## Measurement of the Earth's Magnetic Field

## 1 Exercise 1: The magnetic compass

Look at a compass and note the two ends of the needles. With the compass lying flat on the table, move a rod magnet toward the compass, approaching from a direction perpendicular to the needle and with the ' N ' end aimed at the compass, and as in Figure 1.


Figure 1: Move a rod magnet toward the compass from a direction perpendicular to the needle.

What happens to the needle? Repeat but with the 'S' end of the rod closer to the compass. What happens to the needle now? The compass, we see, is affected by a magnetic field. In fact, one end of the compass is a small magnet. By seeing which end of the rod magnet the needle is attracted to, determine which magnetic pole ( N or S ) the tip of the magnetized end is (remember that opposite poles attract and like poles repel).

## 2 Exercise 2: Perpendicular Magnetic Fields

Bring two rod magnets close to the compass, from directions $90^{\circ}$ apart and both with the South pole aimed toward the compass, as shown in Figure 2. Watch the compass needle as you move one


Figure 2: Two rod magnets approach compass from perpendicular directions
magnet further and closer to the compass. Then move the other magnet further away and closer in. What direction does the compass needle point when one magnet is very close to the compass and the other is further? Why does the compass needle behave the way that it does?

In Exercise 2, you used two magnetic fields to affect the compass needle. The compass needle, though, only knows the direction of the total magnetic field, which is the vector sum of the two individual magnetic fields. In the space below, sketch the two magnetic fields and the total magnetic field that the compass experiences. Be sure to draw the two individual magnetic fields perpendicular, and for demonstration purposes, make these arrows have different lengths. This then represents Exercise 2 when the magnets are not the same distance from the compass.

Since the two individual magnetic fields are perpendicular they act like vector components of the total magnetic field and there is a simple mathematical relation relating the magnitudes of the components for a given direction of the total field. In the triangle that you drew, above, in the vertex between the total magnetic field and the horizontal arrow, draw an angle represented by the greek letter $\theta$. Then, $\tan (\theta)=B_{y} / B_{x}$, where $B_{x}$ represents the magnitude of the horizontal magnetic field.

## 3 The Earth's Magnetic Field

Remove the magnets from the table and leave the compass lying flat on the table. Take note of the direction that the ends of the needle are pointing. Move the compass to a new position on the table and let the needle settle. Do the ends of the needles point in the same direction as before? Rotate the compass and let the needle settle. What direction do they point now?
Clearly there is a preferred direction of the compass needle. Why is that? This is because there is a magnetic field all throughout the room, and outside the room. The Earth, itself, is a magnet, and hence all magnetic substances on the Earth are affected by the Earth's magnetic field. And, we make use of this in the design of a compass to be used for finding our way in the wilderness. The magnetic end of the compass should always point in the general direction of the Earth's North rotational pole, where the Earths' South Magnetic pole is. (That's right, the Earth's South magnetic pole is located near the Earth's North rotational pole.)

## 4 Measuring the Earth's Magnetic Field

In this experiment, we will measure the strength of the Earth's magnetic field in our lab room. We will do this using an electric current running through a pair of coils and a compass in a set-up called a tangent galvonometer.

The direction of the Earth's magnetic field, actually, is not only in the horizontal plane. Here, in Schenectady, in addition to pointing northward, it also points downward. However, the compass operates only in the horizontal plane (so that gravity doesn't affect the needle), and so we will use the compass to first measure just the horizontal component of the Earth's magnetic field. We will then use a dip needle, which is a compass designed to work in the vertical plane, to convert to the total magnetic field.

### 4.1 The tangent galvonometer

We place a small magnetic compass at the center of a pair of circular coils of wire, separated by a distance equal to their radius, making an arrangement called Helmholtz Coils, as shown in Figure 3. The electric current in the coils produces a known magnetic field at the center where the compass is located.


Figure 3: The experimental setup using a tangent galvonometer

To calculate the magnetic field at the center of the Helmholtz Coils, consider the equation for the magnetic field on the axis of a coil of wire with current I discussed in class. Use a distance $z=R / 2$. The Helmholtz Coils involve two sets of coils, each with N turns. The B from all these turns and both sets of coils must all be equal in magnitude and direction. The equation for the total magnetic field in the center, then, is:

$$
\begin{equation*}
B_{\text {coils }}=\frac{\mu_{0}}{4 \pi} 2 \frac{N 2 \pi R^{2} I}{\left((R / 2)^{2}+R^{2}\right)^{3 / 2}}=\frac{8 N \mu_{0} I}{R \sqrt{125}} \text { Tesla. } \tag{1}
\end{equation*}
$$

The coils in this experiment have $N=130$ turns and a radius of $R=15 \mathrm{~cm}$.
Now, we turn the coils so that their axis is perpendicular to the Earth's magnetic field. Then, the magnetic field of the coil $B_{\text {coil }}$ is perpendicular to the Earth's magnetic field. Since we are, at this point, only concerned with the horizontal component of the Earth's magnetic field, we will call this $B_{H}$. We have, then, the same arrangement as the triangle you drew in Exercise 2. The compass will point in the direction of the total magnetic field, which results from adding the magnetic field of the Earth and the magnetic field of the coils, which are perpendicular to each other.

Therefore, we need only to measure the angle $\theta$, shown in Figure 4, which is the angle between the direction of the compass and the direction of magnetic north, and we can infer the horizontal component of the earth's magnetic field $B_{H}$ from

$$
\begin{equation*}
\tan \theta=\frac{B_{\text {coil }}}{B_{H}} \tag{2}
\end{equation*}
$$



Figure 4: The compass needle deflects an angle $\theta$ away from magnetic North due to the magnetic field of the coil, $B_{\text {coil }}$ which is perpendicular to the horizontal component of the Earth's magnetic field, $B_{H}$.

Finally, we will need to include the vertical component of the Earth's magnetic field. We will use a dip needle to measure the dip angle, $\alpha$, shown in Figure 5. The total $B$ field vector, then, is related to the horizontal component $B_{H}$ and the dip angle by

$$
\begin{equation*}
B=\frac{B_{H}}{\cos \alpha} . \tag{3}
\end{equation*}
$$



Figure 5: The total Earth's magnetic field $B$, the horizontal component $B_{H}$, and the dip angle, $\alpha$.

### 4.2 Procedure

Attached at the back is a sample data table with spaces. To facilitate making the same calculations five times, it will be easier to process in Excel. Therefore, you may prefer to open Excel and insert your data there into a table with the same columns.


Figure 6: Initial set up of compass and coils.

1. Place the compass at the very center of the coils, let the needle settle, and turn the coil so that the north-south direction is perpendicular to the axis of the coils, as in Figure 6
2. Carefully rotate the compass until the ends of the compass needle are aligned with $0^{\circ}$ and $180^{\circ}$ on the compass scale.
3. Turn on the power supply and turn up the power until the compass deflects through $30^{\circ}$ and note the reading on the am-meter Turn off the current, and perform the measurement again to see if it is repeatable. Read the angle at both ends of the compass needle to ensure that your reading is not affected by a poor viewing perspective.
4. Switch the leads entering to the power supply to change the direction of the current in the coils and repeat the measurement.
5. Record the average current value, with uncertainty, in your data table in the row for $30^{\circ}$.
6. Repeat for angles of $40,45,50$ and $60^{\circ}$.
7. Use a dip needle (shown in Figure 7) to measure the dip angle of the Earth's magnetic field. Record this value as $\alpha$ in your data table.


Figure 7: Dipneedle.

## 5 Analysis

1. Use Equation 1 to calculate the magnitude of $B_{\text {coils }}$, magnetic field produced by the current in the coils.
2. Using Equations 2, calculate the magnitude of $B_{H}$ for each current setting.
3. Use Equation 3 to calculate the magnitude of the total magnetic field of the Earth.
4. Calculate the average of the values you obtained for $B_{\text {earth }}$ from the five trials and enter the result on the line provided on the data sheet. Calculate the uncertainty in the average ( $\sigma / \sqrt{N}$, where $\sigma$ is the standard deviation) and list it as well.
5. Compare your value for $B_{\text {earth }}$ with accepted values (as can be found by Googling "Magnetic Field Earth Schenectady") and comment on the agreement.

## 6 Data

Table 1: Measurements of compass deflection angle and current and inferred values of the horizontal component and total magnitude of Earth's magnetic field.

| Trial | $\theta(\mathrm{deg})$ | $\mathrm{I}(\mathrm{A})$ | $B_{\text {coil }}(\mathrm{T})$ | $B_{H}(\mathrm{~T})$ |
| :---: | :---: | :---: | :---: | :---: |
| Uncertainties |  |  |  |  |
| 1 | $30^{\circ}$ |  |  |  |
| 2 | $40^{\circ}$ |  |  |  |
| 3 | $45^{\circ}$ |  |  |  |
| 4 | $50^{\circ}$ |  |  |  |
| 5 | $60^{\circ}$ |  |  |  |

Average value (with uncertainty) of $B_{\text {earth }}=$ $\qquad$ $\pm$ $\qquad$ Tesla

Dip Angle = $\qquad$ degrees

Average value (with uncertainty) of $B_{\text {earth }}=$ $\qquad$ $\pm$ $\qquad$ Tesla

