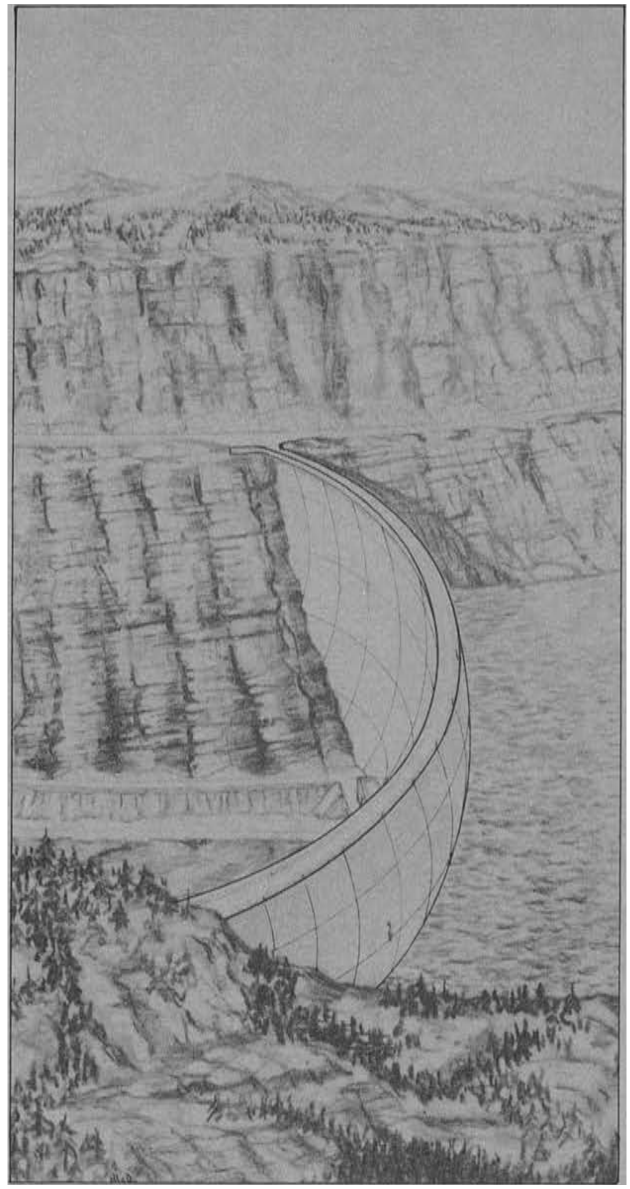
A WATER RESOURCES TECHNICAL PUBLICATION

ENGINEERING MONOGRAPH No. 36



Guide for Preliminary Design of Arch Dams

UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF RECLAMATION

A WATER RESOURCES TECHNICAL PUBLICATION

Engineering Monograph No. 36

Guide for Preliminary Design of Arch Dams

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United States Department of the Interior •

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Preface

IN THEIR PLANNING of water resources development projects, Bureau of Reclamation engineers may be required to prepare preliminary layouts and estimates of arch dams. Such preliminary studies aid the project planners in evaluating the feasibility of arch dams and their relationship to other types of dams in the project plans in terms of comparative economy, availability of construction materials, and other technical considerations.

In the past, there has been no convenient method of preparing preliminary designs of arch dams. This monograph provides a rapid method to aid engineers in preparing such designs. The engineers will find the empirical formulas and charts developed here useful as a guide in preliminary layouts. They will also find that the formulas are useful in reanalyzing previous preliminary designs to obtain more accurate estimates of dimensions and volumes of arch dams being considered in project plans.

Summary

THE EMPIRICAL FORMULAS developed in this monograph for use by project planners or other interested persons will assist them in estimating volumes and basic dimensions for arch dams on future projects and in updating past estimates.

The information contained here is to be used only as a guide for the preliminary layout of concrete arch dams.

Topographic dimensions necessary to compute thicknesses, projections, and volume are structural height, H, and horizontal distances between abutments including estimated excavation to sound rock at crest elevation, L_1, and 15 percent of H above base, L_2. Generally, results from the suggested formulas are conservative as compared with values computed more precisely.

Empirical formulas derived from a statistical analysis of existing concrete arch dam data are:

$$T_c = 0.01[H + 1.2L_1];$$

$$T_{0.45} = 0.95T_B;$$

$$T_B = \sqrt[3]{0.0012HL_1L_2\left(\frac{H}{400}\right)^{\frac{H}{400}}}$$

$$USP_{CREST} = 0.0;$$

$$USP_{BASE} = 0.67T_B;$$

$$USP_{0.45H} = 0.95T_B;$$

$$V = 0.000002H^2L_2\left[\frac{(H+0.8L_1)^2}{L_1-L_2}\right] + 0.0004HL_1[H+1.1L_1].$$

Acknowledgments

John R. Brizzolara and Kenneth G. Bell of the Stress Analysis Unit, Concrete Dams Section, Dams Branch, Division of Design, were major contributors to this monograph. Mr. Brizzolara assembled major portions of the statistical data and Mr. Bell developed the volumetric formulas and prepared the nomographs. The monograph was prepared under the supervision of M. D. Copen, Head of the Stress Analysis Unit.

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Definitions

CONCRETE ARCH DAM TERMINOLOGY is not universal and physical terms used in this guide may, in some instances, seem ambiguous. For this reason, words, phrases, and symbols used in this guide referring to arch dams are defined.

Double curvature	Continuously curved in plan and elevation.
Single curvature	Curved in plan only.
Depth of excavation	Depth from ground surface to sound rock as determined from available geological sources (geologists, geologic maps, cores, etc.).
Foundation	Total mass of sound rock supporting the dam.
Abutment	The rock mass which supports the horizontal elements, such as canyon walls.
Base	Bottom surface of vertical element resting on foundation.
Extrados	Curved upstream surface of horizontal arch elements.
Intrados	Curved downstream surface of horizontal arch elements.
Crest	Top of dam.
Axis	Vertical reference surface, cylindrical in plan and coincident with the extrados at crest elevation.
Axis radius	Radius of axis equal to extrados radius at crest elevation.
Central angle	Angle at extrados center formed by lines extended to arch abutments.
Crown cantilever	Vertical section positioned about midway between abutments and whose base is generally the lowest elevation of the dam.
Structural height	Vertical distance from crest of dam to lowest point of foundation.
Thickness	Horizontal distance between upstream and downstream faces of dam on line normal to extrados.
Upstream projection	Horizontal distance from extrados to axis on line normal to extrados.
Downstream projection.	Horizontal distance from intrados to axis on line normal to extrados (thickness=upstream projection +downstream projection).

H =structural height.
T_c =thickness at crest.
T_B =thickness at base.
$T_{0.45H}$=thickness of crown cantilever at $0.45H$ above base.
USP =upstream projection.
DSP =downstream projection.
L_1 =straight line distance at crest elevation between abutments assumed excavated to sound rock.
L_2 =straight line distance, at $0.15H$ above base, between abutments assumed excavated to sound rock.
V =estimated volume of dam.

Introduction

THIS MONOGRAPH is a guide which presents formulas and charts for preliminary design of arch dams. Included are the procedure and examples for preparing a preliminary layout of a concrete arch dam. Figure 1 shows the topographic map used in example 1. The figure indicates the required measurements for determining the volume of the dam and dimensions of the crown cantilever.

From statistical analyses of arch dam geometrical properties, empirical formulas were developed for the rapid determination of initial physical dimensions for design and volume estimates. The more complicated formulas are expressed in nomographic charts, shown in figures 2, 3, 4, and 5. Figure 6 illustrates a plan and figure 7 illustrates the crown cantilever and lines of centers for example 1.

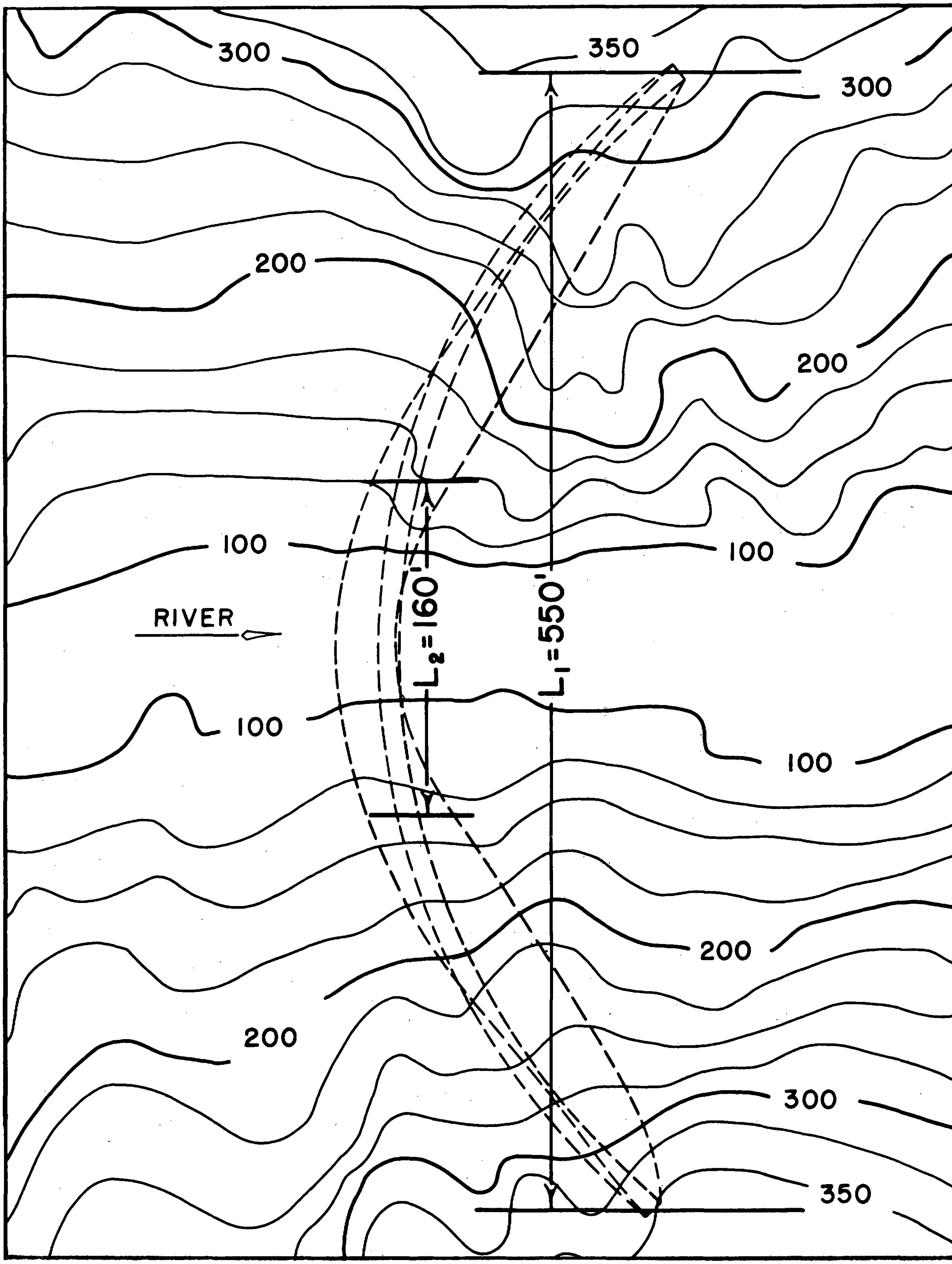


FIGURE 1.—*Damsite topography and required dimensions for nomographs and layout of dam. For example 1: H=estimated structural height (290 feet); L_1=chord length, crest elevation (550 feet); L_2=chord length, 0.15H above base (160 feet); and crest assumed at elevation 325.*

Statistical Data

DATA USED in the statistical analysis were taken from analytical studies currently on file in the Stress Analysis Unit. Data used in preparation of this guide include:

a. Structural height.
b. Length, thickness, and central angle of arch at crest of dam.
c. Axis radius.
d. Length and central angle of lowest theoretical arch in analysis.
e. Thickness and upstream projection at base of crown cantilever.
f. Volume of dam.
g. Sustained modulus of elasticity of concrete and rock.
h. Approximate loaded foundation area.
i. Profile of dam developed along axis.

Analyses

STATISTICAL ANALYSES of tabulated data were based on observation, experience, and intuition. Combinations of height, thicknesses, projections, and distances were incorporated with constant or variable coefficients in linear, nonlinear, or exponential equations to determine proper relationships for the basic dimensions and volume.

A foremost consideration in developing empirical formulas for concrete arch dams is the shape of the canyon. Initially, the tabulated data were separated into two general canyon shapes, U and V. Several formulas were developed in part for each shape, considering arc length at crest and structural height. These observations disclosed to some degree the relative importance of the selected variables. Including another variable, the canyon width near the base, enable both canyon shapes to be handled with a single formula. The most satisfactory elevation for measuring the lower canyon width is 15 percent of the structural height above the base.

Difficulties experienced in arriving at simple equations were due in part to the inclusion of data for both single and double curvature dams. Although a greater number of single curvature dams have been designed, data from the more efficient double curvature concrete dams were used to develop the final empirical formulas. As more double curvature arch dams are designed, more data may become available for refinements in the formulas and nomographs.

Results

THE MOST IMMEDIATE INFORMATION necessary for the design of a concrete arch dam for a reconnaissance study is an estimate of the volume and a general plan. As a result of the statistical analyses, empirical formulas were developed for computing the volume of concrete in a dam and for sufficient dimensions for a crown cantilever to produce an adequate shape. Dimensions, in feet, required for solving the equations are: H, the structural height (which is the vertical distance from the crest of the dam to the lowest assumed point of foundation); L_1, the straight line distance at crest elevation between abutments, assumed excavated to sound rock; and L_2, the straight line distance at $0.15H$ between abutments, assumed excavated to sound rock.

Crown Cantilever—Thicknesses

Thicknesses which are necessary for shaping the crown cantilever are at the crest, T_c; at the base, T_B; and at $0.45H$ above the base, $T_{0.45H}$. Formulas for computing each of the thicknesses are:

a. Crest thickness, in feet,

$$T_c=0.01[H+1.2\ L_1];$$

b. Base thickness, in feet,

$$T_B=\sqrt[3]{0.0012\ H\ L_1\ L_2\left(\frac{H}{400}\right)^{\frac{H}{400}}}\ ;$$

c. Thickness at $0.45H$, in feet,

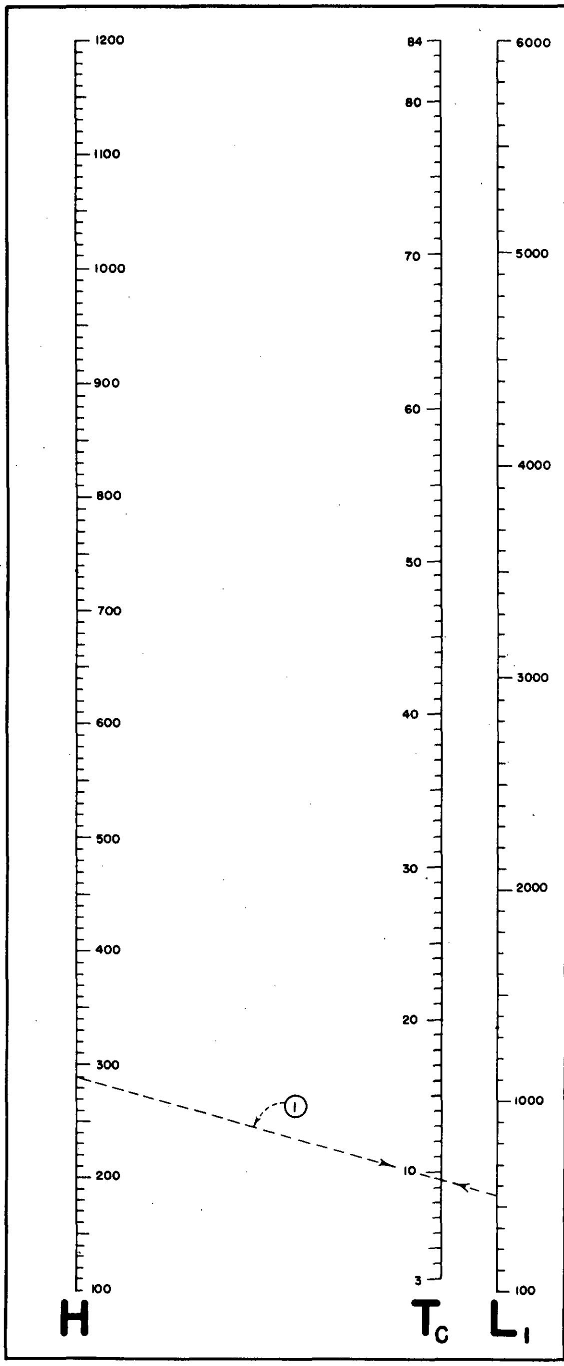
$$T_{0.45H}=0.95\ T_B.$$

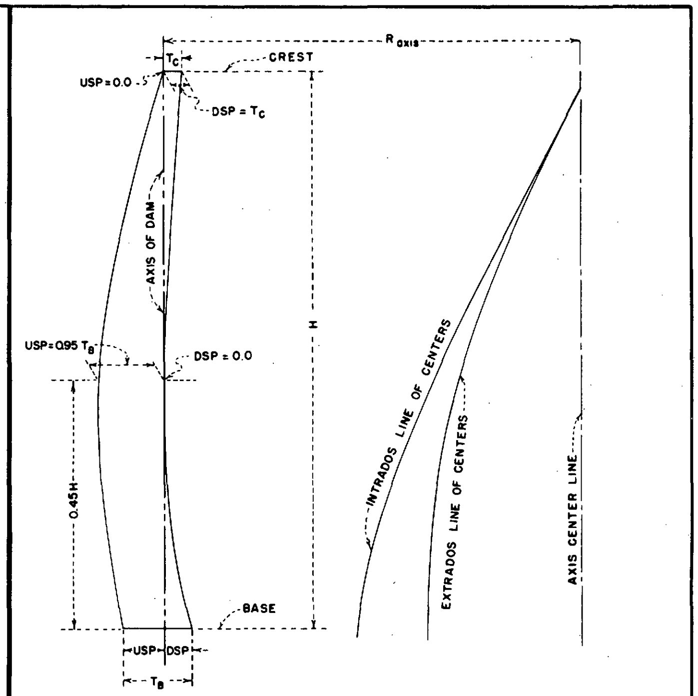
Nomographs for estimating the crest and base thicknesses are shown in figures 2 and 3. The crest thickness is found by intersecting the T_c-scale with a straight line from H to L_1 in figure 2. Thickness of the base is obtained from figure 3 by the following procedure:

1. Mark on the S_1-scale the intersection of a straight line between known values L_1 and L_2.
2. The base thickness is read from the T_B-scale at the intersection of a straight line between S_1 and the known value H.

Crown Cantilever—Projections

Upstream and downstream projections are horizontal distances used to locate the extrados and intrados relative to the reference surface, the axis, as shown in figure 2. Projections related to the preceding three thicknesses are defined as follows:





NOTES

Dimensions required for using charts

- **H** = Estimated structural height of dam from crest to base including assumed depth of excavation.
- **L_1** = Straight line distance at crest elevation between abutments including estimated excavation to sound rock.
- **L_2** = Straight line distance between abutments including estimated excavation to sound rock at 0.15H above the base.

Procedure

Crest thickness, T_C, on Figure 2

1. Intersect T_C with a straight line from H to L_1

Base thickness, T_B, on Figure 3

1. Intersect S_1 with a straight line from L_1 to L_2
2. Intersect T_B with a straight line from H to S_1

Upstream projections:

At crest, USP = 0.0
At 0.45H, USP = $0.95T_B$
At base, USP = $0.67T_B$

Downstream projections:

At crest, DSP = T_C
At 0.45H, DSP = 0.0
At base, DSP = $0.33T_B$

FIGURE 2.—*Nomograph for obtaining crest thickness and projections on crown cantilever.*

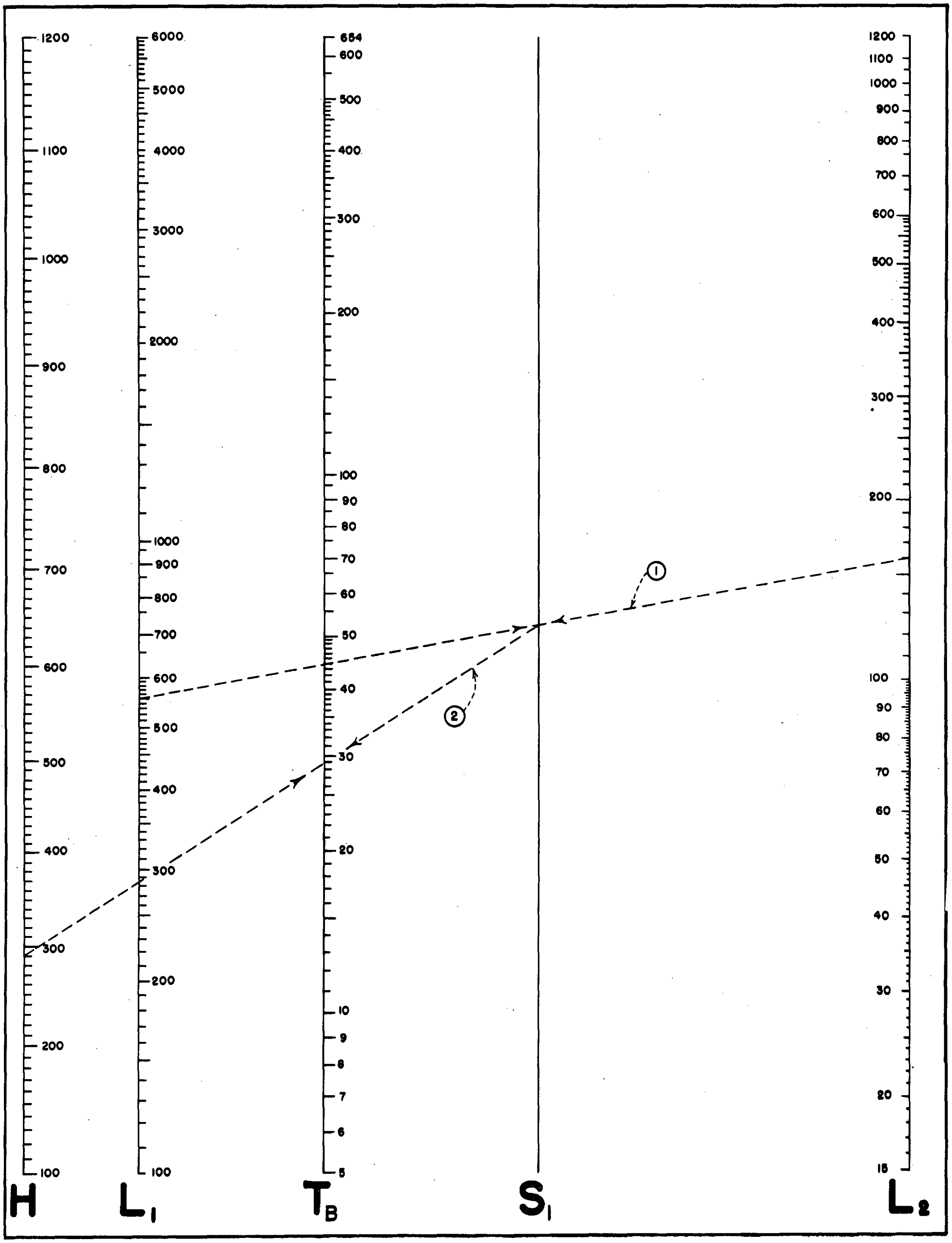


FIGURE 3.—*Nomograph for obtaining base thickness and projections on crown cantilever. See notes and drawing on figure 2.*

a. Crest

$$USP=0.0$$
$$DSP=T_c$$

b. base

$$USP=0.67\ T_B$$
$$DSP=0.33\ T_B$$

c. At $0.45H$

USP=maximum upstream projection=$0.95\ T_B$
DSP=minimum downstream projection=0.0.

Volume

The formula for computing the volume in cubic yards is:

$$V=V_1+V_2,$$

where

$$V_1=0.000002\ H^2\ L_2\left[\frac{(H+0.8\ L_1)^2}{L_1-L_2}\right]$$

and

$$V_2=0.0004\ H\ L_1[H+1.1\ L_1].$$

Formulas for V_1 and V_2 are graphically represented in figures 4 and 5. The procedure for estimating the volume from these nomographs is:

To obtain V_1,

On figure 4–A,

1. Obtain a value of U by intersecting the U-scale with a straight line between known values L_1 and L_2.
2. The value of W is obtained by intersecting the W-scale with a straight line from L_1 to known value H.

On figure 4–B,

3. Mark on the S_1-scale the intersection of a straight line between H and L_2.
4. Indicate on the S_2-scale the intersection of a straight line between U and W (values for U and W are the scaled values from figure 4–A).
5. V_1 is now found by intersecting the V_1-scale with a straight line between the marks on the S_1- and S_2-scales.

To obtain V_2,

On figure 5–A,

1. Drawing a straight line between values H and L_1 intersecting the X-scale.

On figure 5–B,

2. Mark on the S_3-scale its intersection with a straight line between H and L_1.
3. V_2 is found at the intersection of the V_2-scale by a straight line between the values of X and S_3. (The X value is scaled from figure 5–A.)

To obtain V,

V, the total volume, is the sum of V_1 and V_2.

Limitations and Accuracy

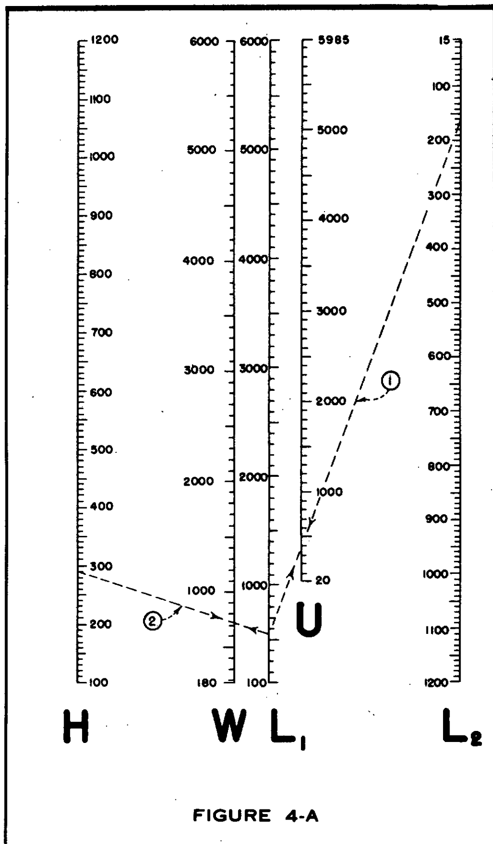
Upper and lower limits on the known data are:

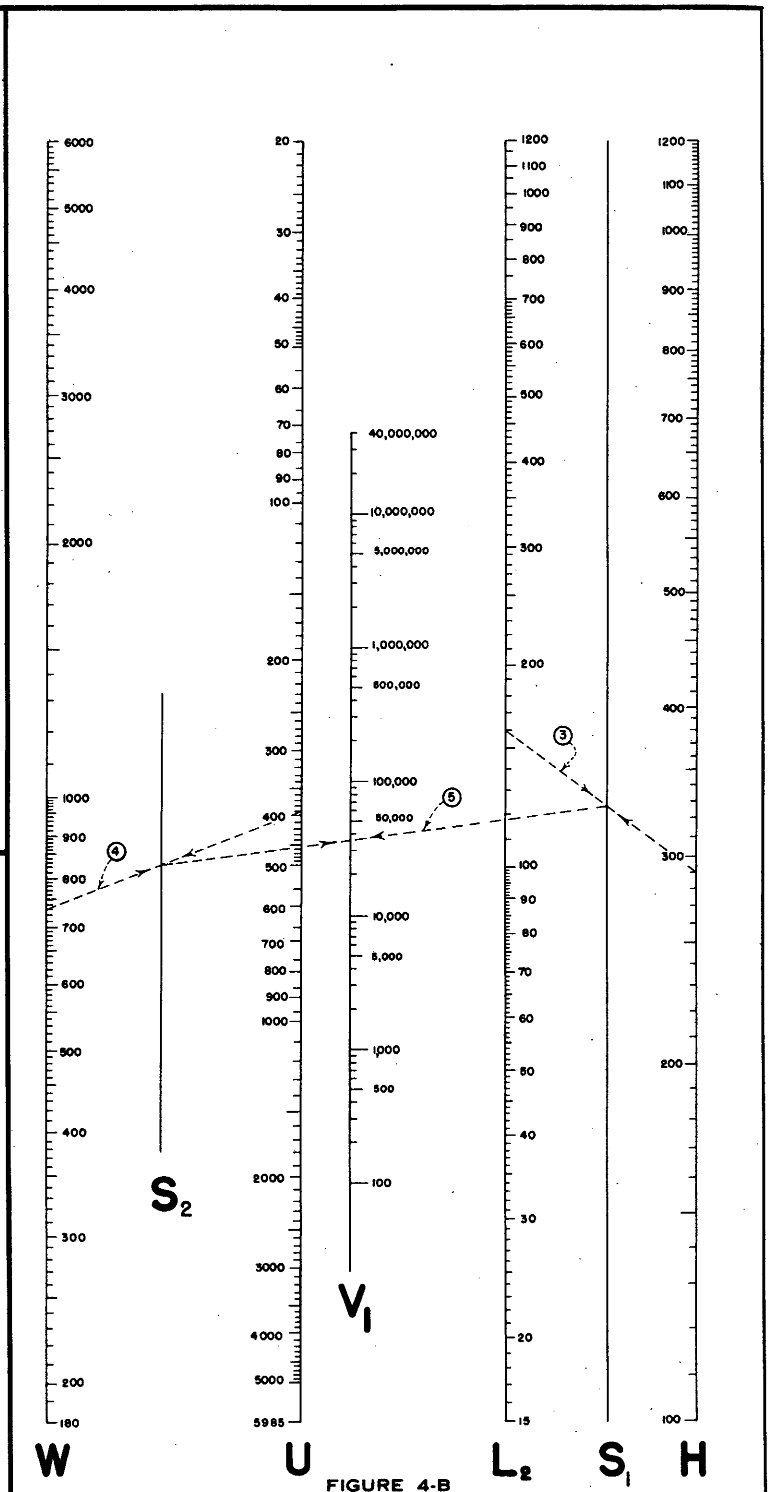
$$100\leq H\leq 1{,}200$$
$$100\leq L_1\leq 6{,}000$$
$$15\leq L_2\leq 1{,}200.$$

Upper and lower limits of the results are shown in the nomographs. The minimum thickness of 3 feet in figure 2 is an arbitrary lower limit; factors other than stress become determining considerations for very thin dams.

Accuracy of the formulas for volumes and thicknesses is within 10 percent of designed values for 75 percent of the double curvature dams studied. However, formulas have been adjusted to assure, for the most part, conservative quantities and dimensions.

Numerical results from the formulas or nomographs are solely for preliminary design of concrete arch dams—that is, for estimating the quantity of mass concrete and computing thicknesses for initial layouts or cost estimates. The final design *must* be prepared by specialists in the design and analysis of arch dams.





NOTES

Dimensions required for using charts.

- **H = Estimated structural height of dam from crest to base including assumed depth of excavation.**
- **L_1 = Straight line distance at crest elevation between abutments including estimated excavation to sound rock.**
- **L_2 = Straight line distance between abutments including estimated excavation to sound rock at 0.15H above the base.**

Procedure for using charts

On Figure 4-A

1. **Intersect U with a straight line from L_1 to L_2**
2. **Intersect W with a straight line from H to L_1**

On Figure 4-B

3. **Intersect S_1 with a straight line from H to L_2**
4. **Intersect S_2 with a straight line from U to W (Values for U and W are obtained from Figure 4-A)**
5. **Intersect V_1 with a straight line from S_1 to S_2**

FIGURE 4.—*Nomograph for obtaining V_1.* ($V = V_1 + V_2$).

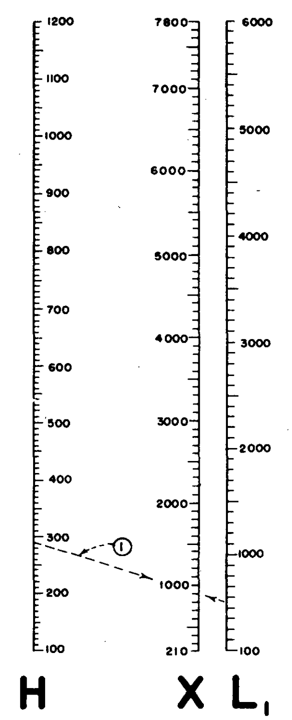


FIGURE 5-A

NOTES

Dimensions required for using charts.

- **H** = Estimated structural height of dam from crest to base including assumed depth of excavation.
- **L_1** = Straight line distance at crest elevation between abutments including estimated excavation to sound rock.
- **L_2** = Straight line distance between abutments including estimated excavation to sound rock at 0.15H above the base.

Procedure for using charts

On Figure 5-A

1. Intersect X with a straight line from H to L_1

On Figure 5-B

2. Intersect S_3 with a straight line from H to L_1
3. Intersect V_2 with a straight line from S_3 to X (Value for X is obtained from Figure 5-A)

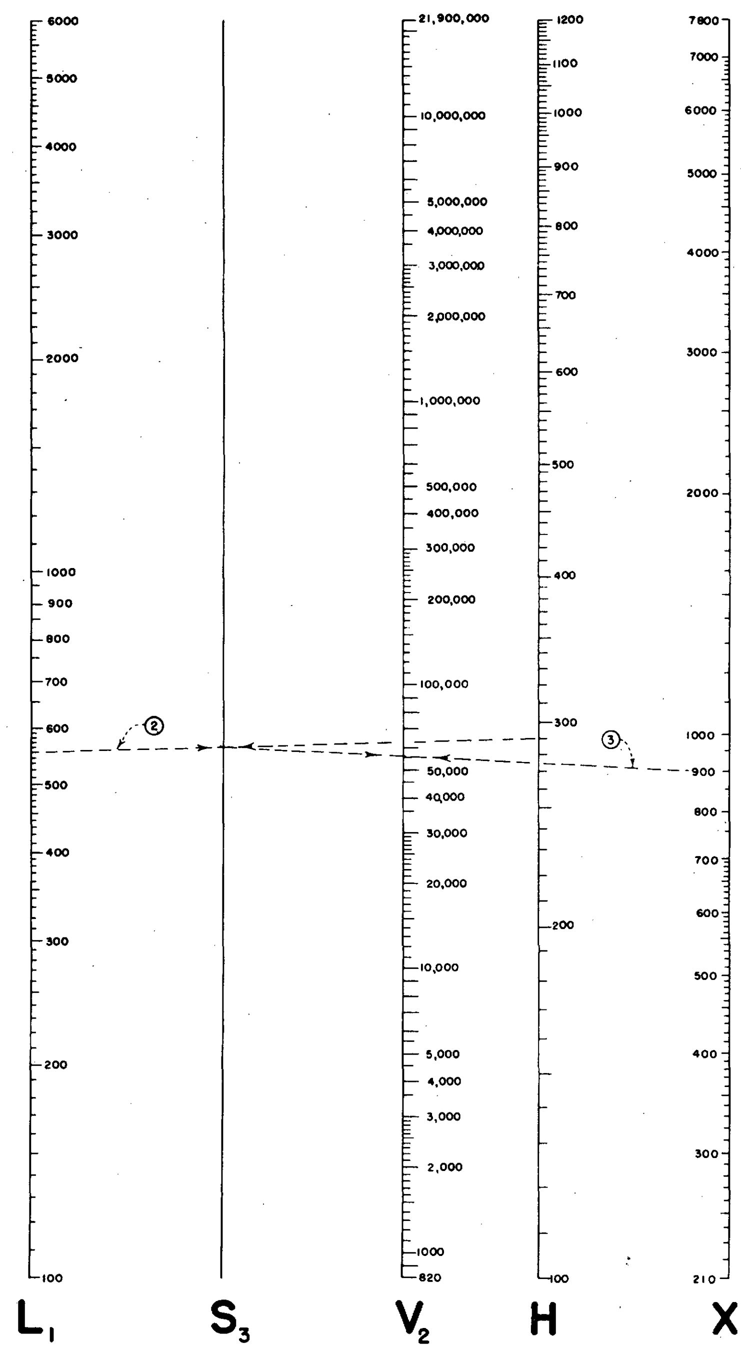


FIGURE 5-B

FIGURE 5.—*Nomograph for obtaining V_2. ($V = V_1 + V_2$).*

Examples

TWO EXAMPLES ARE PRESENTED to demonstrate the procedure for using the formulas. Known data are from dams designed and analyzed by the Stress Analysis Unit. Actual values used in the design studied are indicated following the computed values.

A typical damsite is shown in figure 1 from which were measured the dimensions L_1 and L_2. The estimated structural height, H, together with L_1 and L_2 are illustrated in the nomographs, figures 2, 3, 4, and 5. A layout for example 1 utilizing procedures outlined in this guide, is shown in plan in figure 6 and in elevation in figure 7.

Example 1

From the estimated structural height, $H=290$ feet, and the measured chord lengths from figure 1, $L_1=550$ feet and $L_2=160$ feet, find thicknesses and projections on the crown cantilever and volume.

a. At crest

Thickness:

$$T_c=0.01[H+1.2L_1]$$
$$=0.01[290+(1.2)(550)]$$
$$=9.5 \text{ feet} \quad \ldots\ldots \quad 10.0 \text{ feet.}$$

Upstream projection:

$$USP_c=0.0 \quad \ldots\ldots \quad 0.0 \text{ foot.}$$

Downstream projection:

$$DSP_c=9.5 \text{ feet} \quad \ldots\ldots \quad 10.0 \text{ feet.}$$

b. At base

Thickness:

$$T_B=\sqrt[3]{0.0012\, H\, L_1\, L_2\left(\frac{H}{400}\right)^{\frac{H}{400}}}$$
$$=\sqrt[3]{(0.0012)(290)(550)(160)\left(\frac{290}{400}\right)^{\frac{290}{400}}}$$
$$=28.9 \text{ feet} \quad \ldots\ldots \quad 27.8 \text{ feet.}$$

Upstream projection:

$$USP_B=0.67\ T_B$$
$$=(0.67)(28.9)$$
$$=19.4 \text{ feet} \quad \ldots\ldots \quad 20.5 \text{ feet.}$$

Downstream projection:

$$DSP_B=0.33\ T_B$$
$$=(0.33)(28.9)$$
$$=9.5 \text{ feet} \quad \ldots\ldots \quad 7.3 \text{ feet.}$$

c. At $0.45H$

Thickness:

$$T_{0.45H}=0.95\ T_B$$
$$=(0.95)(28.9)$$
$$=27.5 \text{ feet} \quad \ldots\ldots \quad 25.7 \text{ feet.}$$

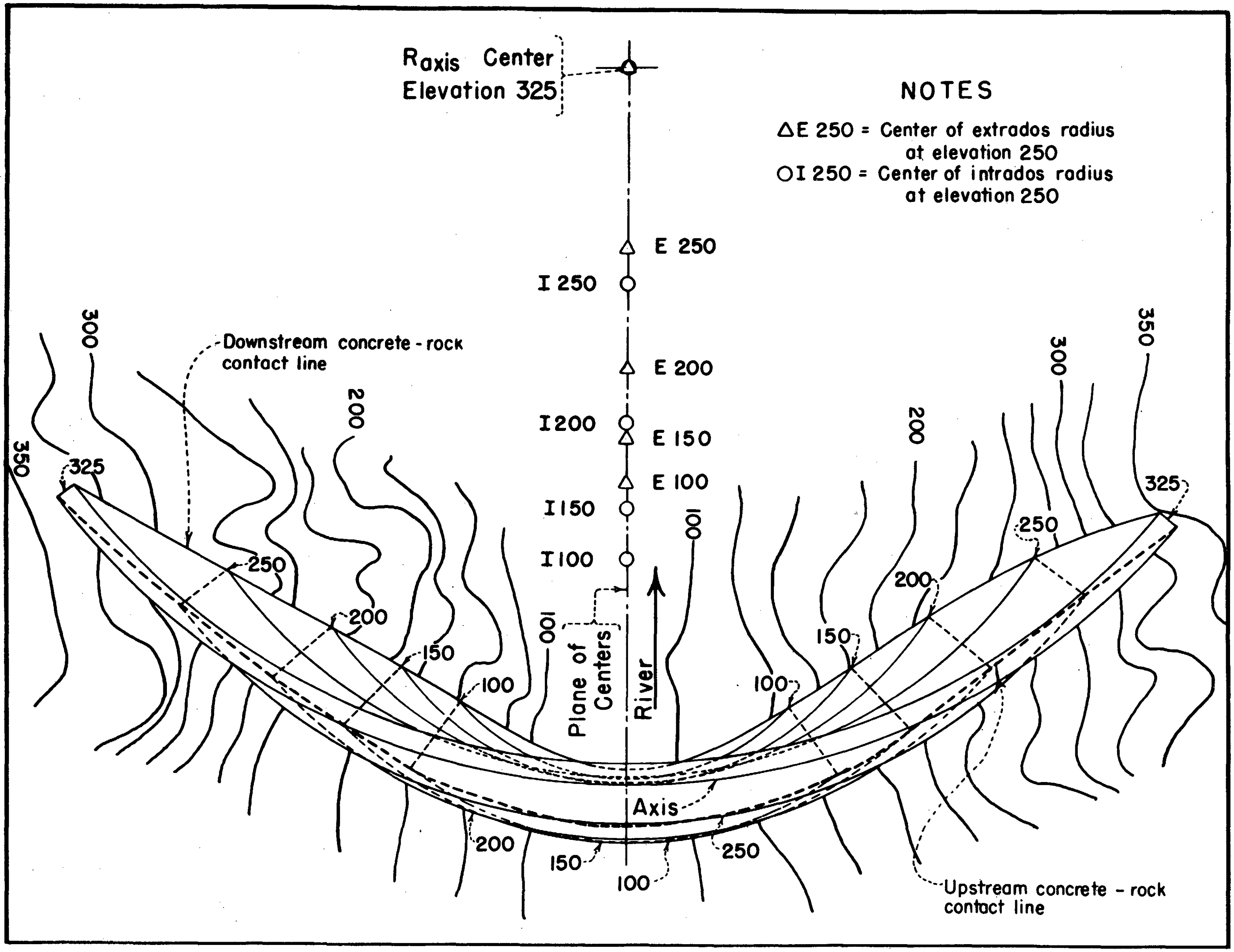


FIGURE 6.—*Plan for a preliminary design.*

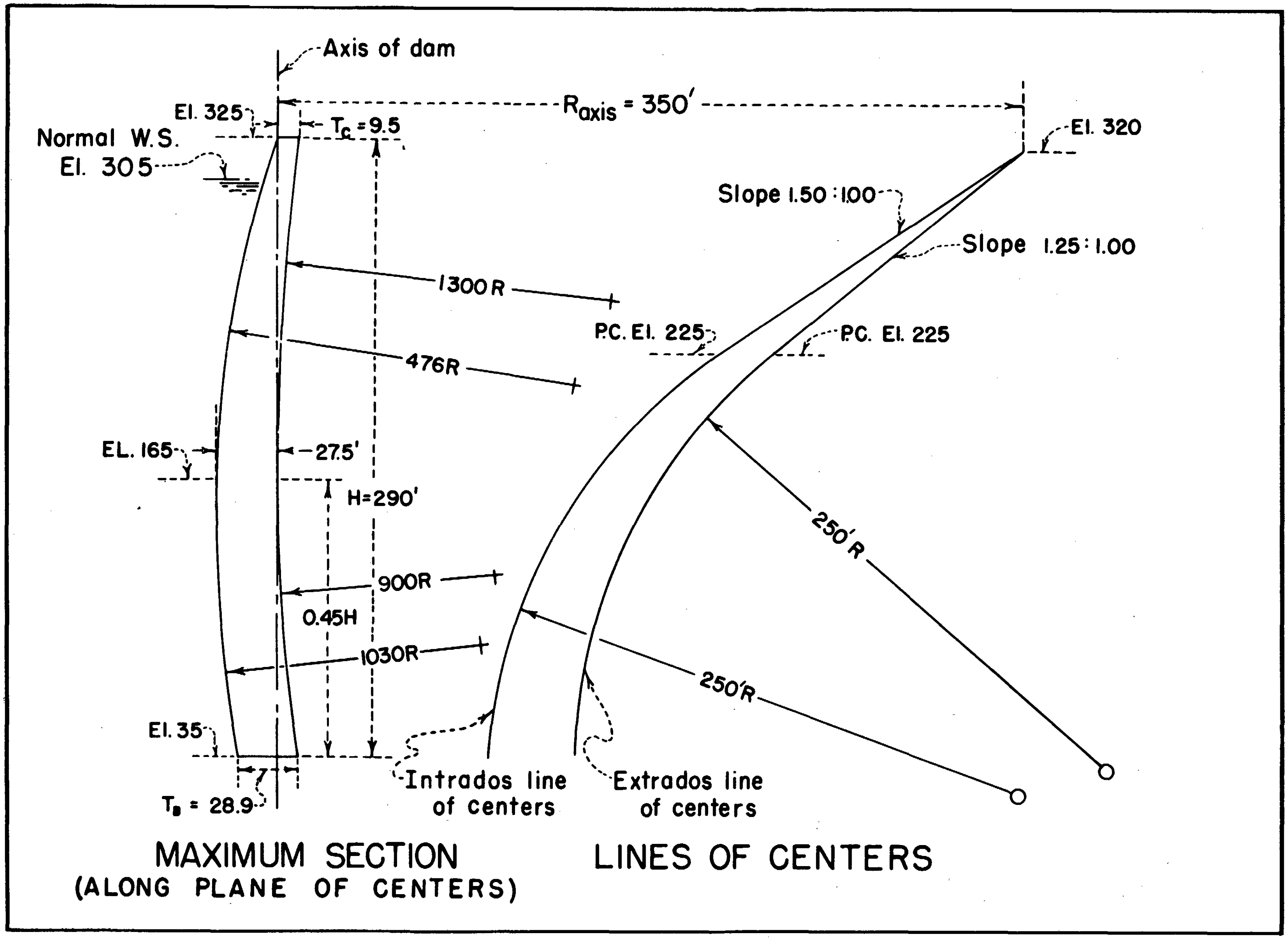


FIGURE 7.—*Crown cantilever and lines of centers for a preliminary design.*

Upstream projection:

$USP_{0.45H}=0.95\ T_B$

$=(0.95)(28.9)$

$=27.5$ feet---------- 30.0 feet.

Downstream projection:

$DSP_{0.45H}=0.0$ foot---------- -4.3 feet.

d. Volume

$$V=V_1+V_2,$$

where

$$V_1=0.000002H^2L_2\left[\frac{(H+0.8L_1)^2}{L_1-L_2}\right]$$

$$=0.000002(290)^2(160)\left[\frac{\{290+(0.8)(550)\}^2}{550-160}\right]$$

$=36{,}800$ cubic yards,

and

$V_2=0.0004HL_1[H+1.1L_1]$

$=0.0004(290)(550)[290+(1.1)(550)]$

$=57{,}100$ cubic yards.

Then,

$V=36{,}800+57{,}100=93{,}900$ cubic yards------ 88,300 cubic yards.

Example 2

Given: $H=736$ feet, $L_1=1{,}350$ feet, and $L_2=400$ feet.

Find: T, USP, and DSP at crest, base, and $0.45H$ and volume.

a. Thicknesses

At crest:

$T_c=0.01[H+1.2L_1]$

$=0.01[736+(1.2)(1{,}350)]$

$=23.6$ feet---------------- 25.0 feet.

At base:

$$T_B=\sqrt[3]{0.0012\ H\ L_1\ L_2\left(\frac{H}{400}\right)^{\frac{H}{400}}}$$

$$=\sqrt[3]{0.0012(736)(1{,}350)(400)\left(\frac{736}{400}\right)^{\frac{736}{400}}}$$

$=113.6$ feet------------- 110.0 feet.

At $0.45H$:

$T_{0.45H}=0.95\ T_B$

$=(0.95)(113.6)$

$=107.9$ feet---------- 100.0 feet.

b. Upstream projections

At crest:

$USP_c=0.0$---------------------- 0.0.

At base:

$USP_B=0.67\ T_B$

$=(0.67)(113.6)$

$=76.1$ feet------------ 65.0 feet.

c. Downstream projections

At crest:

$DSP_c=T_c$

$=23.6$ feet------------ 25.0 feet.

At base:

$DSP_B=0.33\ T_B$

$=(0.33)(113.6)$

$=37.5$ feet------------ 45.0 feet.

At $0.45H$:

$DSP_{0.45H}=0.0$---------------------- 0.0.

d. Volume

$$V=V_1+V_2,$$

where

$$V_1=0.000002H^2L\left[\frac{(H+0.8L_1)^2}{L_1-L_2}\right]$$

$$=0.000002(736)^2(400)\left[\frac{\{736+(0.8)(1{,}350)\}^2}{1{,}350-400}\right]$$

$=1{,}504{,}000$ cubic yards,

and

$V_2=0.0004HL_1[H+1.1L_1]$

$=0.0004(736)(1{,}350)[736+(1.1)(1{,}350)]$

$=883{,}000$ cubic yards.

Then,

$V=1{,}504{,}000+883{,}000=2{,}387{,}000$ cubic yards ----------------- 2,240,000 cubic yards.

Layout

BECAUSE IDEAL DAMSITES are virtually nonexistent, a final design for an arch dam is the result of judicious evaluation and selection of physical properties which best satisfy site conditions, stress requirements, and design criteria. This final design is arrived at by several cycles of layout, analysis, evaluation, and improvement. The initial layout in the series is based on results of formulas in this guide and the judgment of the designer. A procedure for making the initial layout is as follows:

a. From the structural height, H, and chord lengths, L_1 and L_2, describe the crown cantilever using equations or nomographs in this guide.

b. Compute the axis radius: $R_{AXIS}=0.6L_1$.

c. From all available geological information on the damsite, estimate the depth of excavation to sound rock.

d. Draw on vellum overlaying a topographic map of the site a circular arc, with the axis radius connecting abutments at the crest elevation.[1] This arc should be so oriented that the angle of incidence to each abutment is approximately equal.

e. On the axis, locate a point about midway between abutments and in the riverbed (crown cantilever). A line on the drawing connecting this point and the axis center may be used for the plane of centers. On this vertical plane, extrados and intrados centers are located for drawing circular arcs which represent contour lines on the faces of the dam. The system of centers for each face must form smooth and continuous curves to produce a satisfactory line of centers.

f. The extrados and intrados centers at each selected elevation should be spaced on the plane of centers to produce a variation in the ratio of abutment thickness to crown thickness of 1.0:1.0 at the crest to about 1.5:1.0 at mid-height and 1.1:1.0 at the riverbed.

g. Contour lines on the dam pass through the faces of the crown cantilever and terminate at the abutments. For reasons of expediency, the contours should be selected at convenient elevations on both faces, equally spaced wherever possible, and at intervals not greater than 100 feet nor less than 20 feet in elevation.

h. Abutments are drawn radial from the extrados center. The perimetrical contact of dam and foundation should be smooth and continuous.

i. A tangent to each contour on the downstream face at the abutment line should make an angle not less than 30° with a line generally parallel to the canyon wall at that elevation.

[1] The R_{AXIS} should be lengthened if either the arc fails to make contact with the abutment at the estimated depth of excavation, or if the subtended central angle exceeds 120°.

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