Redesigning the Calculus Sequence for Engineering Students

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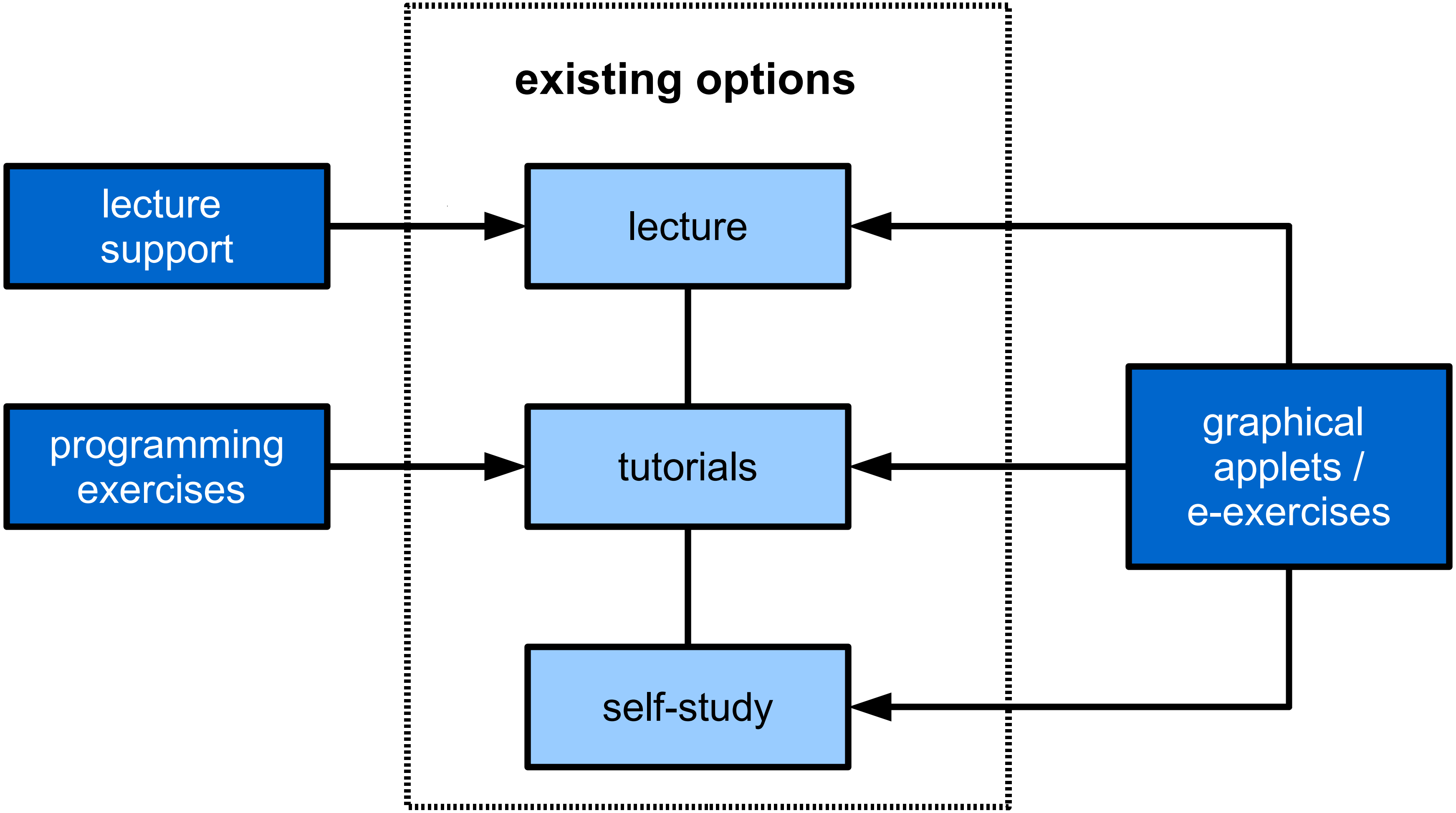
Focus: calculus sequence for first-year engineering students

Background: evolution of the mathematical skills required of modern day practicing engineers

Aim: helping students
- to acquire a deeper understanding of calculus concepts
- to become acquainted with engineering computer software
- to learn how to interpret computer solutions

Proposed Structure



Materials under Development

lecture support:
- provides visualization of mathematical results
- adds vitality and meaning to the lecture
- contributes to the learning and memorization process

graphical applets:
- may be referred to at any given time without supervision
- provide mathematical background, interactivity and feedback
- allow several modes (demo / exercises)

programming exercises:
- are designed to intensify student engagement
- introduce programming concepts and numerical processes
- provide numerical evidence for results from calculus course

Implementation example: the Taylor formula is

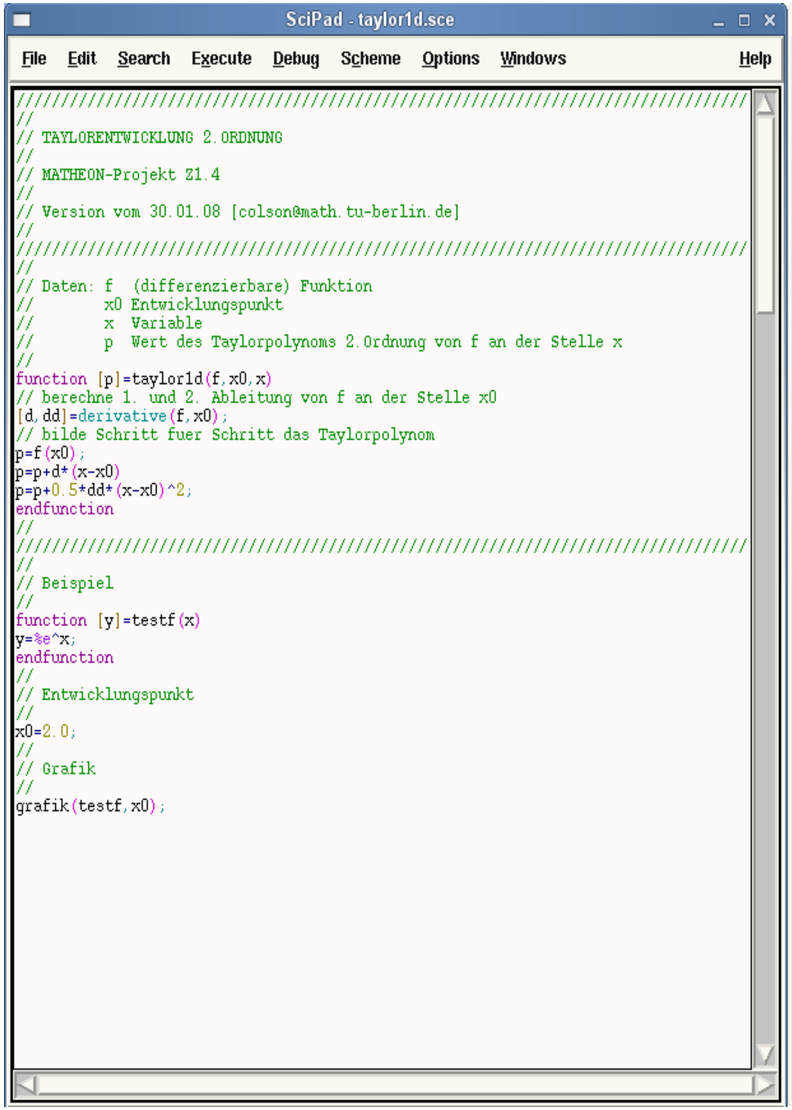
a result at the core of the calculus sequence...

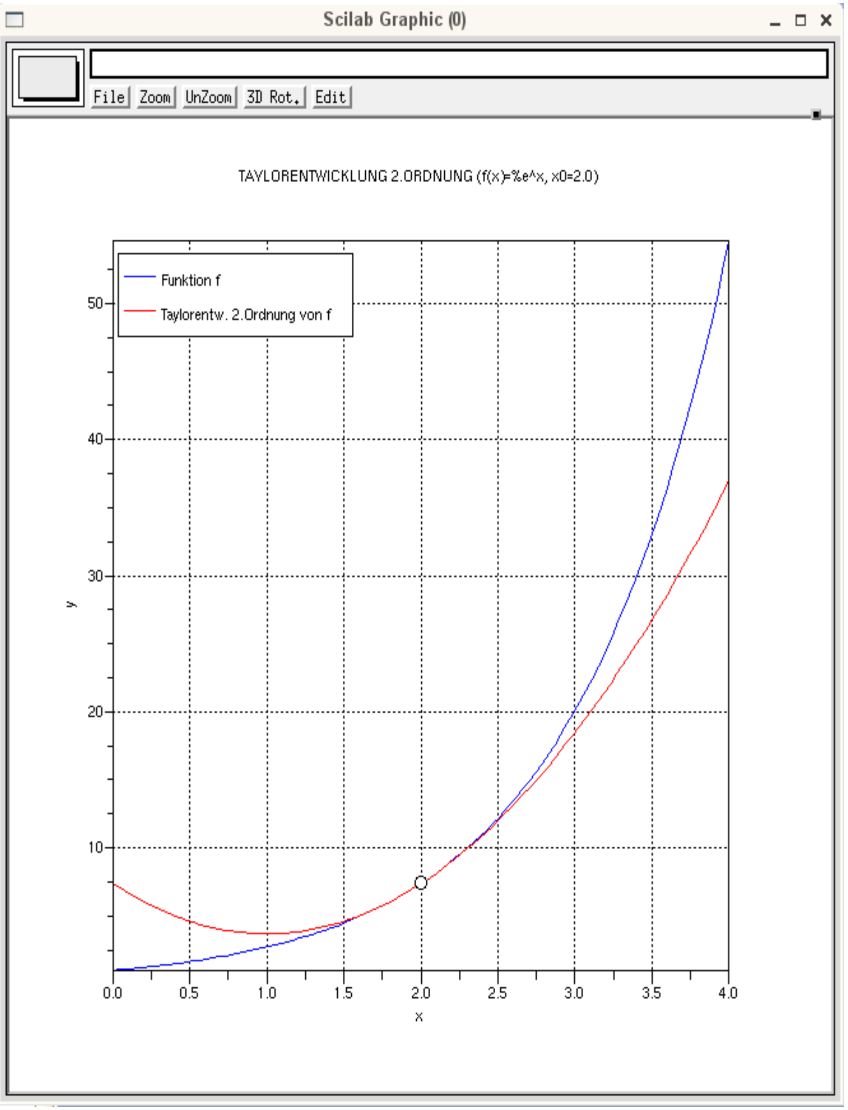
- elementary approximation result
- justification of discretization procedures
- motivation for the multivariate generalization

... and a source of difficulties for many students

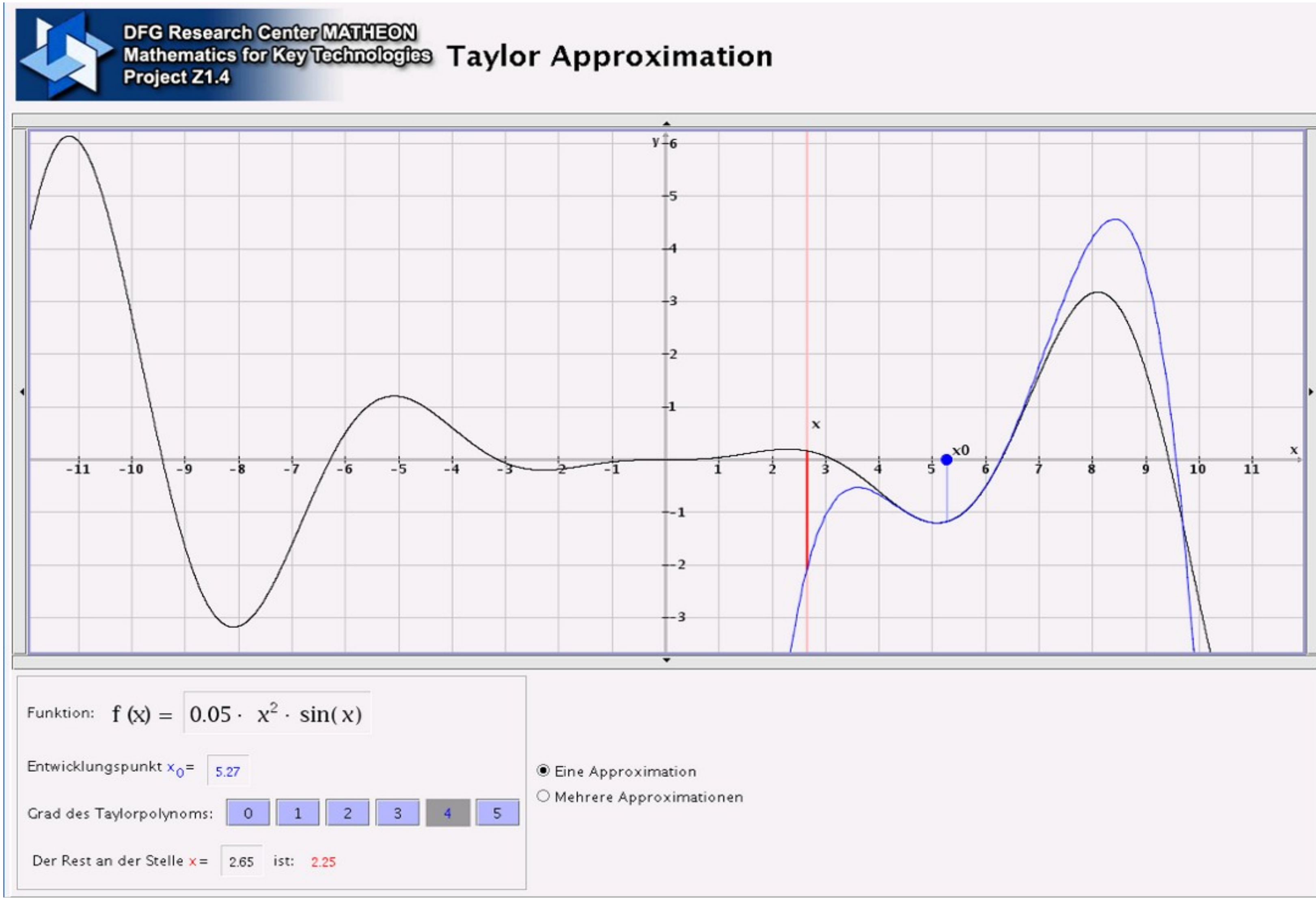
- requires several objects (function, expansion point, degree)
- produces several objects (polynomial, remainder)
- requires bounding the remainder
- the underlying idea remains obscure

Lecture support (using open source software Scilab)

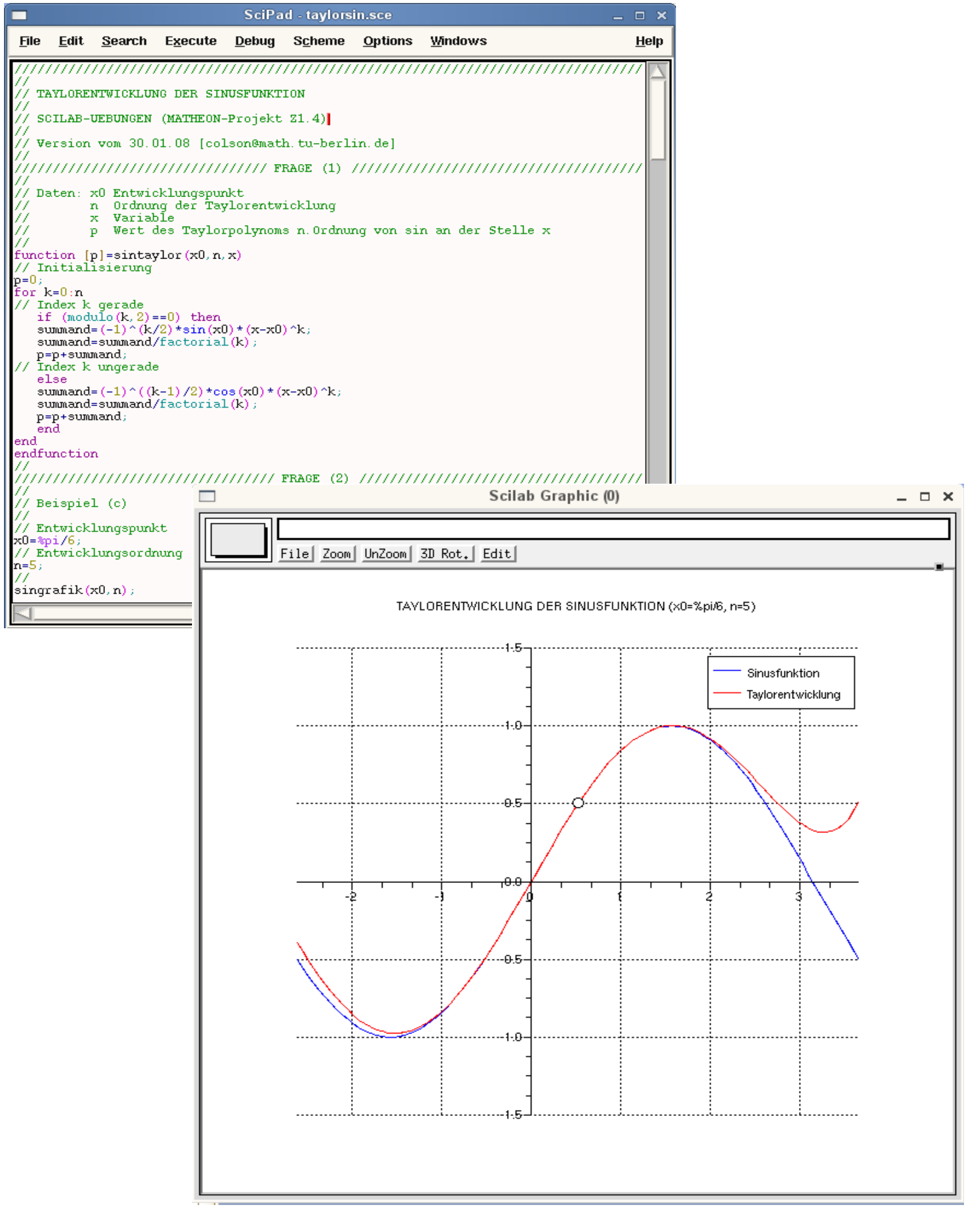




Graphical applet (using open source Java code)



Programming exercise (using software Scilab)



TAYLORENTWICKLUNG DER SINUSFUNKTION

(1) Schreibe eine Scilab-Funktion `sintaylor`, die den Wert des n-ten Taylorpolynoms der Sinusfunktion um x_0 berechnet.

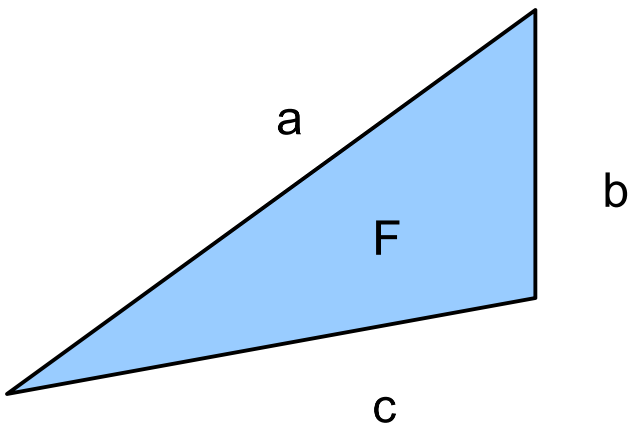
(2) Binde die Datei `taylorsingrafik.sci` ein.
Der Aufruf `taylorsingrafik(x0,n)` erzeugt eine grafische Darstellung der Sinusfunktion und des unter (1) berechneten Polynoms.
Erzeuge Grafiken für die folgenden Fälle:
(a) $x_0 = \pi/2$, $n = 2$;
(b) $x_0 = \pi/4$, $n = 3$;
(c) $x_0 = \pi/6$, $n = 5$;
Prüfe dadurch, ob deine Funktion `sintaylor` richtig programmiert ist.

An emphasis of the programming exercises is on the **need to validate and interpret computer solutions**.

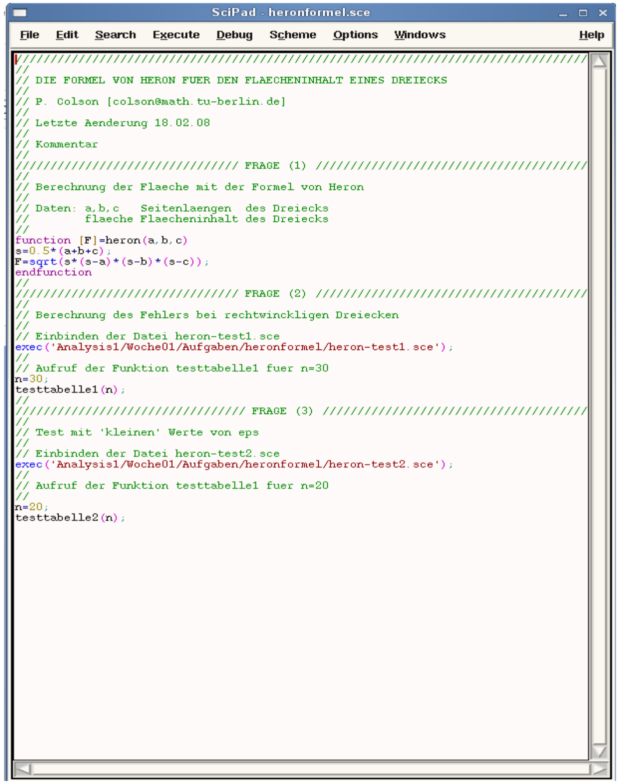
To illustrate this point, the students are asked to compute the surface F of a triangle with side lengths a, b and c using **Heron's formula**

$$F = \sqrt{s\,(s-a)\,(s-b)\,(s-c)},$$

where $s = (a+b+c)/2$.



Heron's formula is easy to implement but delivers false results for small interior angles!




```
Verhalten der Scilab-Funktion [flaeche]=heron(a,b,c)

Heron-Formel F=sqrt(s*(s-a)*(s-b)*(s-c)),

s:=0.5*(a+b+c)

a = 1
b = eps
c = 0.5*eps

mit eps=10^(-n)

n      eps           Fehler in %
===    =======       ===========
1      1.0e-01          0.0
2      1.0e-02          0.0
3      1.0e-03          0.0
4      1.0e-04          0.0
5      1.0e-05          0.0
6      1.0e-06          0.0
7      1.0e-07          0.0
8      1.0e-08          0.0
9      1.0e-09          0.0
10     1.0e-10          0.0
11     1.0e-11          0.0
12     1.0e-12          0.0
13     1.0e-13          0.1
14     1.0e-14          2.3
15     1.0e-15         11.2
16     1.0e-16        100.0
17     1.0e-17        100.0
18     1.0e-18        100.0
19     1.0e-19        100.0
20     1.0e-20        100.0
```

This provides the students with an example of a numerically instable formula.

Current phase of the project:
- development
- testing
- implementation in selected tutorials

Full-scale implementation and evaluation:
- summer term 2009 (single variable calculus)
- winter term 2009/2010 (multivariate calculus)

Future developments:
embedding the materials into an environment containing:
- background information on undergraduate mathematics
- a guide to engineering applications of calculus notions
- challenging material for motivated students