Department of Physics

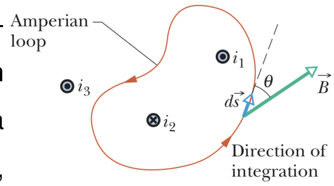
United States Naval Academy

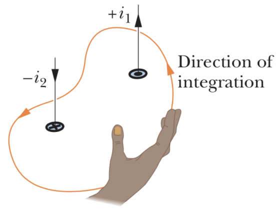
Lecture 23: Ampére's Law, Solenoids and Toroids

Learning Objectives

- Apply Ampére's law to a loop that encircles current, and for more than one current within an Amperian loop, determine the net current to be used in Ampere's law. Apply Ampére's law to a long straight wire with uniform distributed current.

Ampére's Law: In Lectures 1 through 7, we discussed two approaches for calculating the electric field, Coulomb's law and Gauss' law. There are also two prominent approaches for calculating the magnetic field produced by a current. The first approach, discussed extensively in Lecture 22 is commonly referred to as Biot-Savart Law. This law treats each small piece of a wire as a separate source of $\vec{B}$ and is similar in spirit to Coulomb's law. The other approach, based on ***Ampére's law***, is mathematically less demanding and very useful when the magnetic field lines have a *simple symmetry*. This approach is similar to Gauss' law for electric fields, which is most useful when the electric field is highly symmetric. Ampere's law states that



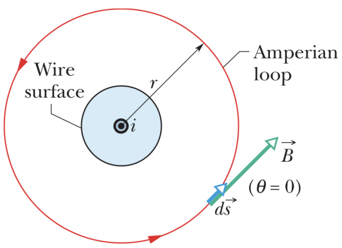


$$\oint \vec{B}\cdot d\vec{s} = \mu_o i_{enc}$$

The loop on the integral sign means that the scalar (dot) product $\vec{B}\cdot d\vec{s}$ is to be integrated around a closed loop, called an *Amperian loop*. The current i_{enc} is the net current encircled by that closed loop. Ampére's law thus relates the magnetic field on the perimeter of a region to the current that passes through the region.

Sign and direction of $\vec{B}$: For the integration, arbitrarily assume $\vec{B}$ to be generally in the direction of integration (as shown), then curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign. For example, the figure shows

$$\oint B\,\cos\theta\,ds = \mu_o(i_1 - i_2)$$



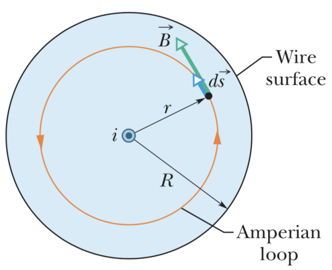
Magnetic Field Outside a Long Straight Wire with Current: We can apply Ampére's to a long straight wire that carries current i directly out of the page. Since the magnetic field $\vec{B}$ produced by the current has the same magnitude at all points that are the same distance r from the wire, we can draw an amperian loop with radius r. For $\theta = 0^\circ$ (see top diagram on left), we write

$$\oint \vec{B}\cdot d\vec{s} = \oint B\,\cos\theta\,ds = B\oint ds = B(2\pi r) = \mu_o i_{enc}$$

Note that $\oint ds$ is the summation of all the line segment lengths ds around the circular loop; this is simply the circumference $2\pi r$ of the loop and the enclosed current is just i; i.e., $i_{enc} = i$. Thus,

$$B = \frac{\mu_o i}{2\pi r}$$

Magnetic Field Inside a Long Straight Wire with Current: The magnetic field $\vec{B}$ produced by a long straight wire of radius R that carries a uniformly distributed current i directly out of the page, must be cylindrically symmetrical. For this, the right hand side of Ampére's Law provides that



$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = B(2\pi r).$$ The enclosed current in this case is $$i_{enc} = i\frac{\pi r^2}{\pi R^2}$$

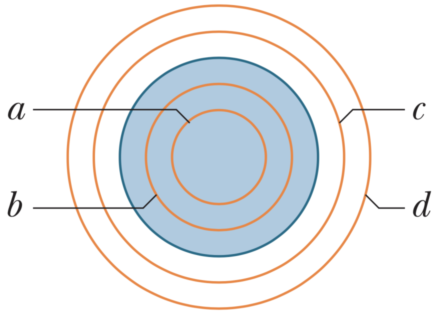
Thus, inside the wire, the magnitude B of the magnetic field is proportional to r;

$$B = \left(\frac{\mu_o i}{2\pi R}\right) r$$

It is zero at the center, and is maximum at $r = R$ (the surface).

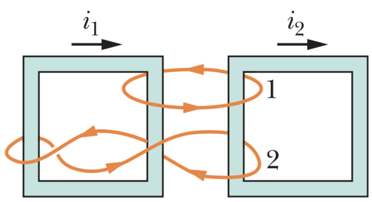
Question 1: CH29-Q7

The figure below shows four circular Amperian loops $(a,\, b,\, c,\, d)$ concentric with a wire whose current is directed out of the page. The current is uniform across the wire's circular cross section (the shaded region). Rank the loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ around each, greatest first.



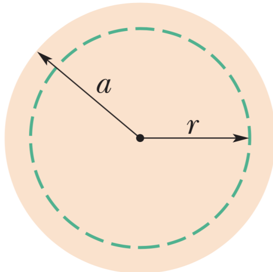
Problem 0: CH29-P44

The figure below shows two closed paths wrapped around two conduct- ing loops carrying currents $i_1 = 5.0$ A and $i_2 = 3.0$ A.What is the value of the integral $\oint \vec{B} \cdot d\vec{s}$ for (a) path 1 and (b) path 2?



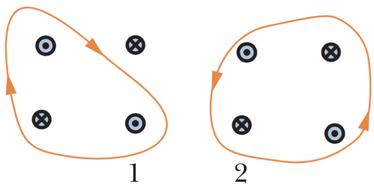
Problem 1: CH29-P43

The figure below shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2.00$ cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?



Problem 2: CH29-P43

Each of the eight conductors in the figure below carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d\vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2?



Problem 3: CH29-P48

In this figure, a long circular pipe with outside radius $R = 2.6$ cm carries a (uniformly distributed) current $i = 8.00$ mA into the page. A wire runs parallel to the pipe at a distance of $3.00R$ from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.