol{v = v_0 + a_c t}} \qquad (8)$$

2- $v = \frac{ds}{dt}$

$\int_{s_0}^{s} ds = \int_{t_0}^{t} v\,dt$, $v = v_0 + a_c t$

$\int_{s_0}^{s} ds = \int_{t_0}^{t} (v_0 + a_c t)\,dt$, at $t_0 = 0$, $s = s_0$

$$\therefore \quad \boxed{\boldsymbol{s = s_0 + v_0 t + \frac{1}{2} a_c t^2}} \qquad (9)$$

3- $vdv = a_c ds$, $s = s_0$ at $t = 0$

$\int_{v_0}^{v} vdv = a_c \int_{s_0}^{s} ds$

$$\boxed{\boldsymbol{v^2 = v_0^2 + 2a_c(s - s_0)}} \qquad (10)$$

B. Acceleration given as a function of time ($a = f(t)$):

$$a = f(t) = dv/dt$$

$$\int_{v_0}^{v} dv = \int_0^t a\, dt$$

$$\boldsymbol{v = v_0 + \int_0^t f(t)\, dt} \qquad (11)$$

Also, if $v = f(t)$

$$\int_{s_0}^{s} ds = \int_0^t v\, dt$$

$$\boldsymbol{s = s_0 + \int_0^t f(t)\, dt} \qquad (12)$$

C. Acceleration given as a function of velocity ($a = f(v)$):

$$a = \frac{dv}{dt} \Longrightarrow dt = \frac{dv}{a}$$

$$\boldsymbol{t = \int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)}} \qquad (13)$$

Or, $vdv = ads \Longrightarrow vdv = f(v)ds$

$$\boldsymbol{s = s_o + \int_{v_0}^{v} \frac{vdv}{f(v)}} \qquad (14)$$

D. Acceleration as a function of displacement ($a = f(s)$)

$$vdv = ads \Longrightarrow \int_{v_0}^{v} vdv = \int_{s_0}^{s} f(s)ds$$

$$\boldsymbol{v^2 = v_o^2 + 2\int_{s_0}^{s} f(s)ds} \qquad (15)$$

<u>Notes</u>:

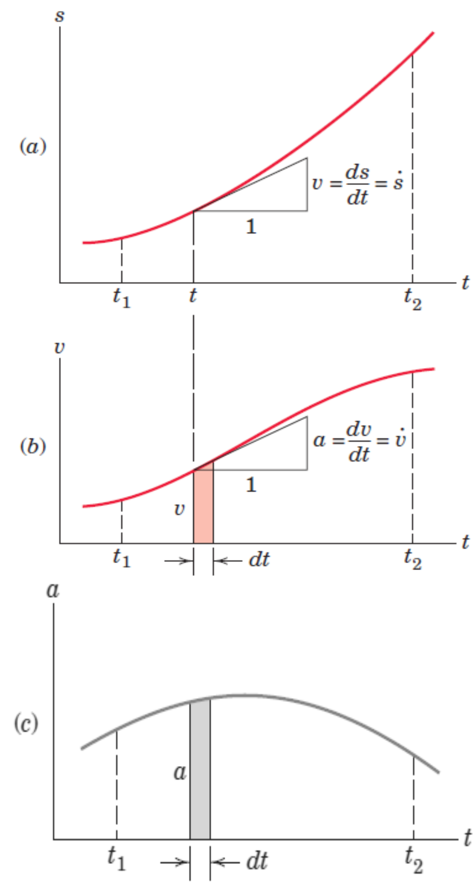
1. If the particles (car, bicycle, train, …etc.) start from rest, then
$v_0 = 0$, $s_0 = 0$, $t_0 = 0$

2. If the particle is stopped or take off , then $v_{final} = 0$

3. $a = 9.81 \text{m/s}^2$ or $32.2\ \text{ft/s}^2$ (when the body falls to the earth). Neglected the air resistant

Grapher interpretation:

1- The s-t , v-t and a-t graphes



$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\, dt \qquad \text{or} \qquad s_2 - s_1 = (\text{area under } v\text{-}t \text{ curve})$$

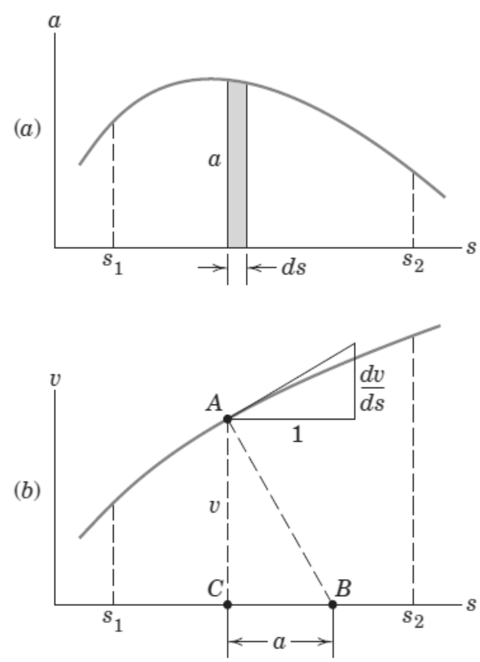
$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\, dt \qquad \text{or} \qquad v_2 - v_1 = (\text{area under } a\text{-}t \text{ curve})$$

2- The $v - s$ and $a - s$ graphics

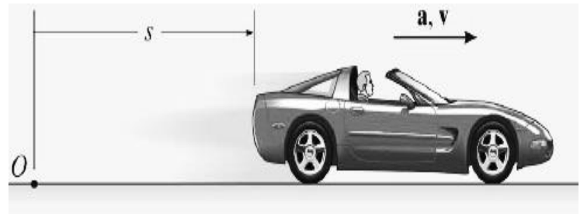
$$\int_{v_1}^{v_2} v\, dv = \int_{s_1}^{s_2} a\, ds \qquad \text{or} \qquad \frac{1}{2}({v_2}^2 - {v_1}^2) = (\text{area under } a\text{-}s \text{ curve})$$

$$a = v\left(\frac{dv}{ds}\right)$$

Accel. = velocity ∗ (slop of $v - s$ graph)



Ex. (1): The car in moves in a straight line such that for a short time its velocity is defined by $v = (3t^{\,2} + \ 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3\ s$. When $t = 0,\ s = 0$



Sol.

$$v = f(t) = 3t^{\,2} + \ 2t$$

$$\left(\overset{+}{\rightarrow}\right) \qquad v = \frac{ds}{dt} = 3t^{\,2} + \ 2t$$

$$\int_{s_0}^{s} ds = \int_{t_0}^{t} (3t^{\,2} + \ 2t)dt \qquad , \ t_0 = 0,\ s_0 = 0$$

$$s = t^{\,3} + t^{\,2}$$

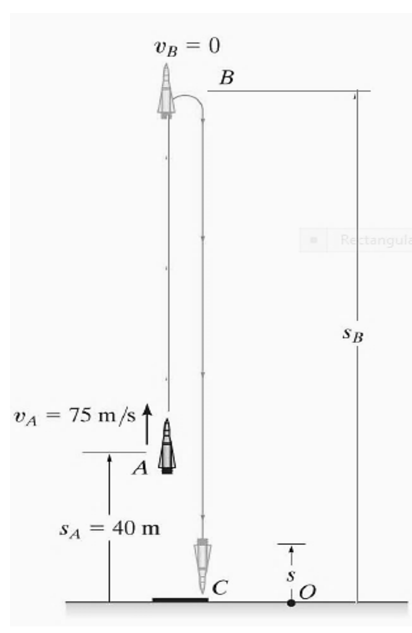
When $t = 3$ sec. , $\qquad s = (3)^{\,3} + (2)^{\,2} = 36$ ft *Ans.*

$$\left(\overset{+}{\rightarrow}\right) \qquad a = \frac{dv}{dt} = 6t + 2$$

When $t = 3$ sec. $\Longrightarrow$ $\quad a = 6(3) + 2 = 20\ \text{ft}/s^2$ *Ans.*

***Note**: The formulas for constant acceleration cannot be used to solve this problem, because the acceleration is a function of time.*

Ex. (2): During a test a rocket travels upward at 75 m/s, and when it is 40m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.



Sol.

$v_A = +75$ m/s at $t = 0$

At maximum height $s = s_B$, $v_B = 0$ (at rest)

$a = -9.81\ \text{m}/s^2$ ($a =$ constant)

Path A-B

$$(+\uparrow) \quad v_B^2 = v_A^2 + 2a(s_B - s_A)$$

$0 = (75)^2 + 2(-9.81)(s_B - 40)$

$\therefore \quad s_B = 327 \text{ m}$ *Ans.*

Path B-C

$(+\uparrow) \quad v_C^2 = v_B^2 + 2a(s_C - s_B)$

$v_C^2 = 0 + 2(9.81)(0 - 327)$

$\therefore \quad v_C = -80.1 = 80.1 \text{ m/s} \downarrow$ *Ans.*

(The negative root was chosen since the rocket is moving downward)

Or, Path A- C

$(+\uparrow) \quad v_C^2 = v_A^2 + 2a(s_C - s_A)$

$v_C^2 = (75)^2 + 2(-9.81)(0 - 40)$

$\therefore \quad v_C = -80.1 = 80.1 \text{ m/s} \downarrow$ *Ans.*

Ex. (3): The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)\text{ft}$, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6 s time interval. Hint: Plot the path to determine the total distance traveled.

Sol.:

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft}$$ **Ans.**

$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$t = 1$ s and $t = 5$ s

The position of the particle at $t = 0$ s, 1 s and 5 s are

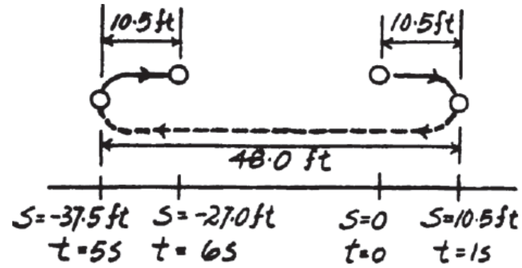
$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5\text{ ft}$$

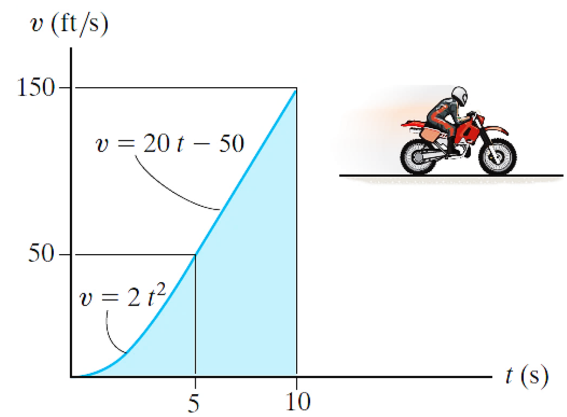
$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5\text{ ft}$$ ***Ans.***

From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0\text{ ft}$$



Ex. (4): A motorcyclist travels along a straight road with the velocity described by the graph. Construct the s-t and a-t graphs. At $t = 0,\ v = 0$



Sol.:

s–t Graph:

For the time interval $0 \le t < 5\text{s}$,

the initial condition is $s = 0$ when $t = 0$

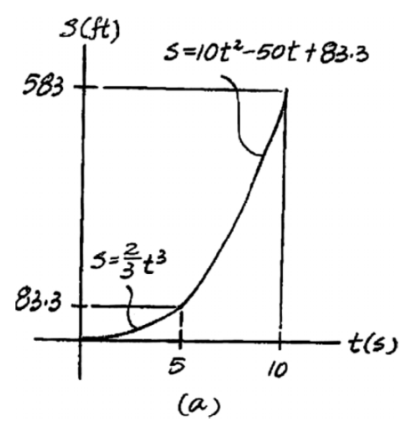
$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t 2t^2 dt$$

$$s = \left(\frac{2}{3}t^3\right)\text{ft}$$

When $t = 5$s,

$$s = \frac{2}{3}(5^3) = 83.33\text{ ft} = 83.3\text{ ft}$$



For $5\text{s} < t \le 10\text{s}$

the initial condition is $s = 83.33$ ft when $t = 5$s

$(\xrightarrow{+})$ $ds = v\,dt$

$$\int_{83.33\text{ ft}}^{s} ds = \int_{5\text{ s}}^{t} (20t - 50)dt$$

$$s\Big|_{83.33\text{ ft}}^{s} = (10t^2 - 50t)\Big|_{5\text{ s}}^{t}$$

$$s = (10t^2 - 50t + 83.33)\text{ ft}$$

When $t = 10$ s,

$$s|_{t=10\text{s}} = 10(10^2) - 50(10) + 83.33 = 583\text{ ft}$$

The $s-t$ graph is shown in Fig. a.

a–t Graph: For the time interval $0 \le t < 5$ s,

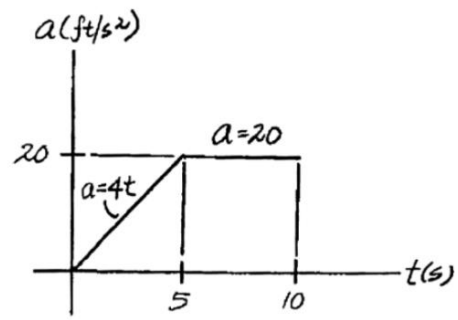
$(\xrightarrow{+})$ $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(2t^2) = (4t)\text{ ft/s}^2$

When $t = 5$s,

$$a = 4(5) = 20\text{ ft/s}^2$$

For the time interval $5\text{ s} < t \le 10$ s,

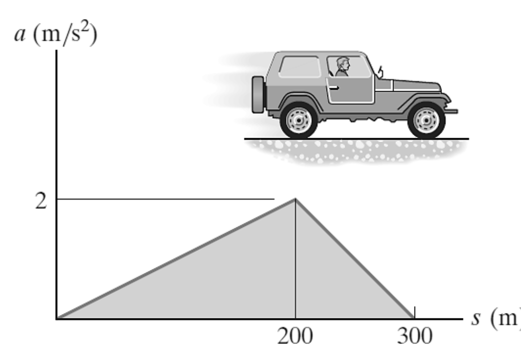
$(\xrightarrow{+})$ $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(20t - 50) = 20\text{ ft/s}^2$



(b)

The $a-t$ graph is shown in Fig. b.

Ex. (5): The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the $v-s$ graph. At $s = 0,\ v = 0$



Sol.:

***a−s* Graph**: The function of acceleration a in terms of s for the interval $0\text{ m} \le s < 200\text{ m}$ is

$$\frac{a}{s} = \frac{2}{200} \qquad a = (0.01s)\text{ m/s}^2$$

For the interval $200 \text{ m} < s \leq 300 \text{ m}$,

$$\frac{a}{300 - s} = \frac{2}{300 - 200} \qquad a = (-0.02s + 6)\ \text{m/s}^2$$

v–s Graph: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0\ m \leq s < 200\ m}$,

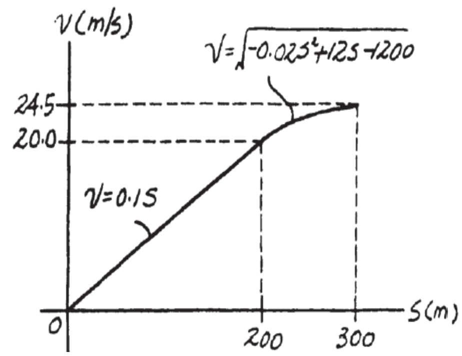
$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s)\ \text{m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0$ m/s

For the interval $\mathbf{200\ m < s \leq 300\ m}$,



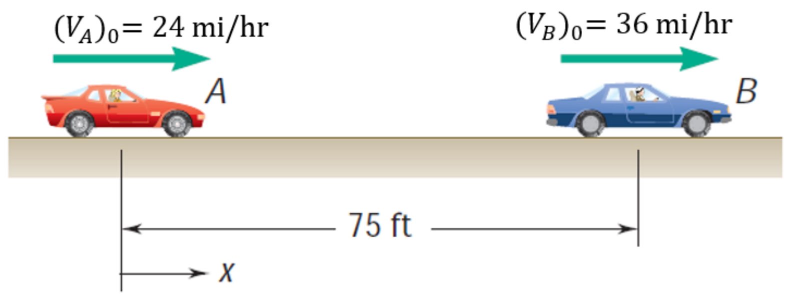
$$vdv = ads$$

$$\int_{20.0\text{ m/s}}^{v} vdv = \int_{200\text{ m}}^{s} (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right)\ \text{m/s}$$

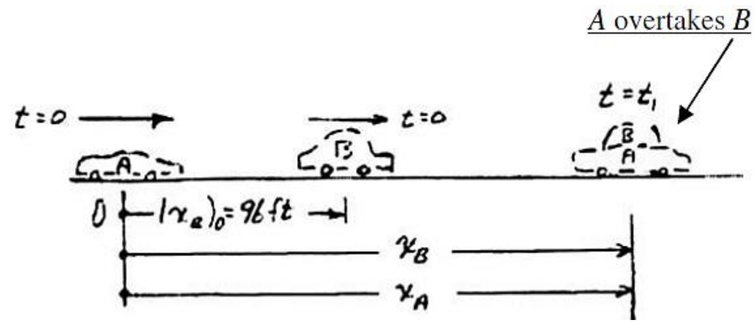
At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5$ m/s

Ex. (6): Automobiles A and B are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile A has a constant acceleration of $1.8\ \text{ft/s}^2$ and that B has a constant deceleration of $1.2\ \text{ft/s}^2$, determine (a) when and where A will overtake B, (b) the speed of each automobile at that time.



Sol.:

$$a_A = +1.8\ \text{ft/s}^2 \quad a_B = -1.2\ \text{ft/s}^2$$
$$(v_A)_0 = 24\ \text{mi/h} = 35.2\ \text{ft/s}$$
$$(v_B)_0 = 36\ \text{mi/h} = 52.8\ \text{ft/s}$$



Motion of Auto. **A**: $v_A = (v_A)_0 + a_A t = 35.2 + 1.8t$ (1)

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2 \quad (2)$$

Motion of Auto. ***B***: $v_B = (v_B)_0 + a_B t = 52.8 - 1.2t$ (3)

$$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2 \quad (4)$$

(a) A overtakes B at $t = t_1$

$$x_A = x_B: \quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$$

$$1.5t_1^2 - 17.6t_1 - 75 = 0$$

$$t_1 = -3.22\ \text{s} \quad \text{and} \quad t_1 = 15.0546 \qquad t_1 = 15.05\ \text{s} \blacktriangleleft$$

From Eq. (2): $x_A = 35.2(15.05) + 0.9(15.05)^2$ $\qquad x_A = 734\ \text{ft} \blacktriangleleft$

(b) Velocities when $t_1 = 15.0546\ s$

$v_A = 35.2 + 1.8(15.05) = 62.29\ \text{ft/s}\ \rightarrow$ ***Ans.***

$v_B = 52.8 - 1.2(15.05) = 34.74\ \text{ft/s}\ \rightarrow$ ***Ans.***