# 12.748 Lecture 2
Cosmic Abundances, Nucleosynthesis and Element Origins

Our intent is to shed further light on the reasons for the abundance of the elements. We made progress in this regard when we discussed nuclear stability and nuclear binding energy explaining the zigzag structure, along with the broad maxima around Iron and Lead. The overall decrease with mass, however, arises from the creation of the universe, and the fact that the universe started out with only the most primitive building blocks of matter, namely electrons, protons and neutrons. The initial explosion that created the universe created a rather bland mixture of just hydrogen, a little helium, and an even smaller amount of lithium. The other elements were built from successive generations of star formation and death.


Cosmic Abundances of the Elements
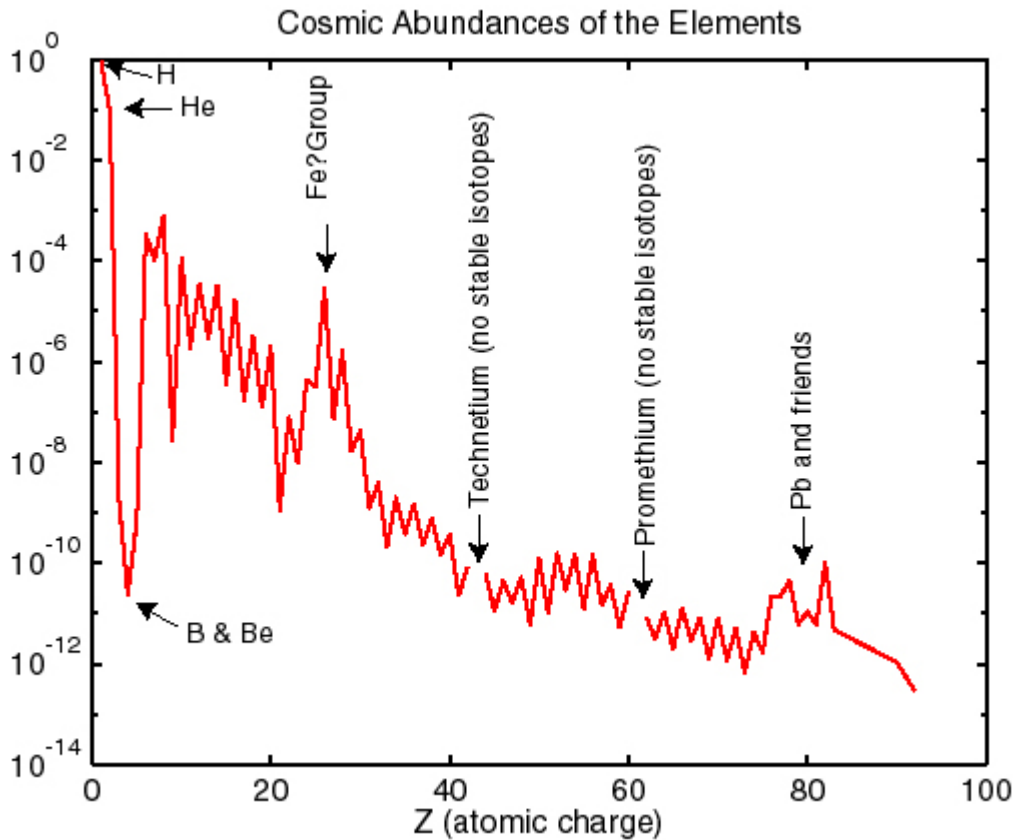
The Origin of the Universe

It is generally accepted that the universe began with a bang, not a whimper, some 15 billion years ago. The two most convincing lines of evidence that this is the case are observation of

- the Hubble expansion: objects are receding at a rate proportional to their distance
- the cosmic microwave background (CMB): an almost perfectly isotropic black-body spectrum

In the initial fraction of a second, the temperature is so hot that even subatomic particles like neutrons, electrons and protons fall apart into their constituent pieces (quarks and gluons). These temperatures are well beyond anything achievable in the universe since. As the fireball expands, the temperature drops, and progressively higher-level particles are manufactured. By the end of the first second, the temperature has dropped to a mere billion degrees or so, and things are starting to get a little more normal: electrons, neutrons and protons can now exist. The temperature is still high enough to encourage nuclear fusion, so that protons and neutrons combine to form ^{4}He. The chemical evolution of the fireball beyond about the first second is sketched below.

Initially just protons and neutrons exist, but when temperature and pressure conditions become right (after about a
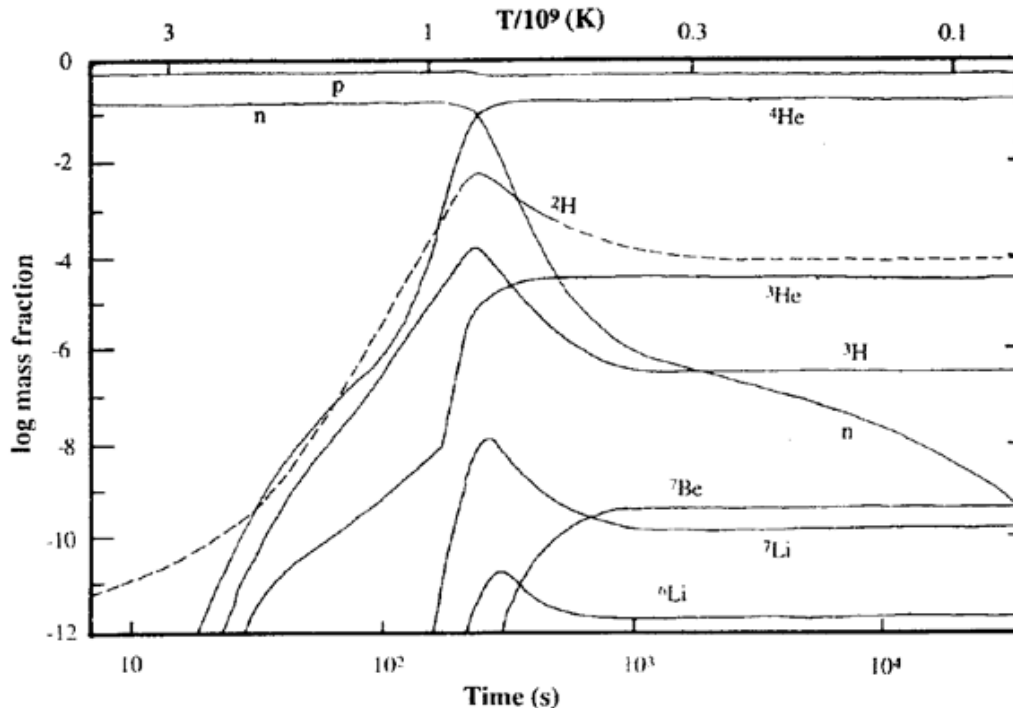
minute or so) first He, and then Be and Li are manufactured. These abundances stabilize quickly, because the temperature (and the pressure) has dropped below critical fusion levels (a few hundred million degrees or so) so that after the first half-hour, things are pretty well set. The few remaining neutrons free-flight beta-decay away (non-bound neutrons are unstable, they β-decay with a half-life of 11 minutes). Thus what we see after the first few moments of creation is a distribution dominated the lightest elements: hydrogen, helium and just a dash of Be and Li. Beyond that, there is little else, and as you can see from the cosmic abundance graph, we've not covered anything beyond Z = 4 without falling off the bottom of the chart. The rest of the elements come from another source…

Birth, Life and Death of Stars

Having done all this work, we have created only the first three elements in the periodic table. The remainder, it turns out, are fabricated in multiple generations of stars. We will outline the life cycle of a star, explaining stellar evolution in terms of nuclear fusion and other factors, and discuss how stars die, and others are born from their ashes. It turns out that the bulk of nucleosynthesis occurs in the final few moments of a star's life, so we will dwell more on their demise than anything. We will then talk about nucleosynthetic processes and their signatures. Finally, we will return to our friend the neutrino (who's more important than you might imagine!).

Stellar Birth

Here's a recipe for creating your own star: start with a "compressed" gas cloud, with as much as 1,000 atoms per cc of space[1]. Typically such cloud would be of order 3 to 4 x 10^{14} km across (that's 30-40 light years). This compression might be as a result of a collision between two galaxies or gas clouds, or as a result of some shock wave, perhaps produced by the explosion of another star. The compressed gas has enough gravitational self-attraction to begin to collapse on itself. This kind of inward collapse occurs on time scales of order centuries to millennia, depending on the nature of the original perturbation. Compression leads to heating: the **virial** theorem says that half of the gravitational potential energy is equipartitioned into heat, the other half into inward kinetic energy.

Compression and heat lead to pressure and temperature. The pressure starts to slow the collapse. This is of limited effectiveness, because the temperature is kept from increasing much above 10^4 K until all of the H and He are ionized. This is an endothermic reaction that cools the gas and hastens its collapse until the gas is completely ionized. Once the gas is completely ionized, the temperature of the proto-star (that's what it is now) continues to rise until it reaches hydrostatic equilibrium. Its radius is about 1 A.U. (the radius of the earth's orbit), or about 100 times larger than a normal star. It is more luminous now than at any other time in its life. Heat is convectively transferred within the body of the star because it is radiatively opaque. The net balance is between outward pressure created by heat, and inward gravitational attraction.
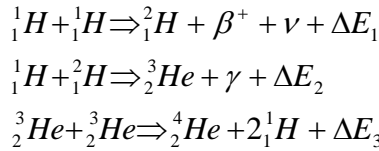
This strategy must ultimately fail, because the star is radiating heat away at an enormous rate. The plot here shows the evolution of the star with its luminosity (vertical axis) plotted against its apparent, surface temperature. After a few tens of thousands of years, the star has become so hot that its thermal energy exceeds the barrier between deuterium nuclei (deuterium is the heavier isotope of hydrogen, whose nucleus consists of one proton and one neutron), and it begins a brief flirtation with nuclear fusion. Temperatures have reached somewhere between 0.5 and 5 million degrees. In addition to deuterium, other light elements, notably Li, Be, and B will burn, a process called "astration". These elements are extremely "flammable" (yummy to stars) and burn very readily. These elements, which exist only in trace amounts, however, are burned very rapidly, usually in a few hundred thousand years (a veritable blink of a cosmic eye) leaving the star poised for further collapse.
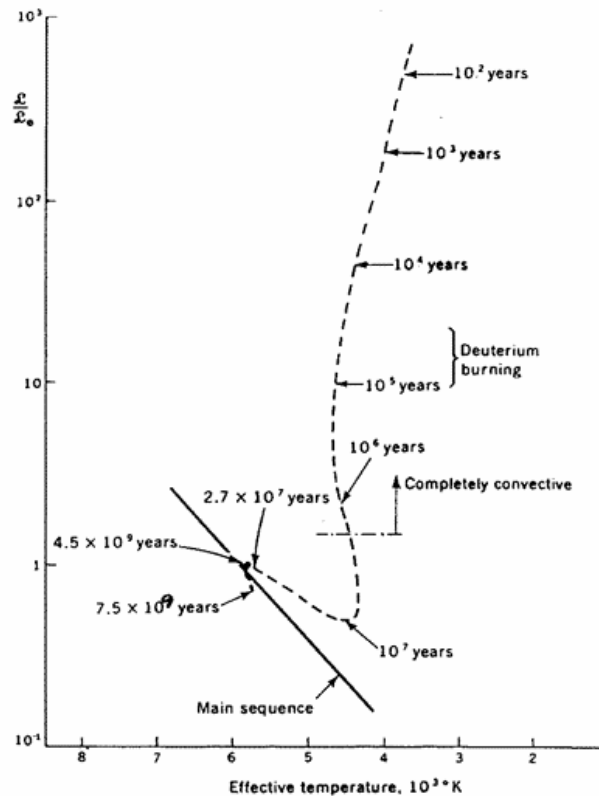
The further collapse raises the internal temperature even higher, reaching tens of millions of degrees. The nuclei (the atoms are long since completely ionized by this point) are banging around at an *enormous* rate. Above 10 million degrees, the kinetic energy is so high that the protons approach one another closely enough to feel the mutual attraction of the strong nuclear force.

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[1] This sounds like a lot, considering that typical interstellar space usually has a density of 0.1 atoms per cc or less. Actually, when you realize that air normally has about 2 to 3 x10^{19} atoms per cc, and ultra-high vacuums in laboratories contain about 10^6 to 10^7 atoms per cc, this isn't very compressed. Still, it's enough to start with.

Nuclear Fusion Begins in Earnest. The easiest reaction is proton-proton burning. This is affectionately known as p-p or the PPI chain. It occurs first because it has the lowest coulomb barrier to overcome: $Z_1 Z_2 = 1$ is as low as you can go. The reactions are:

$$_1^1H + {}_1^1H \Rightarrow {}_1^2H + \beta^+ + \nu + \Delta E_1$$

$$_1^1H + {}_1^2H \Rightarrow {}_2^3He + \gamma + \Delta E_2$$

$$_2^3He + {}_2^3He \Rightarrow {}_2^4He + 2{}_1^1H + \Delta E_3$$
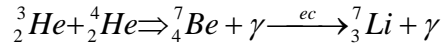
Each cycle yields about 26 MeV (the first two reactions have to occur twice for each execution of the third reaction. Note that not all the energy goes into heating the star. About a fifth of the energy of the first reaction, about 0.25 MeV per execution, is carried away by the neutrino, which doesn't lose its energy in the star. Thus about 2% of the total energy is carried away by neutrinos. Not very much, it may seem, but ultimately significant.
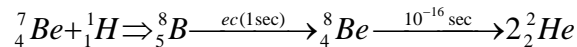
Stellar Evolution

The star now has an enormous energy source, virtually limitless! Yet its doom is sealed. The star must eat itself to stay alive. It can manage this way for many billions of years (depending on its mass, etc), and during this, it becomes increasingly inventive about the way it burns hydrogen. Here are two more reactions:
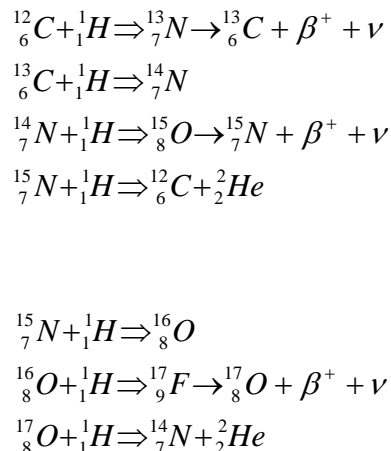
PP-II

$$_2^3He + {}_2^4He \Rightarrow {}_4^7Be + \gamma \xrightarrow{ec} {}_3^7Li + \gamma$$

And PP-III

$$_4^7Be + {}_1^1H \Rightarrow {}_5^8B \xrightarrow{ec(1\sec)} {}_4^8Be \xrightarrow{10^{-16}\sec} 2{}_2^2He$$

They don't look much like hydrogen burning, but the other nuclei (He and Be) are being used as a "carrier" for the proton, so the net reaction is one of proton consumption. Notice that Li, Be and B are very, very flammable to nuclear fusion. The PP-I reaction is by far, the predominant mechanism occurring in our sun.
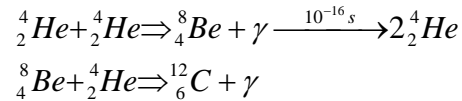
Now the temperature continues to increase, particularly in the core of the star (the outer regions will be cooler), between 20 and 50 million degrees now, another proton burning cycle begins. This requires the building of heavier elements, either within the star itself, or more likely from previous generations of stars. This is called the Carbon cycle, or the CNO bi-cycle:

$$_6^{12}C + {}_1^1H \Rightarrow {}_7^{13}N \rightarrow {}_6^{13}C + \beta^+ + \nu$$

$$_6^{13}C + {}_1^1H \Rightarrow {}_7^{14}N$$

$$_7^{14}N + {}_1^1H \Rightarrow {}_8^{15}O \rightarrow {}_7^{15}N + \beta^+ + \nu$$

$$_7^{15}N + {}_1^1H \Rightarrow {}_6^{12}C + {}_2^2He$$

$$_7^{15}N + {}_1^1H \Rightarrow {}_8^{16}O$$

$$_8^{16}O + {}_1^1H \Rightarrow {}_9^{17}F \rightarrow {}_8^{17}O + \beta^+ + \nu$$

$$_8^{17}O + {}_1^1H \Rightarrow {}_7^{14}N + {}_2^2He$$

The presence of the heavier nuclides (C,N,O) acts as a catalyst/carrier for protons in the process of manufacturing He. The reaction in the net is essentially the same as PP-I. Note again the ubiquitous presence of the neutrino. This time, although the total yield per helium nucleus created is almost identical, a larger fraction of the energy is carried away by neutrinos: about 7%. The price gets a little higher. In a young star like our sun, PP-I is the dominant mechanism by far, with other processes contributing a percent or less to the cause.

Stellar Death

Like people, it's often hard to say when stars begin to die, but the net result is usually easily distinguished. As hydrogen is consumed, and the interior, core temperature of the star continues to build, approaching 100 million degrees, it starts to burn He. Needless to say, this happens within the interior of the star while hydrogen burning proceeds unabated in the outer layers. Because of the higher Z_1Z_2 the requisite temperature to initiate He burning is greater. That is, the kinetic energy (and hence temperature) required to overcome the mutual repulsion of the two like-charged nuclei is proportional to the product of the two charges (Z_1 and Z_2).

$$_{2}^{4}He + _{2}^{4}He \Rightarrow _{4}^{8}Be + \gamma \xrightarrow{10^{-16}s} 2_{2}^{4}He$$

$$_{4}^{8}Be + _{2}^{4}He \Rightarrow _{6}^{12}C + \gamma$$

The first part of the first reaction is actually slightly *endothermic,* but the important factor is that the ^{8}Be nucleus is very short-lived (half-life 2 x 10^{-16} seconds). In order for the second reaction to proceed at a significant rate, the collision rate has to be high enough that another ^{4}He nucleus has to strike it within that short time. This, combined with the Z_1Z_2 effect explains not only the higher temperature threshold in the figure below, but also the very steep slope. The net effect of the steep temperature dependence is that a star that is He burning tends to be unstable: a very slight increase in temperature caused, for example by burning He, results in an abrupt increase in burning. The resultant temperature/pressure surge quickly extinguishes the He burning because it causes a decompression and the star "flashes". The amount of energy produced by the above reaction is only about a quarter of the PP-I reaction and there is not as much He around to burn, so that the energy provided is hardly comparable to the salad days of H burning.

Alpha Ladders: The star is climbing up the nuclear binding energy curve and the returns on its nuclear investments continue to erode. The core of the star continues to collapse, and the temperature rises to several hundred million degrees. It starts to use alpha particles to create the alpha-nuclei:

$$_{6}^{12}C(\alpha,\gamma)_{8}^{16}O$$

$$_{8}^{16}O(\alpha,\gamma)_{10}^{20}Ne$$

$$_{10}^{20}Ne(\alpha,\gamma)_{12}^{24}Mg$$
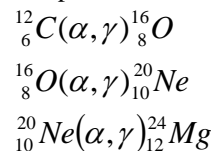
note we're now using typical nuclear short-hand for reactions where the first one is equivalent to

$$_{6}^{12}C + _{2}^{4}He \Rightarrow _{8}^{16}O + \gamma$$
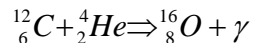
But the above reactions are not very fruitful, because there already *is* a shortage of helium (alpha particles).

The e-Process and the beginning of the end

The lifetime of the star is now measured in hours. Gravitational collapse of the core continues, and the temperature approaches a billion degrees. Core pressure is also very high: the crush of gravity brings the core density to about 10^6 g/cc. The temperature is now sufficiently high to overcome the coulomb barriers of even the heavier elements, and direct reactions proceed amongst them, like

$$_{10}^{20}Ne + _{10}^{20}Ne \Rightarrow _{8}^{16}O + _{12}^{24}Mg$$
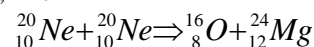
Alpha particles are now being released by (γ,α) reactions: the photons, which are now in thermal equilibrium in the stellar soup have energies high enough to boot alpha particles out of nuclei. The nuclei interact freely, combining when and where they can to move up the binding energy curve, while there is a competing process of breakup by collision with gamma rays. Having burned the lighter, yummier stuff, the star moves onto a less tasty, higher fiber diet: silica burning. This can take it quickly up the ceiling: the maximum in the binding energy curve at Fe-Ni.

Things are *really* hot now. Gamma rays are flying around, in thermal equilibrium with the plasma, smashing into other nuclides, breaking them apart. This is called the *e-process* which is short for *equilibrium process* because the nuclides reach a statistical equilibrium between formation and destruction by gammas. Energy loss must be incessant: building up toward the Fe maximum involves converting protons to neutrons, involving beta decays. The neutrinos steal away about a third of the beta decay energy. The star is bleeding to death.
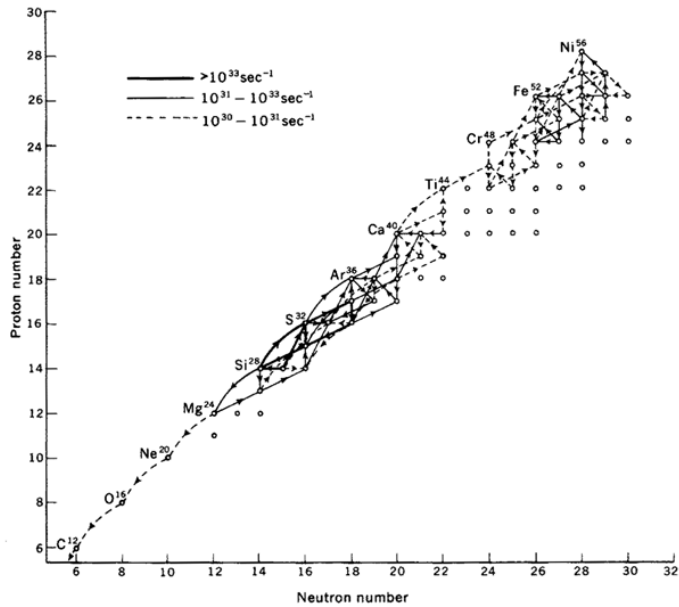
Now bear in mind that this is not happening throughout the entire star, but only in the core. The star has taken on an onion-like character, with progressively cooler shells on the outer edges, where helium, and hydrogen are still being burned.

Bang! You're Dead!

This star has lived for a billion years, but time left is measured in minutes, or even seconds. The inner core has exhausted its nuclear fuel. Reaching the e-process, it has reached the end of the profitable chain of nuclear fusion events. The temperature continues to rise, as thermal increase is the only defense against the relentless crush of gravity.

Disaster strikes. An increase in temperature, whose energy is carried by photons and nuclei alike, begin to fragment the very nuclei whose fusion fueled the star in its last hours. The star begins to eat its own ashes. Now the fragmentation moves *down* the binding energy curve. That is, the processes is *endothermic*. This means that rather than helping the situation, the light fission process *cools the core*. The result is catastrophic: now thermal and gravity forces are working in concert, and the **core collapses.**
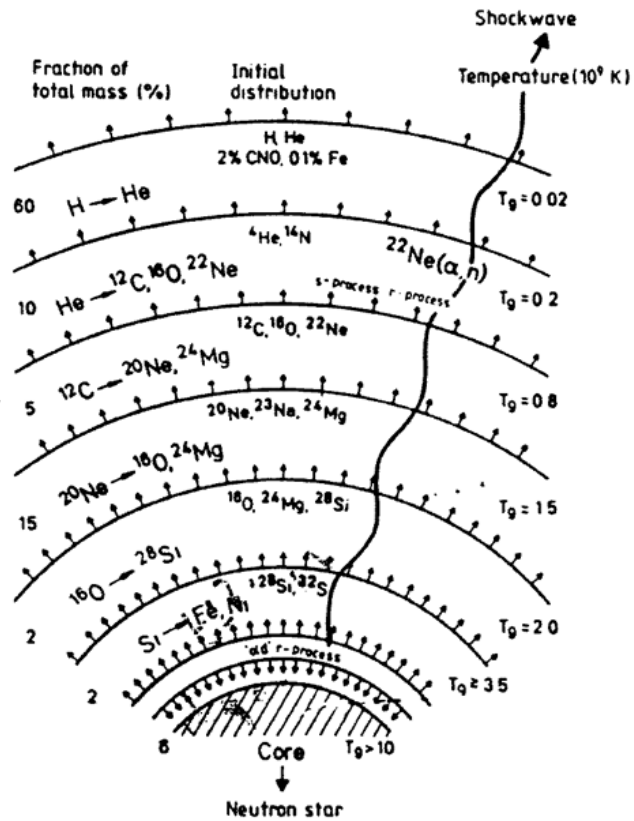
What happens next depends on the mass of the star. For the smallest of stars (one solar mass or smaller), the implosion involves such a small part of the stellar mass, that the entire star shrinks to become a **white dwarf.** This is a stable configuration, where the star quietly shrinks to a fraction of its former size (often shedding part of its outer shell) and quietly lives out the rest of its days, growing dimmer and dimmer. Our sun will likely go this route.

Neutron Stars

For stars about 1.5 times solar mass, something different happens. There is enough material in the core to cause the core to shrink under gravitational self-attraction to the point where the core becomes a giant nucleus. Several things happen at once. First, the material is a roughly equal mixture of neutrons and protons. Having all those protons close to one another is a very unhappy situation. The core, *this **giant nucleus, undergoes a massive beta decay***. That is, all the protons turn into neutrons. The reaction, *en mass* is

$$^1_1H \Rightarrow ^1_0n + \beta^+ + \gamma$$
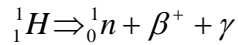
the energy release is enormous. The positrons move outward in a concentric shock wave, pushing through the positively charged nuclei, and trying to blow the star apart. The shock wave moves relentlessly outward through successive onion-skin layer… and dissipates. Nothing happens… what a disappointment - no fireworks! We want our money back!

But wait, there's more! The lowly neutrino, the aloof neutrino, the next best thing to nothing, does something rather amazing. Stellar densities are sufficiently high, and the neutrinos energetic enough, that they deposit energy in the outer shell of the star, and blow it away. The explosion, a Type II supernova, is pushed from behind by neutrinos. The outer 90% of the star is blown away in a rapidly expanding shell of gas in a matter of seconds. Not bad for something with no mass, no charge and very little interaction, mister neutrino.

Now what remains is a neutron star. Its mass is typically about 10% of a solar mass or so. Its density is the same as a nucleus: about a billion tons per cc. What was once a glorious star with a radius of a million km or more is stuffed into a ball that is about 15 km in diameter. The gravitational force at its surface is about a million g (the average person would weigh 150 million pounds on the surface). Just like a skater who pulls in his/her arms while rotating, the neutron star conserves angular momentum and spins up at an enormous rate, typically revolving a hundreds to thousands of times per second. The "core" of the star is a neutron superfluid, which has no viscosity, and infinite conductivity. The rapid implosion traps magnetic field lines within the star, so that they extend straight out of its rotational poles. There is almost certainly a precession, so that the magnetic fields, which trap beams of ionized plasma act like a radio beacon that sweeps past a receiver at a rapid rate: hence the name **pulsars**.

Black Holes

If the star is sufficiently massive, say greater than three solar masses, not even the rigidity of the neutrons is enough to hold gravity back. The thing that gives the neutrons their rigidity is not what you might think: is is the quantum mechanical resistance to having more than one particle in the same energy state (the Pauli Exclusion Principle). Neutrons are fermions, which have to obey the PEP. Needless to say, gravity overwhelms even this last-ditch defense, and the star implodes beyond this, and becomes a **black hole**.

Every gravitating body has an escape velocity, which is dependent on the radius. The escape velocity is the velocity that at body must have (directed outwards) to permanently escape the object's gravitational field. For example, every rocketeer knows that you need to get up to 11 km/s to escape the earth's pull. For a black hole, the gravitational attraction is so strong that inside a certain critical radius, the Schwartzchild Radius, the escape velocity exceeds the speed of light. This means that anything that falls through this horizon, can never come back. Not even light. And especially not graduate students. Kind of like the Joint Program.

There are lots of interesting books about black holes, so we won't dwell on them here, because they aren't really important for nucleosynthesis. We'll only mention that they, along with neutron stars, tend to suck matter of companion stars (a surprising number of stars exist in binary systems) and that the accelerating, active accretionary disks may be locations of interesting nuclear reactions and synthesis. For example, super-sized black holes may fuel **Quasars** (quasistellar objects): luminous features that have brightnesses exceeding *entire galaxies by several orders of magnitude* yet are as small as 1 A.U. in size.

Nucleosynthesis

The S-Process Nuclides

Most of the star's life is spent converting hydrogen into helium. In the last few moments of a star's life, all the other elements are created. During the explosive burning stages, the alpha nuclei are created by successive hops up the valley of stability, but the main mechanism of making elements is through **neutron capture**. Neutrons are uncharged, and no price is paid getting them close enough to feel the nuclear glue. Some neutrons come from the fusion process directly, but more come from the fragmentation of the Fe class nuclides due to photon collisions during the e-process stage. The Fe class nuclides are n-rich relative to the lighter ones (by a little, not a lot) and the fragmentation sheds neutrons. Most of these neutrons tend to be preferentially captured by heavier and nuclides are built up above the Fe maximum, all the way up to ^{210}Po. This is referred to as the **s-process.** The "s" stands for "slow", at least as compared to the explosive phase that follows. Basically neutron capture occurs very slowly, at least relative to beta-decay half-lives near the center of the valley of stability, so that the nuclides travel up the center of the valley. The process stops at ^{210}Po because it alpha decays.

r-Process Nuclides

As the star approaches criticality, the implosion causes a massive destruction of the Fe series nuclides that were built up during the e-process. This is to be distinguished from the on-going destruction that occurs *during* the e-process that fuels the s-process neutron flux. The result is a *massive* flux of neutrons, many orders of magnitude higher. There is a correspondingly massive buildup of nuclides on the high-N side, far from the valley of stability. There is a rain-down of beta-decay toward the valley floor, producing a characteristic isotopic signature. The s-process (slow), and r-process (rapid) neutron capture pathways. The s-process tends to follow the line of stability of the nuclides, whereas the r-process shoots off far away in the ocean of instability:
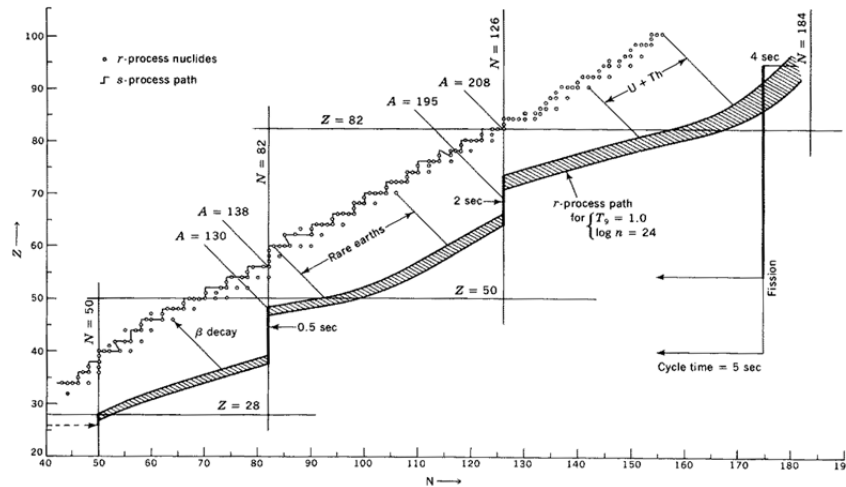

The figure to the right gives yet another view of the process. Note the speculative suggestion of a region of stable, superheavy elements hinted at by quantum-shell theory of the nucleus. Some work involves searching for the putative fission decay products of such nuclei.

Ultimately even the r-process runs out of steam at high masses, where the nuclei begin to spontaneously fission so rapidly that the chain cannot go any further. Note the presence of the **neutron drip line,** which can be tracked if the neutron flux from the core of the star becomes high enough. This is where the nuclei are so unstable that they spontaneously eject neutrons to move toward the stability line.

As with nuclear fission, there are shielding effects which distinguish the s-process nuclides from the r-process ones. Note also that there are actually *four* types of stable nuclides here:

- nuclides produced by s-process only (closest to the center of line of stability, and shielded by other, stable nuclides)
- nuclides produced by r-process only (furthest to the high-N side)
- nuclides (a few) made by both s- and r-process
- nuclides made by *neither*, which must have been produced by either a "p-process" associated with proton fluxes, or multiple α-capture.
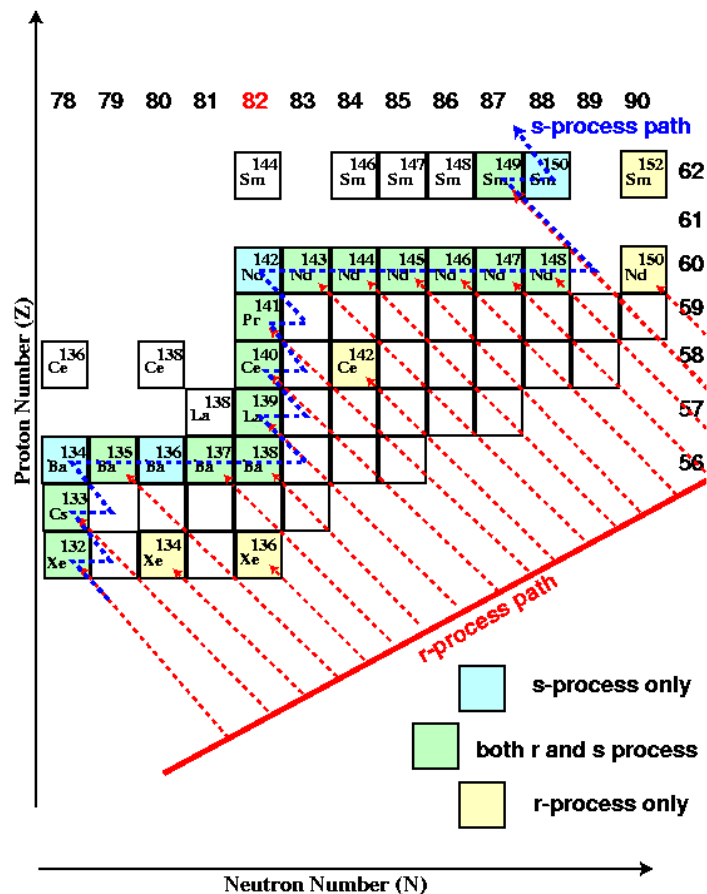

The Smoking Guns

Supernovae explosions are the most dramatic event in the skies. They are characterized by a flash of brilliance that evokes religious feelings in most primitive societies (such as Joint Program Students). One of the most striking observations of our generation is the sight of supernova 1987a. This was one of the most well studied stellar events in the history of man. We even detected the neutrino pulse (all 19 neutrino events detected) which drove the shell off the star, and evidence that a neutron star existed for at least 10 seconds before probably collapsing into a black hole.

The expanding shell of glowing gas, moving outward as a shock wave, interacts with surrounding gas (often blown off by a previous stage of the star), and produces characteristic shells and bubbles in space. For most type II supernovae, the bright flash of the explosion itself is caused not by the release of pent up thermal energy, but by the short-lived radioactive decay of newly created nuclides. There are a variety of highly radioactive isotopes created in the Fe-Ni group during the last stages of e-process (and Si-burning). The decay of luminosity of the star is controlled by their half-lives.

Of equal interest is the new data set emerging in gamma-ray astronomy. Since gamma-rays are energy specific signatures of nuclear decay, these products can be identified unambiguously and directly. One of the key features of these observations is the range of half-lives, which allow us to look at different timing aspects of the process. Moreover, observations of the gamma-ray emission lines from the decay of ^{26}Al (and other isotopes) taken from satellite-based gamma observations, shows *incontrovertible evidence that nucleosynthesis is occurring in our galaxy* at the present time. The half-life of these isotopes are many orders of magnitude smaller than stellar ages, and must have been produced *in situ* in stellar events. Click on the striking map here to get more information.
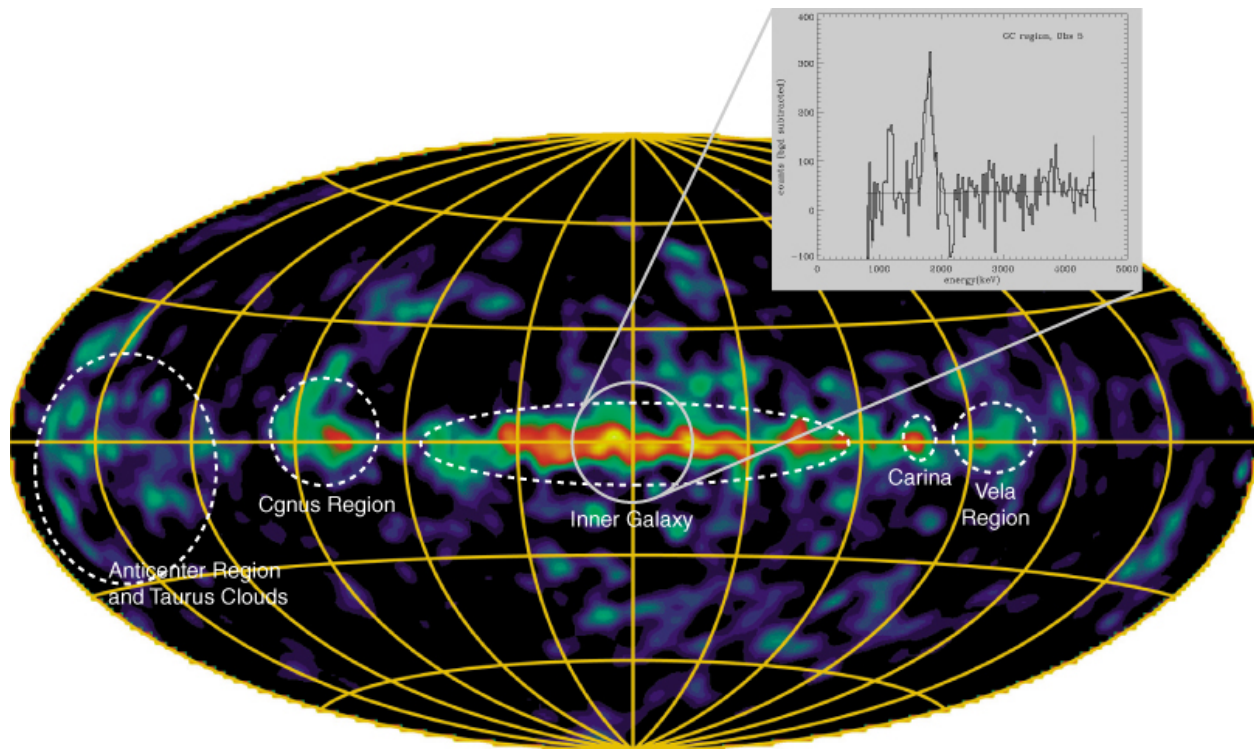
How Old are the Elements?

Our sun and solar system were formed as the result of a supernova shock wave travelling through the remains of other supernovae. The elements that make up the material from which the solar system formed were fabricated in the last few moments of a star's life. How long ago did this happen? How long did this material sit around before the sun and solar system formed? An upper bound for this time is the age of the universe, about 14-15 Ga. The lower bound for this is the age of the earth (about 4.5 Ga). It turns out we can do better than this. Thinking about the formation of the elements, especially the r-process, it can be argued that the major isotopes of U (235 and 238) and Th (232) are produced in roughly equal proportions. More thorough (and complicated) calculations provide us with an estimate of the primordial production ratio of

$$\left(\frac{^{235}U}{^{238}U} \right)_{\mathrm{Pr}imordial} \approx 1.3$$

That is, the lighter isotope was initially more abundant. This makes sense, because the elements are built up from lower masses. However, present day isotopic ratios in the earth appear quite constant at

$$\left(\frac{^{235}U}{^{238}U} \right)_{\mathrm{Pr}esent} = 0.0072$$

The reason for this is quite simple. Both isotopes are radioactive, but the lighter isotope ^{235}U has a shorter half-life, 704 Ma, and has decayed away more rapidly than the heavier isotope ^{238}U, which has a half-life of 4.47 Ga. Over the time since the elements were created, more than 99% of the ^{235}U has decayed, while the ^{238}U is about half gone.

We can derive a simple relationship based on the radioactive decay equation. Here's how:

$$^{235}U = {}^{235}U_0 e^{-\lambda_{235}t}$$

$$^{238}U = {}^{238}U_0 e^{-\lambda_{238}t}$$

Now dividing the first equation by the second:

$$\left(\frac{^{235}U}{^{238}U}\right) = \left(\frac{^{235}U}{^{238}U}\right)_0 e^{-(\lambda_{235}-\lambda_{238})t}$$

Let's call the uranium isotope ratio "R", so that we have

$$R = R_0 e^{-(\lambda_{235}-\lambda_{238})t}$$

$$or$$

$$\frac{R}{R_0} = e^{-(\lambda_{235}-\lambda_{238})t}$$

and can be solved by taking the natural logarithm of both sides:

$$\ln\left(\frac{R}{R_0}\right) = -(\lambda_{235} - \lambda_{238})t$$

$$or$$

$$t = \frac{\ln\left(\frac{R}{R_0}\right)}{\lambda_{238} - \lambda_{235}} = \frac{\ln\left(\frac{0.0072}{1.3}\right)}{1.537x10^{-10} - 9.763x10^{-10}} = \frac{-5.196}{-8.226x10^{-10}} = 6.3 Ga$$

The uncertainty in this number is about 0.2 Ga, which is due to uncertainties in the initial abundance ratio of about 20%.

A similar calculation can be done using the pair ^{232}Th:^{238}U, where the initial and present day ratios are 1.6 and 2.8 respectively. The equations are identical for this pair, but using the different half-lives (4.47 Ga for ^{238}U and 14.0 Ga for ^{232}Th). The result is an estimated age of 5.3 Ga, but this number is considerably more uncertain, about ± 2 Ga, due to the much longer half-lives involved, even thought the primordial ratios are still known to about ±20%. Moreover, there is an additional uncertainty introduced, because chemical/physical processes can also easily separate Th and U during the earth's formation and subsequent evolution.

The big problem, however, is that the ashes from which the solar system condensed are likely a mixture of materials that were fabricated over a range of times. The simple calculations done above must fail because it is probable that the ratios of the isotopes are substantially different from the primordial ratios. One could argue that for a more-or-less continuous formation process (stars exploding all the time), that the inventories of these isotopes would build up until their production would be balanced by radioactive decay. Thus at steady state, one would have

$$P_{235} = \lambda_{235} N_{235}$$

$$P_{238} = \lambda_{238} N_{238}$$

which, using the primordial production ratio of 1.3, we get

$$\left[\frac{N_{235}}{N_{238}} \right]_{initial} = 1.3 \frac{\lambda_{238}}{\lambda_{235}} = 0.20$$

which, if we reapply this number to our original calculation, we get

$$t = \frac{\ln\left(\dfrac{R}{R_0}\right)}{\lambda_{238} - \lambda_{235}} = \frac{\ln\left(\dfrac{0.0072}{0.20}\right)}{1.537 x 10^{-10} - 9.763 x 10^{-10}} = \frac{-3.324}{-8.226 x 10^{-10}} = 4.04 Ga$$

which is actually *younger* than the solar system. The answer is somewhere between the two extremes, because the activity ratio of the two isotopes will not have had time to really reach their "secular" equilibrium. The question now becomes more complex, since we are not asking at what specific time a nucleosynthetic event occurred, but rather over what range times, and what circumstances. For example, the young age above could be a signature of late addition of U isotopes to the earth/solar system, although we have arguments to explain it other ways as well. We've probably reached the limit of what we can realistically do with these isotopes (although that might not stop some people), so we ought to turn to other approaches. Before doing so, we need to consider other lines of evidence.