Evolution of single and binary stars in gravitational N-body simulations

ARI Colloquium

06.05.2021

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Collaborators so far

- **Initialisation of star cluster simulations:** Abbas Askar, Mirek Giersz, Peter Berczik, Agostino Leveque, Rainer Spurzem, Wang Long, Shu Qi, Bhusan Kayastha
- **Direct N-body simulations and high performance computing:** Rainer Spurzem, Peter Berczik, Bhusan Kayastha, Shu Qi, Manuel Arca Sedda
- **Monte-Carlo models of star clusters:** Mirek Giersz, Arkadiusz Hypki, Abbas Askar, Agostino Leveque
- **Stellar evolution:** Jarrod Hurley, Sambaran Banerjee, Mirek Giersz, Rainer Spurzem, Diogo Belloni, Lars Kühlmichel, Peter Berczik, Manuel Arca Sedda
- **Population synthesis vs. N-body simulation:** Li Zhongmu, Bhusan Kayastha, Rainer Spurzem, Peter Berczik, Deng Yangyang
- **Observations of star clusters:** Benjamin Giesers, Stefan Dreizler, Pang Xiaoying
- **Planetary systems in star clusters:** Katja Reichert, Rainer Spurzem

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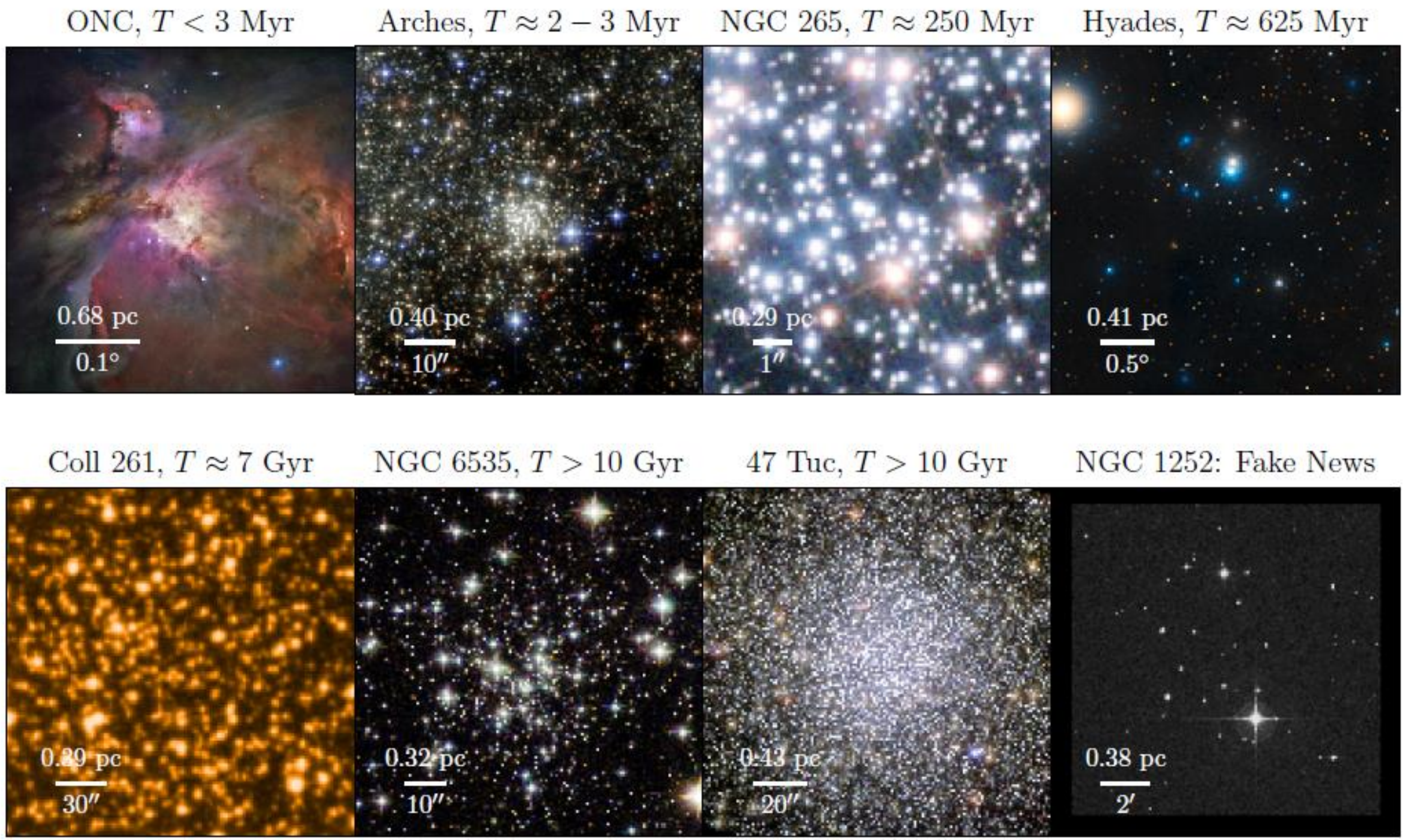
Topics today

- Some general comments on star clusters
- Star cluster simulations
- Current stellar evolution in star cluster simulations
- Future additions in the stellar evolution of star cluster simulations

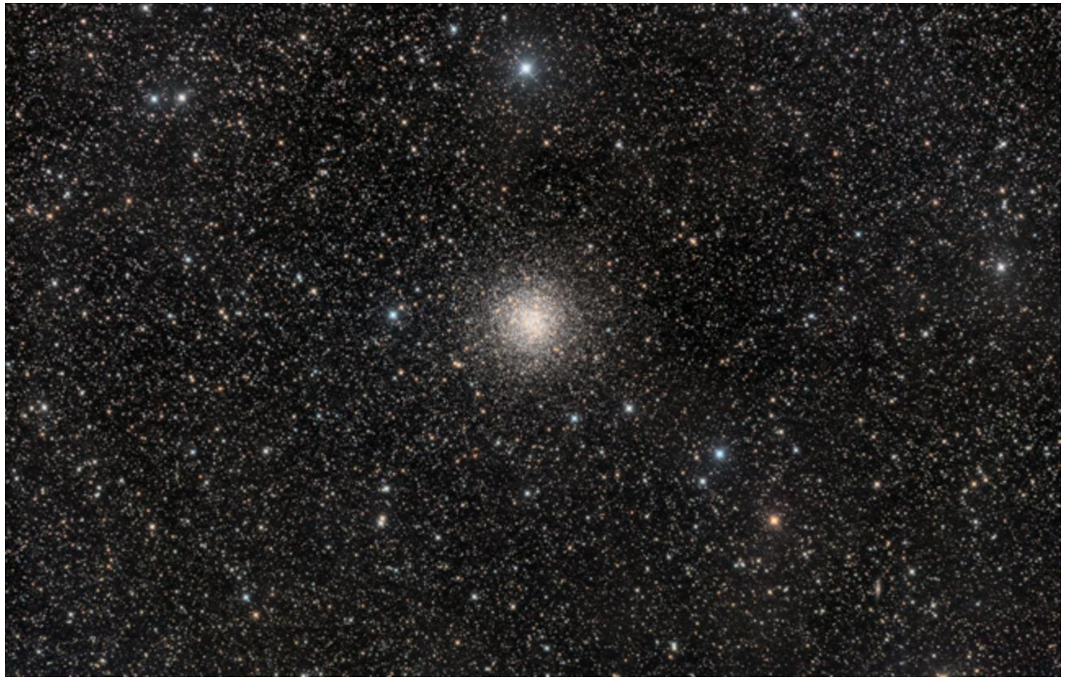
Star clusters



- Star clusters exist on a huge range of mass, size and density scale.
- Cluster formation is central to star formation; almost all stars form in groups or some kind of hierarchies.
- Gaia and massive spectroscopic survey such as MUSE → kinematics and abundance data now accurate enough to trace back field stars to their birth place → resolve structures of star formation.

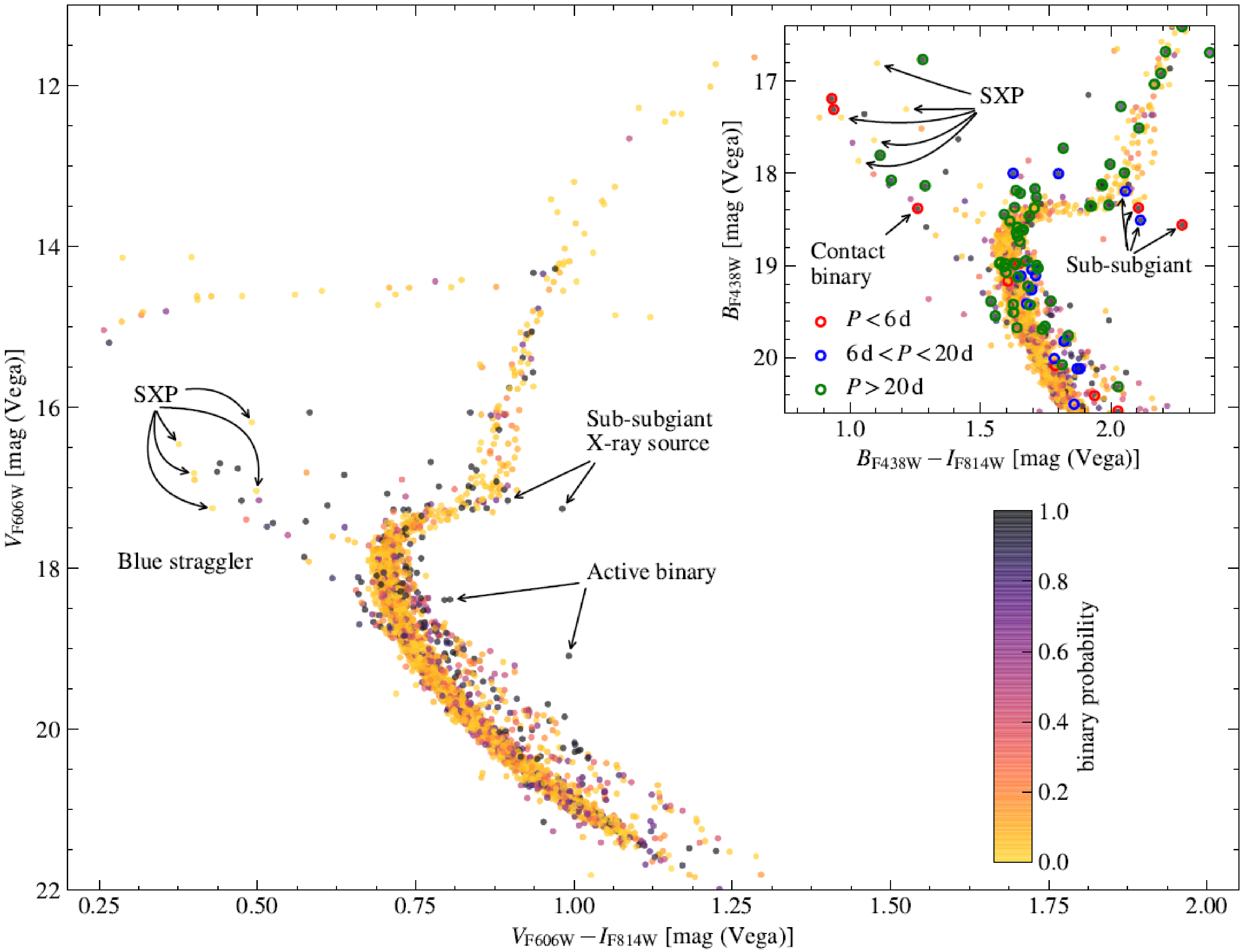
Figure from Krumholz et al. 2019; see also Kruijssen 2014, Baumgardt & Hilker 2018, Bianchini et al. 2018

Globular clusters: NGC 3201



NGC 3201 (Orion Optics UK AG12Orion telescope): http://www.astro-austral.cl/imagenes/starclusters/ngc%203201/info.htm

- Globular clusters (GCs) are extremely old → fossil remnants of early galaxy formation
- Milky Way hosts over 150 of these
- GCs are extremely dense
- Host hierarchical stellar systems, especially in their cores that act as a kind of heat source → delay core-collapse

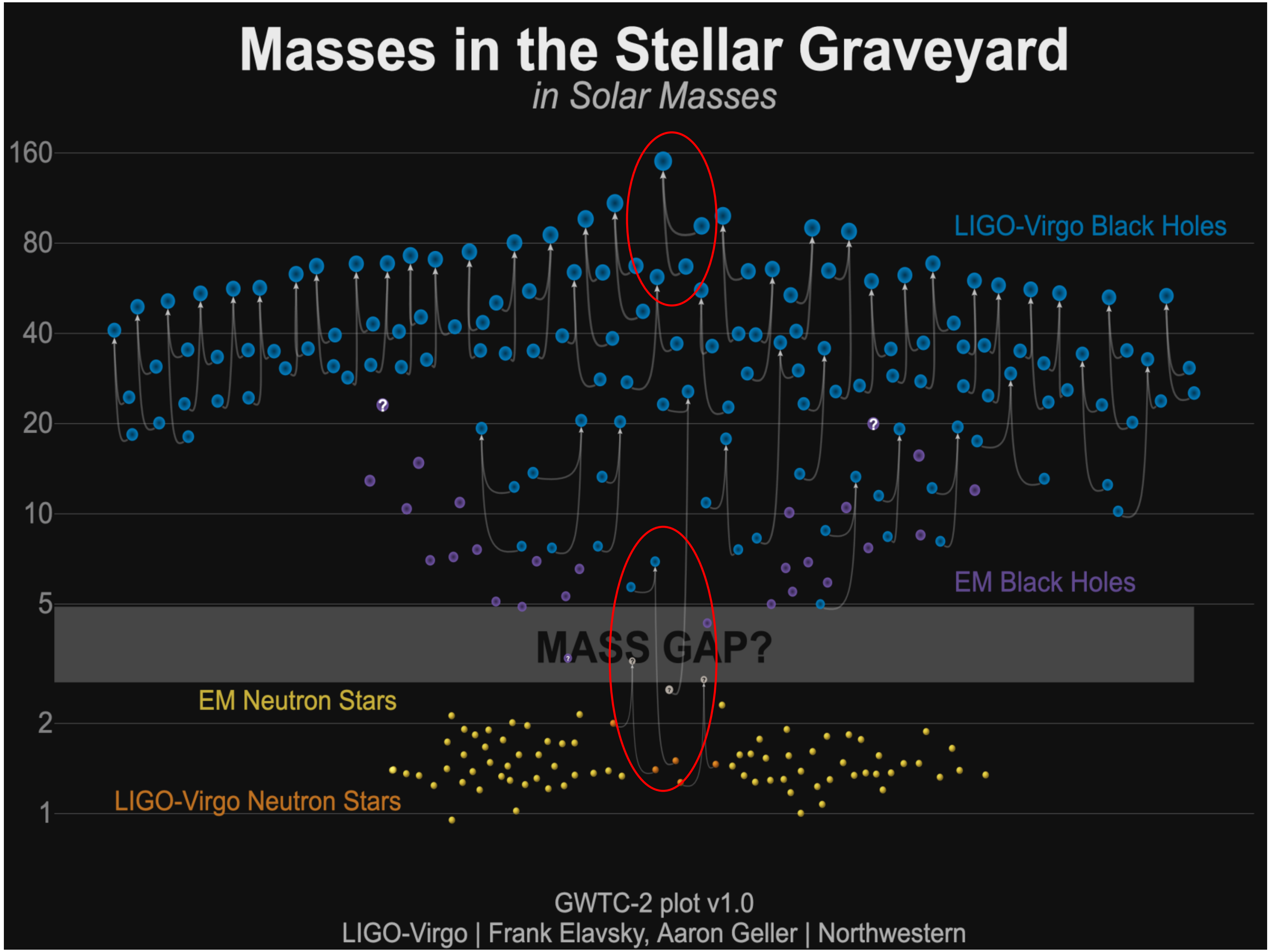


NGC 3201

- Very old and metal-poor GC
- Mass of around $1.49 \times 10^5 M$*
- Negligbly rotating today (Bianchini et al. 2018) → initially rotating much faster?
- Large half mass radius of 6.20 pc → BH subsystem / hard binary subsystem that counters core-collapse?
- Three stellar mass BH binaries found in MUSE data (Giesers et al. 2018, 2019)
- Giesers et al. Also use MOCCA models to cross-check binary fractions

→ DRAGON-II NGC 3201 simulation running!

Figure: Giesers et al. 2019; HST survey: Nardiello et al. 2018; Piotto et al. 2015; See also Askar et al. 2018; Kremer et al. 2018 Giesers et al. 2018, Price-Whelan et al. 2018



Gravitational wave detections

- Dense stellar systems like globular clusters (GCs) are considered to provide conducive environments to form GW progenitors via few-body dynamical interactions (e.g. Downing et al. 2010, Tanikawa 2013, Rodriguez et al. 2016a,b, Arca Sedda & Gualandris 2018, Hong et al. 2020, Wang et al. 2021)
- Closest LIGO/VIRGO detection 40±10 Mpc (NS-NS merger and first detection; Abbott et al. 2017)
- Low significance detection of elusive BH-NS merger (Abbot et al. 2015; see also Arca Sedda 2020, Chattopadhyay et al. 2020b)
- IMBH (>100 M*) detection of mass 142^{+28}_{-16} M* in 2020 (Abbott et al. 2020)

Star cluster simulations: modelling methods

- Two main methods that stand out today after decades of developing and testing them (e.g. Spurzem 1999)

Direct N-body methods

- Orbit integration of the orbits of many particles in a self-gravitating bound star cluster.

→ **Nbody6++GPU** (Spurzem 1999, Nitadori & Aarseth 2012, Wang et al. 2015)

- Currently still best available high accuracy direct N-body simulation code.

Monte-Carlo methods

- Fokker-Planck equation solved by Monte Carlo Markoff chain method (Hénon 1971; Stodolkiewicz 1982, 1986; Giersz 1998; Giersz et al. 2015; Askar et al. 2017; Kremer et al. 2020)

→ **MOCCA** (Fregeau et al. 2004, Hypki & Giersz 2013, Giersz et al. 2013, 2015, Askar 2017, Hong et al. 2020)

- Much faster (only days) than Nbody6++GPU BUT (Giersz et al. 2013b) MOCCA has some shortcomings and cannot model
  - Higher-order multiples
  - Rotation
  - Tidal shocks, complex tides

MOCCA models (MOCCA Survey Database I: Askar et al. 2017) used to find initial conditions of Nbody6++GPU simulations for realistic star clusters (Kamlah et al. 2021) → DRAGON-II of NGC3201

GPU supercomputers

- Need the world‘s fastest computers to run our simulations → GPU high performance computing (HPC) clusters.
- **DRAGON-II simulations (starting with NGC 3201) already running for four months on JUWELS** (using PeTar by Wang et al. 2020a, 2020b; then switch to Nbody6++GPU later; MOCCA simulations also work in progress).
- Also in use are Kepler at ARI-ZAH, binAC at Tübingen, silk nodes at NAOC-CAS, chuck in Poland,…

Some research questions that star cluster simulations are expected to answer nowadays

- What are formation channels of IMBHs and what initial conditions produce multi-generation BH growth? (Giersz et al. 2015, Arca Sedda et al. 2019, di Carlo et al. 2020, Rizzuto et al. 2021, Banerjee 2021)

- What are the formation channels and what is the long-term evolution of blue stragglers, X-ray binaries, cataclysmic variables (CVs) and other special photometric binary stars in dense star cluster environments? (Rappaport et al. 1982, Knigge 2006, Schreiber et al. 2010, Knigge 2011, Zorotovic et al. 2016, Belloni et al. 2018, Breivik et al. 2020)

- What is the effect of multiple populations and initial bulk rotation on the dynamics and stellar content of the star cluster? (theory: Einsel & Spurzem 1999, Fiestas, Spurzem & Kim 2006, Hong et al. 2013; observations: Kamann et al. 2018, 2020, Bianchini et al. 2018, Lahén et al. 2020, Ballone et al. 2021)

- Can we populate the first compact object mass gap between (2-5)M* in star cluster simulations with BH-NS or NS-NS binary mergers? (Arca Sedda 2020, Chattophadhyay et al. 2020a,b, Drozda et al. 2020)

Preceding, groundbreaking simulations with Nbody6++GPU and MOCCA

- DRAGON-I simulations of globular clusters (GC) (Wang et al. 2015, 2016, Shu et al. 2021)
- MOCCA Survey Database I of 2000 GC models (Askar et al. 2017, Morawski et al. 2018)
- Nuclear star cluster harbouring a central accreting SMBH (Panamarev et al. 2019)
- IMBH growth studies (Giersz et al. 2015, Arca Sedda et al. 2019, di Carlo et al. 2020, Rizzuto et al. 2021)

Stellar evolution largely outdated (not standard today compared with other SSE/BSE variants, e.g. CMC/COSMIC (Breivik et al. 2020), MSE (Hamers et al. 2020), MOBSE2 (Giacobbo et al. 2018),….) in above simulations → Need new simulations with new stellar evolution

Current simulations (still running) with updated SSE/BSE:

- New MOCCA Survey Database II (chuck; Giersz et al.)
- DRAGON-II IMBH studies (JUWELS, binAC; Arca Sedda et al.)
- DRAGON-II of NGC 3201 (JUWELS; Spurzem et al.)

Single Stellar Evolution (SSE) & Binary Stellar Evolution (BSE)

- Among the most influential astrophysics codes ever written.
- Implemented in (for example):
  - Star cluster initialisation:
    - **McLuster** (Kuepper et al. 2011, Kamlah et al. 2021, Leveque et al. 2021)
  - Population synthesis:
    - COMPAS (Stevenson et al. 2017)
    - MOBSE2 (Giacobbo et al. 2018)
    - COSMIC2.0 (Breivik et al. 2020)
  - Monte-Carlo methods for N-body problems:
    - **MOCCA** (Fregeau et al. 2004, Hypki & Giersz 2013, Giersz et al. 2013, 2015, Askar 2017, Hong et al. 2020)
    - CMC (Joshi et al. 2000, 2001. Fregeau & Rasio 2007, Kremer et al. 2018, 2019, 2020)
  - Direct N-body simulations:
    - Nbody7 (Banerjee et al. 2019, Banerjee & Belczynski 2020, Banerjee 2021)
    - **Nbody6++GPU** (Spurzem 1999, Nitadori & Aarseth 2012, Wang et al. 2015)
    - PeTar (Wang et al. 2020a, b)

BSE & SSE

- **SSE** (Hurley et al. 2000)
  Comprehensive analytical formulae of stellar mass & metallicity → give luminosity, stellar radii, core mass dependent on age, stellar mass & metallicity.

- **BSE** (Hurley et al. 2002)
  Tidal interaction between stars → circularization & stellar spin synchronization → inspiral and merger of binaries. RLOF, CE evolution integrated.

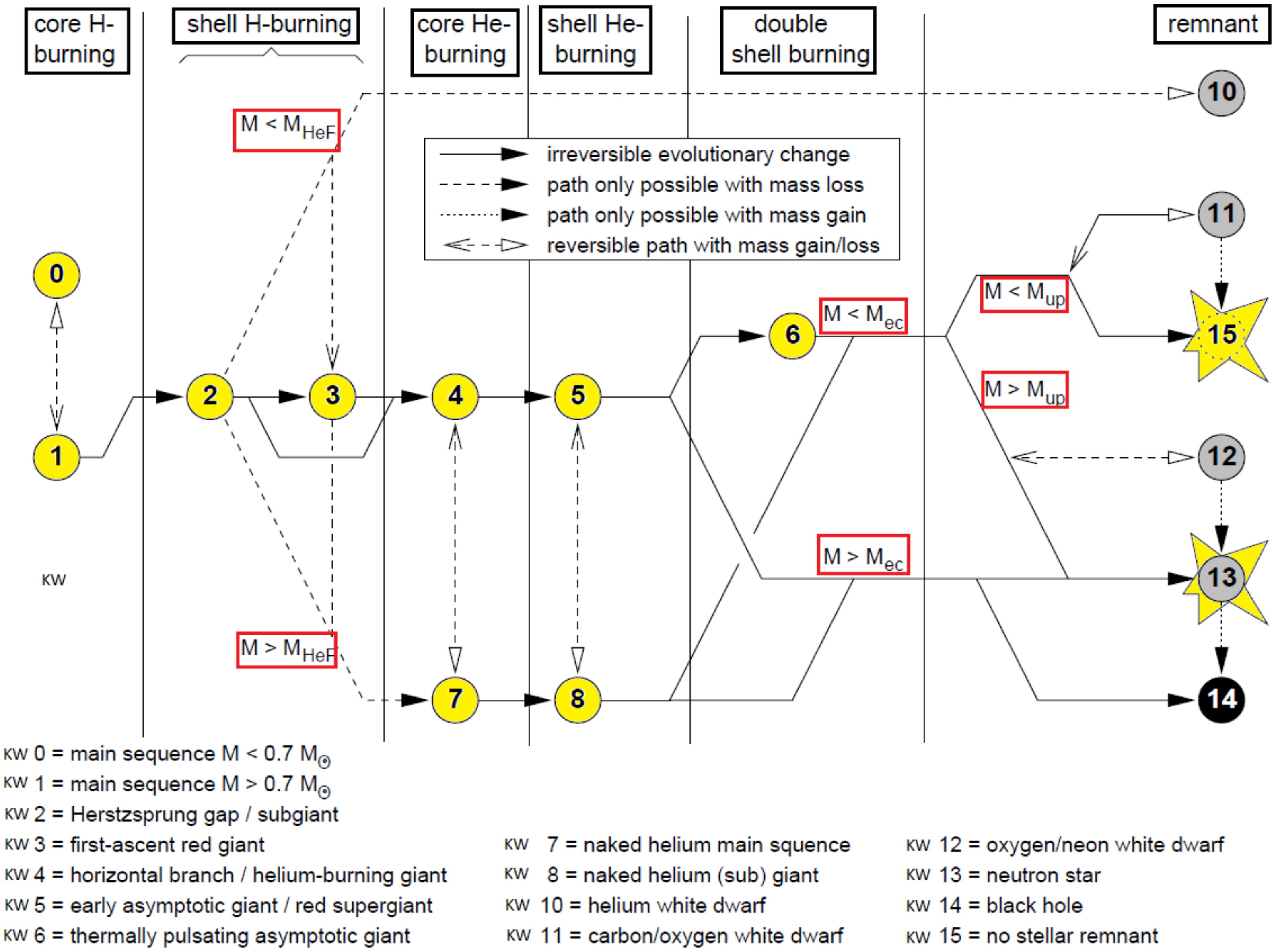


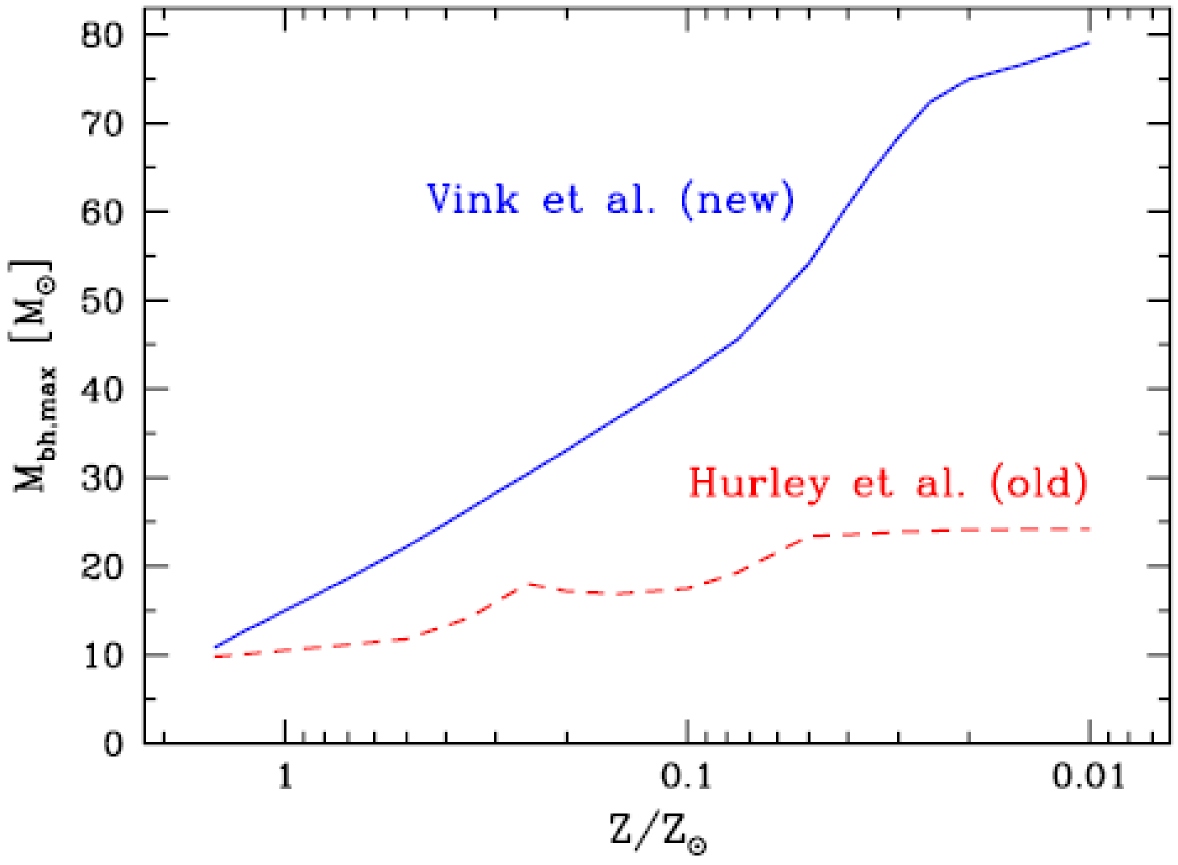
Figure from Hurley et al. (2000); Hurley et al. (2002)

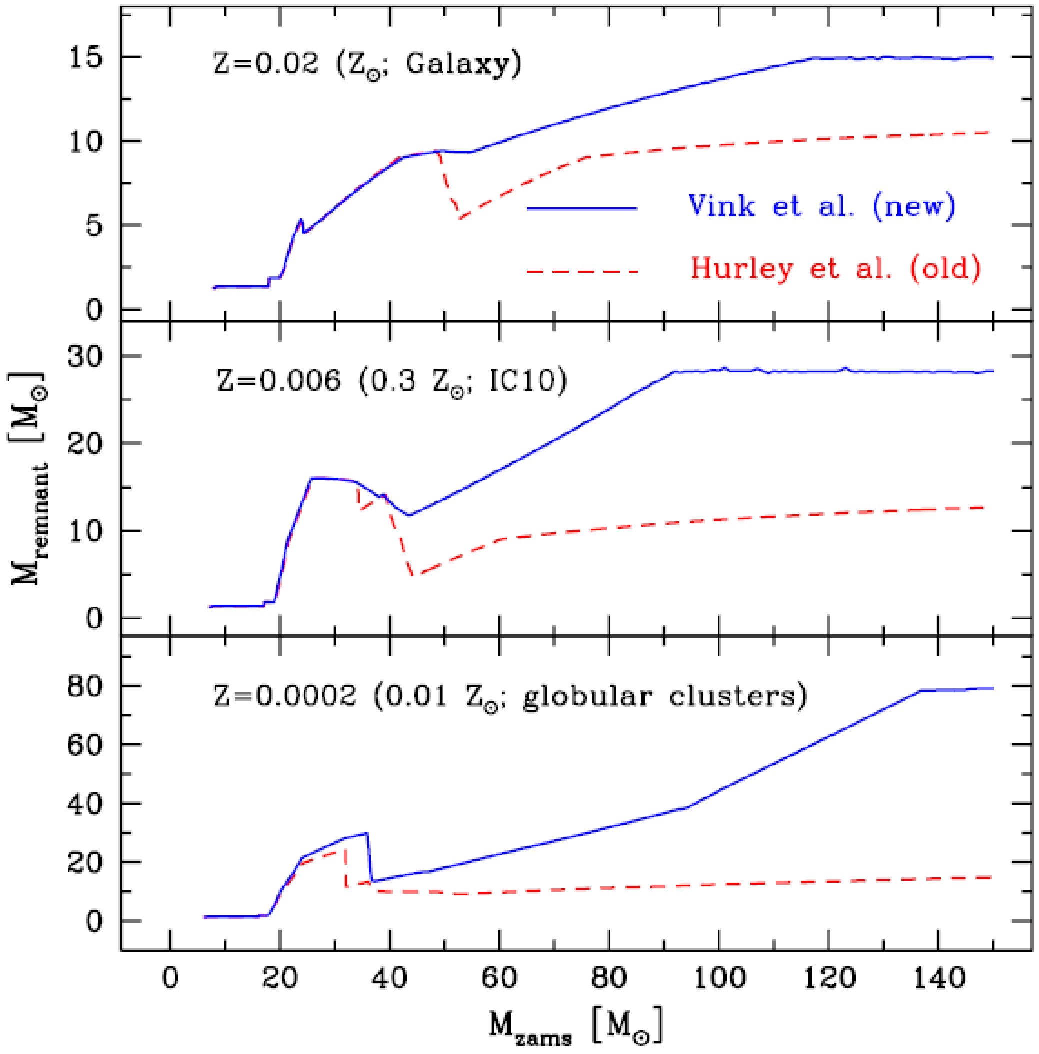
Completed SSE/BSE updates

Our **current** (Level C; Kamlah et al. 2021) code versions of **Nbody6++GPU** and **MOCCA** are able to model (on top of Hurley et al. 2000, 2002):

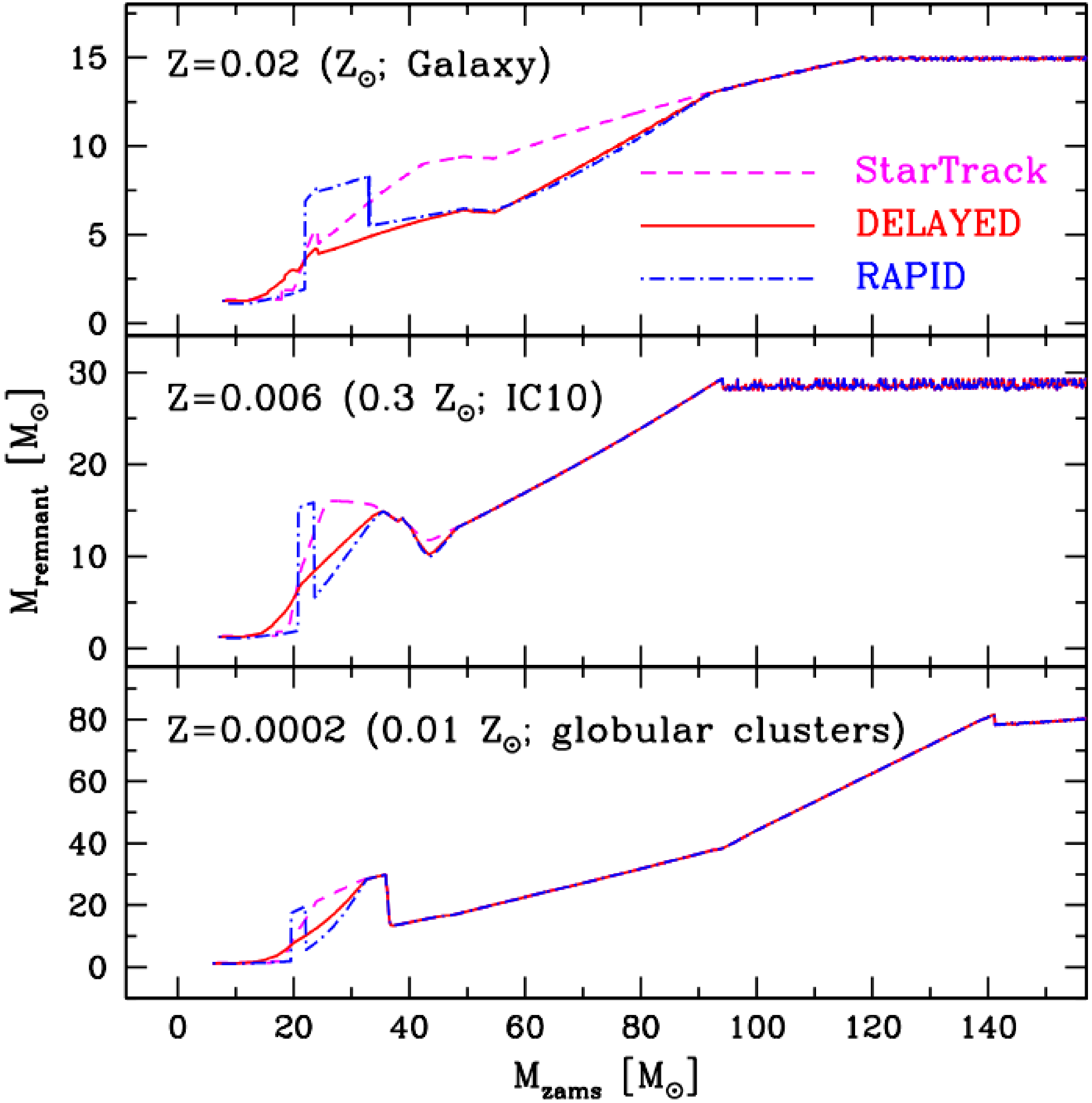
- Metallicity dependent stellar wind mass loss (Vink et al. 2001, Vink & de Koter 2005, Belczynski et al. 2010)
- Rapid and delayed core-collapse SNe (Fryer et al. 2012)
- ECSNe (EIC, AIC, MIC) (Nomoto & Saio 1985 , Ivanova 2008, Leung et al. 2020)
- (P)PISNe (Belczinsky et al. 2016, Spera & Mapelli 2017, Woosley 2017, Leung et al. 2019, Banerjee et al. 2020, Kremer et al. 2020)
- Fallback-scaled natal kicks (Scheck et al. 2004, Fryer & Young 2007; Scheck et al. 2008, Banerjee et al. 2020, Banerjee 2021)
- BH natal spins:
  - Fuller (Fuller & Ma 2019, Banerjee 2021)
  - Geneva (Eggenberger et al. 2008, Ekström et al. 2012, Banerjee 2021)
  - MESA (Spruit 2002, Paxton et al. 2011, 2015, Fuller & Ma 2019, Banerjee 2021)

Stellar winds





Figures from Belczinsky et al. (2010); Vink et al. (2001, 2005), Banerjee et al. (2019), Kremer et al. (2020)



Remnant masses – core-collapse SNe

- Rapid vs. Delayed SNe as extremes for the convection-enhanced neutrino-driven paradigm based on current (2012) knowledge of SNe and gamma-ray burst explosions.
- BUT Envelope stripping in binary stars → significant reduction in BH numbers → BH-NS, BH-BH merger rates reduced significantly

(Schneider et al. 2021; Tauris et al. 2013, 2015, 2017, Breivik et al. 2020)

Figures from Fryer et al. 2012; See also Banerjee et al. 2019, Kamlah et al. 2021

Remnant masses – electron-capture SNe (ECSNe)

- ONe(Mg) WD reaches a critical mass $M = 1.38\mathrm{M}^{*}$ → electron capture is triggered on Mg^{24} and Ne^{20} → WD undergoes collapse from sudden lack of electron pressure.
- Characteristic remnant mass in N-body simulations / (B)PS → $M_{\mathrm{NS}} = 1.26\mathrm{M}^{*}$ (Belczinsky et al. 2008)
- Characteristic natal kick velocity dispersion σ_{ECSNe} uncertain:

  - $\sigma_{\mathrm{ECSNe}} = 30.0\ \mathrm{kms}^{-1}$ (Wang et al. 2016, DRAGON-I)
  - $\sigma_{\mathrm{ECSNe}} = 20.0\ \mathrm{kms}^{-1}$ (Askar et al. 2017, Breivik et al. 2020)
  - $\sigma_{\mathrm{ECSNe}} = 15.0\ \mathrm{kms}^{-1}$ (Giacobbo et al. 2018)
  - $\sigma_{\mathrm{ECSNe}} = 3.0\ \mathrm{kms}^{-1}$ (Gessner & Janka 2018) → DRAGON-II, MOCCA Survey Database II
  - …

- Models typically include three possible pathways that can lead to an ECSNe (≡remnant masses & natal kicks modelled the same in the N-body codes) (Ivanova et al. 2008, Ye et al. 2019):

  - Evolutionary-induced collapse (EIC)
  - Accretion-induced collapse (AIC)
  - Merger-induced collapse (MIC)

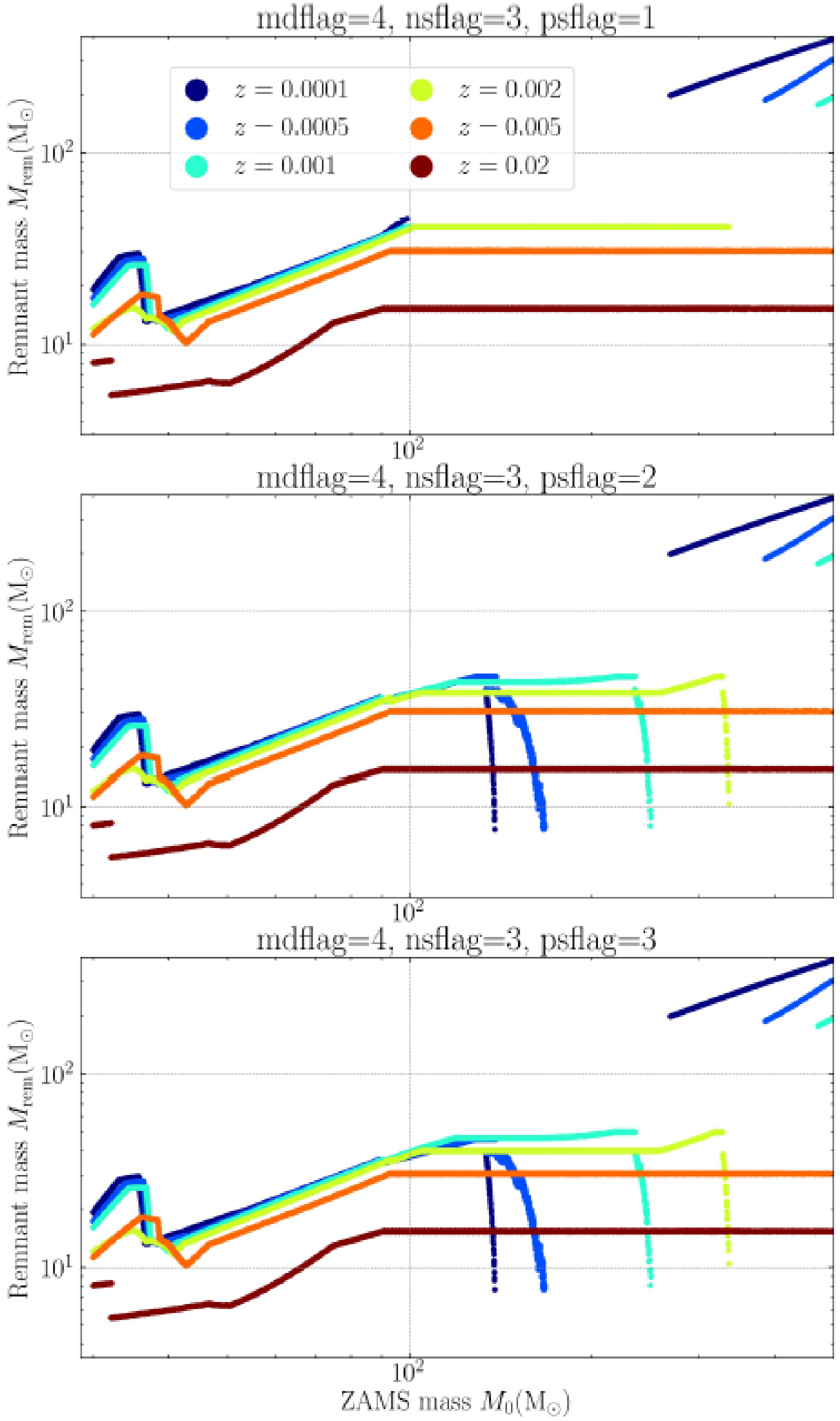
Remnant masses – EIC,AIC,MIC

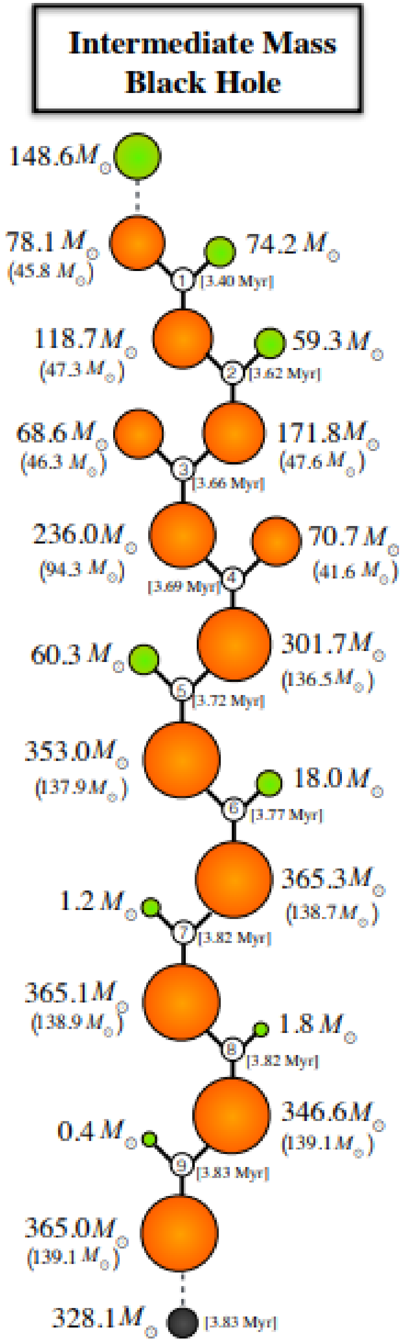
- Evolutionary-induced collapse (EIC):
  - Single zero-age main-sequence (ZAMS) of mass (6-8)M^* (metallicity dependent) (Nomoto 1984, 1987; Kiel et al. 2008)
  - Helium stars with masses between (1.6-2.25)M^* (Hurley et al. 2000)

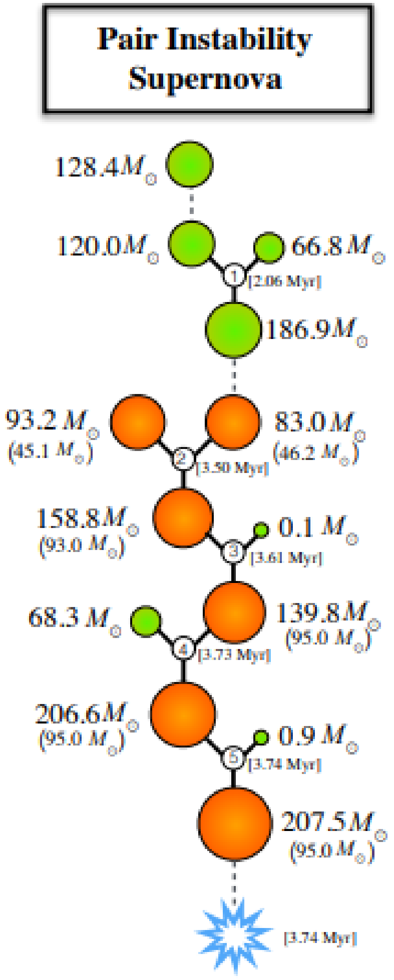
- Accretion-induced collapse (AIC) (Nomoto & Kondo 1991; Saio & Nomoto 2004):
  - ONe(Mg) WD accretes ONe or CO during RLOF → WD collapse if mass larger than ECSNe critical mass (1.38M*) but smaller than maximum NS mass (2.5M* Wang et al. 2016).

- Merger-induced collapse (MIC) (Saio & Nomoto 1985):
  - Product of a merger or collision between two WDs (One(Mg) WDs or CO WDs)

→ All the above paths generally produce NSs in binaries, which can often lead to subsequent RLOF and the production of low-mass X-ray binaries (LMXBs; in GCs see Clark 1975) and millisecond pulsars (MSPs; in GCs see Manchester et al. 2005).





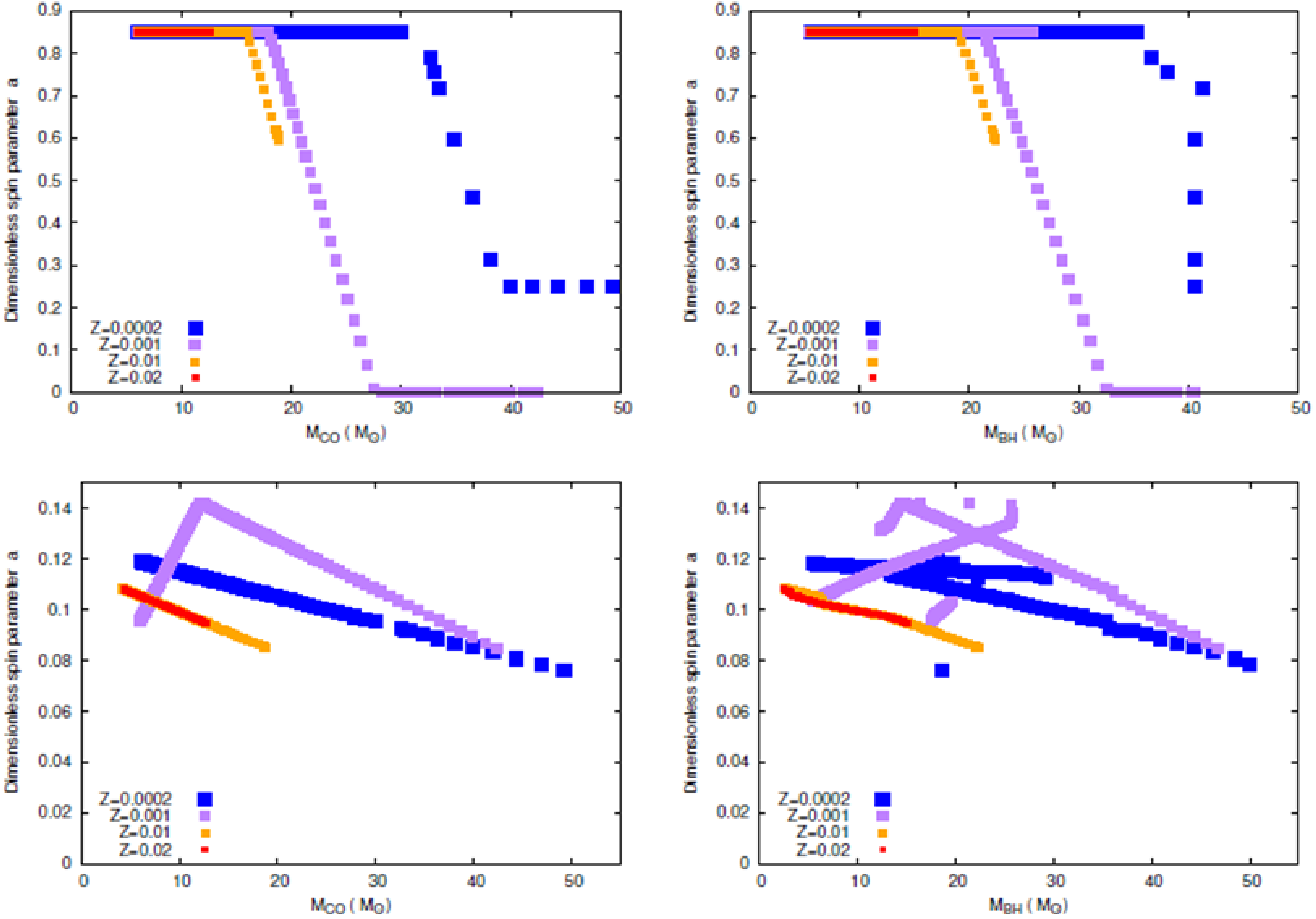


Remnant masses – (pulsational) pair instability SNe

- Theoretical modeling of massive stars predicts a gap in the black hole (BH) mass function above $\sim 40 - 50$ M for BHs formed through single star evolution, arising from (pulsational) pair-instability supernovae → in dense star clusters, dynamical channels may exist that allow construction of BHs with masses in excess of those allowed from single star evolution

Figures: left: Kamlah et al. 2021 & right: Kremer et al. 2020; See also (Woosley 2017; Banerjee et al. 2020, Breivik et al. 2020, Wang et al. 2021).

BH natal spins



$$\vec{a} = \frac{c\vec{S}_{\rm BH}}{GM_{\rm BH}^2}.$$

BUT recent work claims that the Tayler-Spruit magnetic dynamo can essentially extract all of the angular momentum of the proto-remnant core, leading to nearly non-spinning BHs (Fuller et al. 2019)

Figures from Banerjee 2021; See also Belczinsky & Banerjee 2020, Belczinsky et al. 2020

Future additions (work in progress)

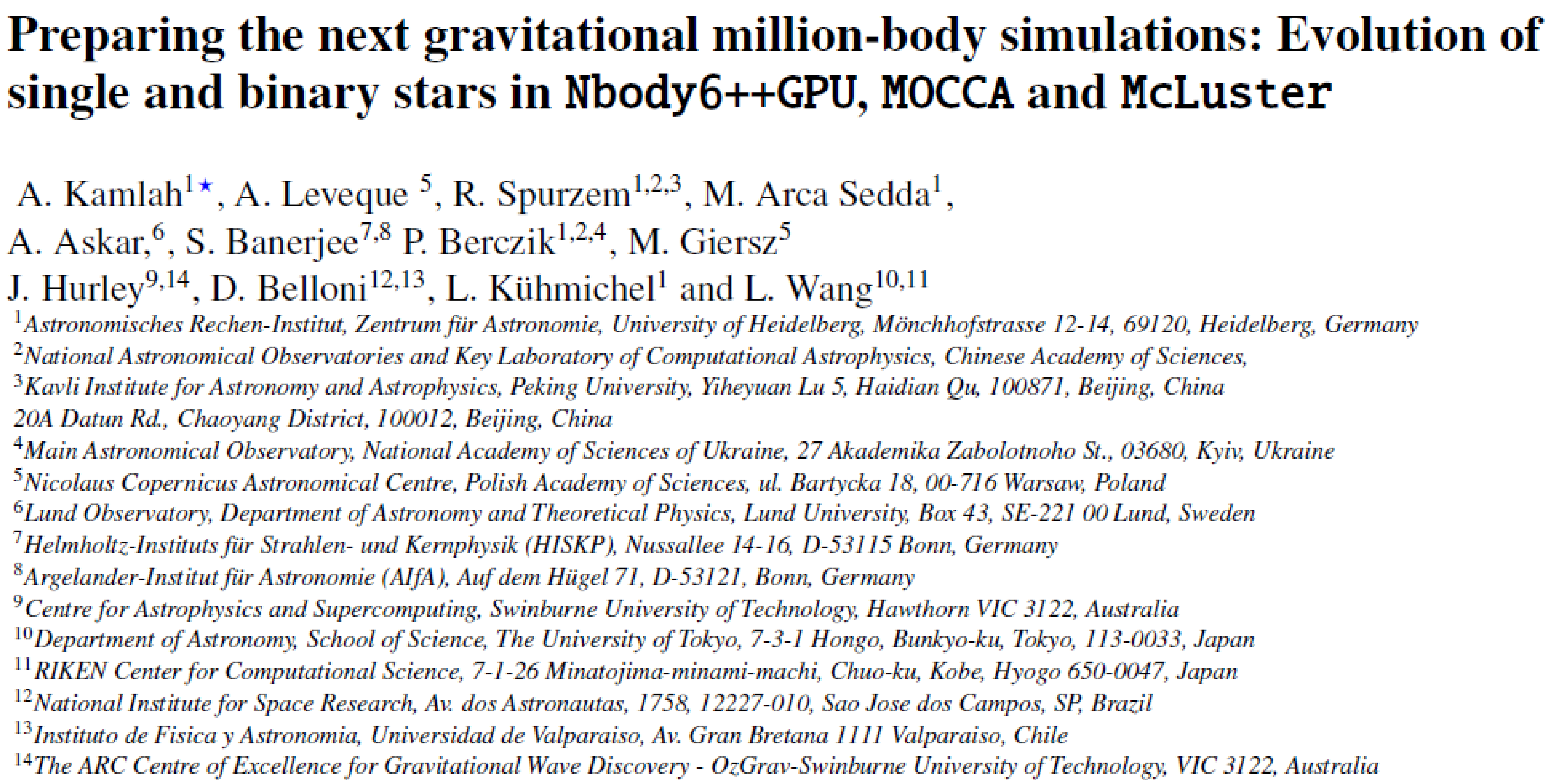
- Our **Level-D** code versions of **Nbody6++GPU** and **MOCCA** will be able to model (*later this year*):
  - Extremely metal-poor massive stars (Pop-III) up to 1280M* (Tanikawa et al. 2020, 2021, Hijikawa et al. 2021)
  - Cataclysmic variables around the orbital period gap (Rappaport et al. 1982, Ritter 2011, Belloni et al. 2018)
  - Pulsar spin evolution (Kiel et al. 2008, Kiel & Hurley 2009, Ye et al. 2019, Breivik et al. 2020)
  - General relativistic merger recoil velocities and merger product spins (e.g. Banerjee et al. 2021)
  - Wind velocity factors (Belczinsky et al. 2008, Breivik et al. 2020, Belloni et al. 2021)
  - Updated magnetic braking (Rappaport et al. 1983, Belczynski et al. 2008, Belloni et al. 2018, 2019, 2020a)
  - Ultra-stripping in binary stars (Tauris et al. 2013, 2015, 2017, Breivik et al. 2020, Schneider et al. 2021)
  - Masses of merger products (Hillier et al. 2001, Barniske et al. 2008 Bestenlehner et al. 2020, Olejak et al. 2020)

Summary: Further SSE/BSE updates
(Level D in Kamlah et al. 2021)

	Nbody6++GPU	MOCCA
BSE extension to 1280 M* (Tanikawa et al. 2020, 2021, Hijikawa et al. 2021)	No	Yes
Cataclysmic variables around the orbital period gap (Ritter 2011, Schreiber et al. 2016, Zorotovic, Belloni et al. 2018	No	Yes
Pulsar spin evolution and magnetic field burying (Kiel 2008, Kiel & Hurley 2009, Ye et al. 2019, Breivik et al. 2020, Chattopadhyay et al. 2020a,b)	No	No
General relativistic merger recoils and spins (Morawski et al. 2018, Belczinsky et al. 2020, Banerjee 2021)	No	Yes
Wind velocity/accretion factors (Belczinsky et al. 2008, Breivik et al. 2020, Belloni et al. 2021)	No	No
Updated Magnetic braking (Rappaport et al. 1983, Belczynski et al. 2008, Belloni et al. 2018, 2019, 2020a)	No	No
Ultra-stripping in binary stars (Tauris et al. 2013, 2015, 2017, Breivik et al. 2020, Schneider et al. 2021)	No	No

Small commercial break

- Everything discussed today (and much more) in our paper:


Preparing the next gravitational million-body simulations: Evolution of single and binary stars in Nbody6++GPU, MOCCA and McLuster

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Summary – SSE/BSE in N-body simulations

- Our **Level-C** code versions of **Nbody6++GPU** and **MOCCA** are able to model:
  - Metallicity dependent stellar wind mass loss (e.g. Vink et al. 2001, Vink & de Koter 2005, Belczynski et al. 2010)
  - Rapid and delayed core-collapse SNe (e.g. Fryer et al. 2012, Banerjee et al. 2019, Kamlah et al. 2021)
  - ECSNe (EIC, AIC, MIC) (e.g. Nomoto 1984, 1987, Nomoto & Saio 1985, Saio & Nomoto 2004, Ivanova 2008, Leung et al. 2020)
  - (P)PISNe (Belczinsky et al. 2016, Spera & Mapelli 2017, Woosley 2017, Leung et al. 2019, Banerjee et al. 2020, Kremer et al. 2020, Kamlah et al. 2021)
  - Fallback-scaled natal kicks (e.g. Scheck et al. 2004, Fryer & Young 2007; Scheck et al. 2008, Banerjee et al. 2020, Banerjee 2021)
  - BH natal spins:
    - Fuller (e.g. Fuller & Ma 2019, Banerjee 2021)
    - Geneva (Eggenberger et al. 2008, Ekström et al. 2012, Banerjee 2021)
    - MESA (Spruit 2002, Paxton et al. 2011, 2015, Fuller & Ma 2019, Banerjee 2021)

- Our **Level-D** code versions of **Nbody6++GPU** and **MOCCA** will be able to model (*later this year*):
  - Extremely metal-poor massive stars (Pop-III) up to 1280M* (e.g. Tanikawa et al. 2020, 2021, Hijikawa et al. 2021)
  - Cataclysmic variables around the orbital period gap (e.g. Rappaport et al. 1982, Ritter 2011, Belloni et al. 2018)
  - Pulsar spin evolution (e.g. Kiel et al. 2008, Kiel & Hurley 2009, Ye et al. 2019, Breivik et al. 2020)
  - General relativistic merger recoil velocities/product spins (e.g. Rezzolla et al. 2008, Belczynski et al. 2020, Banerjee 2021)
  - Wind velocity factors (e.g. Belczynski et al. 2008, Breivik et al. 2020, Belloni et al. 2021)
  - Updated magnetic braking (e.g. Rappaport et al. 1983, Belczynski et al. 2008, Belloni et al. 2018, 2019, 2020a)
  - Ultra-stripping in binary stars (e.g. Tauris et al. 2013, 2015, 2017, Breivik et al. 2020, Schneider et al. 2021)
  - Masses of merger products (e.g. Hillier et al. 2001, Barniske et al. 2008 Bestenlehner et al. 2020, Olejak et al. 2020)

Please, if you any, send me literature on any new progress in the field of stellar winds, remnant masses, stellar mixing, stellar spins, binary evolution,… to albrecht.kamlah@stud.uni-heidelberg.de Thank you so much in advance!

Summary

- Updates in the stellar evolution of Nbody6++GPU and MOCCA make it possible to study the evolution of LIGO/VIRGO GW source progenitor stars and sources themselves according to our best current understanding in our simulations.

- Likewise, these additions in McLuster make it possible to model accurately star cluster initial conditions with multiple stellar populations.

- The following state-of-the-art studies with the updated codes are in progress:
  - DRAGON-II of NGC 3201 (Spurzem et al.)
  - DRAGON-II IMBH growth studies (Arca Sedda et al.)
  - MOCCA Survey Database II (Giersz et al.)

- Many further and important updates are work in progress and are to be completed this year. With these updates we will also start to model 47 Tuc and Omega Cen.

Sources

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